Seminari 8

Matematika za ekonomiste 2

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Sadržaj

prvi zadatak

Zadatak 1

Zadana je funkcija proizvodnje

$$Q(L, K) = 0.24L^{0.45}K^{0.37}$$

u ovisnosti o radu L i kapitalu K.

a) *Provjerite da je Q homogena funkcija i odredite njezin stupanj homogenosti.*

b) *Koristeći Eulerov teorem odredite sumu parcijalnih elastičnosti proizvodnje u odnosu na rad i kapital.*

c) *Odredite sumu parcijalnih elastičnosti direktno bez korištenja Eulerovog teorema.*

d) *Kakav tip prinosa određuje zadana funkcija proizvodnje?*

e) *Za koliko se promijeni količina proizvodnje ako rad i kapital povećamo za 10%?*

Rješenje

$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

$$Q(L, K) = 0.24 L^{0.45} K^{0.37}$$

a)

$$Q(\lambda L, \lambda K) =$$

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$$Q(L, K) = 0.24 L^{0.45} K^{0.37}$$

Rješenje

a)

$$Q(\lambda L, \lambda K) = 0.24 \cdot$$

$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

$$Q(L, K) = 0.24 L^{0.45} K^{0.37}$$

Rješenje

a)

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot$$

Rješenje

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$$= 0.24 \cdot$$

Rješenje

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$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

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$$\begin{aligned} Q(\lambda L, \lambda K) &= 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} = \\ &= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} = \\ &= \lambda^{0.82} \, . \end{aligned}$$

Rješenje

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$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

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Rješenje

a)

$$
\begin{aligned}
Q(\lambda L, \lambda K) &= 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} = \\
&= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} = \\
&= \lambda^{0.82} \cdot 0.24 L^{0.45} K^{0.37} = \\
&= \lambda^{0.82} \cdot Q(L, K)
\end{aligned}
$$

Q je homogena funkcija stupnja homogenosti $\alpha = 0.82$.

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$$= \lambda^{0.82} \cdot Q(L, K)$$

Q je homogena funkcija stupnja homogenosti $\alpha = 0.82$.

b)

$$E_{Q,L} + E_{Q,K} =$$

$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

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$$E_{Q,L} + E_{Q,K} = 0.82$$

$$Q(L,K) = 0.24L^{0.45}K^{0.37}$$

c) $E_{Q,L} + E_{Q,K} =$

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c) $E_{Q,L} + E_{Q,K} =$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L$$

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c) $E_{Q,L} + E_{Q,K} =$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \text{———————}$$

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c) $E_{Q,L} + E_{Q,K} =$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{\qquad\qquad}$$

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c) $E_{Q,L} + E_{Q,K} =$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}}$$

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$$E_{Q,L} = 0.45$$

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$$E_{Q,K} = 0.37$$

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c) $E_{Q,L} + E_{Q,K} = 0.45 + 0.37$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L} = 0.45$$

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c) $E_{Q,L} + E_{Q,K} = 0.45 + 0.37 = 0.82$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

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$$E_{Q,K} = 0.37$$

d) Stupanj homogenosti: $\alpha = 0.82, \quad 0 < \alpha < 1$

$$Q(L,K) = 0.24L^{0.45}K^{0.37}$$

c) $E_{Q,L} + E_{Q,K} = 0.45 + 0.37 = 0.82$

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$$E_{Q,K} = 0.37$$

d) Stupanj homogenosti: $\alpha = 0.82, \quad 0 < \alpha < 1$

Kako je stupanj homogenosti između 0 i 1, zadana funkcija proizvodnje ima padajuće prinose.

e)

$$L \xrightarrow{\text{10\% povećanja rada}}$$

e)

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

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$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}}$$

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$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

e)

$$\boxed{Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)}$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) =$$

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$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

e)

$$\boxed{Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)}$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

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- Promjena proizvodnje:

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$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

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- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

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- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

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$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

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$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

$$\frac{\qquad\qquad\qquad\qquad}{Q(L, K)}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)}$$

$$p = \frac{100y}{x}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

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$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{}{Q(L, K)}$$

$$p = \frac{100y}{x}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K)}{Q(L, K)}$$

$$p = \frac{100y}{x}$$

e)

$$\boxed{Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)}$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

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$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)}$$

$$\boxed{p = \frac{100y}{x}}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$$

$$= \frac{\qquad\qquad\qquad\qquad}{Q(L, K)}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$$

$$= \frac{(1.1^{0.82} - 1) \cdot Q(L, K)}{Q(L, K)}$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$$

$$= \frac{(1.1^{0.82} - 1) \cdot Q(L, K)}{Q(L, K)} = 1.1^{0.82} - 1$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$$

$$= \frac{(1.1^{0.82} - 1) \cdot Q(L, K)}{Q(L, K)} = 1.1^{0.82} - 1 \approx 0.08129$$

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

$$p = \frac{100y}{x}$$

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$$

$$= \frac{(1.1^{0.82} - 1) \cdot Q(L, K)}{Q(L, K)} = 1.1^{0.82} - 1 \approx 0.08129$$

Ako rad i kapital povećamo za 10%, proizvodnja će se povećati za 8.129%.

e)

$$Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$$

$$L \xrightarrow{\text{10\% povećanja rada}} L + 0.1L = 1.1L$$

$$K \xrightarrow{\text{10\% povećanja kapitala}} K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

- Promjena proizvodnje: $Q(1.1L, 1.1K) - Q(L, K)$
- Promjena proizvodnje u postocima:

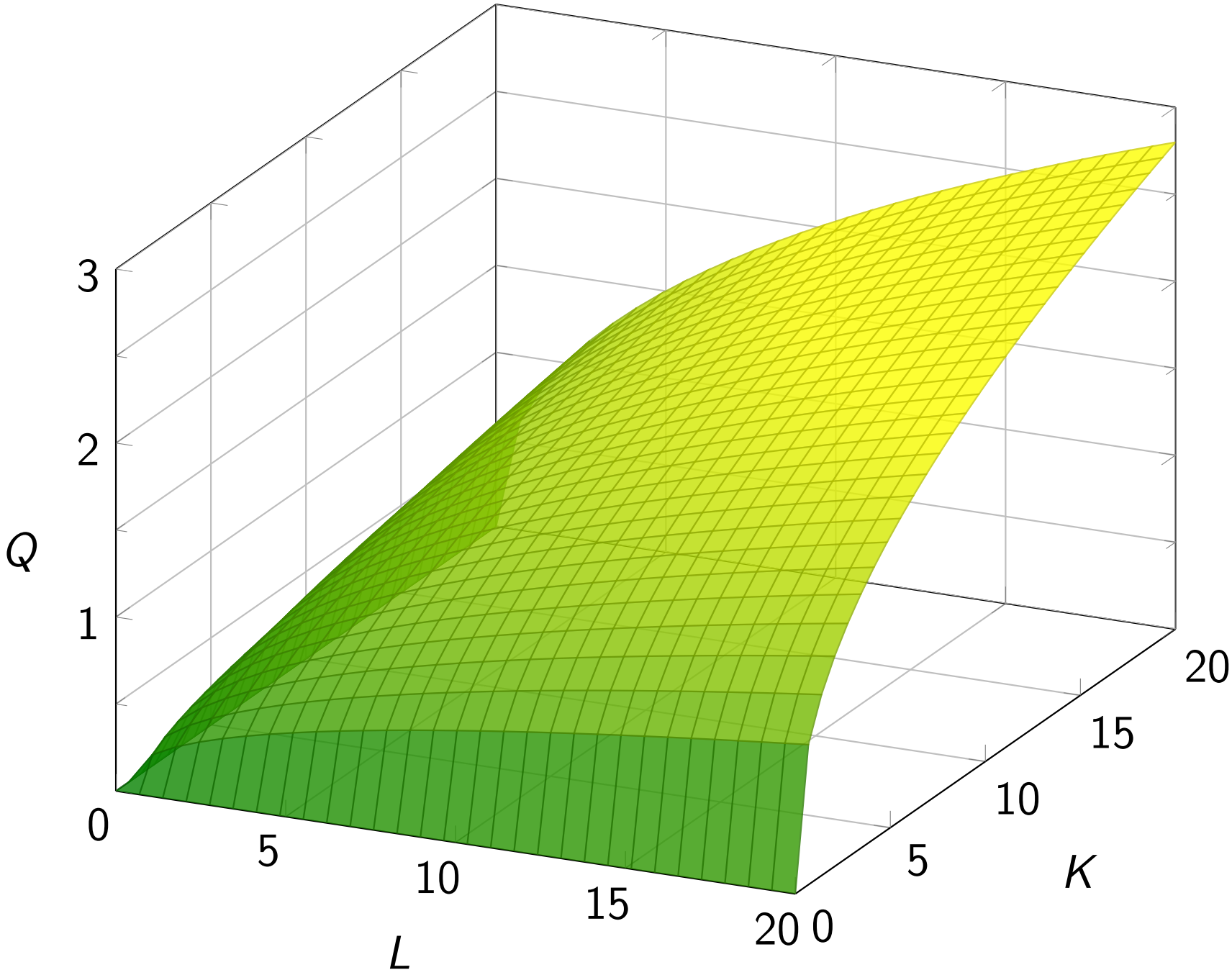
$$p = \frac{100y}{x}$$

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$$

$$= \frac{(1.1^{0.82} - 1) \cdot Q(L, K)}{Q(L, K)} = 1.1^{0.82} - 1 \approx 0.08129$$

Ako rad i kapital povećamo za 10%, proizvodnja će se povećati za 8.129%.

padajući prinosi


3
2
1
0
Q
0
5
10
15
20
L
0
5
10
15
20
K

drugi zadatak

Zadatak 2

Zadana je funkcija proizvodnje

$$Q(L, K) = 2L^{0.25}K^{0.5}$$

u ovisnosti o radu L i kapitalu K.

a) *Odredite funkciju granične produktivnosti rada i interpretirajte rezultat na nivou $L = 10$, $K = 5$.*

b) *Odredite funkciju granične produktivnosti kapitala i interpretirajte rezultat na nivou $L = 10$, $K = 5$.*

c) *Izvedite jednadžbu izokvante $L = L(K)$ na nivou proizvodnje $Q = 30$.*

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L =$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75}$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(10,5) =$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L \quad K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5}$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L \quad K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(\underset{L}{10},\underset{K}{5}) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(\underset{L}{10},\underset{K}{5}) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(\underset{L}{10},\underset{K}{5}) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K =$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(\underset{L}{10},\underset{K}{5}) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L \quad K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5}$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(\underset{L}{10},\underset{K}{5}) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

$$Q_K(10,5) =$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L\ \ K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

$$\overset{L\ \ K}{Q_K(10,5)} = 10^{0.25} \cdot 5^{-0.5}$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L\ \ K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

$$\overset{L\ \ K}{Q_K(10,5)} = 10^{0.25} \cdot 5^{-0.5} = 0.79527\cdots$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L\ \ K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

$$\overset{L\ \ K}{Q_K(10,5)} = 10^{0.25} \cdot 5^{-0.5} = 0.79527\cdots \approx 0.8$$

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$\overset{L\ \ K}{Q_L(10,5)} = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988\cdots \approx 0.2$$

Ako na nivou $L = 10$, $K = 5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_K = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

$$\overset{L\ \ K}{Q_K(10,5)} = 10^{0.25} \cdot 5^{-0.5} = 0.79527\cdots \approx 0.8$$

Ako na nivou $L = 10$, $K = 5$ kapital povećamo za jednu jedinicu, proizvodnja će se povećati za 0.8 jedinica.

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 \ \big/ : 2$$

$$L^{0.25}K^{0.5} = 15$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 \;/ : 2$$

$$L^{0.25}K^{0.5} = 15 \;/^{4}$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 \;/ : 2$$

$$L^{0.25}K^{0.5} = 15 \;/^{4}$$

$$LK^{2} = 50\,625$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 \;/ : 2$$

$$L^{0.25}K^{0.5} = 15 \;/^{4}$$

$$LK^2 = 50\,625$$

$$L = L(K)$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 \;/ : 2$$

$$L^{0.25}K^{0.5} = 15 \;/^{4}$$

$$LK^2 = 50\,625$$

$$L = \frac{50\,625}{K^2}$$

$$L = L(K)$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

c)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 \;/ : 2$$

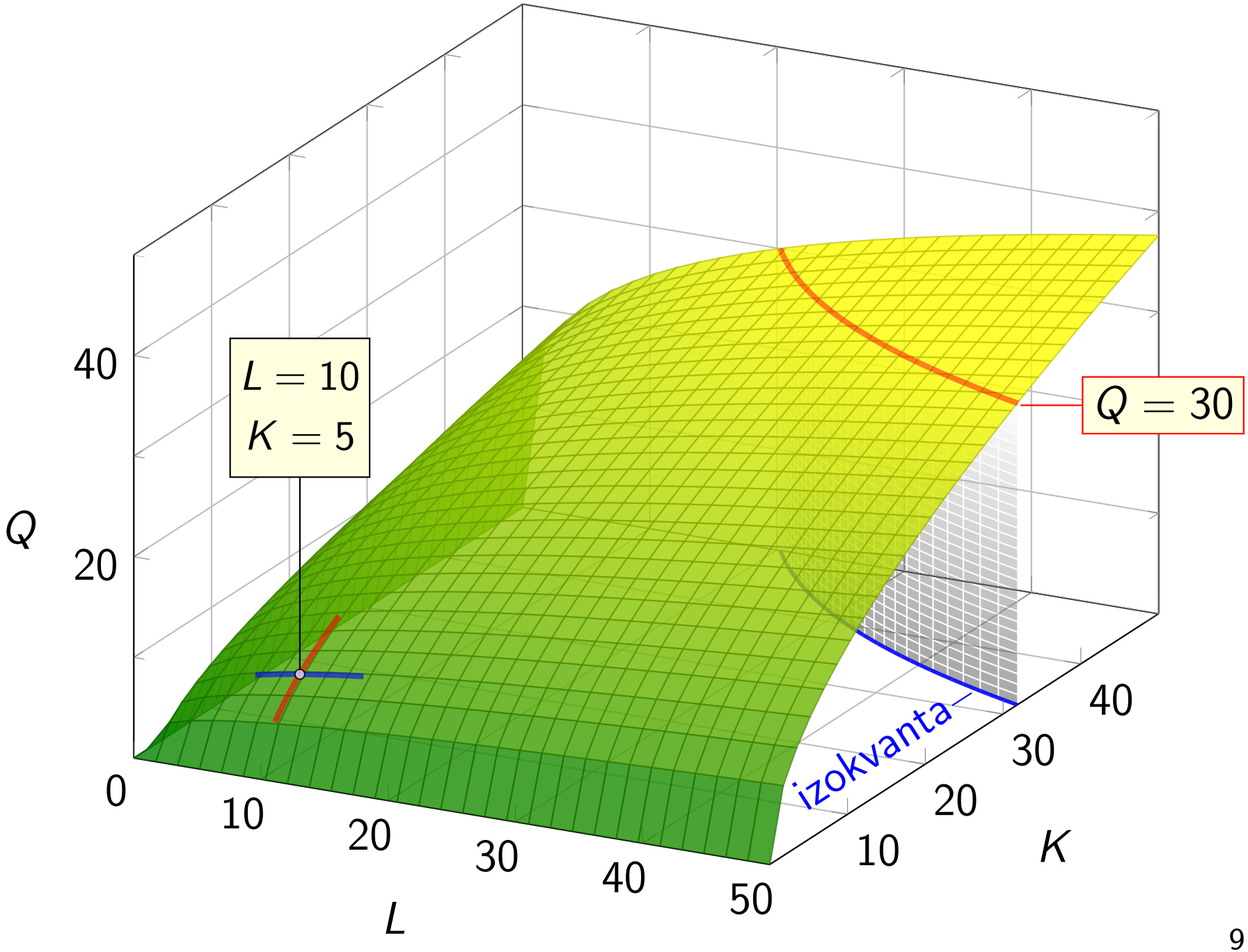
$$L^{0.25}K^{0.5} = 15 \;/^{4}$$

$$LK^2 = 50\,625$$

$$L = L(K)$$

$$L = \frac{50\,625}{K^2}$$

jednadžba izokvante


$L = 10$
$K = 5$
$Q = 30$
izokvanta
Q
40
20
0
10
20
30
40
50
L
10
20
30
40
K

treći zadatak

Zadatak 3

Zadana je funkcija proizvodnje

$$Q(L, K) = 3L^{\frac{1}{2}}K$$

u ovisnosti o radu L i kapitalu K.

a) *Jedna jedinica rada košta 10 €, a jedna jedinica kapitala košta 15 €. Ako poduzeće ima na raspolaganju 20 000 €, pronađite kombinaciju rada i kapitala za koje se uz maksimalno iskorištenje kapaciteta ostvaruje maksimalna proizvodnja. Koliko iznosi maksimalna proizvodnja?*

b) *Na istoj slici prikažite budžetsko ograničenje i izokvantu na nivou maksimalne proizvodnje. Što možete reći o njihovom odnosu?*

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje:

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000 \,/ : 5$$
$$2L + 3K = 4000$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$\begin{aligned} 10L + 15K &= 20\,000 \,/ : 5 \\ 2L + 3K &= 4000 \\ 2L &= 4000 - 3K \end{aligned}$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$\begin{aligned} 10L + 15K &= 20\,000 \,/ : 5 \\ 2L + 3K &= 4000 \\ 2L &= 4000 - 3K \,/ : 2 \\ L &= 2000 - \frac{3}{2}K \end{aligned}$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K \,/ : 2$$

$$\boxed{L = 2000 - \frac{3}{2}K}$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K \,/ : 2$$

$$\boxed{L = 2000 - \frac{3}{2}K}$$

$$Q\left(2000 - \frac{3}{2}K, K\right) =$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000 \,/ : 5$$
$$2L + 3K = 4000$$
$$2L = 4000 - 3K \,/ : 2$$
$$\boxed{L = 2000 - \frac{3}{2}K}$$

$$Q\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} K$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K \,/ : 2$$

$$\boxed{L = 2000 - \frac{3}{2}K}$$

$$Q\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} K$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

Rješenje

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$2000 - \frac{3}{2}K \geqslant 0$$

$$-\frac{3}{2}K \geqslant -2000$$

$$K \leqslant \frac{4000}{3}$$

$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K \,/ : 2$$

$$\boxed{L = 2000 - \frac{3}{2}K}$$

$$Q\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} K$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) =$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} +$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

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$$f'(K) =$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

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$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

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$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \Big[$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

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$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right)\right.$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) -\right.$$

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$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

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$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

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$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

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$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot$$

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$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000\right.$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right.$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

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$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

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$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

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$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

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$$24\,000 - 27K = 0$$

$$K = \frac{8000}{9}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

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$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0	$\frac{8000}{9}$	$\frac{4000}{3}$
f'			
f			

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$			
f					

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

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$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f					

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

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$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$			

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

globalni maksimum

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4 \qquad L = 2000 - \frac{3}{2}K$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

globalni maksimum

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

$$L = 2000 - \frac{3}{2}K$$

$$L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

globalni maksimum

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

$$L = 2000 - \frac{3}{2}K$$

$$L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$L = \frac{2000}{3}$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

globalni maksimum

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

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$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

$$L = 2000 - \frac{3}{2}K$$

$$L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$\boxed{L = \frac{2000}{3}}$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

globalni maksimum

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 \Big/ \cdot 4$$

$$24\,000 - 27K = 0$$

$$\boxed{K = \frac{8000}{9}} \qquad K \approx 888.89$$

$$L = 2000 - \frac{3}{2}K$$

$$L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$\boxed{L = \frac{2000}{3}} \qquad L \approx 666.67$$

	0		$\frac{8000}{9}$		$\frac{4000}{3}$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

globalni maksimum

- Maksimalna proizvodnja

$$Q(L, K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) =$$

- Maksimalna proizvodnja

$$Q(L, K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\overset{L}{\frac{2000}{3}}, \overset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9}$$

- Maksimalna proizvodnja

$$Q(L, K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\underset{L}{\frac{2000}{3}}, \underset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

- Maksimalna proizvodnja

$$Q(L, K) = 3L^{\frac{1}{2}}K$$

$$\overset{L}{\frac{2000}{3}}, \overset{K}{\frac{8000}{9}}$$

$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

b) Izokvanta na nivou maksimalne proizvodnje

- Maksimalna proizvodnja

$$Q(L, K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\overset{L}{\frac{2000}{3}}, \overset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

b) Izokvanta na nivou maksimalne proizvodnje

$$Q = 68\,853.04$$

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

- Maksimalna proizvodnja

$$Q\left(\underset{L}{\frac{2000}{3}}, \underset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

b) Izokvanta na nivou maksimalne proizvodnje

$$Q = 68\,853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

- Maksimalna proizvodnja

$$Q\left(\underset{L}{\frac{2000}{3}}, \underset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

b) Izokvanta na nivou maksimalne proizvodnje

$$Q = 68\,853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$K = \frac{68\,853.04}{3L^{\frac{1}{2}}}$$

- Maksimalna proizvodnja

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\overset{L}{\frac{2000}{3}}, \overset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

b) Izokvanta na nivou maksimalne proizvodnje

$$Q = 68\,853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$K = \frac{68\,853.04}{3L^{\frac{1}{2}}}$$

$$K = K(L)$$

- Maksimalna proizvodnja

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\overset{L}{\frac{2000}{3}}, \overset{K}{\frac{8000}{9}}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

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Budžetsko ograničenje

- Maksimalna proizvodnja

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Budžetsko ograničenje

$$10L + 15K = 20\,000 \,/ : 5$$

- Maksimalna proizvodnja

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Budžetsko ograničenje

$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000 \,/ : 4000$$

- Maksimalna proizvodnja

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

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Budžetsko ograničenje

$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000 \,/ : 4000$$

$$\frac{2L}{4000} + \frac{3K}{4000} = 1$$

- Maksimalna proizvodnja

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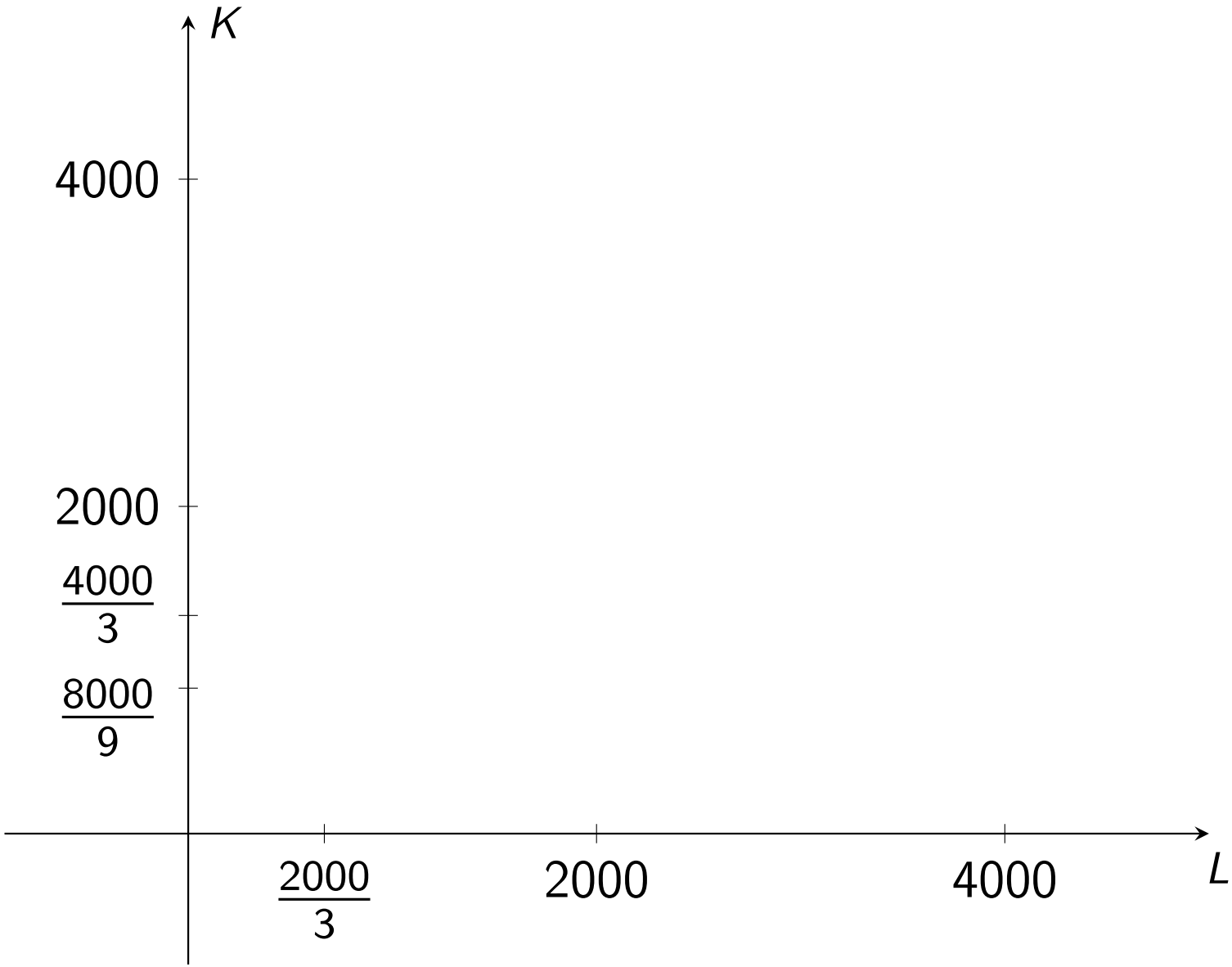
Budžetsko ograničenje

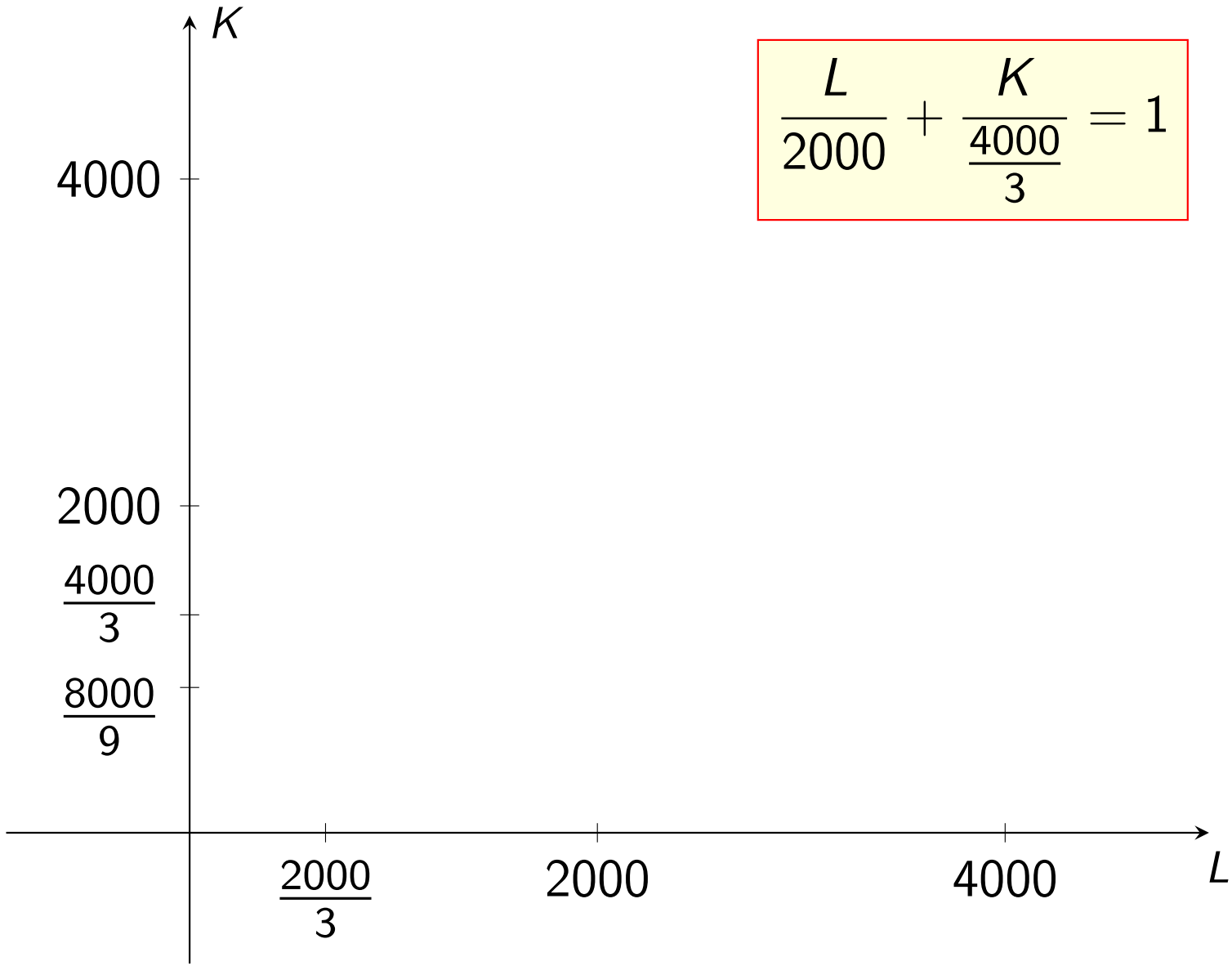
$$10L + 15K = 20\,000 \,/ : 5$$

$$2L + 3K = 4000 \,/ : 4000$$

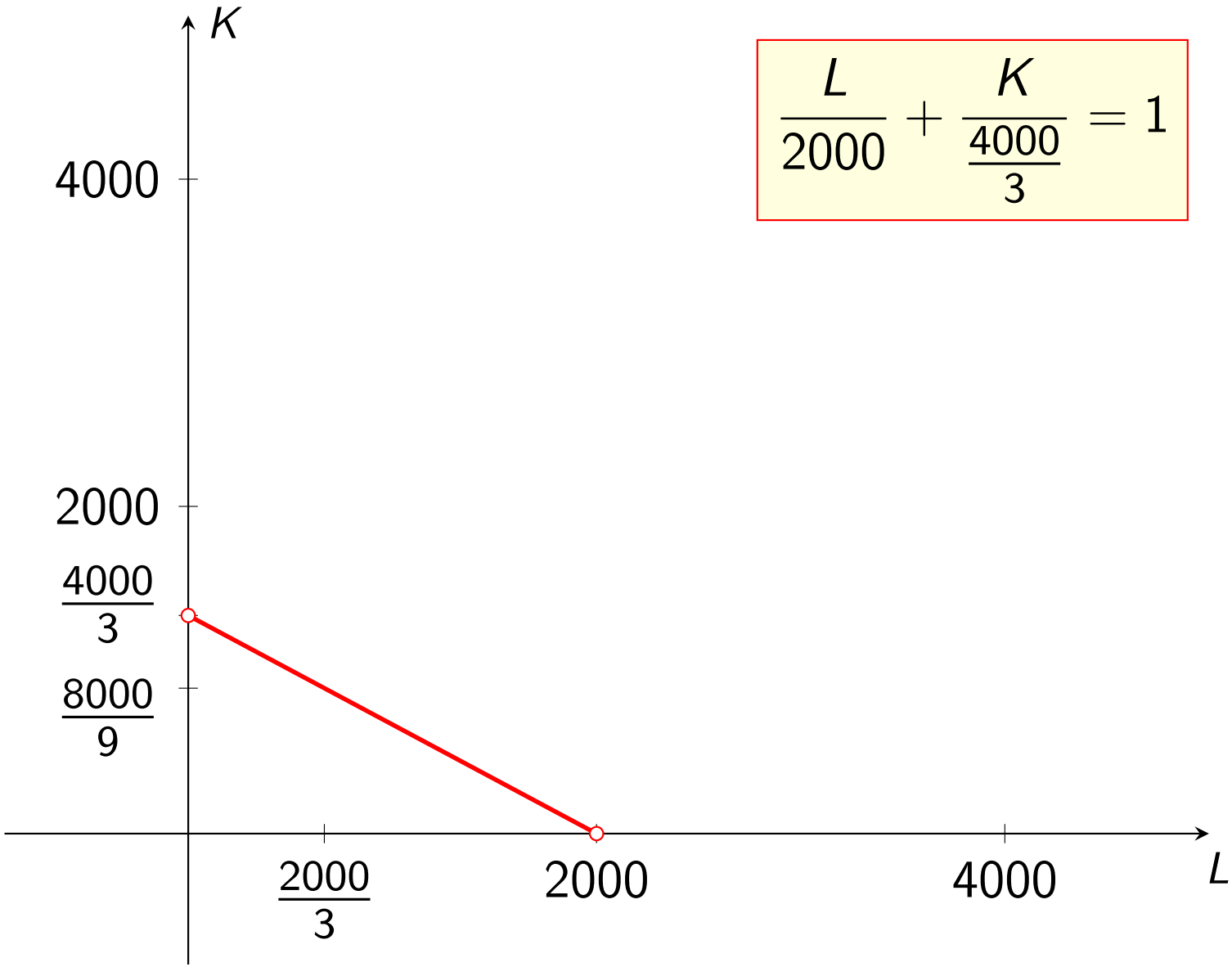
$$\frac{2L}{4000} + \frac{3K}{4000} = 1$$

$$\frac{L}{2000} + \frac{K}{\frac{4000}{3}} = 1$$


K
4000
2000
4000/3
8000/9
2000/3
2000
4000
L



$$\frac{L}{2000} + \frac{K}{\frac{4000}{3}} = 1$$



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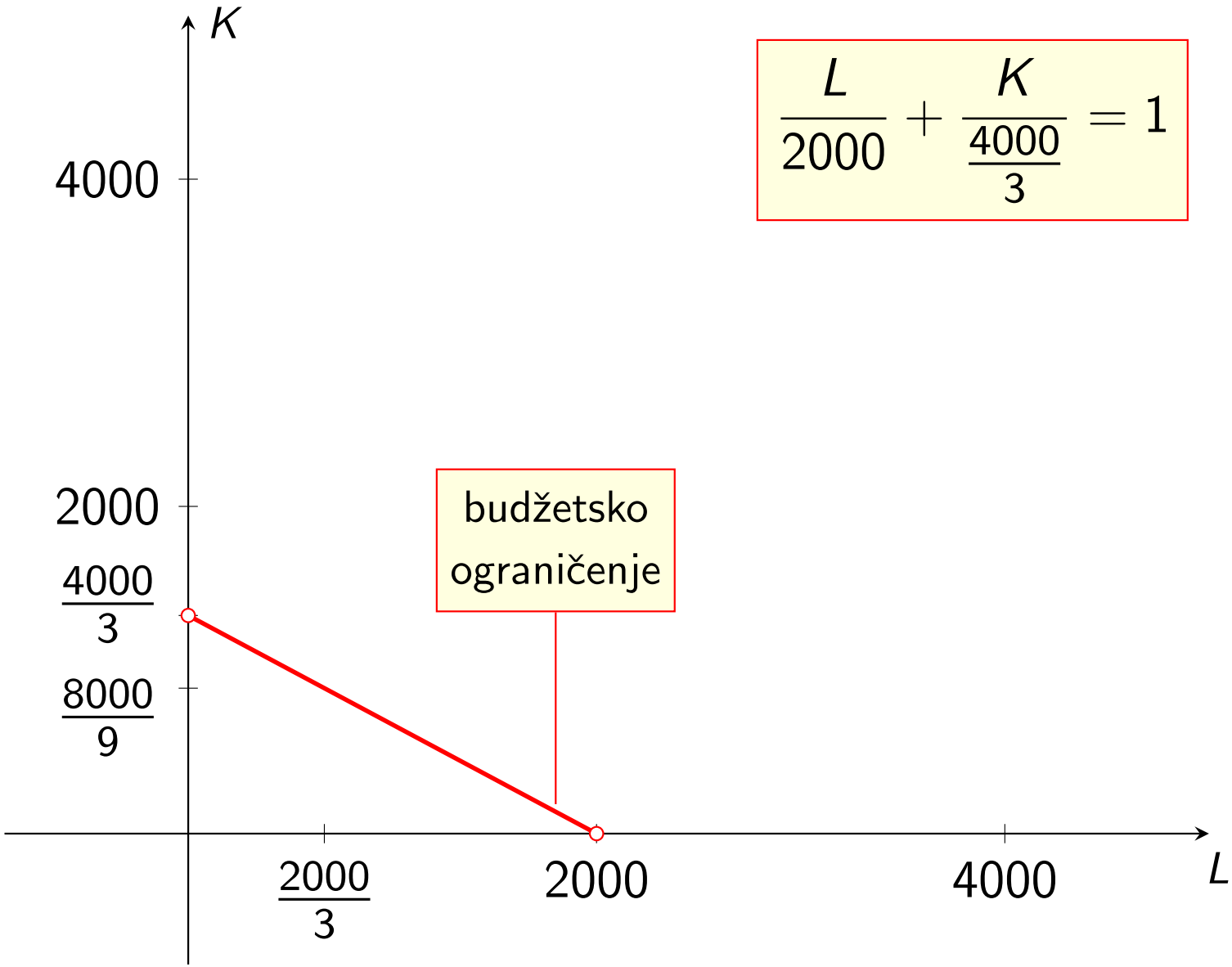
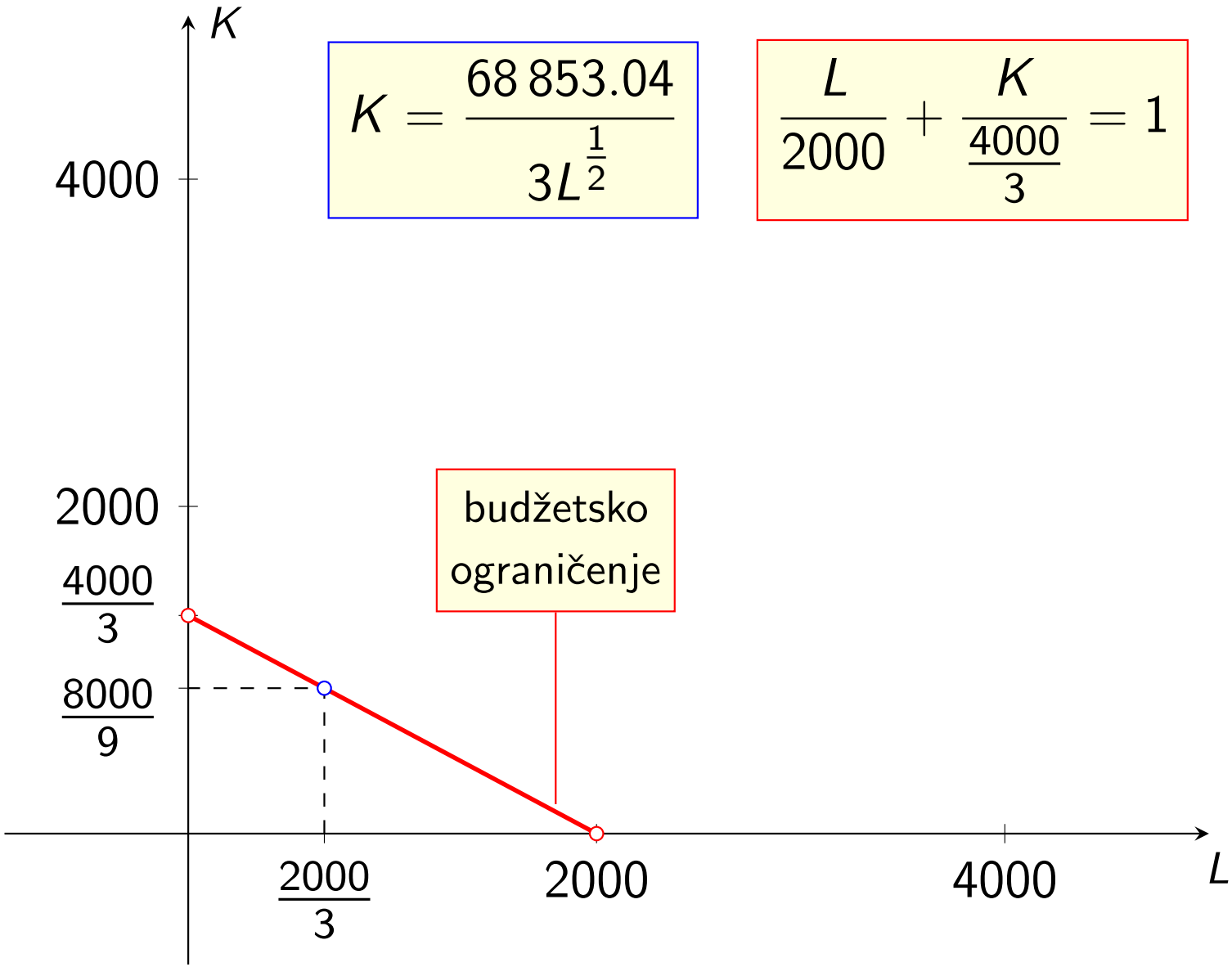
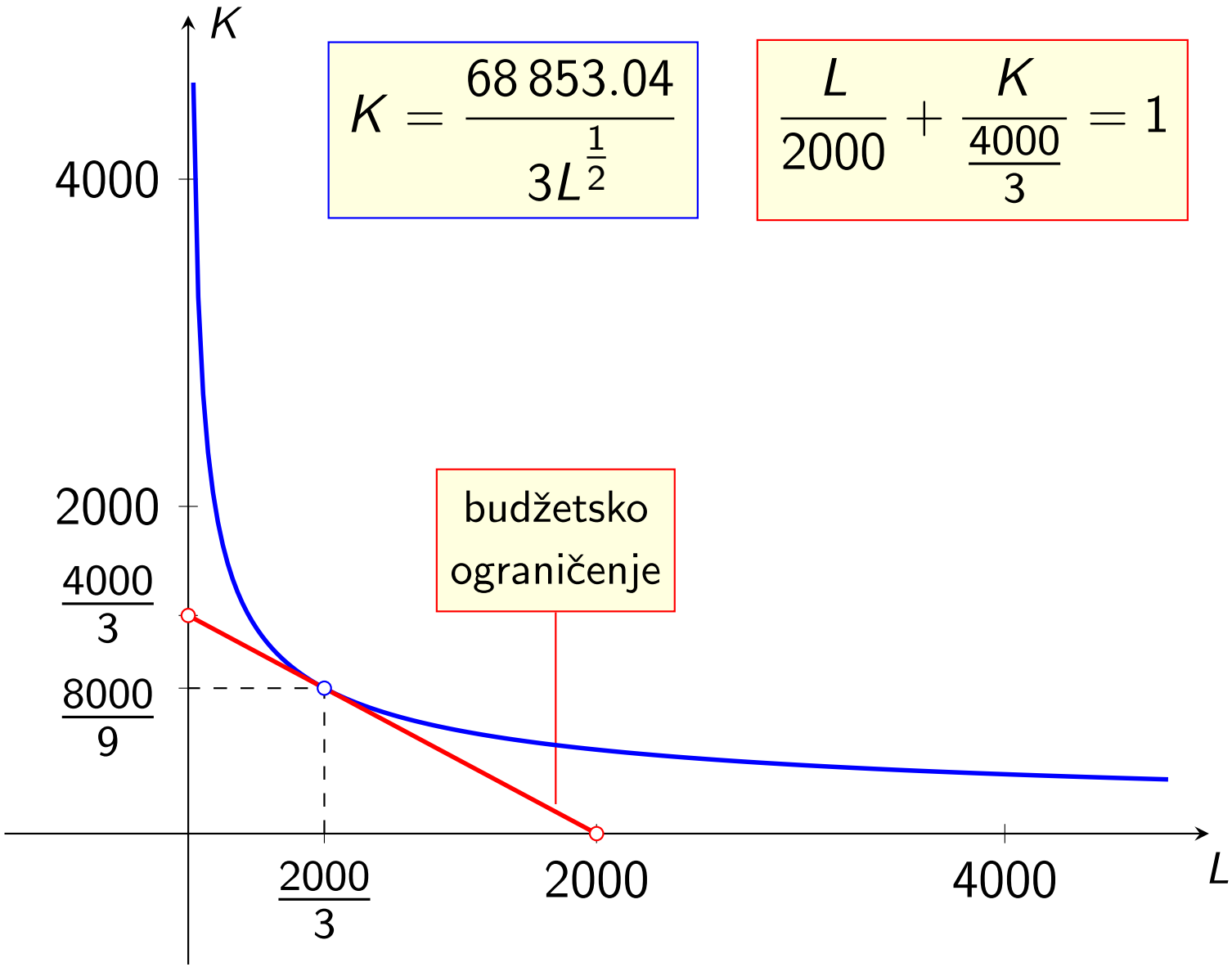


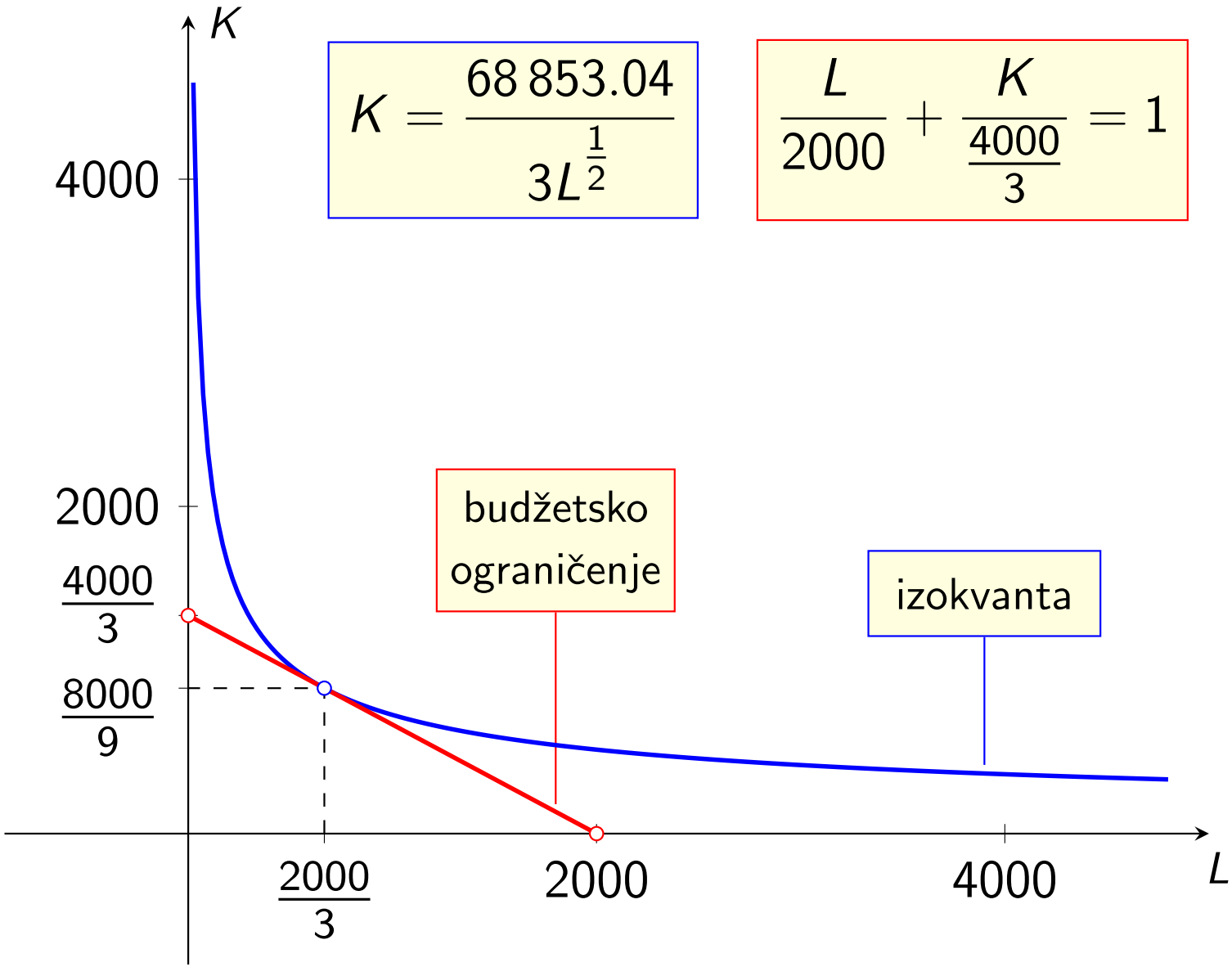
$$\frac{L}{2000} + \frac{K}{\frac{4000}{3}} = 1$$



$$K = \frac{68\,853.04}{3L^{\frac{1}{2}}}$$

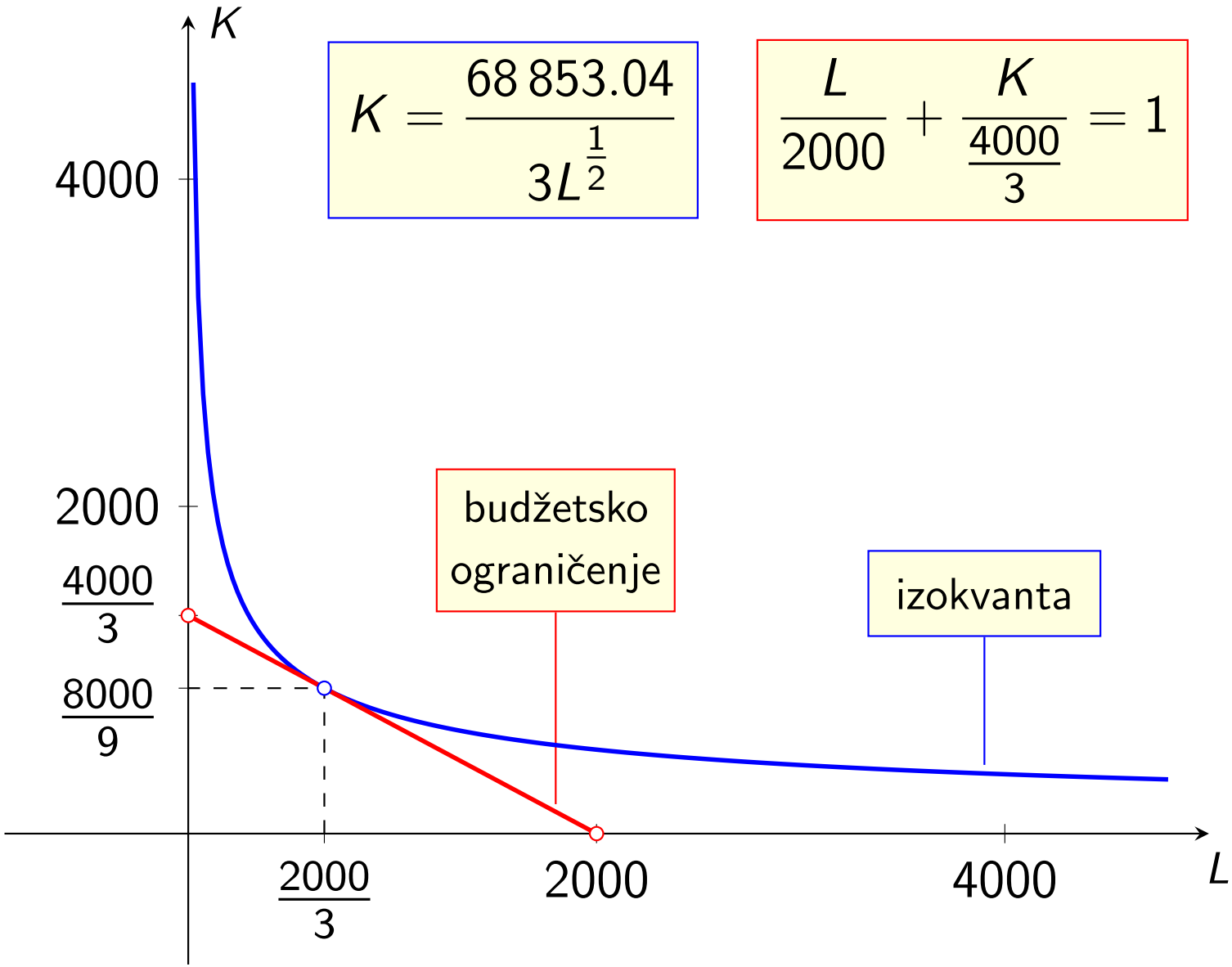
$$\frac{L}{2000} + \frac{K}{\frac{4000}{3}} = 1$$


K
$K = \frac{68\,853.04}{3L^{\frac{1}{2}}}$
$\frac{L}{2000} + \frac{K}{\frac{4000}{3}} = 1$
4000
2000
$\frac{4000}{3}$
$\frac{8000}{9}$
budžetsko ograničenje
$\frac{2000}{3}$
2000
4000
L

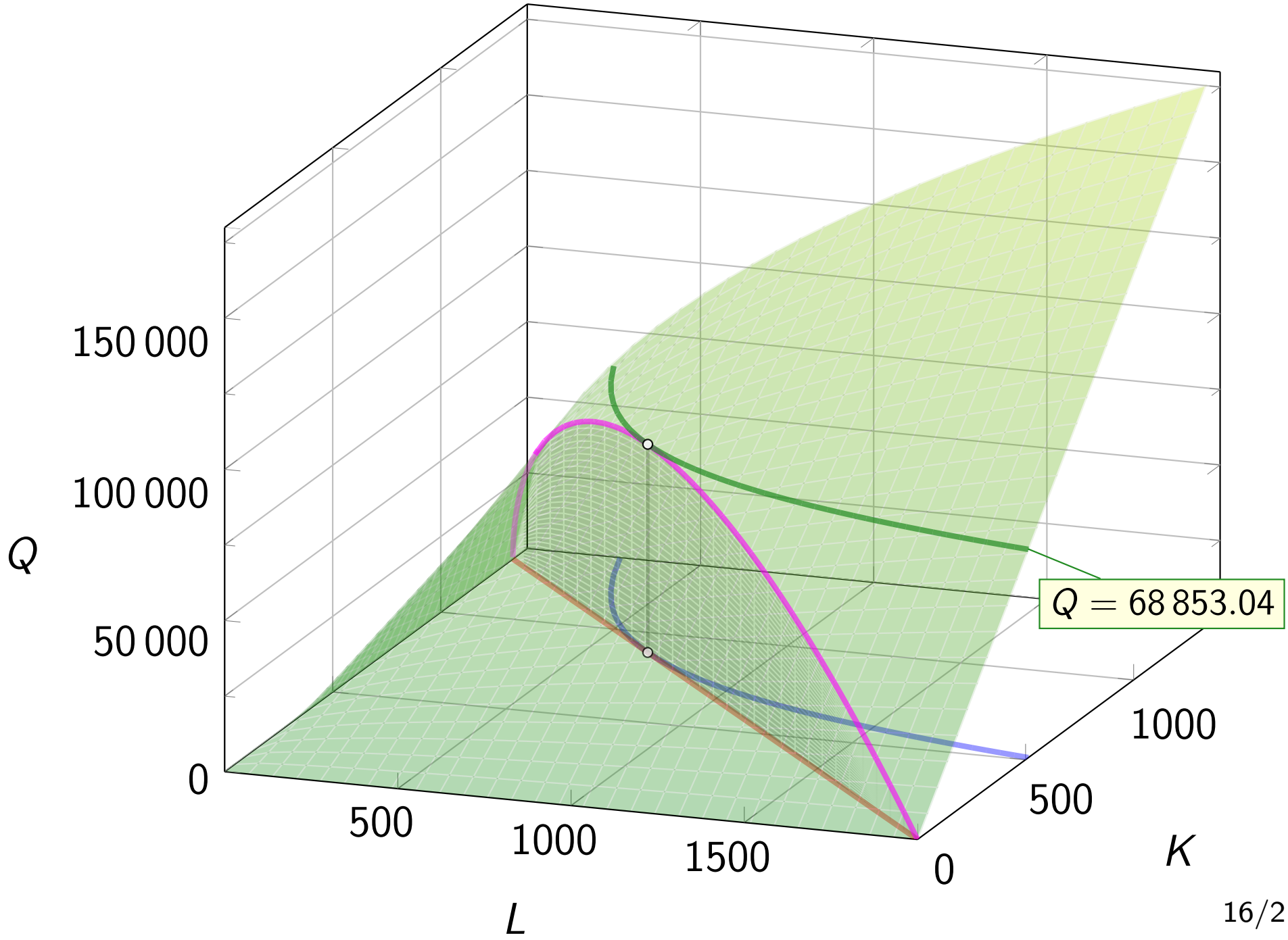


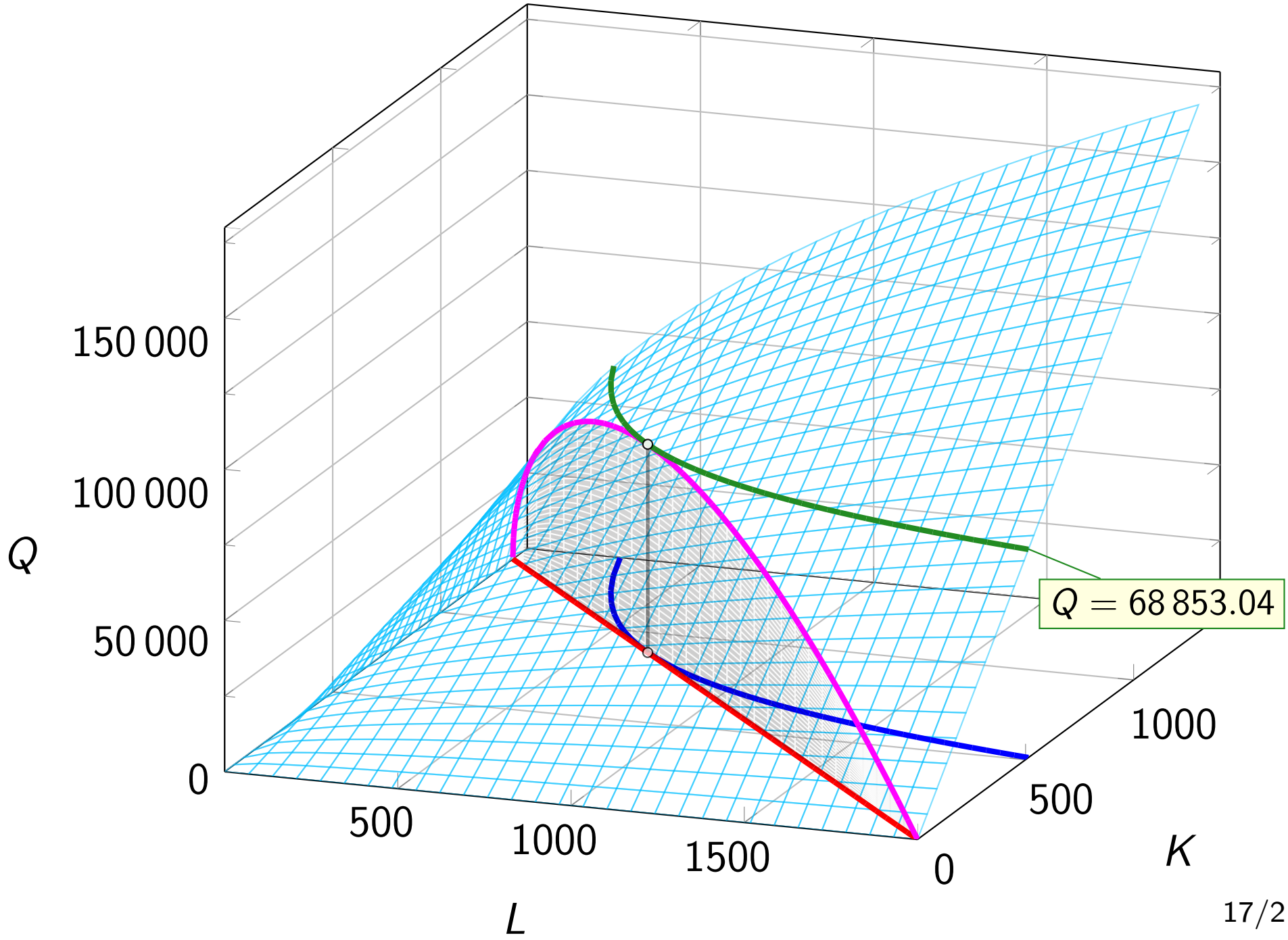
$$K = \frac{68\,853.04}{3L^{\frac{1}{2}}}$$

$$\frac{L}{2000} + \frac{K}{\frac{4000}{3}} = 1$$



Budžetsko ograničenje je tangenta na izokvantu na nivou maksimalne proizvodnje u točki u kojoj se postiže maksimalna proizvodnja.


150 000
100 000
50 000
0
Q
500
1000
1500
L
0
500
1000
K
$Q = 68\,853.04$


150 000
100 000
50 000
0
Q
500
1000
1500
L
0
500
1000
K
$Q = 68\,853.04$

četvrti zadatak

Zadatak 4

Cijena jedinice rada iznosi $1\,€$*, cijena jedinice kapitala iznosi* $2\,€$*, a fiksni troškovi su* $10\,€$*. Funkcija proizvodnje u ovisnosti o radu* L *i kapitalu* K *dana je s*

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}.$$

Na nivou proizvodnje $Q = 8$ *pronađite optimalnu kombinaciju rada i kapitala tako da troškovi budu minimalni. Koliko iznose minimalni troškovi?*

Rješenje

$$Q(L, K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L, K) =$$

Rješenje

$$Q(L, K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L, K) = 1 \cdot L$$

Rješenje

$$Q(L, K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L, K) = 1 \cdot L + 2 \cdot K$$

Rješenje

$$Q(L, K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L, K) = 1 \cdot L + 2 \cdot K + 10$$

Rješenje

$$Q(L,K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$

$$T(L,K) = L + 2K + 10$$

Rješenje

$$Q(L,K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

- Uvjet

Rješenje

$$Q(L,K) = \sqrt{0.5} L K^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$
$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$
$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$
$$L =$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$
$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$
$$L = \frac{8}{\sqrt{0.5}}$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$
$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$
$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

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$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
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- Uvjet

$$Q = 8$$
$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$
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Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$

$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

$$T\left(\frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}, K\right) =$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

- Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$

$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

$$T\left(\frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}, K\right) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

Rješenje

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

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$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$

$$T(L,K) = L + 2K + 10$$

- Uvjet

$$Q = 8$$

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$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

$$T\left(\frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}, K\right) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

$$f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$f'(K) =$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot$$

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$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot \frac{-1}{2} K^{-\frac{3}{2}} + 2$$

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$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

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$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot \frac{-1}{2} K^{-\frac{3}{2}} + 2$$

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$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} = -2$$

$$T(L,K) = L + 2K + 10$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot \frac{-1}{2} K^{-\frac{3}{2}} + 2$$

$$f'(K) = -\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} = -2 \Big/ \cdot \frac{-\sqrt{0.5}}{4}$$

$$T(L,K) = L + 2K + 10$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

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$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2}$$

$$T(L,K) = L + 2K + 10$$

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$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2} \Big/^{-\frac{2}{3}}$$

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$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2} \Big/^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

$$f'$$

$$f' \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} =$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

f'

$$\left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}}$$

f'

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$f' \quad -\frac{4}{\quad} K^{-\frac{3}{2}} + 2 = 0$$

$$f' \quad \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}} = \left(\frac{2^{-\frac{1}{2}}}{2}\right)^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

$$f'$$

$$f'$$

$$\left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}} = \left(\frac{2^{-\frac{1}{2}}}{2}\right)^{-\frac{2}{3}} = \left(2^{-\frac{3}{2}}\right)^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$T(L,K) = L + 2K + 10$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 = 0$$

$$f'$$

$$\left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}} = \left(\frac{2^{-\frac{1}{2}}}{2}\right)^{-\frac{2}{3}} = \left(2^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = 2$$

$$f'$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$T(L,K) = L + 2K + 10$$

$$f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot \frac{-1}{2}K^{-\frac{3}{2}} + 2$$

$$f'(K) = -\frac{4}{\sqrt{0.5}}K^{-\frac{3}{2}} + 2$$

$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

$$-\frac{4}{\sqrt{0.5}}K^{-\frac{3}{2}} + 2 = 0$$

$$-\frac{4}{\sqrt{0.5}}K^{-\frac{3}{2}} = -2 \Big/ \cdot \frac{-\sqrt{0.5}}{4}$$

$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2} \Big/^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$K = 2$$

$$T(L,K) = L + 2K + 10$$

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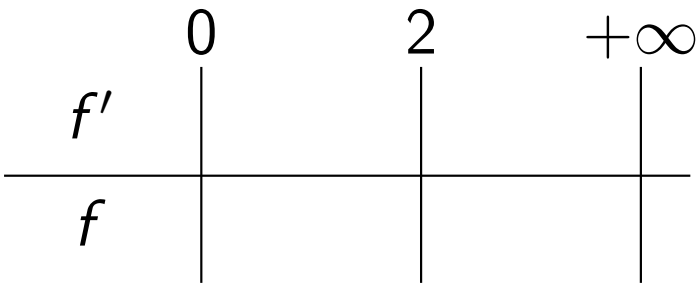

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	0		2		$+\infty$
f'		$-$			
f					

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	0		2		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

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	0		2		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

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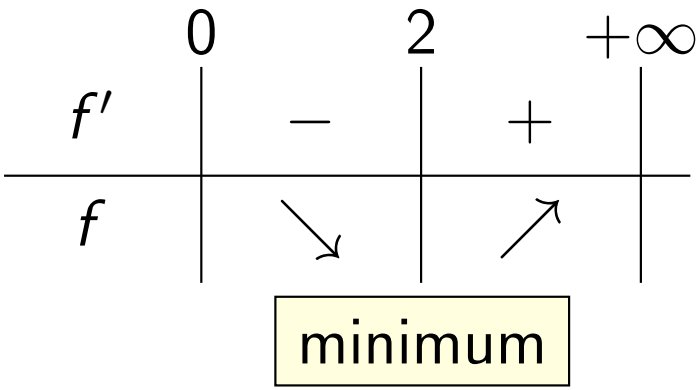

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	0		2		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

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f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

minimum

$$T(8,2) =$$

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	0		2		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

minimum

$$T(8,2) = 8 + 2 \cdot 2 + 10$$

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	0		2		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

minimum

$$T(8,2) = 8 + 2 \cdot 2 + 10 = 22$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

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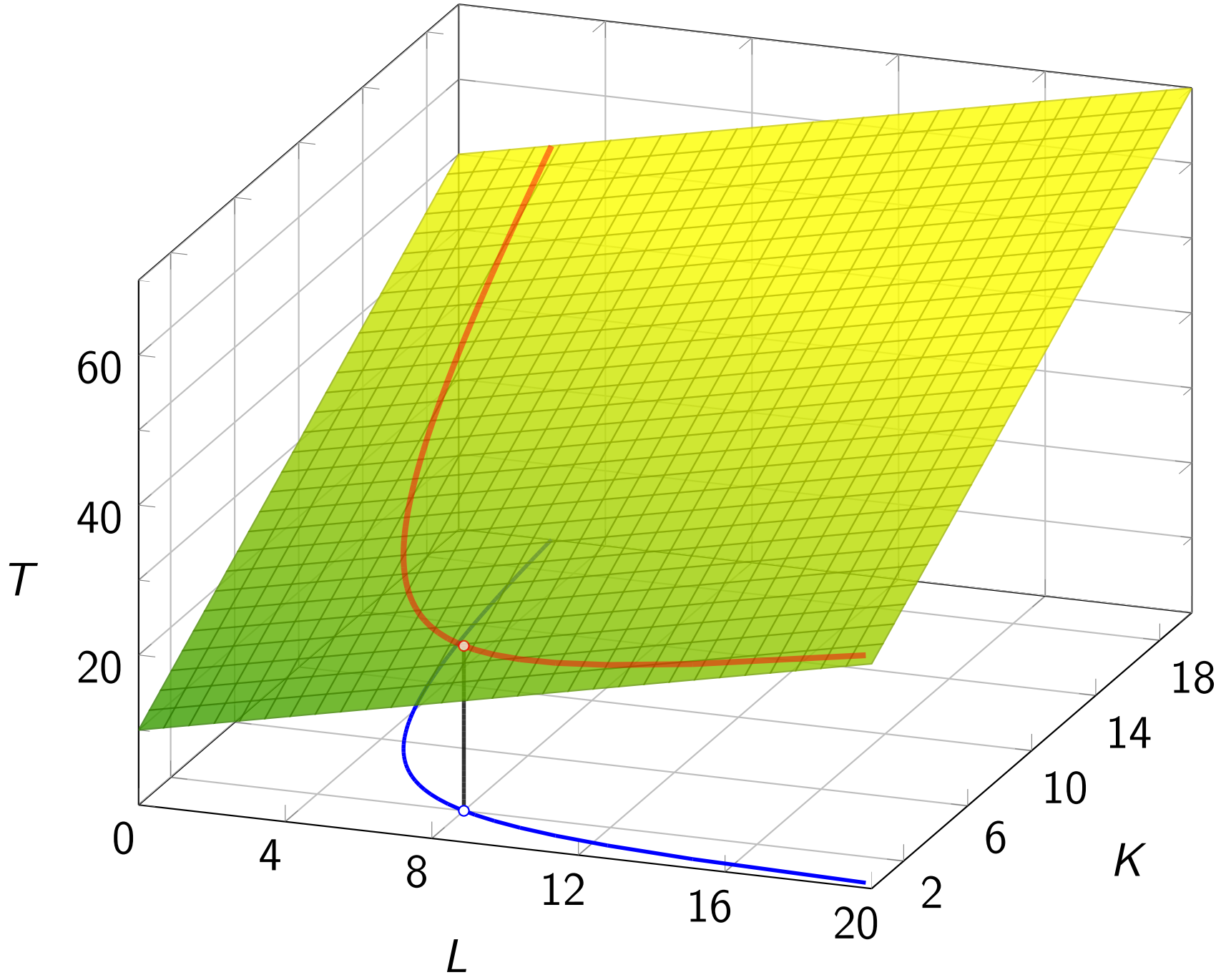
60
40
20
T
0
4
8
12
16
20
L
2
6
10
14
18
K