Seminari 9

Matematika za ekonomiste 2

Damir Horvat

FOI, Varaždin

Sadržaj

prvi zadatak

Zadatak 1

Julija je 19.6.2009. podmirila dug sa zakašnjenjem od 101 *dana plativši ukupno* 7441.40 *kn uz godišnju kamatnu stopu* 7%*. Odredite iznos kojim se taj dug mogao podmiriti 12.5.2009. Obračun kamata je jednostavni i dekurzivni.*

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Rješenje

$$C_{101} = 7441.40, \quad n = 101, \quad p = 7\%$$

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Rješenje

$$C_{101} = 7441.40, \quad n = 101, \quad p = 7\%$$

Iz $C_n = C_0 \left(1 + \frac{pn}{36500}\right)$ slijedi

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$$C_0 = \frac{C_n}{1 + \dfrac{pn}{36500}}$$

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$$C_0 = \frac{C_n}{1 + \frac{pn}{36500}} = \frac{7441.40}{1 + \frac{7 \cdot 101}{36500}}$$

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Početni dug bez kamata iznosi 7300 kn.

Prebrojimo koliko ima dana između datuma

$$12.5.2009. \quad \text{i} \quad 19.6.2009.$$

Prvi dan brojimo, zadnji ne brojimo.

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12.5.2009. $\rightarrow 101 - 38 =$ 63 dana zakašnjenja

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$$C_{63} = 7388.20$$

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$$C_{63} = 7388.20$$

Julija bi platila 7388.20 kn ako bi dug podmirila 12.5.2009.

drugi zadatak

Zadatak 2

Uz koju je mjesečnu kamatnu stopu posuđeno 8000 *kn ako nakon* 32 *mjeseca dužnik treba vratiti* 9500 *kn? Kolika je ekvivalentna godišnja kamatna stopa? Obračun kamata je složeni i dekurzivni.*

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Rješenje

- $C_0 = 8000, \quad n = 32, \quad C_{32} = 9500$

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Rješenje

- $C_0 = 8000, \quad n = 32, \quad C_{32} = 9500$
- Koristimo formulu $C_n = C_0 r^n$.

$$C_{32} = C_0 \cdot r_{\mathrm{mj}}^{32}$$

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$$r_{\mathrm{mj}}^{32} = \frac{C_{32}}{C_0}$$

$$
\begin{aligned}
C_{32} &= C_0 \cdot r_{\mathrm{mj}}^{32} \\
r_{\mathrm{mj}}^{32} &= \frac{C_{32}}{C_0} \\
r_{\mathrm{mj}} &= \sqrt[32]{\frac{C_{32}}{C_0}}
\end{aligned}
$$

$$C_{32} = C_0 \cdot r_{\mathrm{mj}}^{32}$$

$$r_{\mathrm{mj}}^{32} = \frac{C_{32}}{C_0}$$

$$r_{\mathrm{mj}} = \sqrt[32]{\frac{C_{32}}{C_0}}$$

$$r_{\mathrm{mj}} = \sqrt[32]{\frac{9500}{8000}}$$

$$C_{32} = C_0 \cdot r_{\mathrm{mj}}^{32}$$

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$$r_{\mathrm{mj}} = \sqrt[32]{\frac{C_{32}}{C_0}}$$

$$r_{\mathrm{mj}} = \sqrt[32]{\frac{9500}{8000}}$$

$$r_{\mathrm{mj}} = 1.0053847665\cdots$$

$$r_{\mathrm{mj}} = 1 + \frac{p_{\mathrm{mj}}}{100}$$

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$$p_{\mathrm{mj}} = 100(r_{\mathrm{mj}} - 1)$$

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$$r_{\mathrm{mj}} = 1 + \frac{p_{\mathrm{mj}}}{100}$$

$$p_{\mathrm{mj}} = 100(r_{\mathrm{mj}} - 1)$$

$$p_{\mathrm{mj}} = 0.5384767\%$$

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$$r_{\mathrm{god}} = r_{\mathrm{mj}}^{12}$$

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$$p_{\mathrm{god}} = 100(r_{\mathrm{god}} - 1)$$

$$p_{\mathrm{god}} = 6.65657\%$$

Nekoliko napomena

- kvartalni dekurzivni kamatni faktor

$$r_{\mathrm{kv}} = r_{\mathrm{mj}}^3 \quad \text{ili} \quad r_{\mathrm{kv}} = \sqrt[4]{r_{\mathrm{god}}} \quad \text{ili} \quad r_{\mathrm{kv}} = \sqrt{r_{\mathrm{pgod}}}$$

- polugodišnji dekurzivni kamatni faktor

$$r_{\mathrm{pgod}} = r_{\mathrm{mj}}^6 \quad \text{ili} \quad r_{\mathrm{pgod}} = \sqrt{r_{\mathrm{god}}} \quad \text{ili} \quad r_{\mathrm{pgod}} = r_{\mathrm{kv}}^2$$

- mjesečni dekurzivni kamatni faktor

$$r_{\mathrm{mj}} = \sqrt[12]{r_{\mathrm{god}}} \quad \text{ili} \quad r_{\mathrm{mj}} = \sqrt[3]{r_{\mathrm{kv}}} \quad \text{ili} \quad r_{\mathrm{mj}} = \sqrt[6]{r_{\mathrm{pgod}}}$$

- godišnji dekurzivni kamatni faktor

$$r_{\mathrm{god}} = r_{\mathrm{mj}}^{12} \quad \text{ili} \quad r_{\mathrm{god}} = r_{\mathrm{pgod}}^2 \quad \text{ili} \quad r_{\mathrm{god}} = r_{\mathrm{kv}}^4$$

treći zadatak

Zadatak 3

Zadana je glavnica od 1200 *kn i godišnja kamatna stopa* 5%.

a) *Odredite vrijednost glavnice nakon* 8 *mjeseci uz konformno ukamaćivanje.*
b) *Odredite vrijednost glavnice nakon* 8 *mjeseci uz relativno mjesečno ukamaćivanje.*

Obračuna kamata je složeni i dekurzivni.

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Rješenje

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$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

Rješenje

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$$C_8 = 1200 \cdot \sqrt[12]{1.05}^8 = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \frac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100}$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100}$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100} = 1 + \frac{5}{1200}$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100} = 1 + \frac{5}{1200} = \frac{241}{240}$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100} = 1 + \frac{5}{1200} = \frac{241}{240}$$

Korištenjem formule $C_n = C_0 r^n$ dobivamo

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100} = 1 + \frac{5}{1200} = \frac{241}{240}$$

Korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \left(\frac{241}{240}\right)^8 = 1240.59$$

četvrti zadatak

Zadatak 4

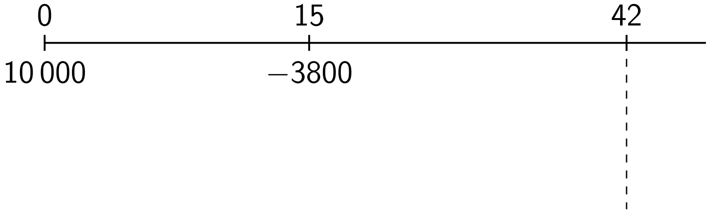
Stipe uplati 10 000 *kn, a nakon* 15 *mjeseci podigne* 3800 *kn.*

a) *Koliko novaca ima Stipe tri i pol godine nakon prve uplate?*
b) *Nakon koliko će mjeseci, u odnosu na zadnje stanje, Stipe ponovo raspolagati s* 10 000 *kn?*
c) *Koliko bi novaca morao podići četiri godine nakon prve uplate tako da bi pet godina nakon prve uplate imao polovicu iznosa kojeg je uplatio?*

Godišnja kamatna stopa je 11.1%.

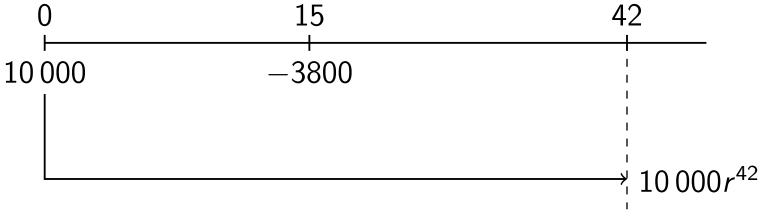
Rješenje

a)



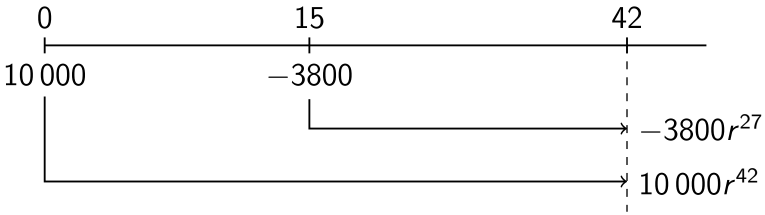
Rješenje

a)



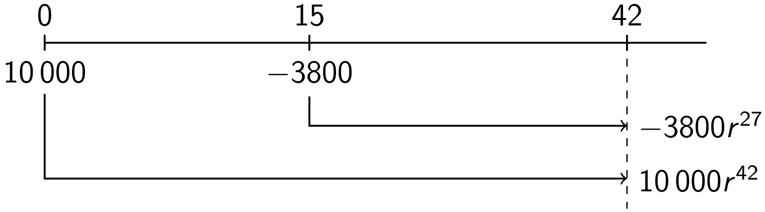
Rješenje

a)



Rješenje

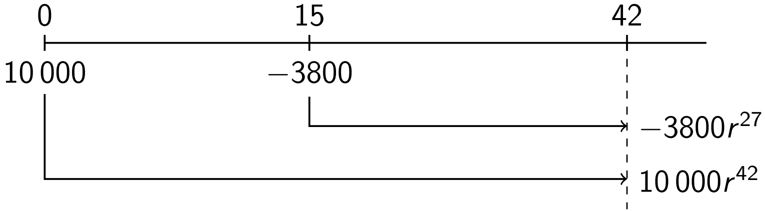
a)



$$C_{42} = 10\,000r^{42} - 3800r^{27}$$

Rješenje

a)



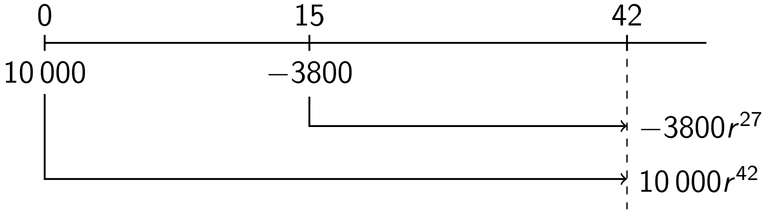
$$\boxed{r = \sqrt[12]{1.111}}$$

$$C_{42} = 10\,000 r^{42} - 3800 r^{27}$$

$$C_{42} = 10\,000 \cdot \sqrt[12]{1.111}^{\,42} - 3800 \cdot \sqrt[12]{1.111}^{\,27}$$

Rješenje

a)



$$\boxed{r = \sqrt[12]{1.111}}$$

$$C_{42} = 10\,000r^{42} - 3800r^{27}$$

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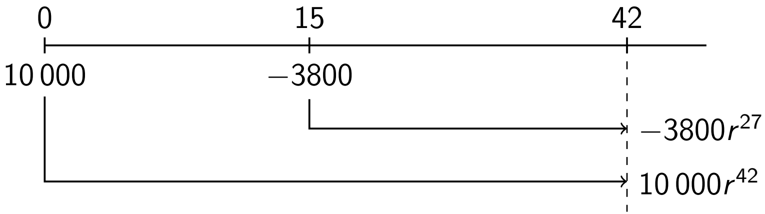
$$C_{42} = 9638.88$$

Rješenje

a)



$$\boxed{r = \sqrt[12]{1.111}}$$

$$C_{42} = 10\,000 r^{42} - 3800 r^{27}$$

$$C_{42} = 10\,000 \cdot \sqrt[12]{1.111}^{\,42} - 3800 \cdot \sqrt[12]{1.111}^{\,27}$$

$$C_{42} = 9638.88$$

Tri i pol godine nakon prve uplate Stipe ima 9638.88 kn.

b)

$$C_{42} r^n = 10\,000$$

b)

$$C_{42} r^n = 10\,000$$

$$r^n = \frac{10\,000}{C_{42}}$$

b)

$$C_{42} r^n = 10\,000$$

$$r^n = \frac{10\,000}{C_{42}} \quad \Big/ \log$$

$$n \log r = \log \frac{10\,000}{C_{42}}$$

b)

$$
\begin{aligned}
C_{42} r^n &= 10\,000 \\
r^n &= \frac{10\,000}{C_{42}} \quad \Big/ \log \\
n \log r &= \log \frac{10\,000}{C_{42}} \\
n &= \frac{\log \frac{10\,000}{C_{42}}}{\log r}
\end{aligned}
$$

b)

$$
\begin{aligned}
C_{42} r^n &= 10\,000 \\
r^n &= \frac{10\,000}{C_{42}} \quad \Big/ \log \\
n \log r &= \log \frac{10\,000}{C_{42}} \\
n &= \frac{\log \frac{10\,000}{C_{42}}}{\log r} \\
n &= \frac{\log \frac{10\,000}{9638.88}}{\log \sqrt[12]{1.111}}
\end{aligned}
$$

b)

$$
\begin{aligned}
C_{42} r^n &= 10\,000 \\
r^n &= \frac{10\,000}{C_{42}} \;\Big/ \log \\
n \log r &= \log \frac{10\,000}{C_{42}} \\
n &= \frac{\log \dfrac{10\,000}{C_{42}}}{\log r} \\
n &= \frac{\log \dfrac{10\,000}{9638.88}}{\log \sqrt[12]{1.111}} \\
n &= 4.19
\end{aligned}
$$

b)

$$C_{42} r^n = 10\,000$$

$$r^n = \frac{10\,000}{C_{42}} \Big/ \log$$

$$n \log r = \log \frac{10\,000}{C_{42}}$$

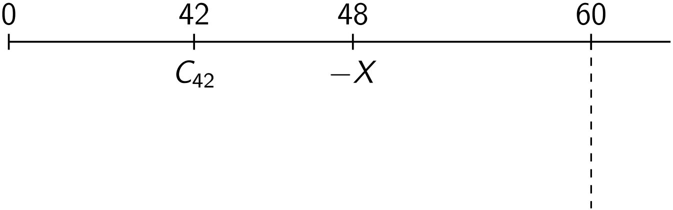
$$n = \frac{\log \frac{10\,000}{C_{42}}}{\log r}$$

$$n = \frac{\log \frac{10\,000}{9638.88}}{\log \sqrt[12]{1.111}}$$

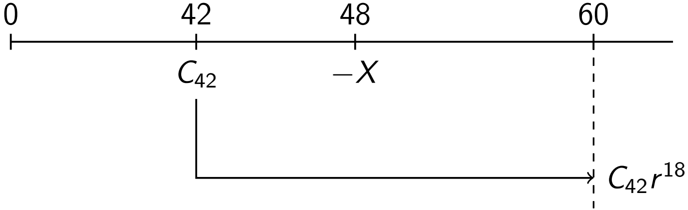
$$n = 4.19$$

Stipe će ponovo raspolagati s 10 000 kn nakon 5 mjeseci od zadnjeg stanja.

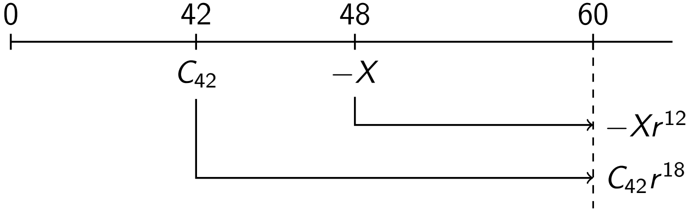
c)



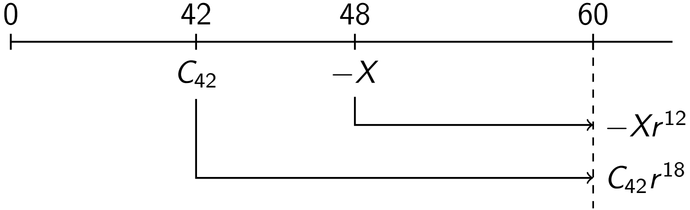
c)



c)

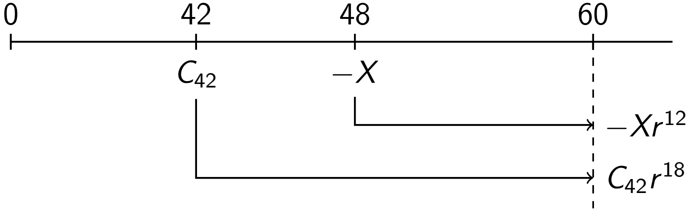


c)



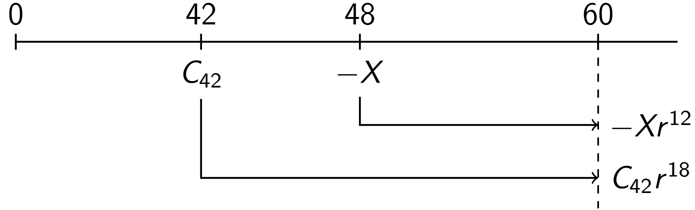
$$C_{42}r^{18} - Xr^{12} = 5000$$

c)



$$C_{42}r^{18} - Xr^{12} = 5000$$

$$Xr^{12} = C_{42}r^{18} - 5000$$

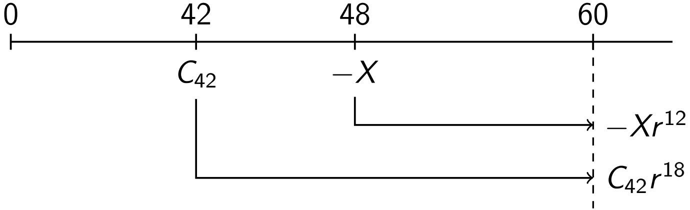
c) 

$$C_{42}r^{18} - Xr^{12} = 5000$$

$$Xr^{12} = C_{42}r^{18} - 5000$$

$$X = \frac{C_{42}r^{18} - 5000}{r^{12}}$$

c)



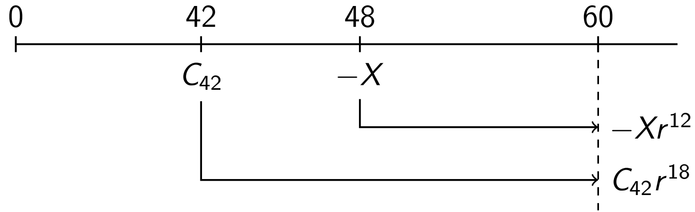
$$C_{42}r^{18} - Xr^{12} = 5000$$

$$Xr^{12} = C_{42}r^{18} - 5000$$

$$X = \frac{C_{42}r^{18} - 5000}{r^{12}}$$

$$X = \frac{9638.88 \cdot \sqrt[12]{1.111}^{\,18} - 5000}{\sqrt[12]{1.111}^{\,12}}$$

c)



$$C_{42}r^{18} - Xr^{12} = 5000$$

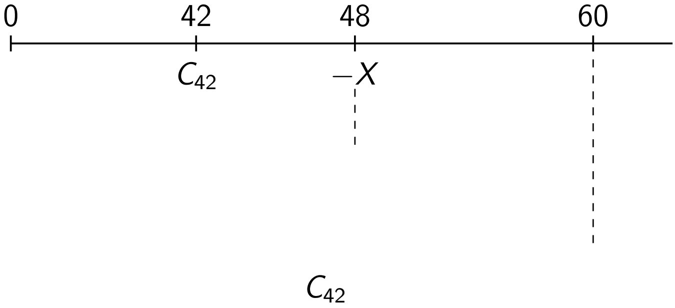
$$Xr^{12} = C_{42}r^{18} - 5000$$

$$X = \frac{C_{42}r^{18} - 5000}{r^{12}}$$

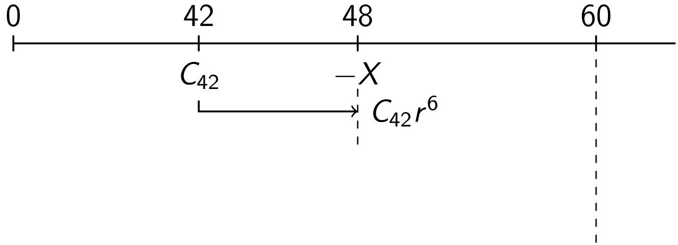
$$X = \frac{9638.88 \cdot \sqrt[12]{1.111}^{\,18} - 5000}{\sqrt[12]{1.111}^{\,12}}$$

$$X = 5659.32$$

Drugi način razmišljanja

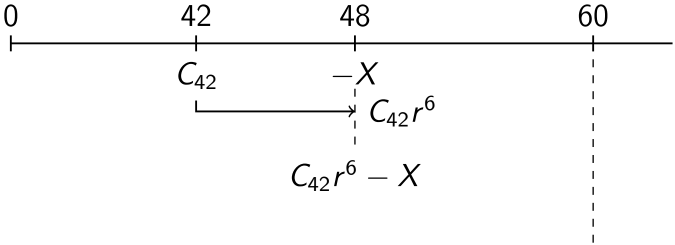


Drugi način razmišljanja



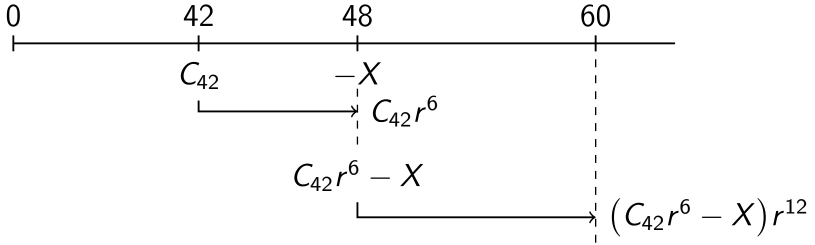
$C_{42} r^6$

Drugi način razmišljanja



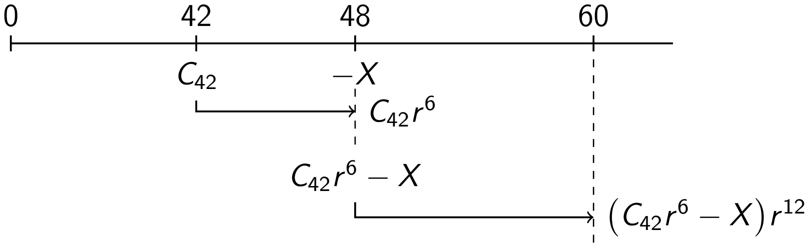
$$C_{42}r^6 - X$$

Drugi način razmišljanja



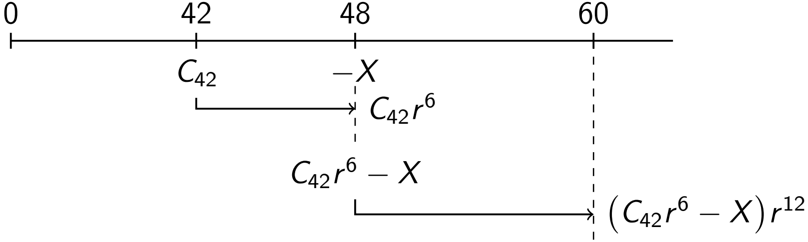
$$\left(C_{42}r^6 - X\right)r^{12}$$

Drugi način razmišljanja



$$\left(C_{42}r^6 - X\right)r^{12} = 5000$$

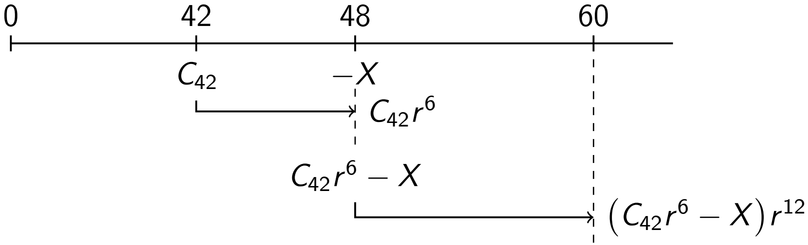
Drugi način razmišljanja



$$\left(C_{42}r^6 - X\right)r^{12} = 5000$$

$$C_{42}r^{18} - Xr^{12} = 5000$$

Drugi način razmišljanja



$$\left(C_{42}r^6 - X\right)r^{12} = 5000$$

$$C_{42}r^{18} - Xr^{12} = 5000$$

$$\vdots$$

$$X = 5659.32$$

peti zadatak

Zadatak 5

Martina uplati nepoznati iznos. Nakon 8 *mjeseci podigne polovinu tog iznosa, a* 5 *kvartala nakon toga uplati još* $\frac{1}{5}$ *tog nepoznatog iznosa.*

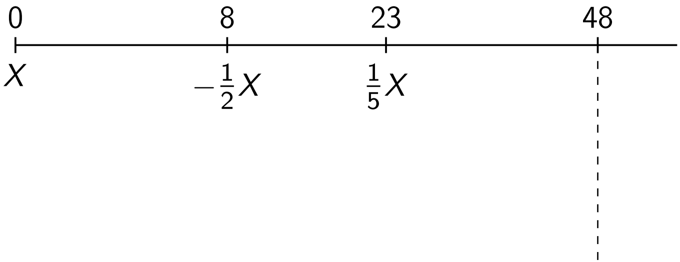
a) *Koliki je iznos uplaćen ako četiri godine nakon prve uplate Martina ima* 8000 *kn? Skicirajte tijek novca!*

b) *Nakon koliko će kvartala u odnosu na prvu uplatu Martina raspolagati s dvostruko većim iznosom od prve uplate?*

Godišnja dekurzivna kamatna stopa je 7.25%.

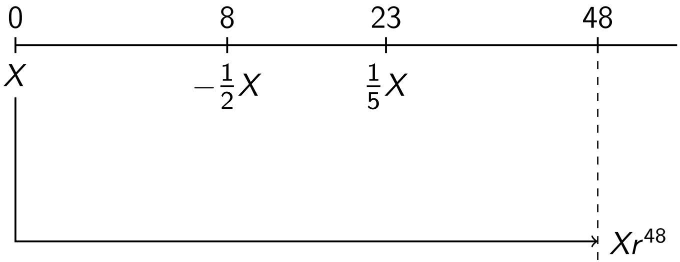
Rješenje

a)



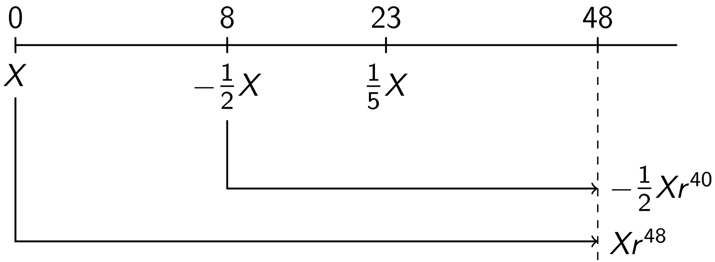
Rješenje

a)



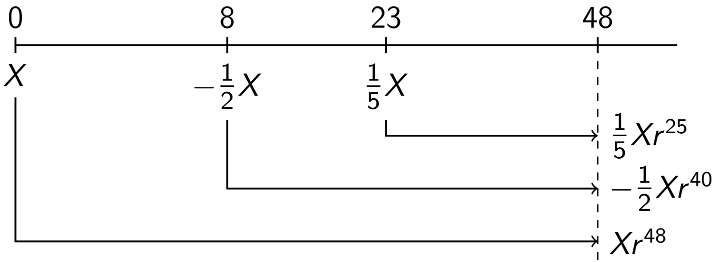
Rješenje

a)



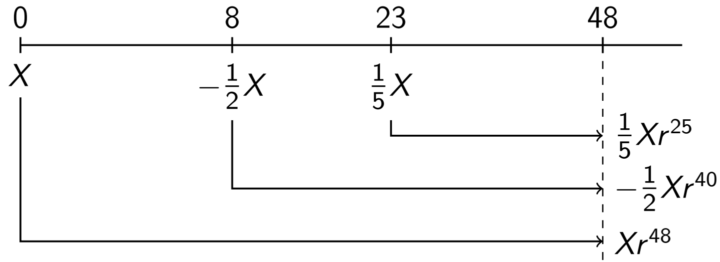
Rješenje

a)



Rješenje

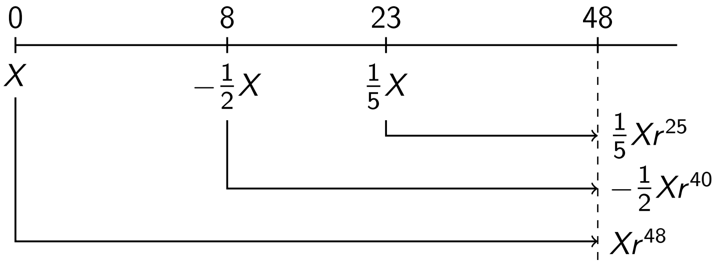
a)



$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

Rješenje

a)

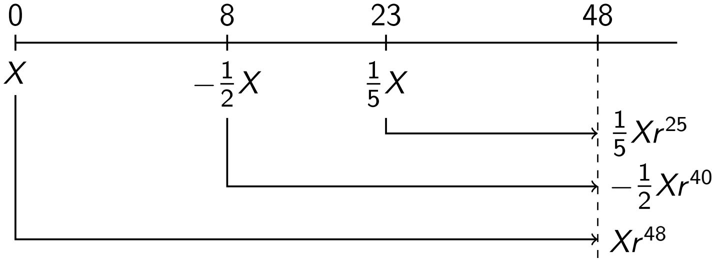


$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

Rješenje

a)



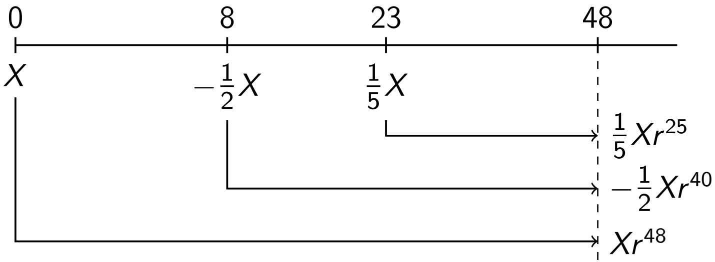
$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

$$X = \frac{8000}{r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}}$$

Rješenje

a)



$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

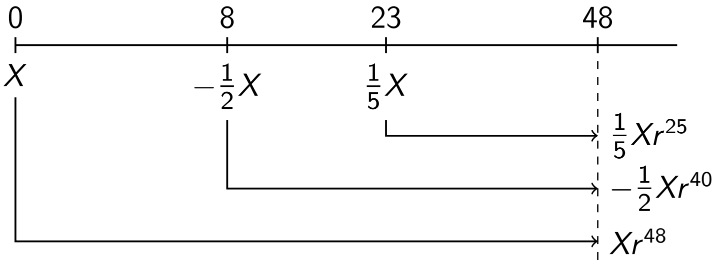
$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

$$X = \frac{8000}{r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}}$$

$$\boxed{r = \sqrt[12]{1.0725}}$$

Rješenje

a)



$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

$$X = \frac{8000}{r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}}$$

$$\boxed{r = \sqrt[12]{1.0725}}$$

$$X = 8666.44$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^{\,8} = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^{\,n} = 2X$$

$$\sqrt[4]{1.0725}^{\,n} = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = \frac{\log \frac{8666.44}{4000}}{\log \sqrt[4]{1.0725}}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = \frac{\log \frac{8666.44}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = 44.19$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = \frac{\log \frac{8666.44}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = 44.19$$

Martina će raspolagati s dvostruko većim iznosom od uplaćenog nakon $45 + 16 = 61$ kvartala od prve uplate.

šesti zadatak

Zadatak 6

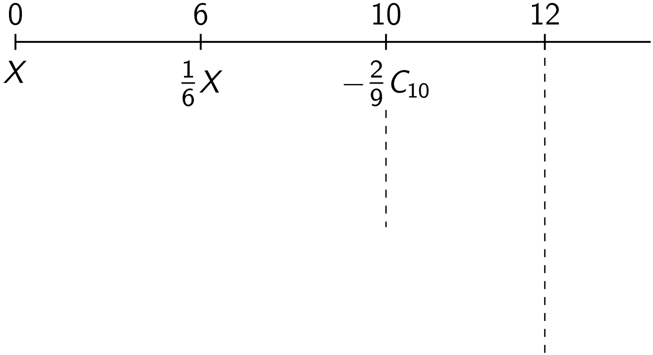
Viktorija uplati nepoznati iznos. Nakon pola godine uloži još šestinu tog iznosa, a četiri mjeseca nakon toga podigne dvije devetine svote s kojom raspolaže u tom trenutku.

a) *Koliki je početni ulog ako na kraju godine Viktorija na računu ima* $2950\,kn$*, a godišnja kamatna stopa je* 8.25%*?*

b) *Nakon koliko će polugodišta, u odnosu na zadnje stanje, Viktorija raspolagati s* $4500\,kn$*?*

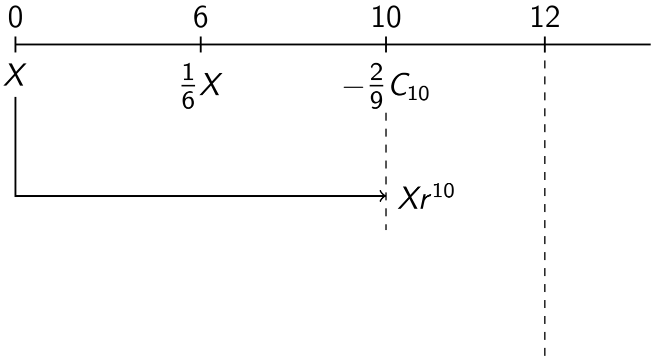
Rješenje

a)

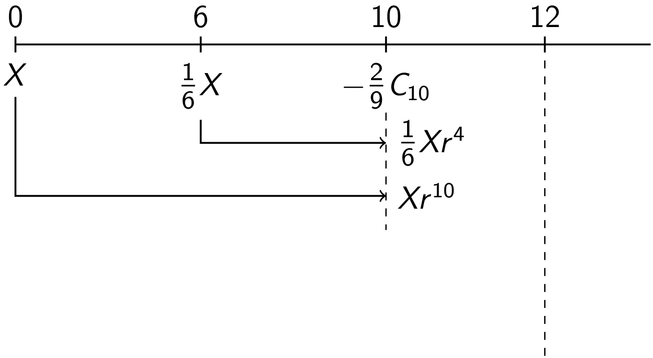


Rješenje

a)

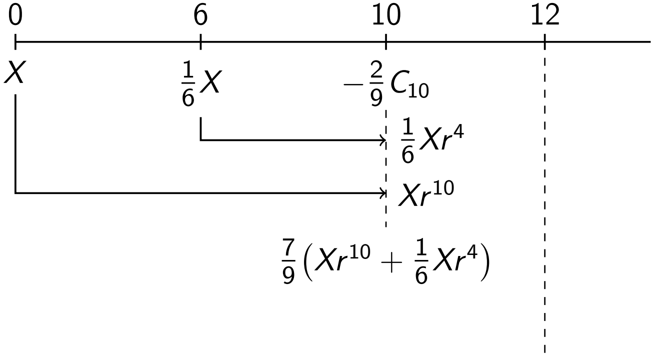


Rješenje

a) 

Rješenje

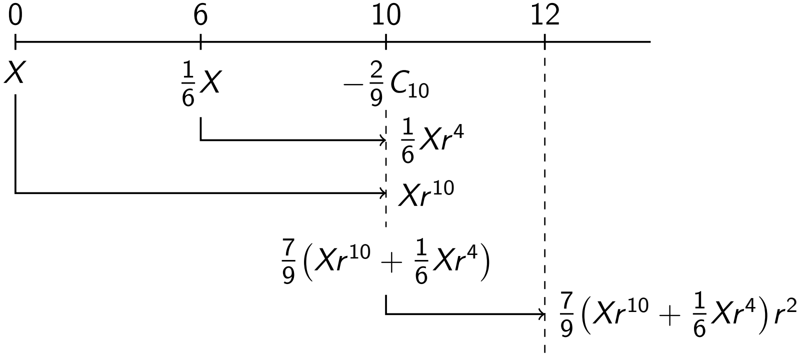
a)



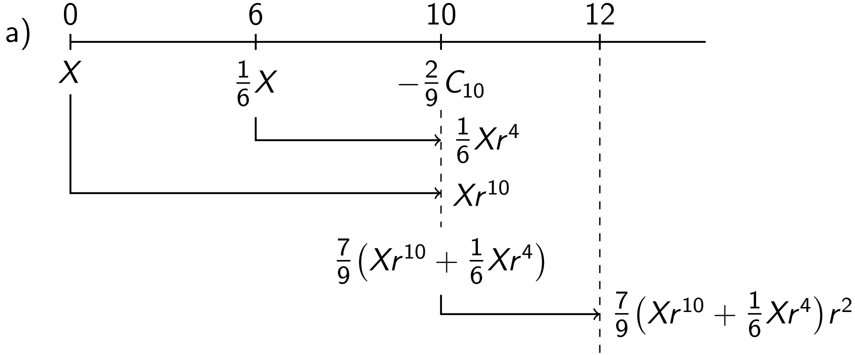
0
6
10
12
X
$\frac{1}{6}X$
$-\frac{2}{9}C_{10}$
$\frac{1}{6}Xr^4$
Xr^{10}
$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)$

Rješenje

a)


0
6
10
12
X
$\frac{1}{6}X$
$-\frac{2}{9}C_{10}$
$\frac{1}{6}Xr^4$
Xr^{10}
$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^4\right)$
$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^4\right)r^2$

Rješenje



$$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^4\right)r^2 = 2950$$

$$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^{4}\right)r^{2}=2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^{4}\right)r^{2}=2950$$

$$\frac{7}{9}Xr^{12}+\frac{7}{54}Xr^{6}=2950$$

$$X\left(\frac{7}{9}r^{12}+\frac{7}{54}r^{6}\right)=2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$X\left(\frac{7}{9}r^{12} + \frac{7}{54}r^{6}\right) = 2950$$

$$X = \frac{2950}{\frac{7}{9}r^{12} + \frac{7}{54}r^{6}}$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$X\left(\frac{7}{9}r^{12} + \frac{7}{54}r^{6}\right) = 2950$$

$$\boxed{r = \sqrt[12]{1.0825}}$$

$$X = \frac{2950}{\frac{7}{9}r^{12} + \frac{7}{54}r^{6}}$$

$$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^{4}\right)r^{2}=2950$$

$$\frac{7}{9}Xr^{12}+\frac{7}{54}Xr^{6}=2950$$

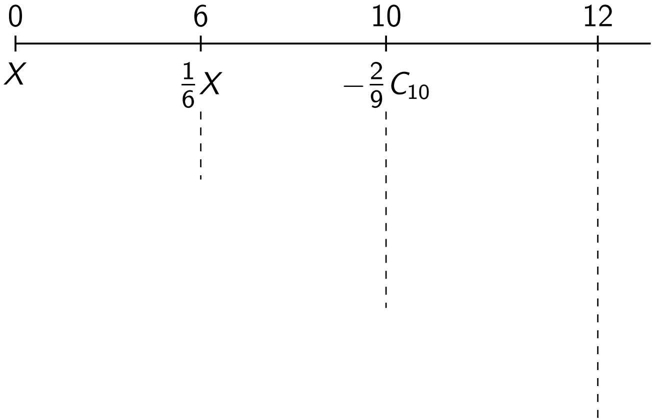
$$X\left(\frac{7}{9}r^{12}+\frac{7}{54}r^{6}\right)=2950$$

$$\boxed{r=\sqrt[12]{1.0825}}$$

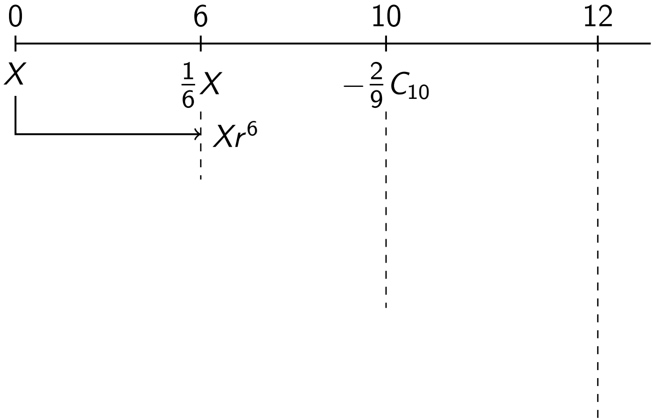
$$X=\frac{2950}{\frac{7}{9}r^{12}+\frac{7}{54}r^{6}}$$

$$X=3020.02$$

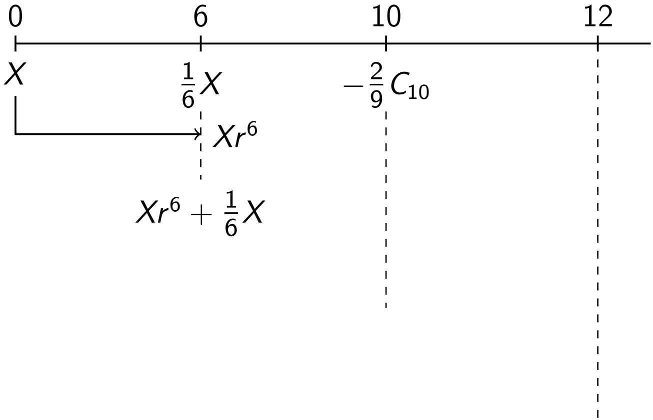
Drugi način razmišljanja



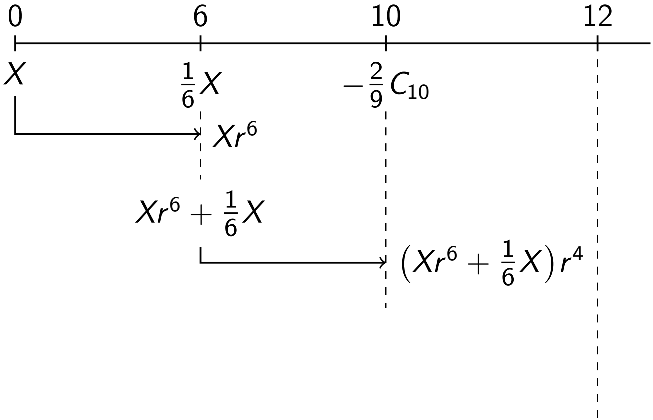
Drugi način razmišljanja



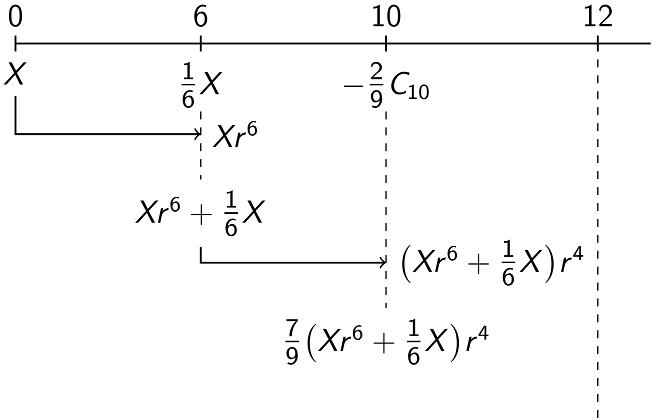
Drugi način razmišljanja



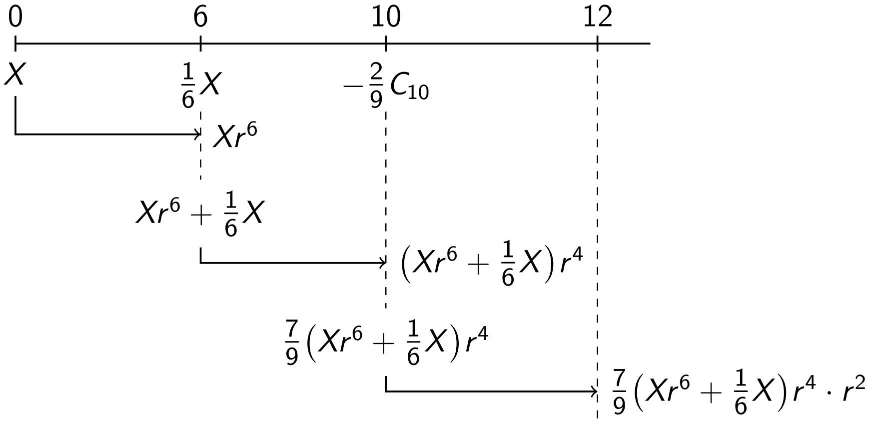
Drugi način razmišljanja



Drugi način razmišljanja

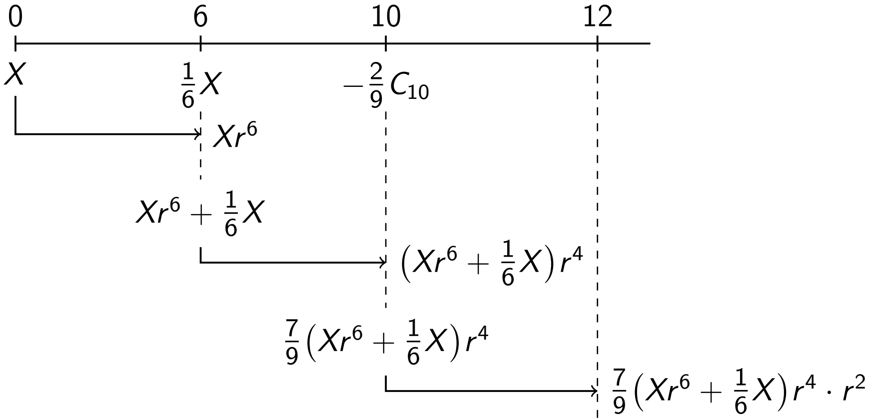


Drugi način razmišljanja



$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

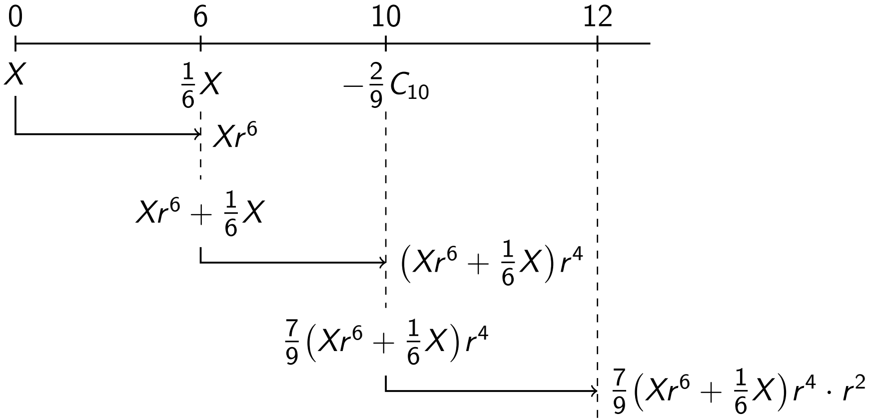
Drugi način razmišljanja



$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2 = 2950$$

Drugi način razmišljanja

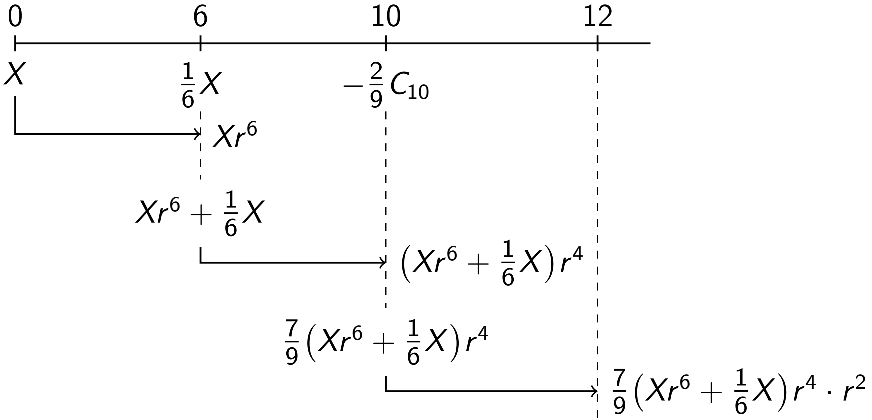


$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2 = 2950$$

$$\vdots$$

Drugi način razmišljanja



$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2 = 2950$$

$$\vdots$$

$$X = 3020.02$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

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$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950}$$

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$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950} \;\Big/ \log$$

$$n \log \sqrt{1.0825} = \log \frac{90}{59}$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950} \Big/ \log$$

$$n \log \sqrt{1.0825} = \log \frac{90}{59}$$

$$n = \frac{\log \frac{90}{59}}{\log \sqrt{1.0825}}$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950} \Big/ \log$$

$$n \log \sqrt{1.0825} = \log \frac{90}{59}$$

$$n = \frac{\log \frac{90}{59}}{\log \sqrt{1.0825}}$$

$$n = 10.65$$

b)

$$
\begin{aligned}
2950 \cdot \sqrt{1.0825}^{\,n} &= 4500 \\
\sqrt{1.0825}^{\,n} &= \frac{4500}{2950} \Big/ \log \\
n \log \sqrt{1.0825} &= \log \frac{90}{59} \\
n &= \frac{\log \frac{90}{59}}{\log \sqrt{1.0825}} \\
n &= 10.65
\end{aligned}
$$

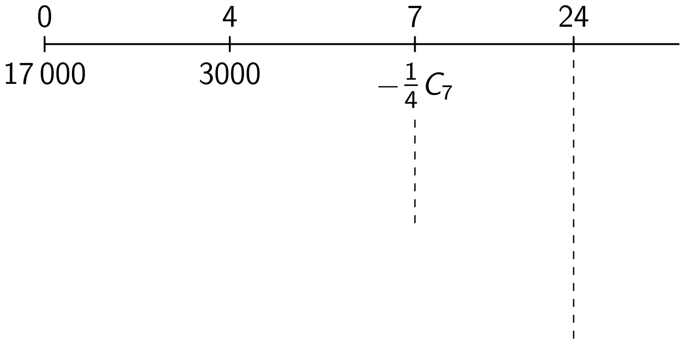
Viktorija će raspolagati s 4500 kn nakon 11 polugodišta od zadnjeg stanja.

sedmi zadatak

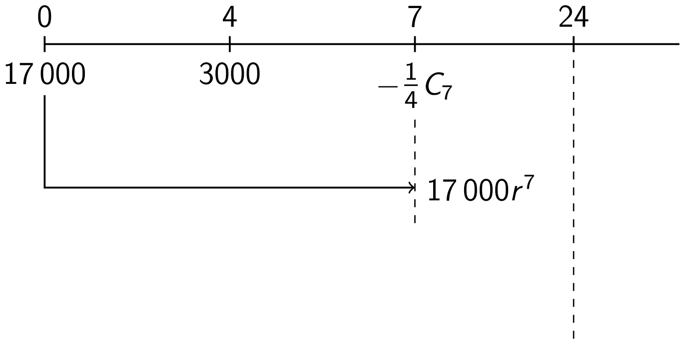
Zadatak 7

Netko uloži $17\,000$ *kn uz mjesečnu kamatnu stopu* 1.02%. *Nakon četiri mjeseca uloži još* 3000 *kn, a tri mjeseca poslije podigne četvrtinu iznosa s kojim raspolaže u tom trenutku. S kojom svotom raspolaže dvije godine nakon prve uplate?*

Rješenje



Rješenje



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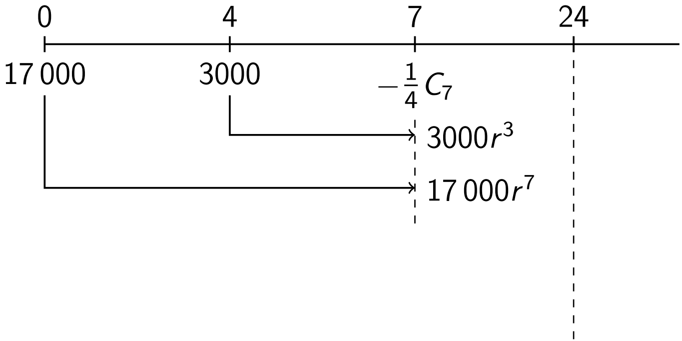
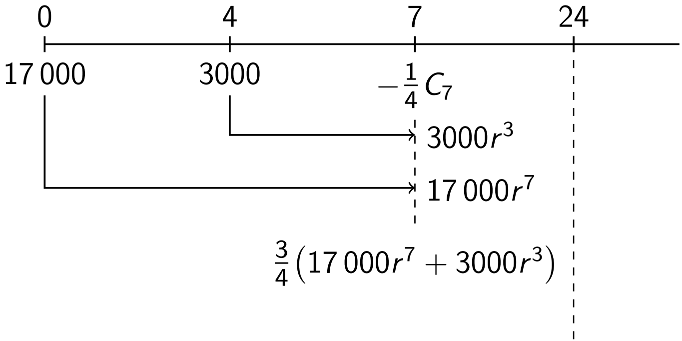
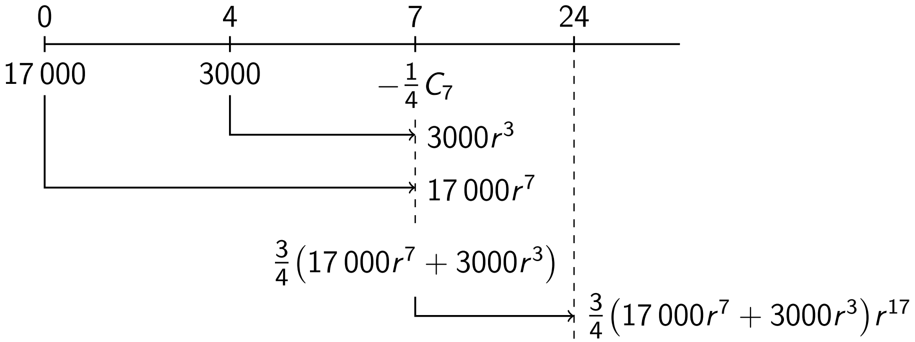


Rješenje

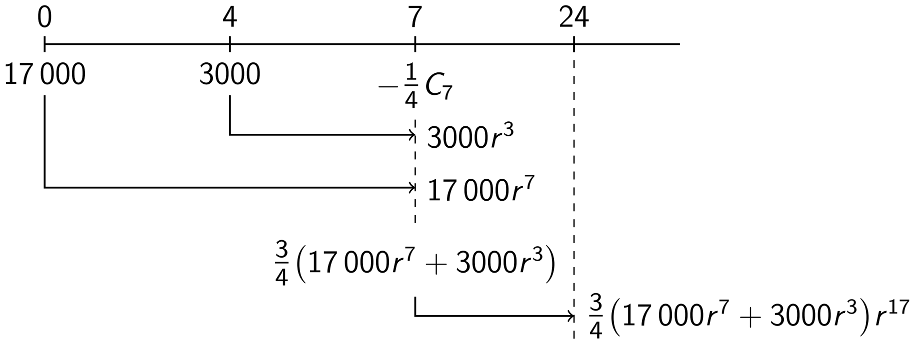


0
4
7
24
17 000
3000
$-\frac{1}{4}C_7$
$3000r^3$
$17\,000r^7$
$\frac{3}{4}\left(17\,000r^7 + 3000r^3\right)$

Rješenje

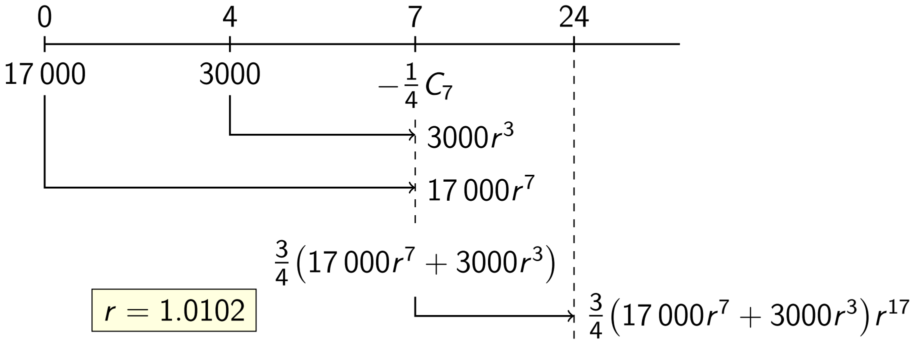

0
4
7
24
17 000
3000
$-\frac{1}{4}C_7$
$3000r^3$
$17\,000r^7$
$\frac{3}{4}\left(17\,000r^7 + 3000r^3\right)$
$\frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$

Rješenje



$$C_{24} = \frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$$

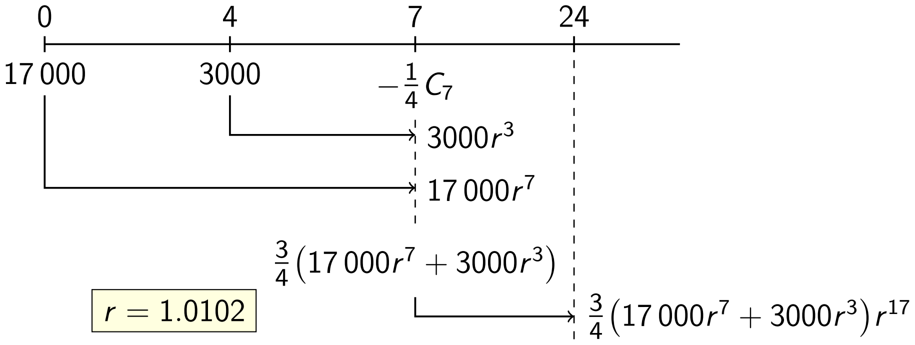
Rješenje



$$C_{24} = \frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$$

$$C_{24} = \frac{3}{4}\left(17\,000 \cdot 1.0102^7 + 3000 \cdot 1.0102^3\right) \cdot 1.0102^{17}$$

Rješenje



$$C_{24} = \frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$$

$$C_{24} = \frac{3}{4}\left(17\,000 \cdot 1.0102^7 + 3000 \cdot 1.0102^3\right) \cdot 1.0102^{17}$$

$$C_{24} = 19\,022.55$$