Seminari 5

Matematika za ekonomiste 2

Damir Horvat

FOI, Varaždin

Sadržaj

Newton-Leibnizova formula

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

peti zadatak

Newton-Leibnizova formula

Newton-Leibnizova formula

Teorem

Ako je f neprekidna funkcija na otvorenom intervalu I i F bilo koja primitivna funkcija funkcije f na I, tada za svaki $[a, b] \subseteq I$ vrijedi

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

Newton-Leibnizova formula

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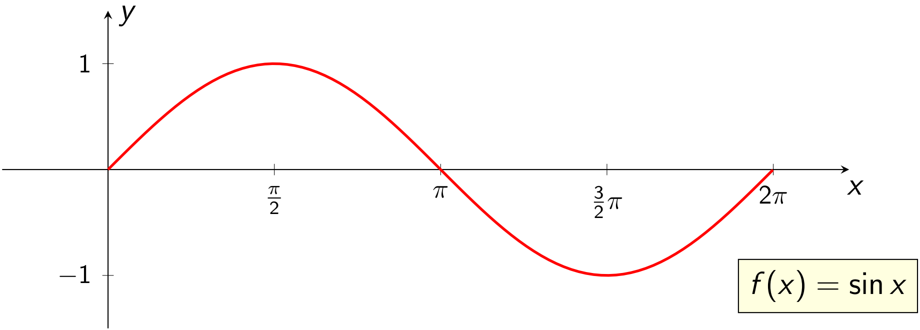
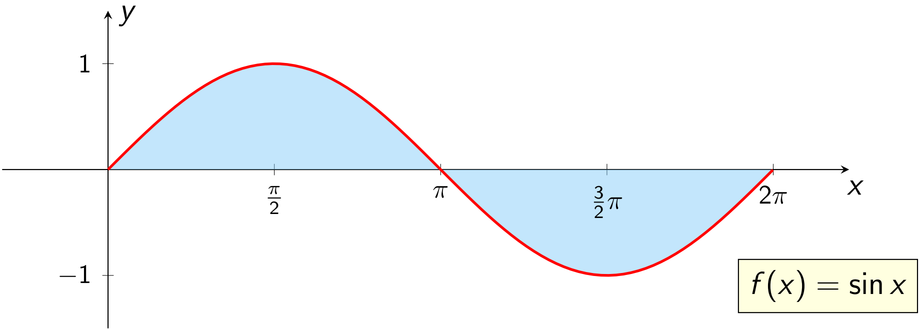
$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a) = F(x)\Big|_a^b \qquad F'(x) = f(x),\ x \in [a, b]$$



y
1
−1
$\frac{\pi}{2}$
π
$\frac{3}{2}\pi$
2π
x
$f(x) = \sin x$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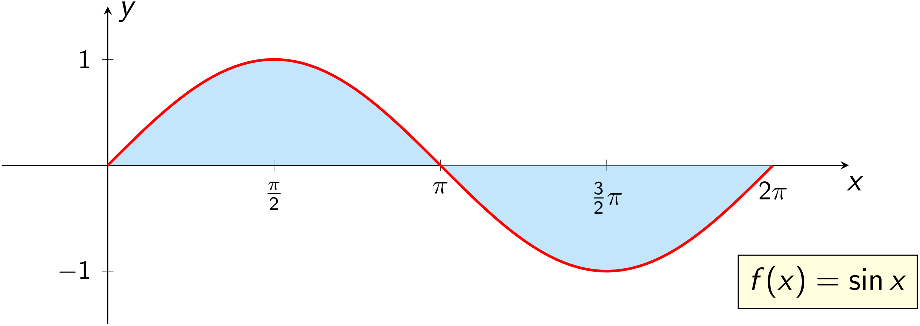
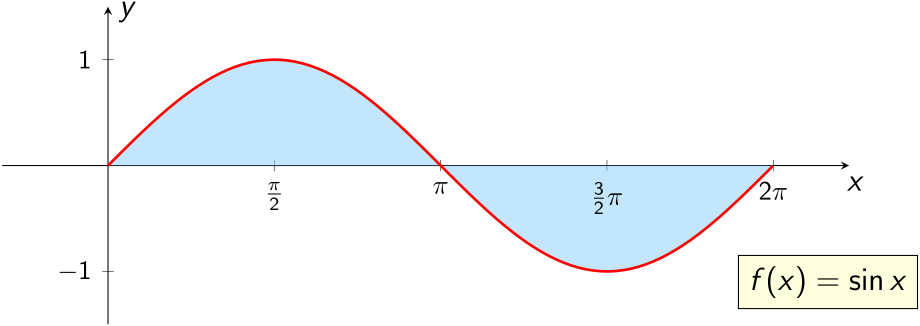


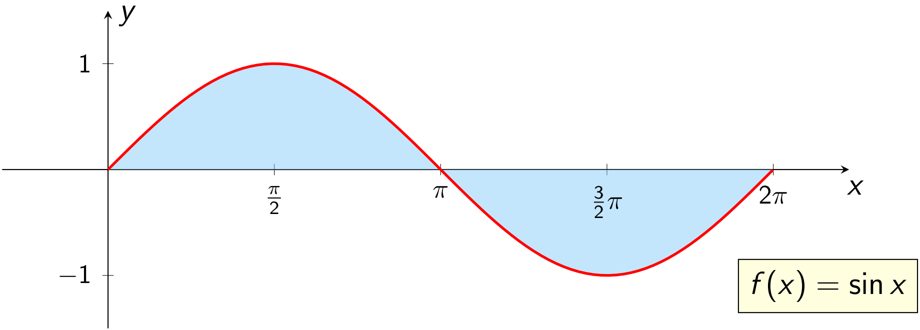
Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x =$$



Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi}$$



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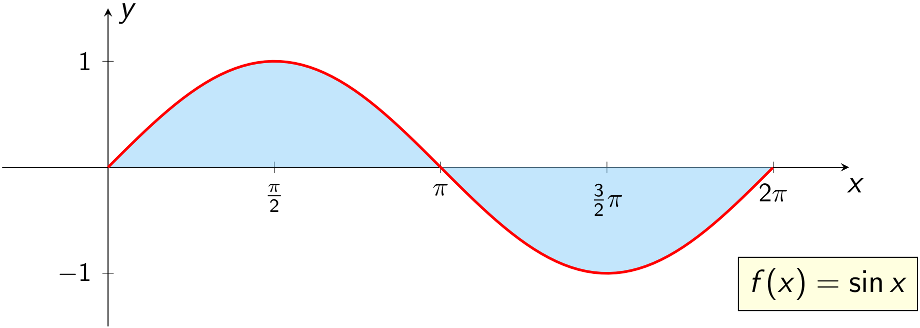
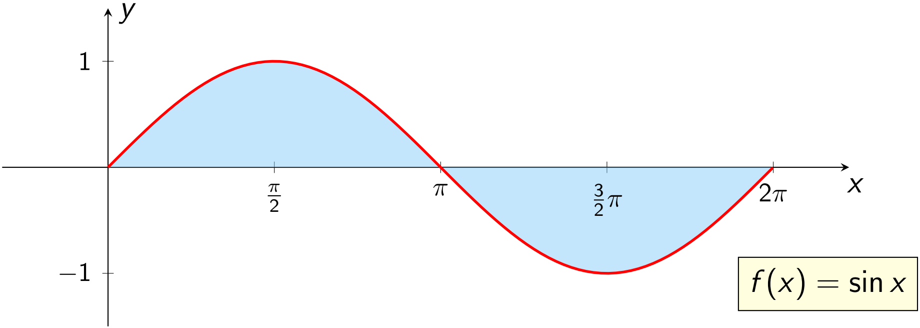


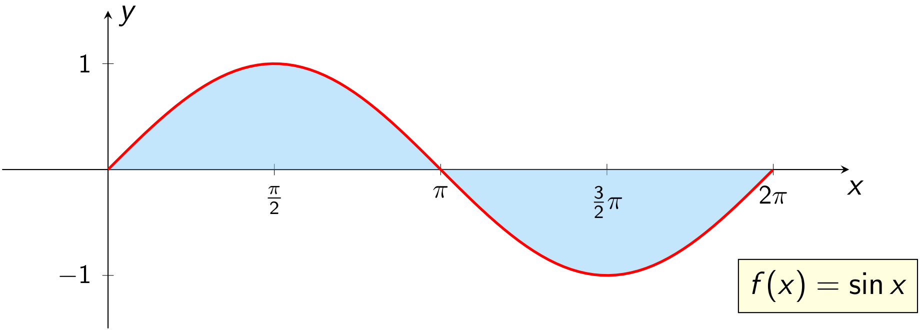
Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi -$$



Vrijednost integrala na segmentu $[0, 2\pi]$

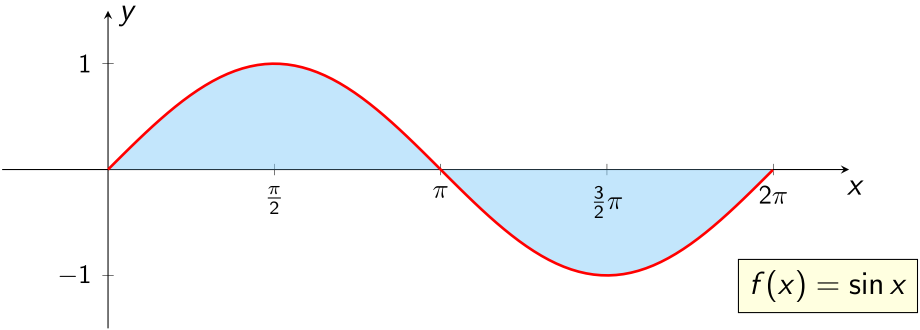
$$\int_0^{2\pi} \sin x \,\mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi - \left(-\cos 0\right)$$



Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi - \left(-\cos 0\right) =$$

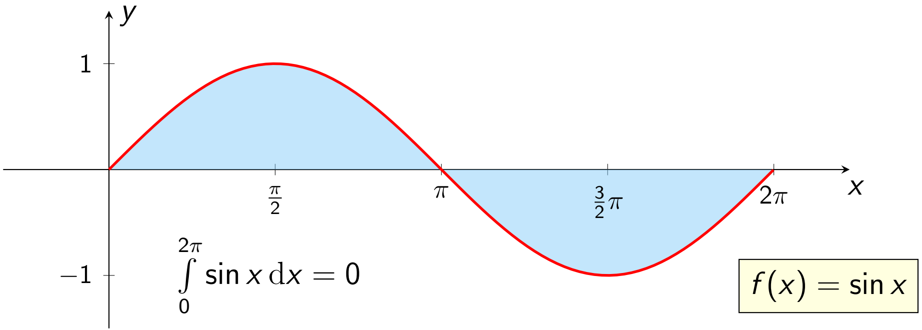
$$= -1 - (-1)$$



Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi - \left(-\cos 0\right) =$$

$$= -1 - (-1) = -1 + 1$$



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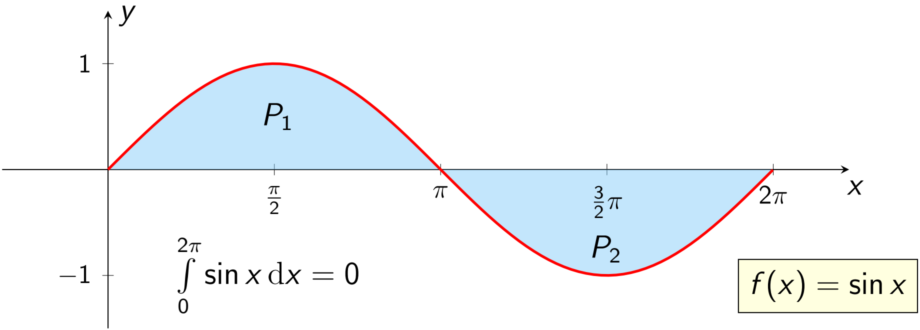
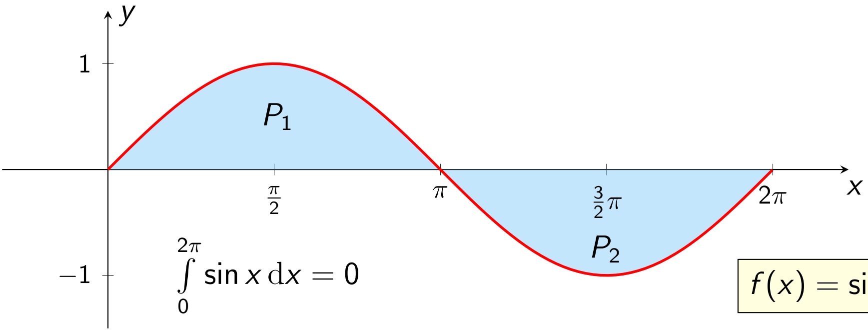


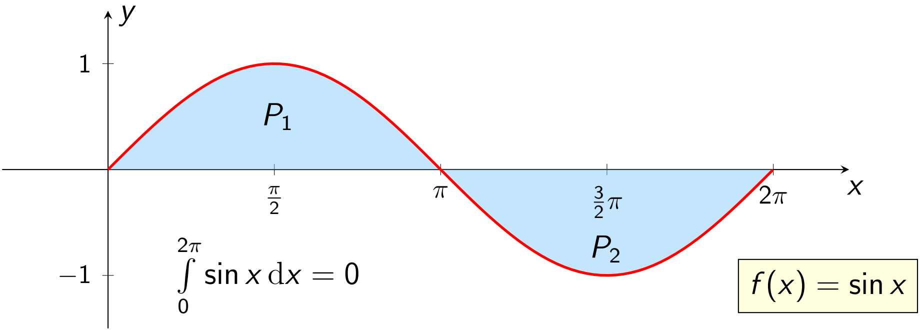
Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$



$f(x) = \sin x$

Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_1 = \int_0^{\pi} \sin x \, \mathrm{d}x =$$



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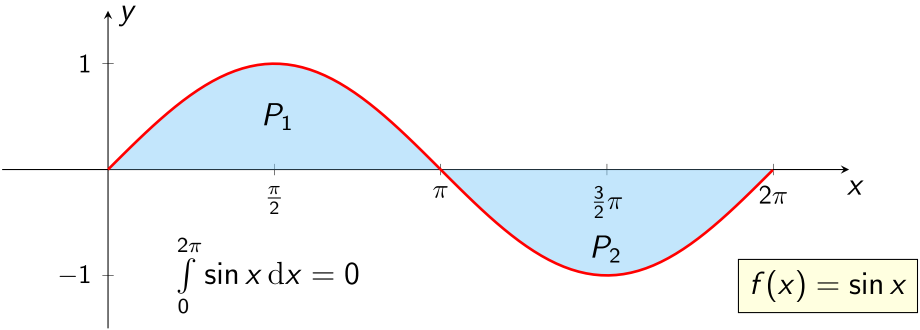

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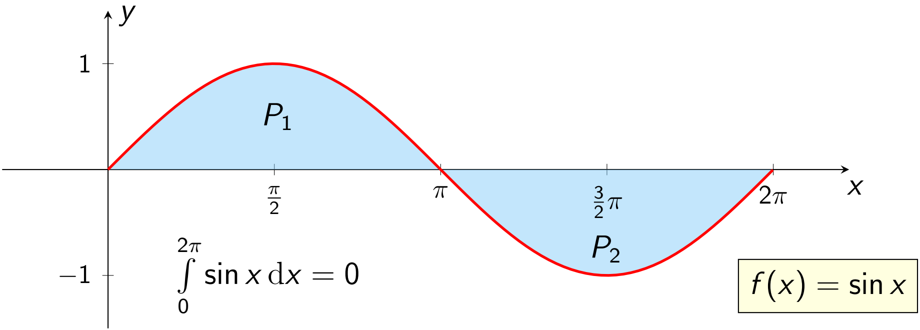

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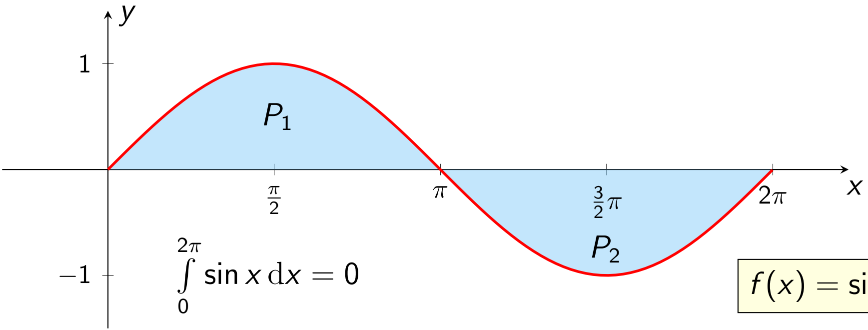

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = 0$$

$$\boxed{f(x) = \sin x}$$

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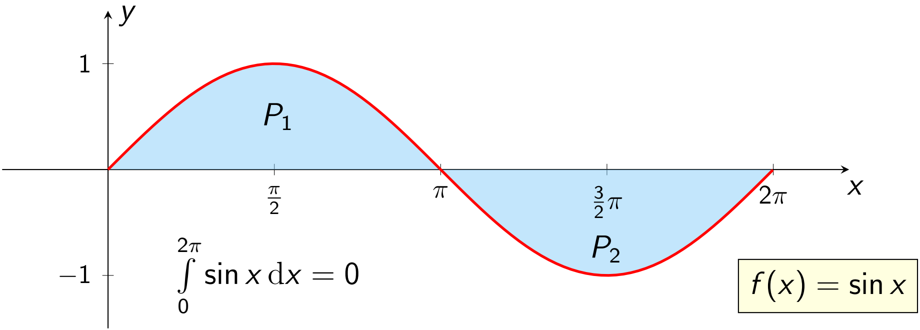

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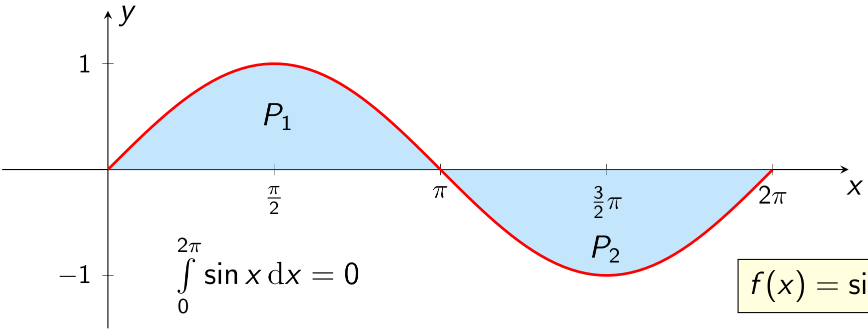
$$= -(-1) - (-1)$$



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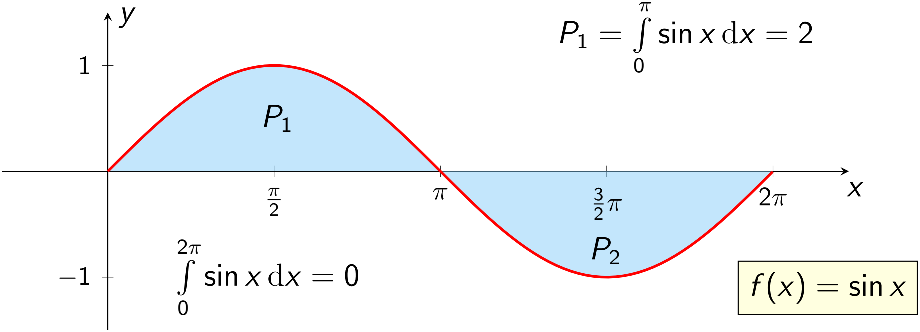

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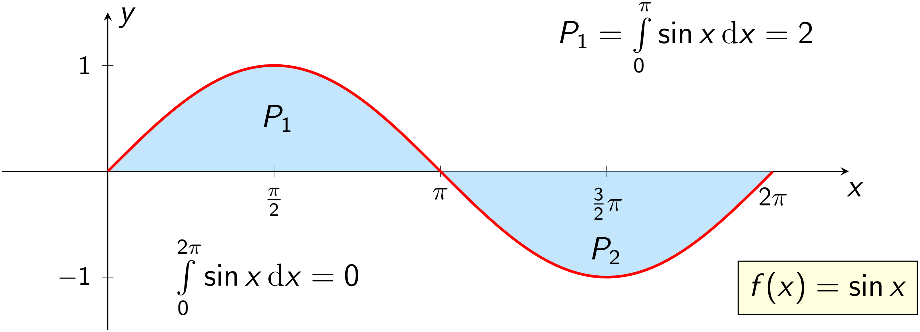
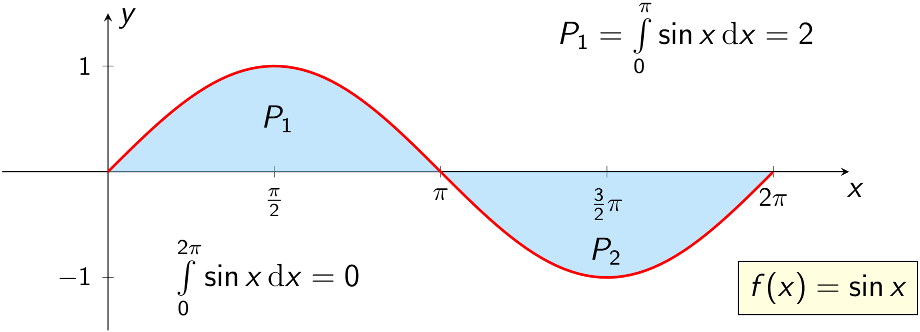


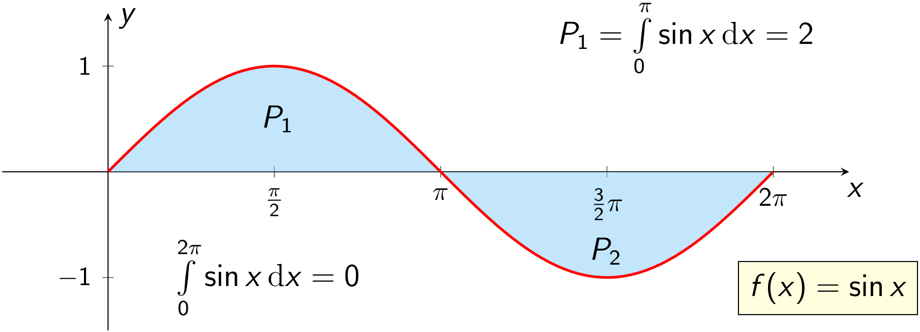
Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \, \mathrm{d}x =$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int\limits_{\pi}^{2\pi} \sin x \, \mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi}$$



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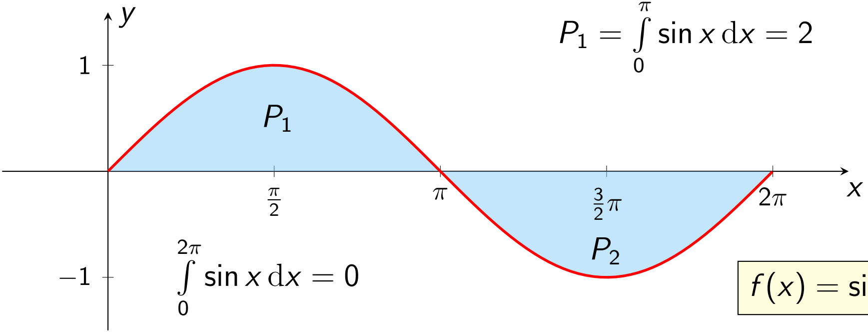

$$P_1 = \int_0^{\pi} \sin x \,\mathrm{d}x = 2$$

$$\int_0^{2\pi} \sin x \,\mathrm{d}x = 0$$

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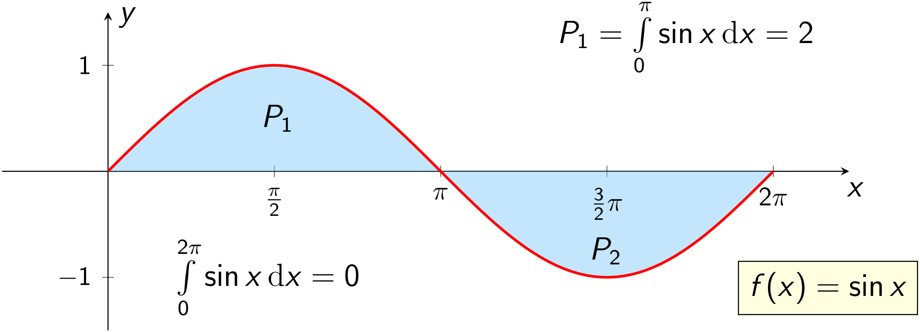
Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \,\mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi} = \cos x\Big|_{\pi}^{2\pi} =$$

$$= \cos 2\pi$$



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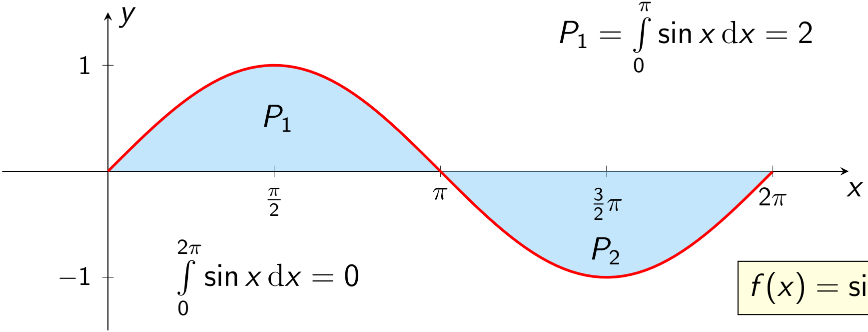

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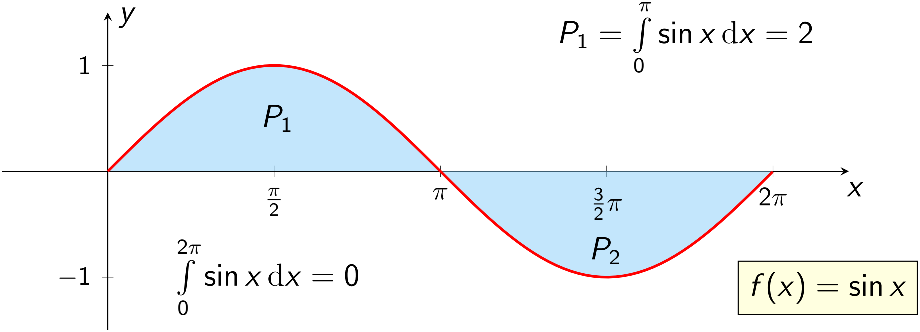
$$P_2 = -\int_{\pi}^{2\pi} \sin x \, \mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi} = \cos x \Big|_{\pi}^{2\pi} =$$

$$= \cos 2\pi - \cos \pi$$



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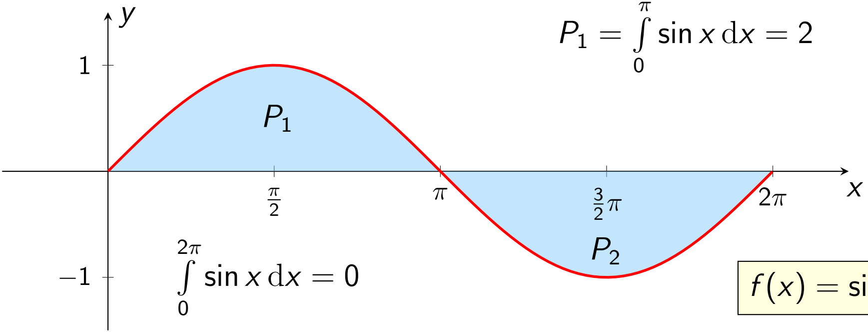
$$= \cos 2\pi - \cos \pi = 1 - (-1)$$



$$P_1 = \int_0^{\pi} \sin x \, \mathrm{d}x = 2$$

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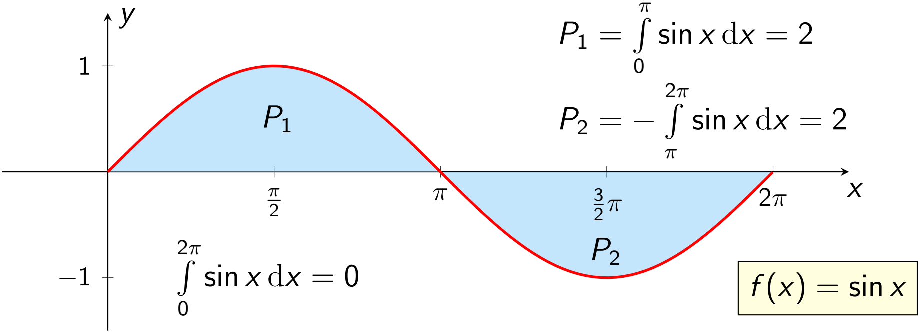
$$= \cos 2\pi - \cos \pi = 1 - (-1) = 1 + 1$$



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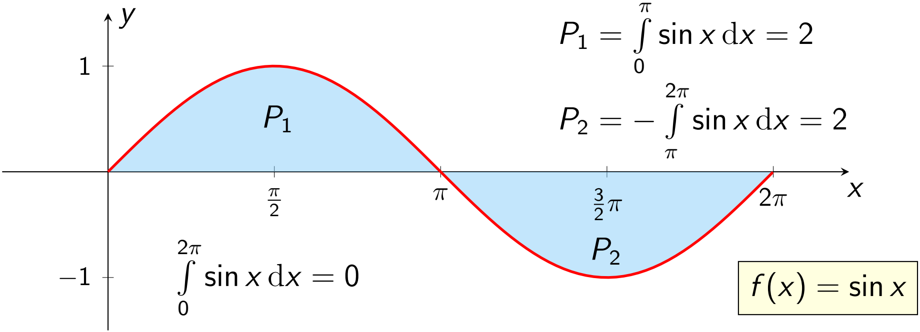
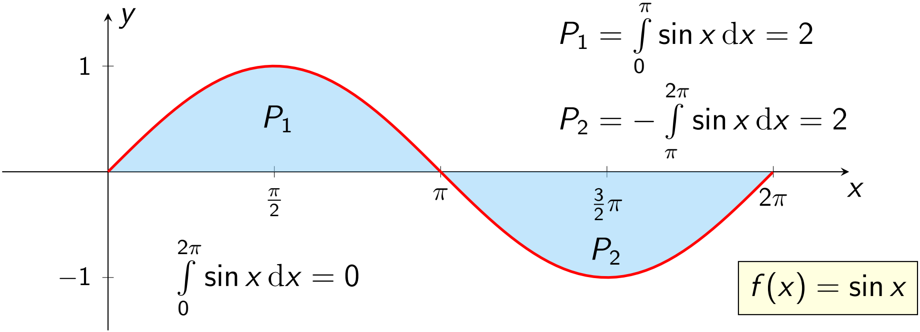


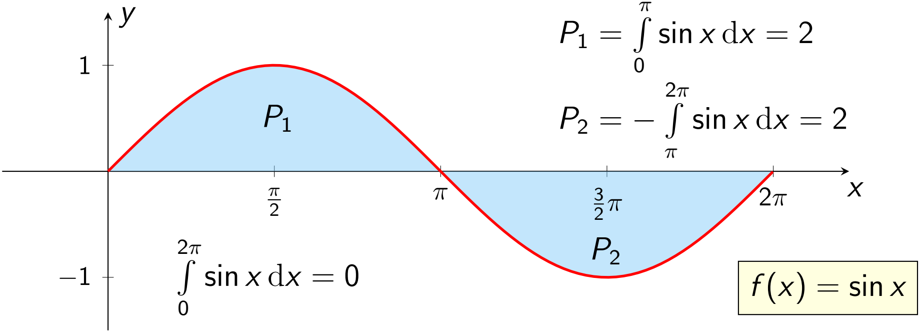
Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P = P_1 + P_2$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P = P_1 + P_2 = 2 + 2$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P = P_1 + P_2 = 2 + 2 = 4$$

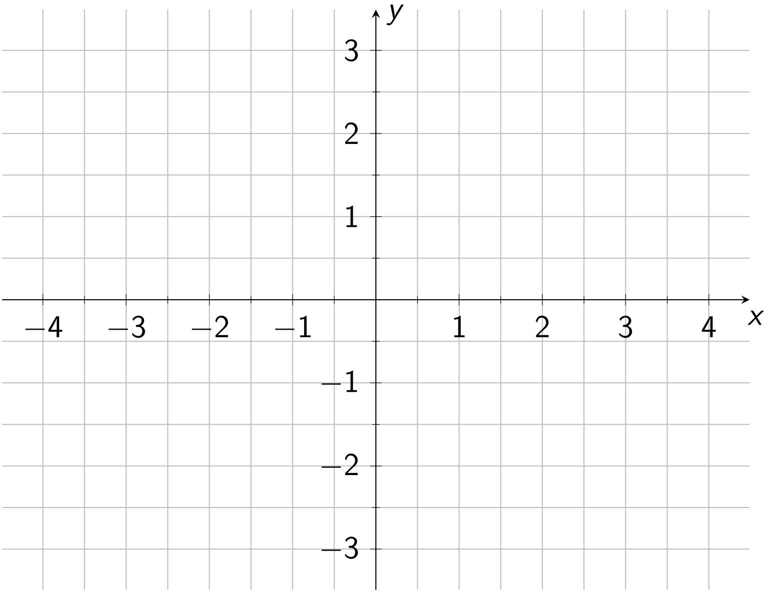
prvi zadatak

Zadatak 1

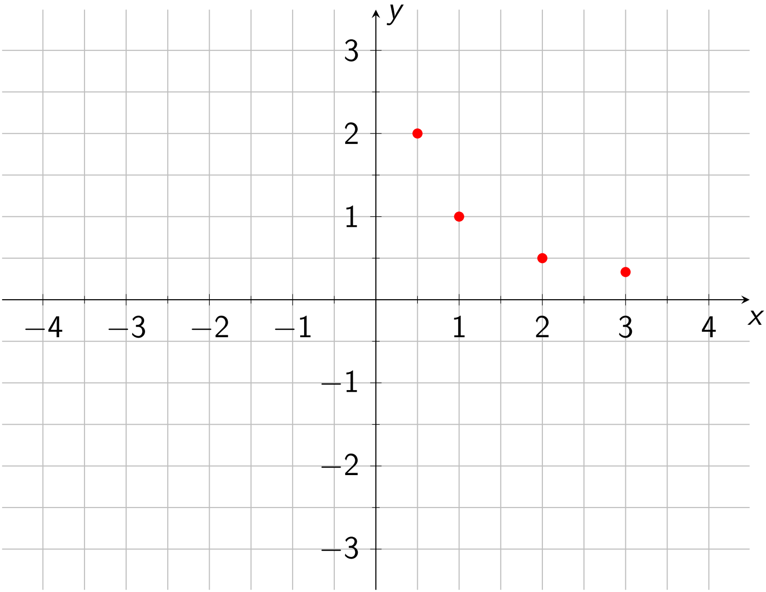
Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = x, \quad y = 0, \quad x = 3.$$

Rješenje



Rješenje



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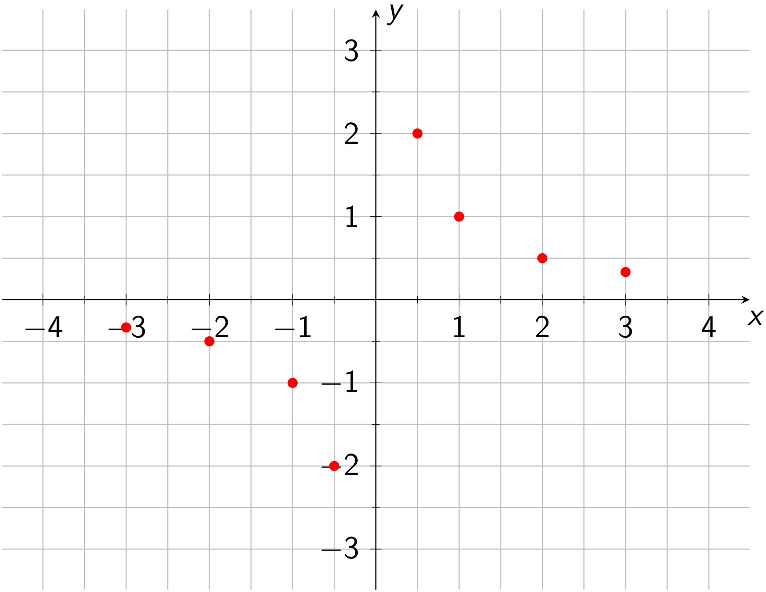

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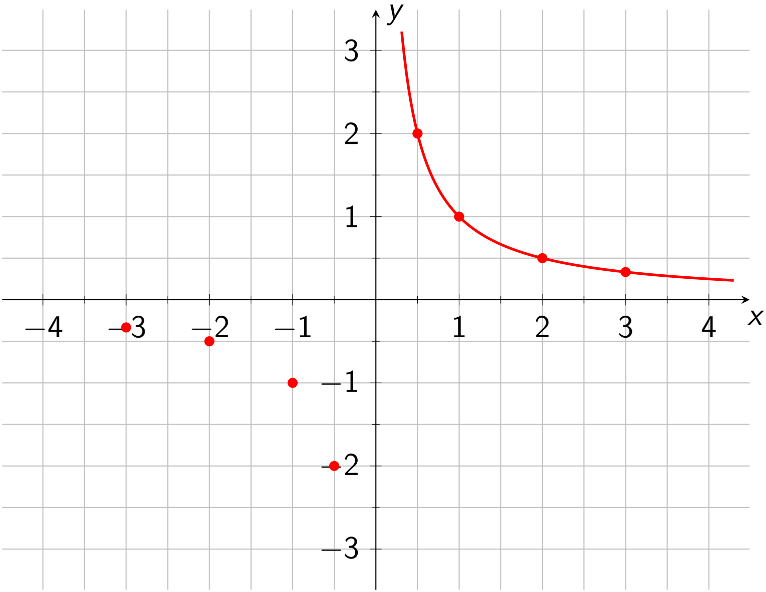

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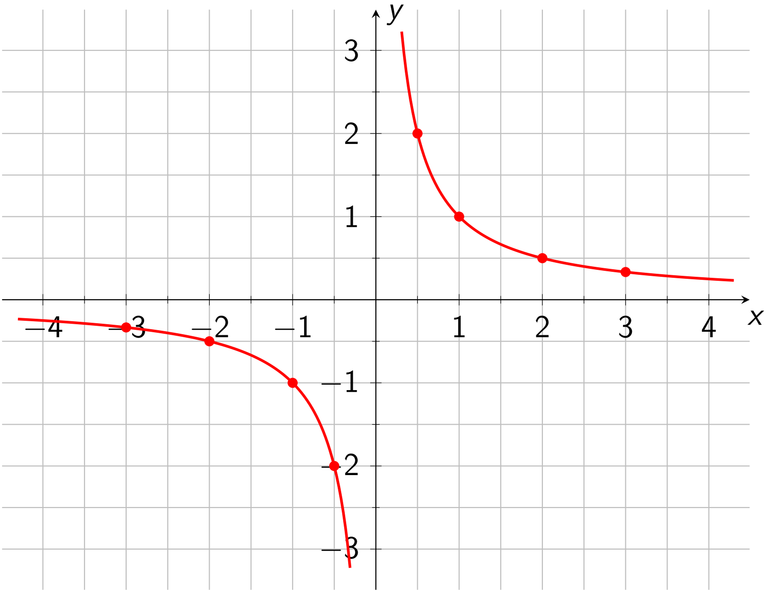

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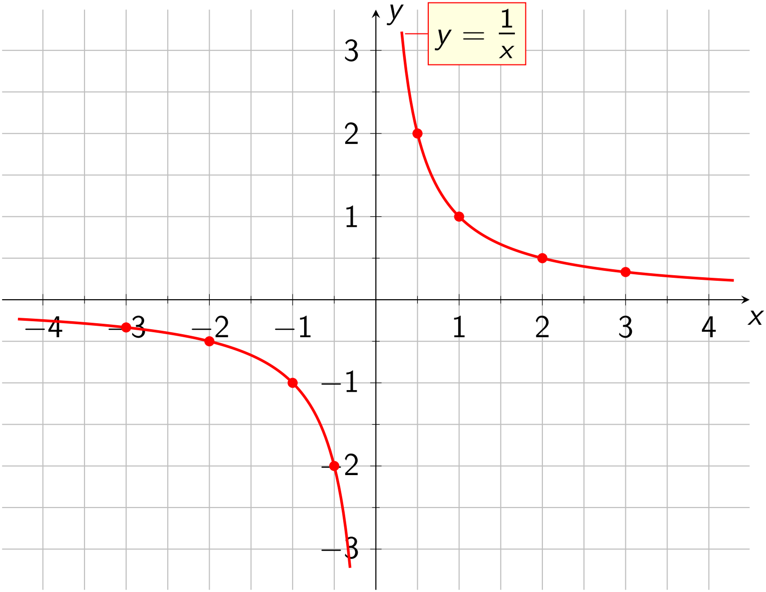

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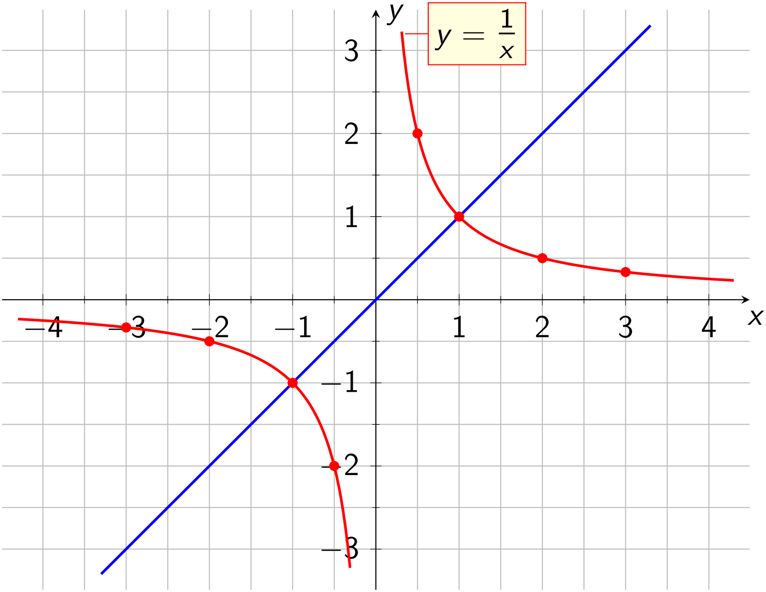

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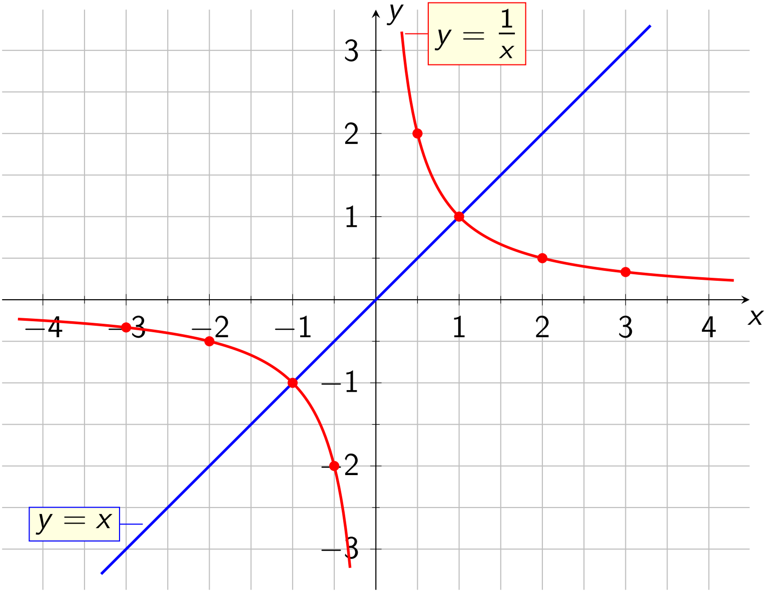

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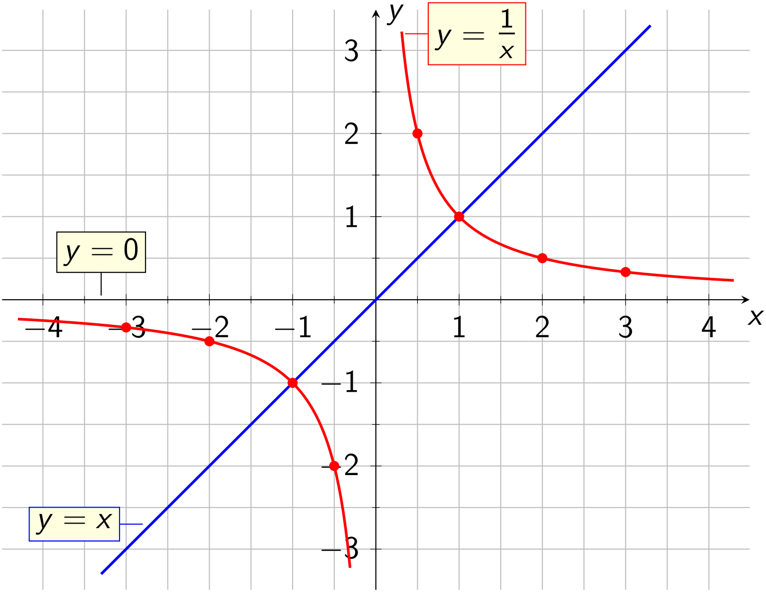

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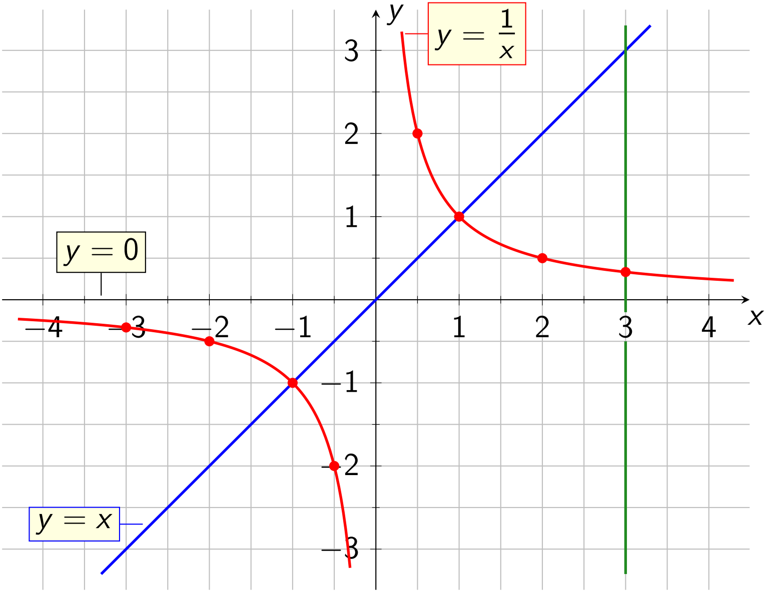

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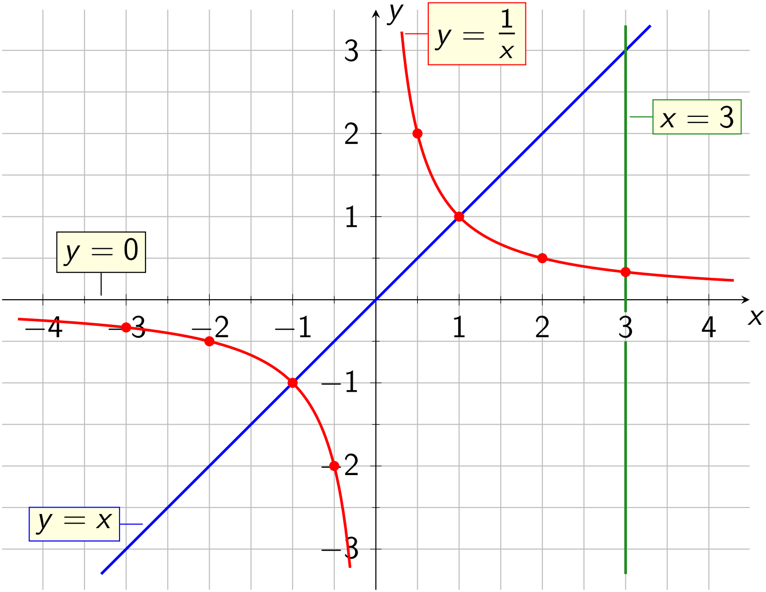

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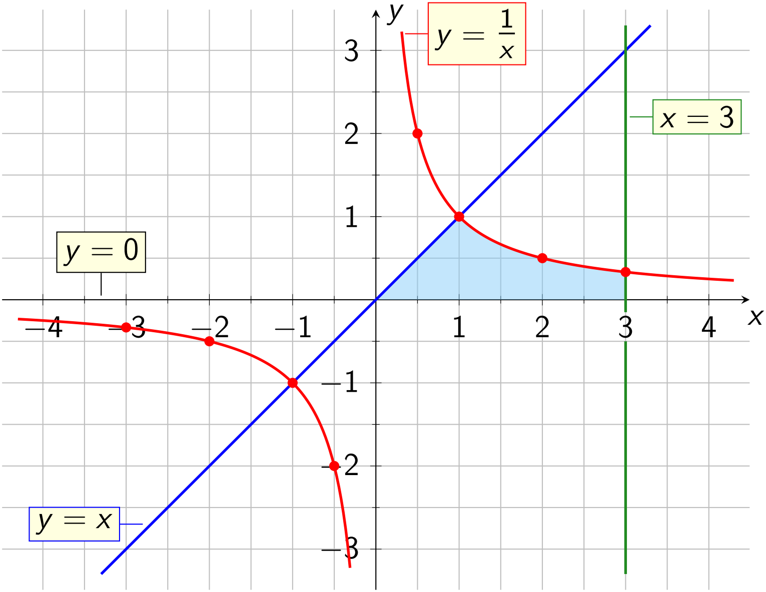

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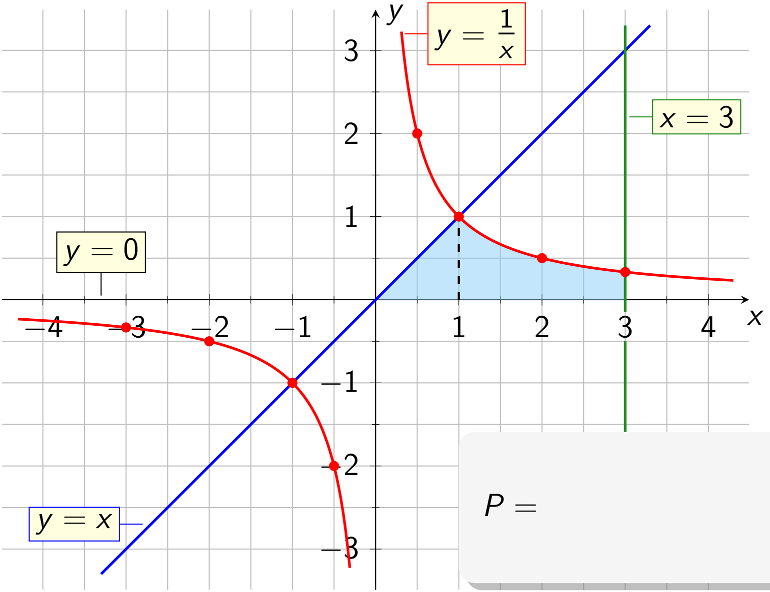
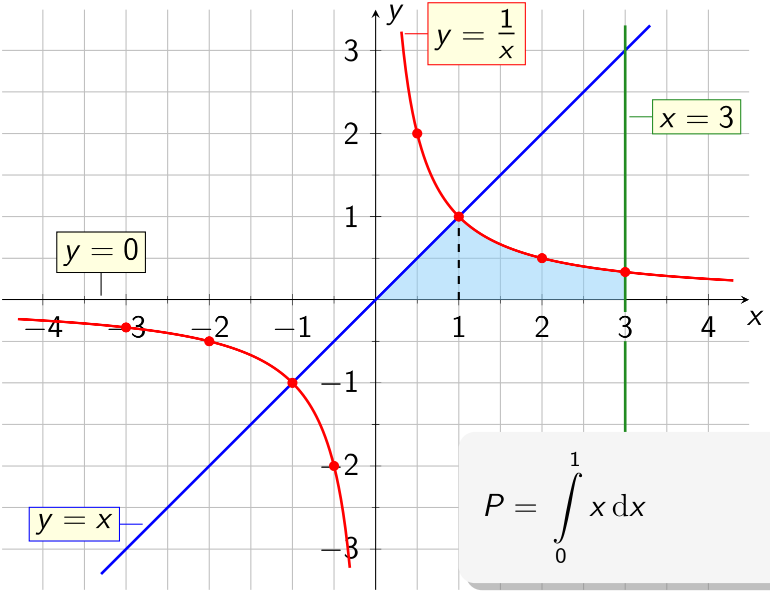


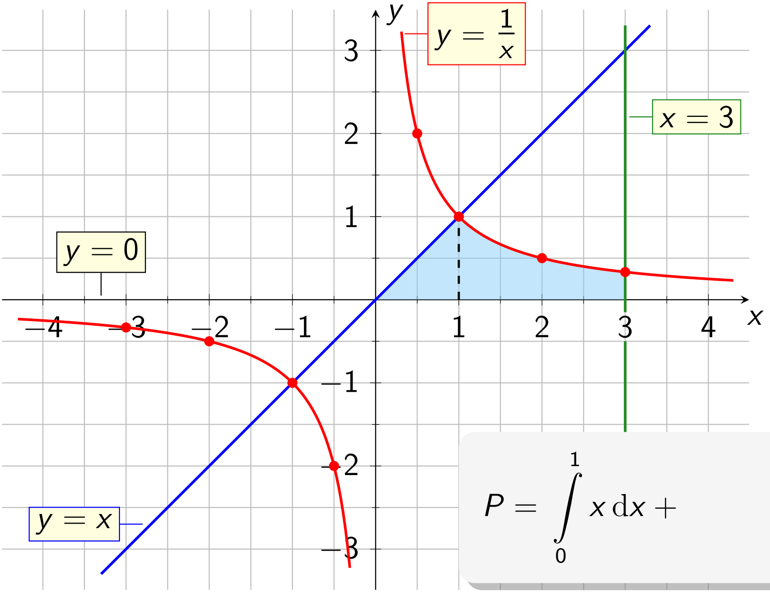
$P =$

Rješenje



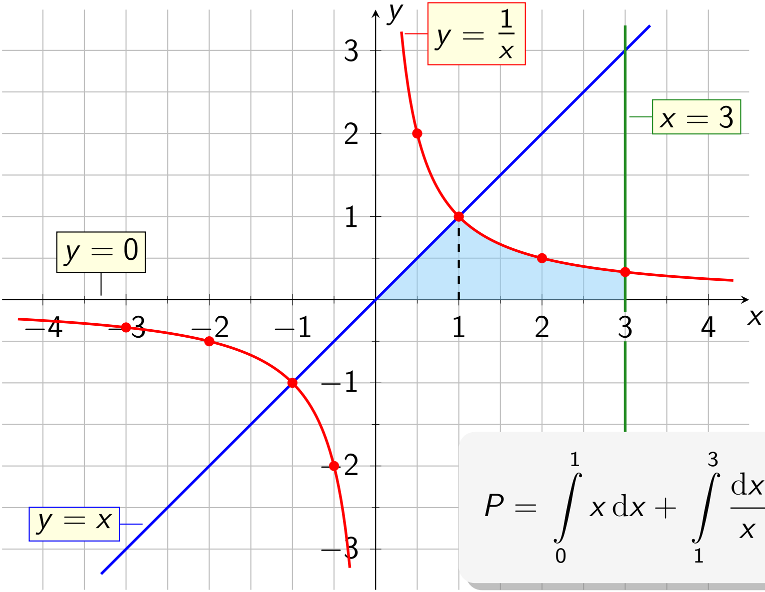
$$P = \int_0^1 x \, \mathrm{d}x$$

Rješenje



$$P = \int_0^1 x \, \mathrm{d}x +$$

Rješenje



$$P = \int_0^1 x \,\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x}$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x}$$

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$$= \left(\qquad \right)$$

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$$= \left(\frac{1}{2} \qquad \right)$$

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$$= \left(\frac{1}{2} - 0\right) + (\ln 3 - \ln 1) = \frac{1}{2} + \ln 3$$

drugi zadatak

Zadatak 2

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

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- Presjek pravca i parabole

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Rješenje

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Rješenje

- Presjek pravca i parabole

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$

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Rješenje

$$ax^2 + bx + c = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$T_1(2, 4)$$
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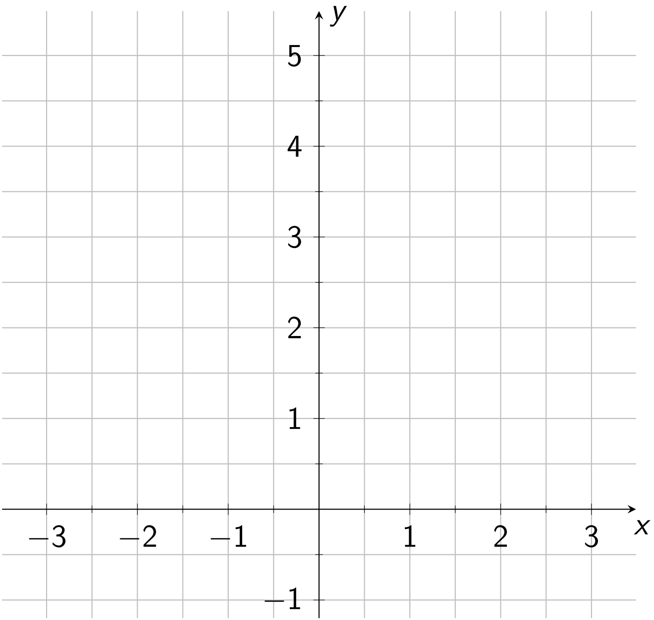
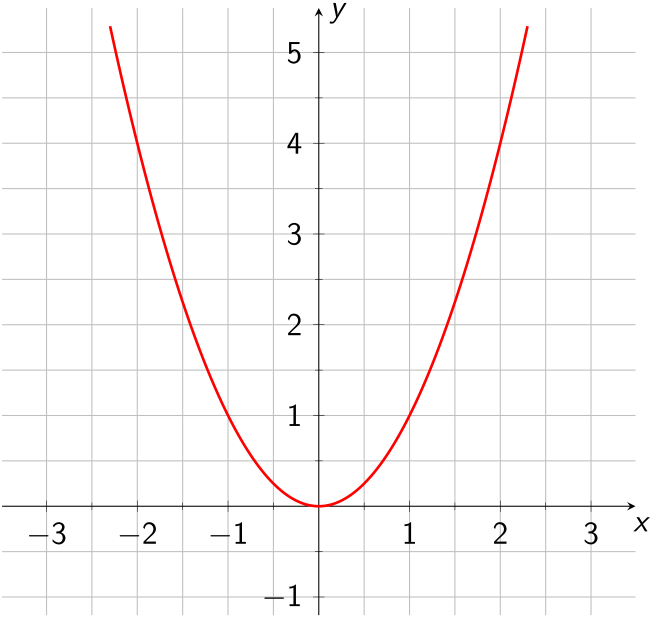
točke presjeka

$T_1(2, 4)$

$T_2(-1, 1)$


y
5
4
3
2
1
−1
−3
−2
−1
1
2
3
x



y
5
4
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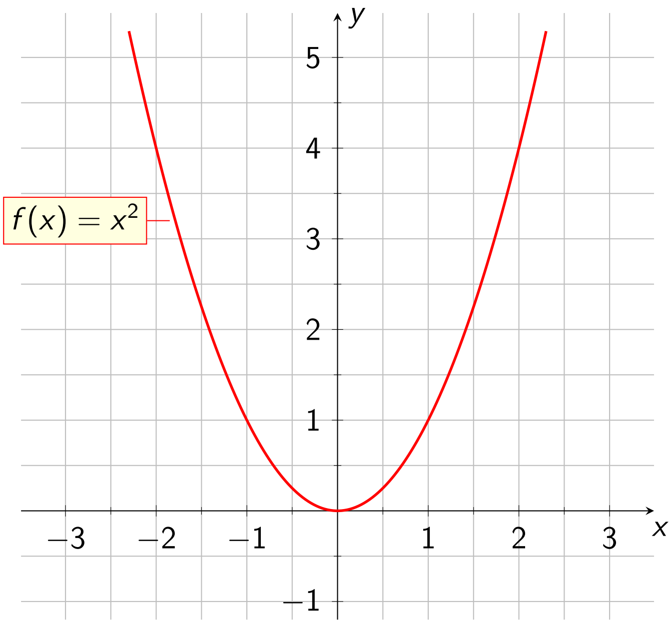
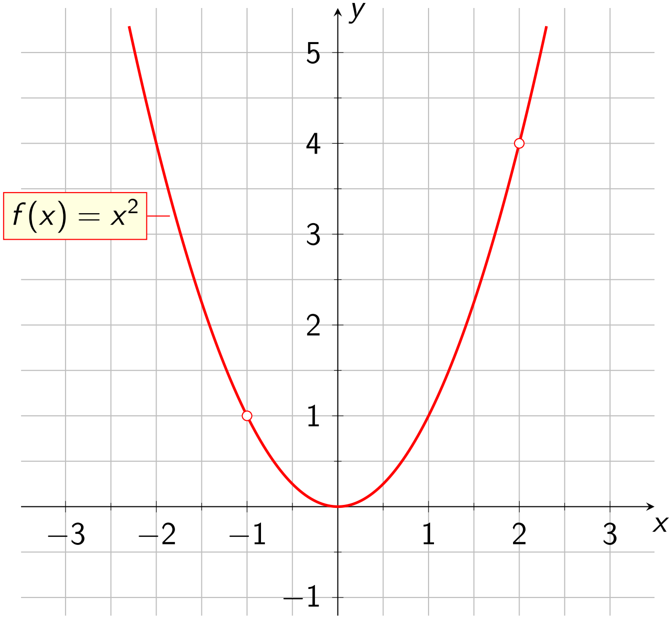
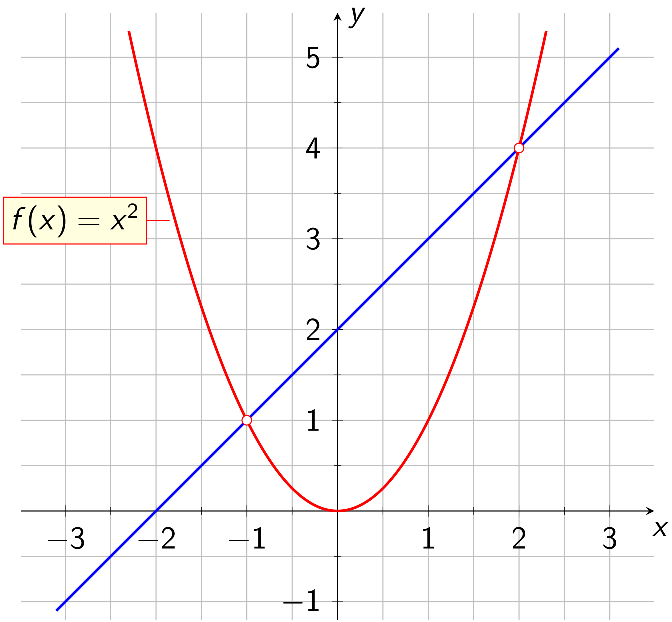
−1
1
2
3
x

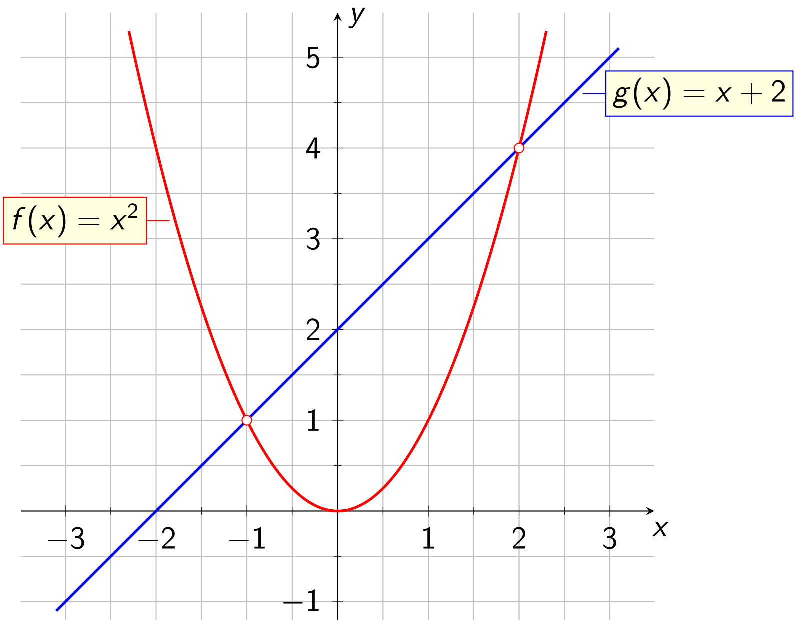


y
5
4
3
2
1
−1
x
−3
−2
−1
1
2
3
$f(x) = x^2$

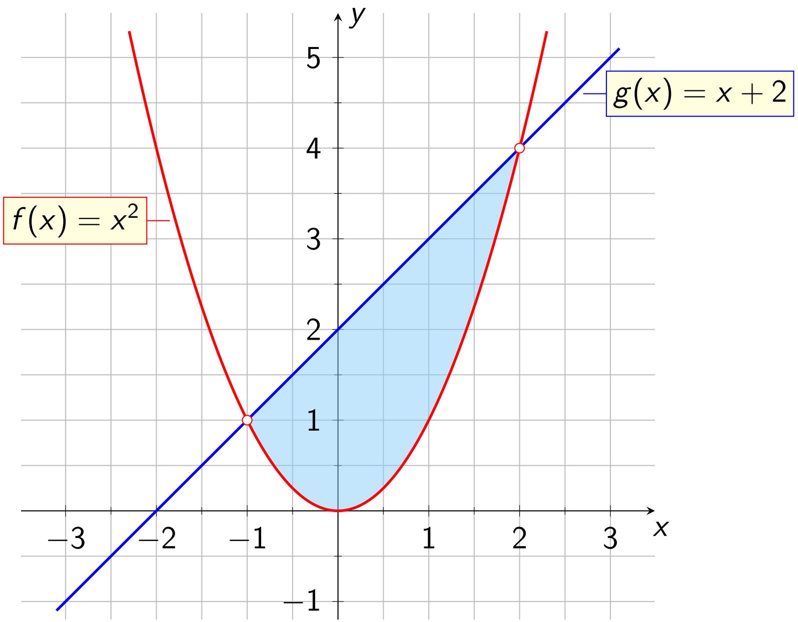


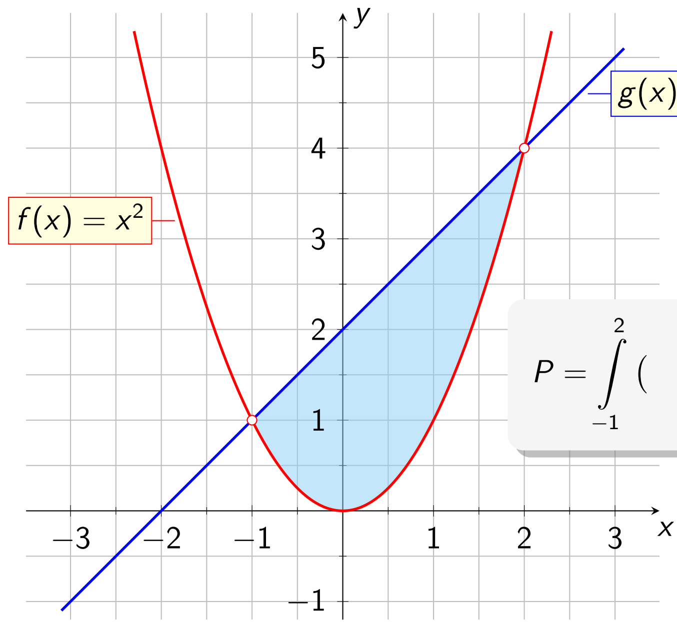
y
5
4
3
2
1
−1
$f(x) = x^2$
−3
−2
−1
1
2
3
x


y
5
4
$f(x) = x^2$
3
2
1
−3
−2
−1
1
2
3
x
−1

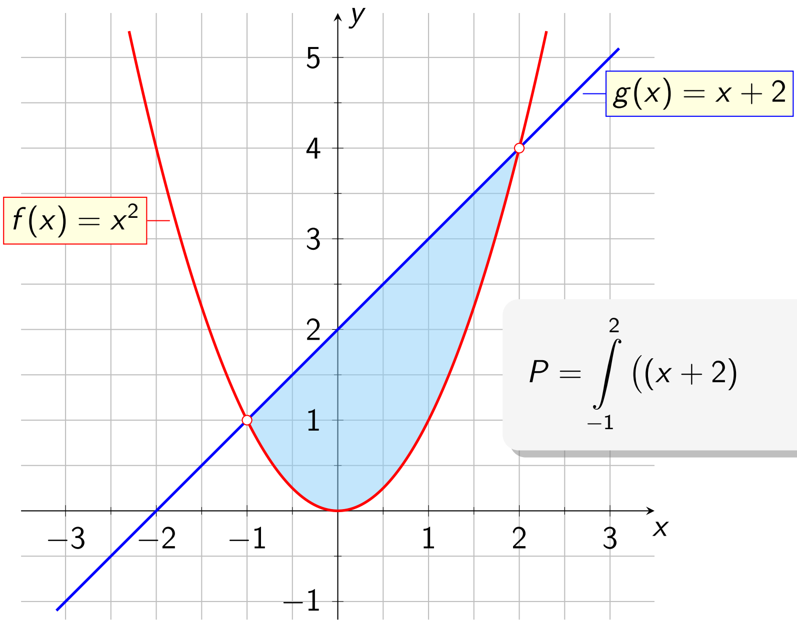


y
x
$f(x) = x^2$
$g(x) = x + 2$
5
4
3
2
1
−1
−3
−2
−1
1
2
3

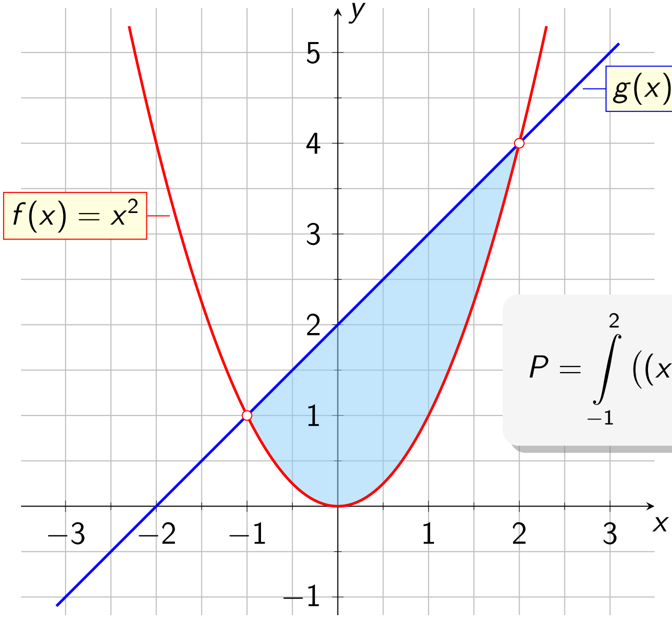

y
5
4
3
2
1
−1
$g(x) = x + 2$
$f(x) = x^2$
−3
−2
−1
1
2
3
x



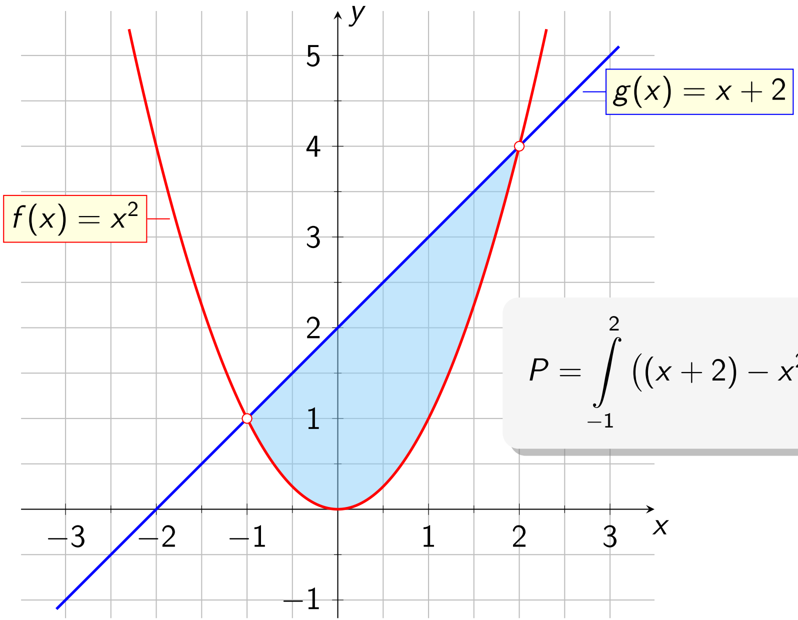
$$P = \int_{-1}^{2} \Big(\qquad \qquad \Big) \mathrm{d}x$$



$$P = \int_{-1}^{2} \left((x+2) \qquad \right) \mathrm{d}x$$



$$P = \int_{-1}^{2} \left((x+2) - \quad \right) \mathrm{d}x$$



$$P = \int_{-1}^{2} \left((x+2) - x^2 \right) \mathrm{d}x$$

$$P = \int\limits_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \Bigg($$

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$$= \left(-\frac{2^3}{3}\right.$$

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$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_{-1}^{2}=$$

$$=\left(-\frac{2^3}{3}+\frac{2^2}{2}+2\cdot 2\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right) \mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right) \mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right) \Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right)$$

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$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \Bigg($$

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$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

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$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3}\right.$$

$$P=\int\limits_{-1}^{2}\left((x+2)-x^2\right)\mathrm{d}x=\int\limits_{-1}^{2}\left(-x^2+x+2\right)\mathrm{d}x=$$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Bigg|_{-1}^{2}=$$

$$=\left(-\frac{2^3}{3}+\frac{2^2}{2}+2\cdot 2\right)-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2\cdot(-1)\right)=$$

$$=\left(-\frac{8}{3}+\frac{4}{2}\right.$$

$$P=\int_{-1}^{2}\left((x+2)-x^2\right)\mathrm{d}x=\int_{-1}^{2}\left(-x^2+x+2\right)\mathrm{d}x=$$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_{-1}^{2}=$$

$$=\left(-\frac{2^3}{3}+\frac{2^2}{2}+2\cdot 2\right)-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2\cdot(-1)\right)=$$

$$=\left(-\frac{8}{3}+\frac{4}{2}+4\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \Bigg($$

$$P = \int\limits_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int\limits_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2}\right.$$

$$P=\int\limits_{-1}^{2}\left((x+2)-x^2\right)\mathrm{d}x=\int\limits_{-1}^{2}\left(-x^2+x+2\right)\mathrm{d}x=$$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Bigg|_{-1}^{2}=$$

$$=\left(-\frac{2^3}{3}+\frac{2^2}{2}+2\cdot 2\right)-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2\cdot(-1)\right)=$$

$$=\left(-\frac{8}{3}+\frac{4}{2}+4\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3}$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2}$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2$$

$$P = \int\limits_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int\limits_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2 = \frac{9}{2}$$

treći zadatak

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \frac{1}{x}$ i $y = 4$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \frac{1}{x}$ i $y = 4$

  $$\frac{1}{x} = 4$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \,\Big/ \cdot x$$

$$4x = 1$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \dfrac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \;\Big/ \cdot x$$
$$4x = 1$$
$$x = \frac{1}{4}$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \;\Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

$$2^{x-1} = 4$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \,\Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

$$\boxed{(3, 4)}$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja $y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$
$$4x = 1$$
$$x = \frac{1}{4}$$
$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja $y = 2^{x-1}$ i $y = 4$

$$2^{x-1} = 4$$
$$x - 1 = \log_2 4$$
$$x = 2 + 1$$
$$x = 3$$
$$\boxed{(3, 4)}$$

- Presjek krivulja $y = 2^{x-1}$ i $y = \frac{1}{x}$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

$$\boxed{(3, 4)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = \frac{1}{x}$$

$$2^{x-1} = \frac{1}{x}$$

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

$$\boxed{(3, 4)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = \frac{1}{x}$$

$$2^{x-1} = \frac{1}{x}$$

pogađamo rješenje

Zadatak 3

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

$$\boxed{(3, 4)}$$

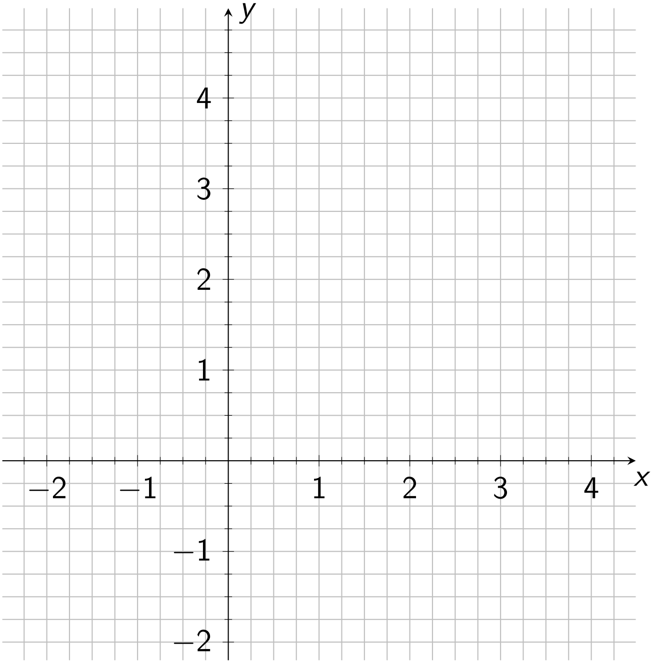
- Presjek krivulja

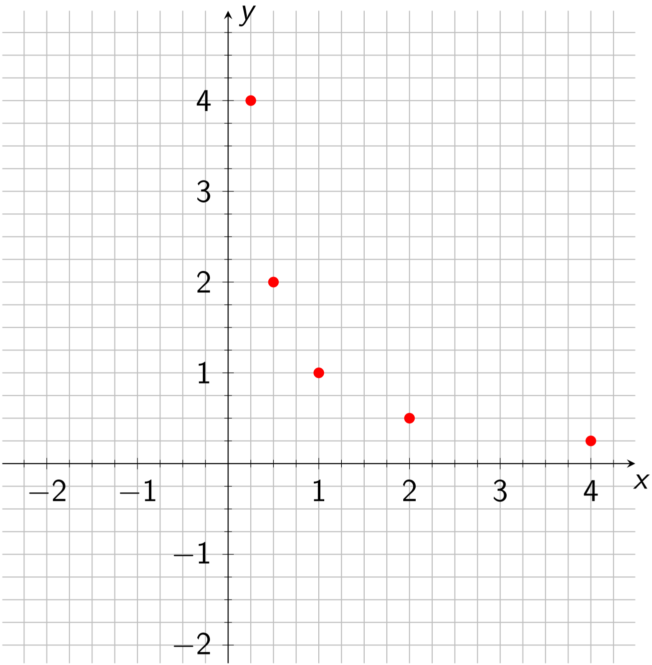
$$y = 2^{x-1} \text{ i } y = \frac{1}{x}$$

$$2^{x-1} = \frac{1}{x}$$

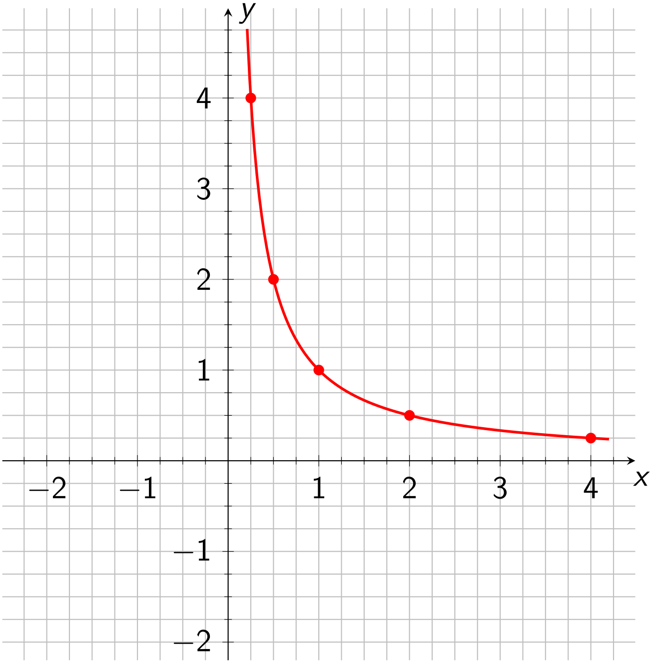
pogađamo rješenje

$$\boxed{(1, 1)}$$

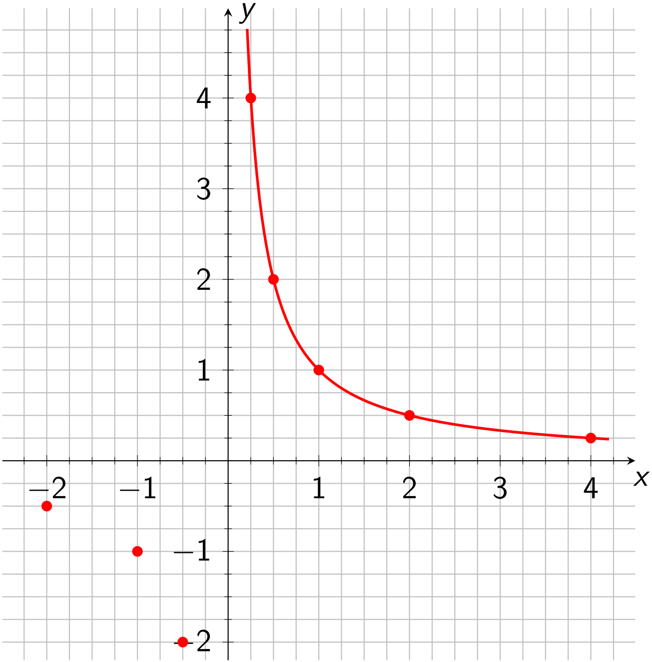

y
4
3
2
1
−2
−1
1
2
3
4
x
−1
−2



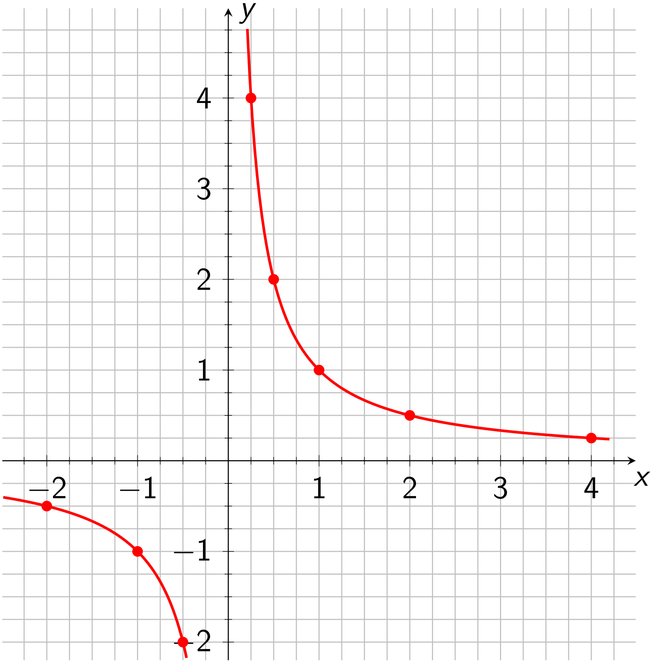
$$y = \frac{1}{x}$$



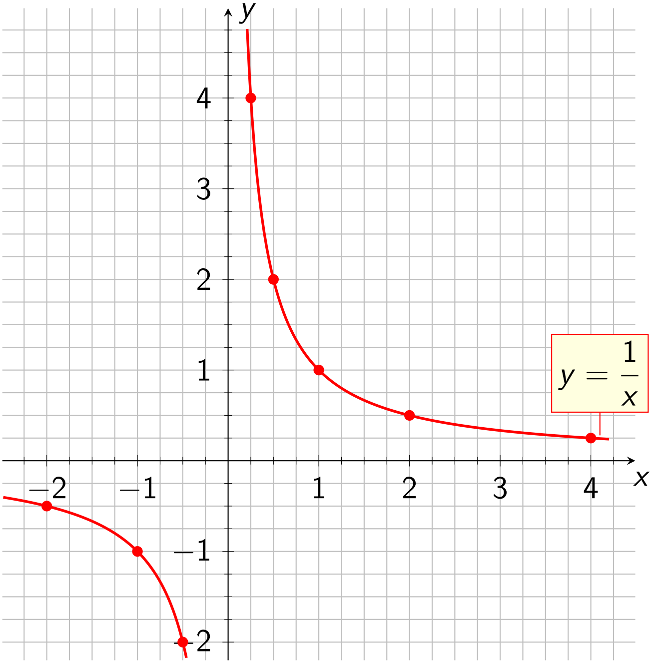
$$y = \frac{1}{x}$$



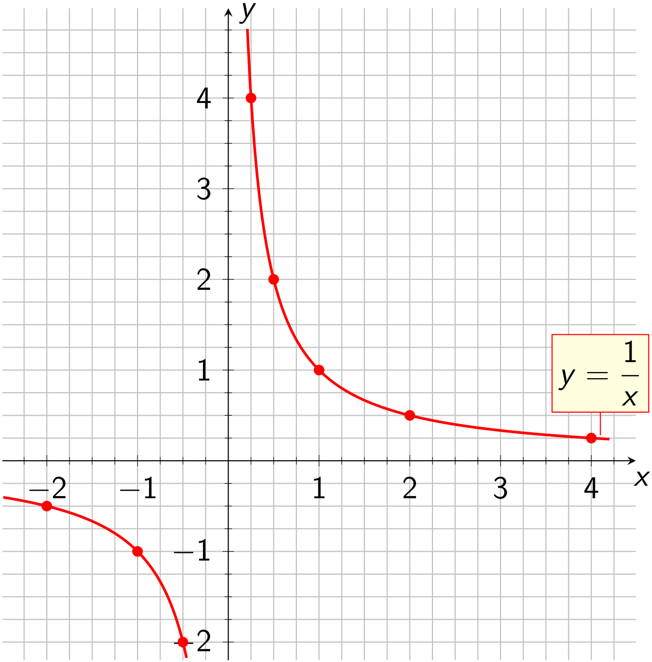
$$y = \frac{1}{x}$$



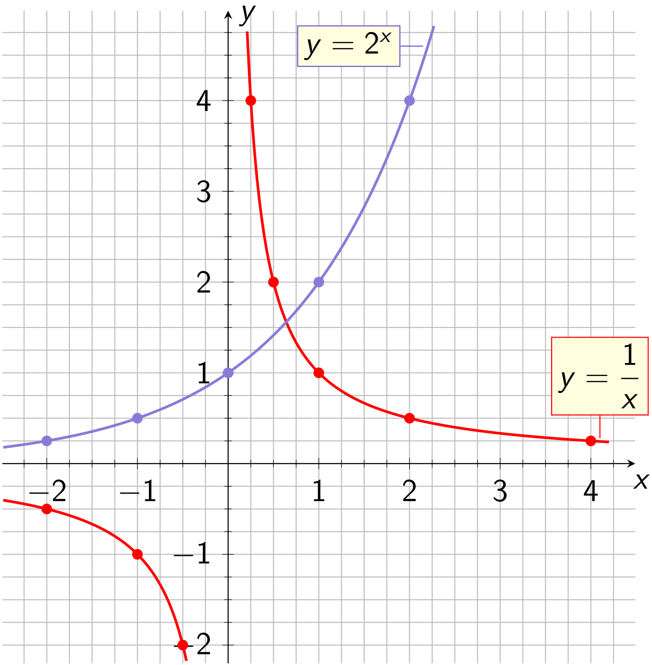
$$y = \frac{1}{x}$$



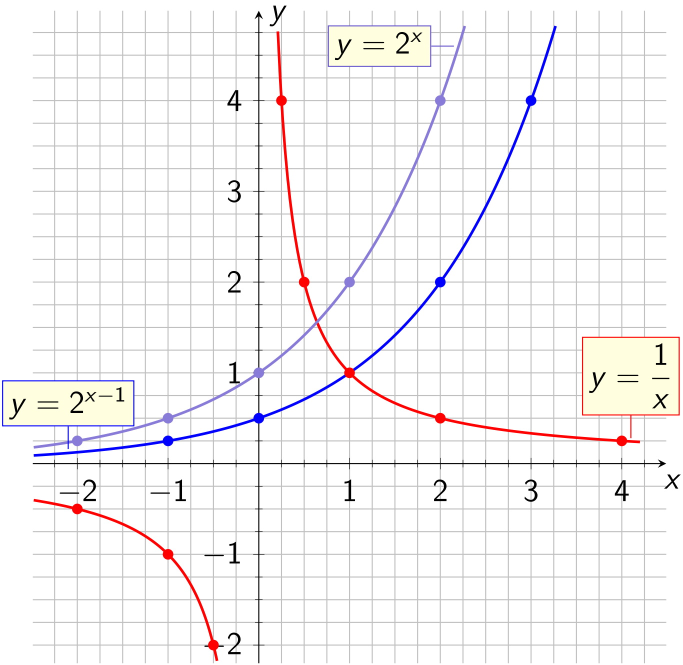
y
x
4
3
2
1
−1
−2
−2
−1
1
2
3
4
$y = \frac{1}{x}$



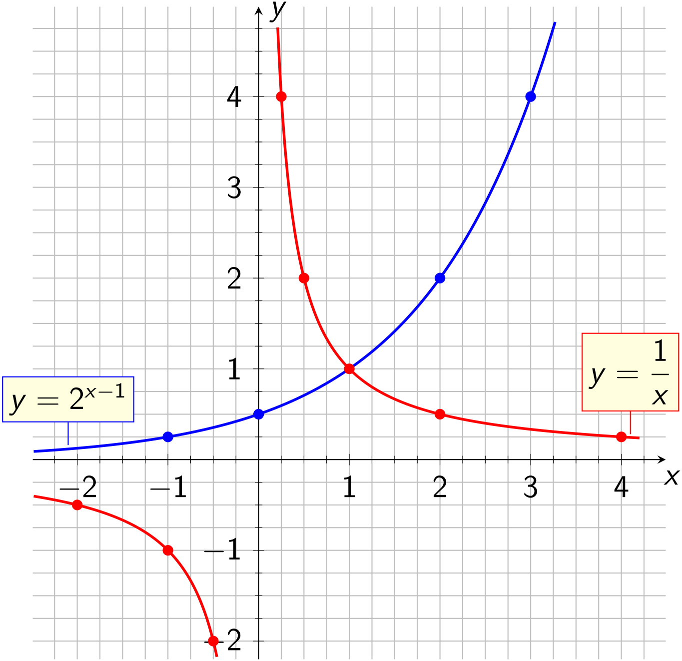
$y = 2^{x-1}$

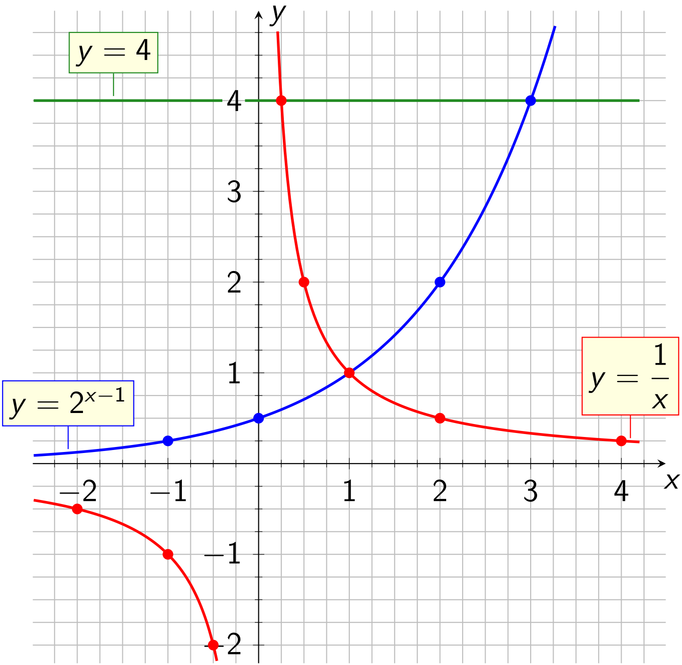


$y = 2^{x-1}$

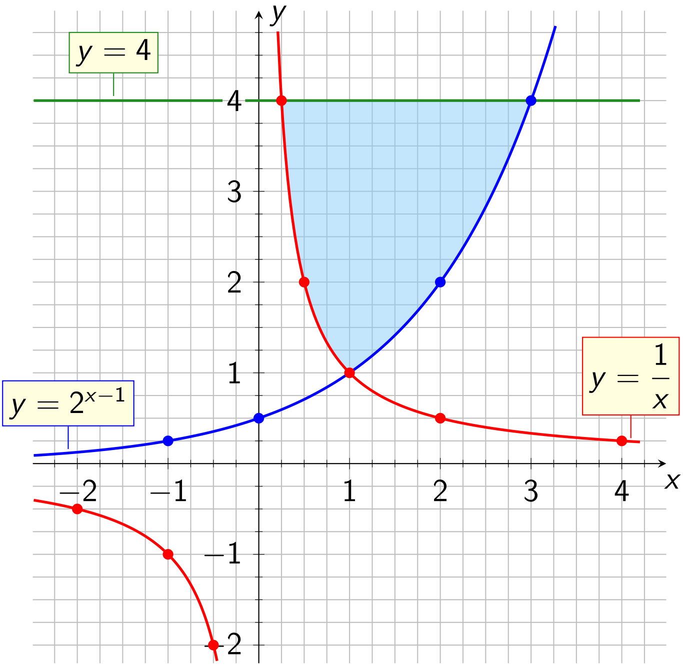


$y = 2^{x-1}$

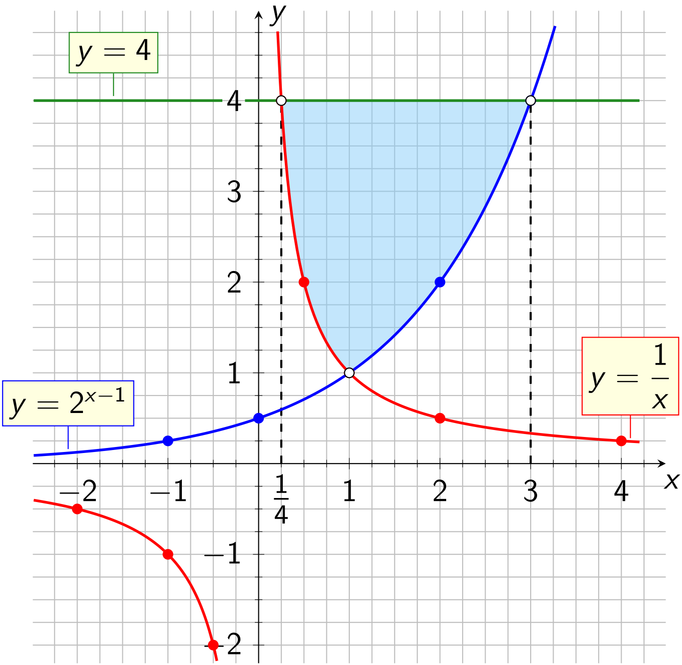

y
x
4
3
2
1
−1
−2
−2
−1
1
2
3
4
$y = 2^{x-1}$
$y = \frac{1}{x}$



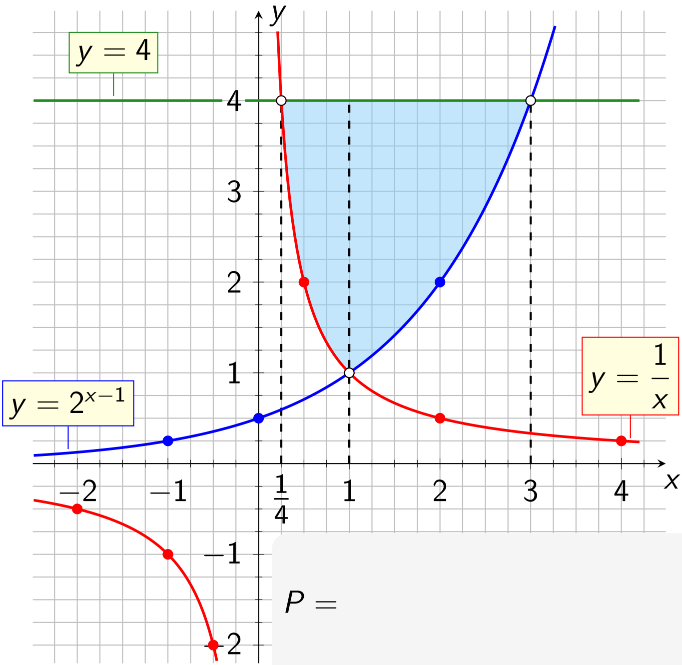
y
x
y = 4
y = 2^{x−1}
y = 1/x
4
3
2
1
−1
−2
−2
−1
1
2
3
4



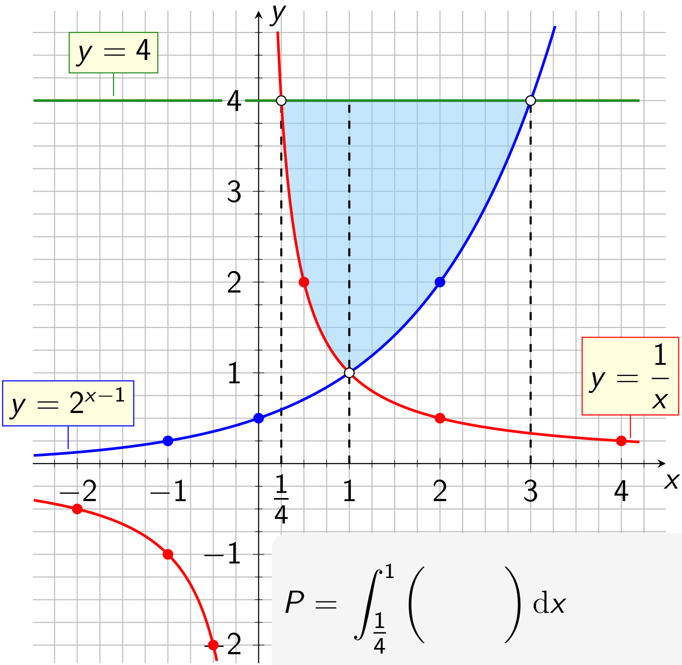
y
x
y = 4
y = 2^{x−1}
y = 1/x
4
3
2
1
−1
−2
−2
−1
1
2
3
4



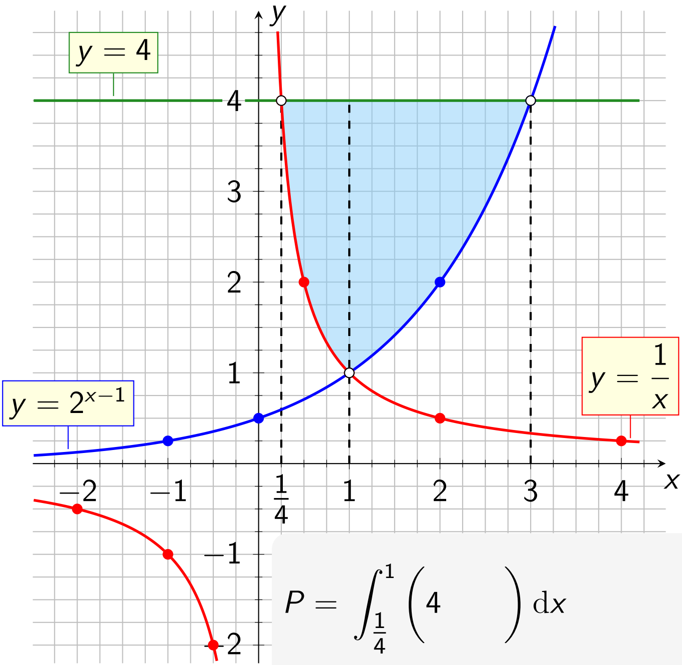
$y = 4$
$y = 2^{x-1}$
$y = \frac{1}{x}$
x
y
−2
−1
$\frac{1}{4}$
1
2
3
4
−1
−2
1
2
3
4



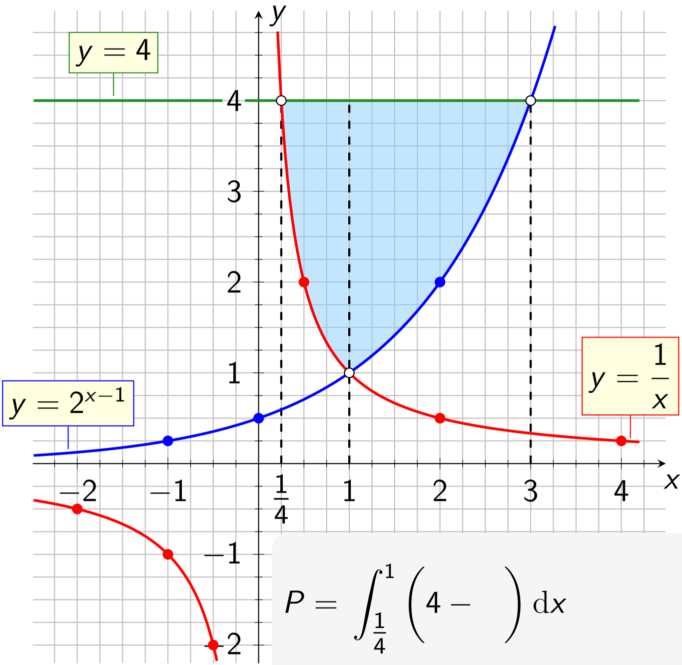
$P =$



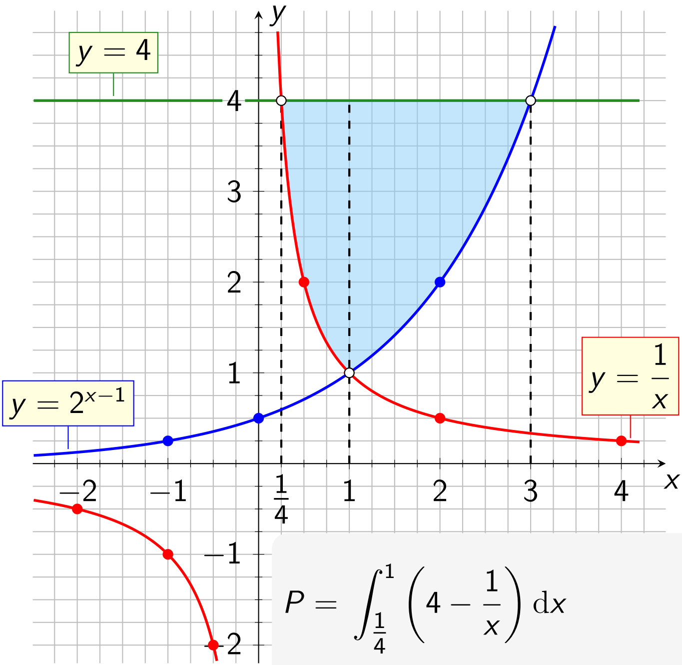
$$P = \int_{\frac{1}{4}}^{1} \left(\qquad \right) \mathrm{d}x$$



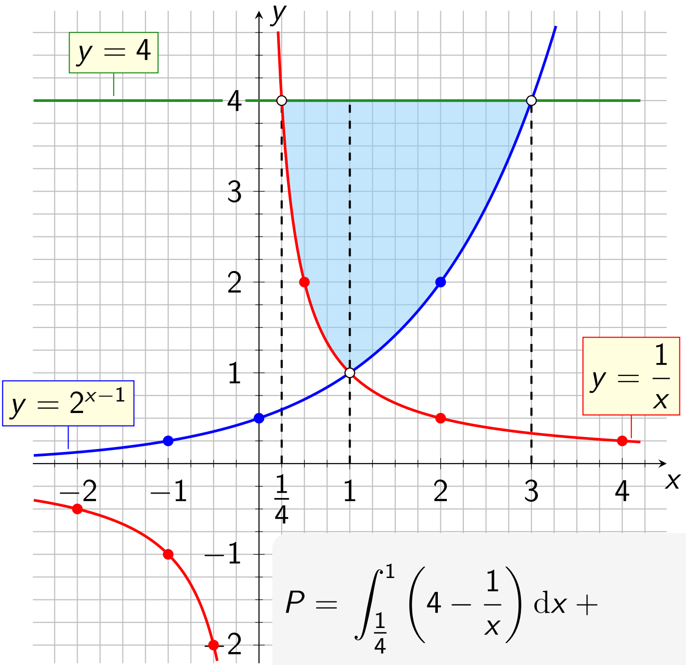
$$P = \int_{\frac{1}{4}}^{1} \left(4 \qquad \right) \mathrm{d}x$$



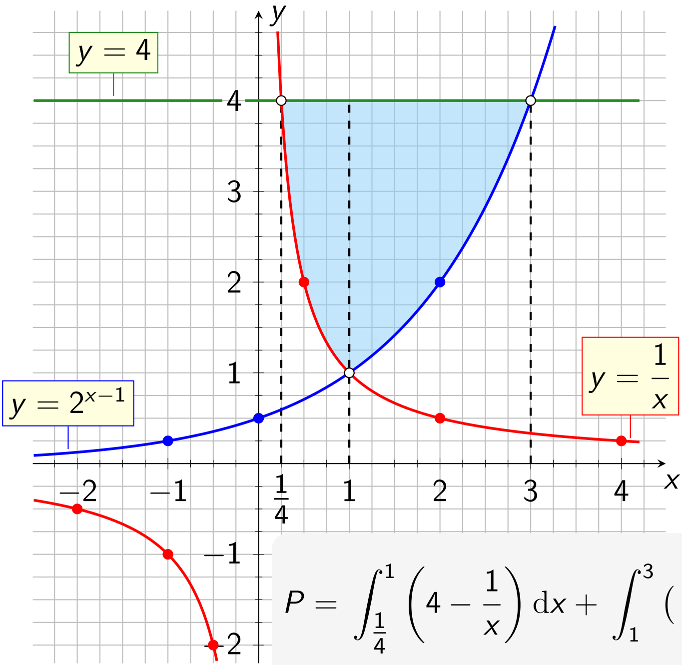
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \quad \right) \mathrm{d}x$$



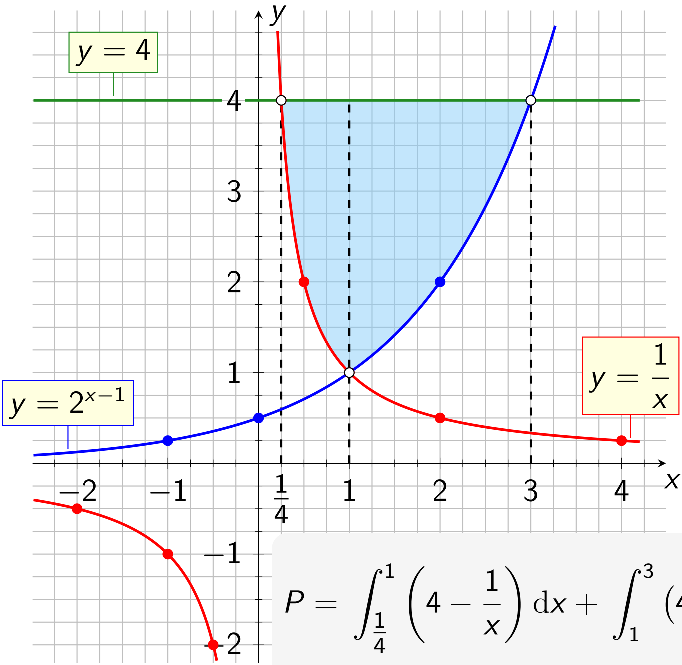
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x$$



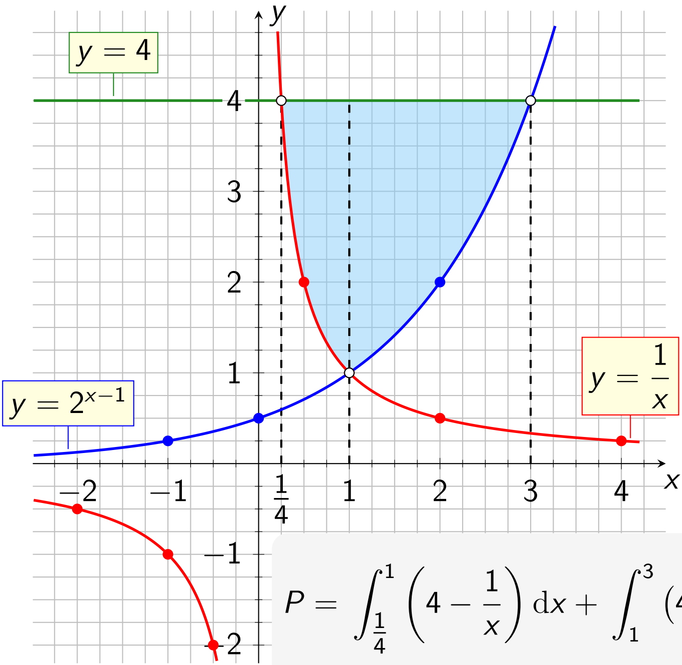
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x +$$



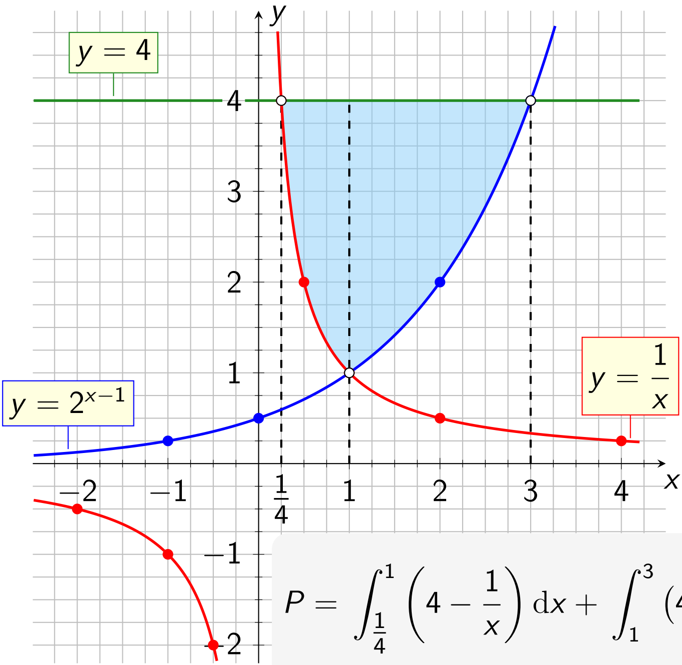
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} (\qquad\qquad) \,\mathrm{d}x$$



$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 \qquad\qquad \right) \mathrm{d}x$$



$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - \qquad \right) \mathrm{d}x$$



$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x$$

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$$= ($$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x -$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right)$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right)\Big|_{\frac{1}{4}}^{1}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Bigg($$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \Bigg(4x -$$

$$\boxed{\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Bigg(4x -$$

$$\int 2^{x-1} \,\mathrm{d}x =$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \Big[\; x - 1 = t$$

$$\boxed{\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \,\mathrm{d}x = \Big[\; x - 1 = t \;/\,'$$

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$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \,\mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\, ' \\ \mathrm{d}x \end{array} \right.$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

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$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \,\mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\,' \\ \mathrm{d}x = \mathrm{d}t \end{array}\right] =$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \, /\, ' \\ \mathrm{d}x = \mathrm{d}t \end{array} \right] = \int$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\, ' \\ \mathrm{d}x = \mathrm{d}t \end{array}\right] = \int 2^t$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\,' \\ \mathrm{d}x = \mathrm{d}t \end{array}\right] = \int 2^t \, \mathrm{d}t$$

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$$\int 2^{x-1} \,\mathrm{d}x = \begin{bmatrix} x - 1 = t \,/\, ' \\ \mathrm{d}x = \mathrm{d}t \end{bmatrix} = \int 2^t \,\mathrm{d}t =$$

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$$= \frac{2^t}{\ln 2}$$

$$\boxed{\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$= \frac{2^t}{\ln 2} + C = \frac{2^{x-1}}{\ln 2} + C$$

$$\boxed{\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Bigg(4x -$$

$$\int 2^{x-1}\,\mathrm{d}x = \left[\begin{array}{c} x-1 = t \,/\,' \\ \mathrm{d}x = \mathrm{d}t \end{array}\right] = \int 2^t \,\mathrm{d}t =$$

$$= \frac{2^t}{\ln 2} + C = \frac{2^{x-1}}{\ln 2} + C, \quad C \in \mathbb{R}$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right)\Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right.$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\, ' \\ \mathrm{d}x = \mathrm{d}t \end{array} \right] = \int 2^t \, \mathrm{d}t =$$

$$= \frac{2^t}{\ln 2} + C = \frac{2^{x-1}}{\ln 2} + C, \quad C \in \mathbb{R}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$= \left(4x - \ln|x|\right) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \Bigg($$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \Big((4 - \ln 1)$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \Bigg((4 - \ln 1) -$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right.$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right)$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right)\Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln\frac{1}{4}\right)\right) + \Bigg($$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right)\right.$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right) \Bigg|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Bigg|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln\frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) -\right.$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right.$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right)$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right) =$$

$$= \left(3 + \ln \frac{1}{4}\right)$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right)\Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln\frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right) =$$

$$= \left(3 + \ln\frac{1}{4}\right) +$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right) =$$

$$= \left(3 + \ln \frac{1}{4}\right) + \left(8 - \frac{3}{\ln 2}\right)$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ln|x|\right) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left(\left(4 - \ln 1\right) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right) =$$

$$= \left(3 + \ln \frac{1}{4}\right) + \left(8 - \frac{3}{\ln 2}\right) =$$

$$= 11 + \ln \frac{1}{4} - \frac{3}{\ln 2}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|) \Big|_{\frac{1}{4}}^{1} + \left(4x - \frac{2^{x-1}}{\ln 2}\right) \Big|_{1}^{3} =$$

$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right) + \left(\left(12 - \frac{4}{\ln 2}\right) - \left(4 - \frac{1}{\ln 2}\right)\right) =$$

$$= \left(3 + \ln \frac{1}{4}\right) + \left(8 - \frac{3}{\ln 2}\right) =$$

$$= 11 + \ln \frac{1}{4} - \frac{3}{\ln 2}$$

$P \approx 5.28562$

četvrti zadatak

Zadatak 4

Zadana je funkcija graničnih troškova $T_G = (1 + Q)e^{-Q}$.

a) *Odredite za koliko se promijene troškovi ako se proizvodnja s dva proizvoda poveća na pet proizvoda.*
b) *Odredite funkciju troškova ako fiksni troškovi iznose* 100 *novčanih jedinica.*

Rješenje

a)

$$T_G = T'$$

Rješenje

a)
$$T_G = T'$$
$$T' = (1 + Q)e^{-Q}$$

Rješenje

$$\boxed{\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x}$$

a)

$$T_G = T'$$

$$T' = (1+Q)e^{-Q}$$

$$T = \int (1+Q)e^{-Q}\,\mathrm{d}Q$$

Rješenje

$$\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x$$

a)

$$T_G = T'$$

$$T' = (1+Q)e^{-Q}$$

$$T = \int (1+Q)e^{-Q}\,\mathrm{d}Q = \int$$

Rješenje

$$\boxed{\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x}$$

a)

$$T_G = T'$$

$$T' = (1+Q)e^{-Q}$$

$$T = \int (1+Q)e^{-Q}\,\mathrm{d}Q = \int (1+Q)$$

Rješenje

$$\boxed{\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x}$$

a)

$$T_G = T'$$

$$T' = (1+Q)e^{-Q}$$

$$T = \int (1+Q)e^{-Q}\,\mathrm{d}Q = \int (1+Q)\cdot$$

Rješenje

$$\boxed{\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x}$$

a)
$$T_G = T'$$

$$T' = (1+Q)e^{-Q}$$

$$T = \int (1+Q)e^{-Q}\,\mathrm{d}Q = \int (1+Q)\cdot\left(-e^{-Q}\right)'$$

Rješenje

$$\boxed{\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x}$$

a)
$$T_G = T'$$

$$T' = (1+Q)e^{-Q}$$

$$T = \int (1+Q)e^{-Q}\,\mathrm{d}Q = \int (1+Q)\cdot\left(-e^{-Q}\right)'\,\mathrm{d}Q$$

Rješenje

$$\int f'(x)g(x)\,\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\,\mathrm{d}x$$

a)

$$T_G = T'$$

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$$T(5) - T(2) =$$

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Troškovi se promijene za približno 0.49 novčanih jedinica.

$$T'(Q) = (1+Q)e^{-Q}$$

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Troškovi se promijene za približno 0.49 novčanih jedinica.

$\boxed{\text{Ova promjena troškova vrijedi za svaki } C \in \mathbb{R}.}$

b)

$$T(Q) = (-Q-2)e^{-Q} + C, \quad C \in \mathbb{R}$$

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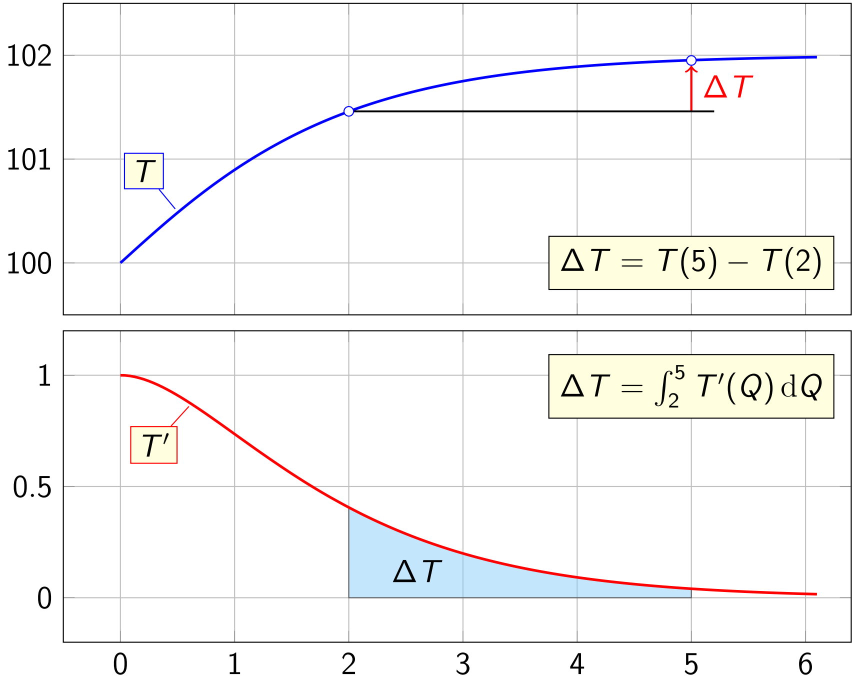
$$T(Q) = (-Q-2)e^{-Q} + 102$$



102
101
100
T
ΔT
$\Delta T = T(5) - T(2)$
1
0.5
0
T'
$\Delta T = \int_2^5 T'(Q)\,\mathrm{d}Q$
ΔT
0
1
2
3
4
5
6

peti zadatak

Zadatak 5

Odredite funkciju potražnje $q(p)$ *za koju je* $E_{q,p} = -2p$ *i* $q(0) = 2$.

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Rješenje

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Rješenje

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q'$$

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Rješenje

$$y = y(x)$$
$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

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$$\frac{\mathrm{d}q}{q}$$

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$$\boxed{\begin{aligned} y &= y(x) \\ y' &= \frac{\mathrm{d}y}{\mathrm{d}x} \end{aligned}}$$

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$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$y = y(x)$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$\ln|q|$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$\ln|q| =$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$y = y(x)$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$y = y(x)$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$\ln|q| = -2p +$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$\boxed{\begin{aligned} y &= y(x) \\ y' &= \frac{\mathrm{d}y}{\mathrm{d}x} \end{aligned}}$$

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$y = y(x)$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

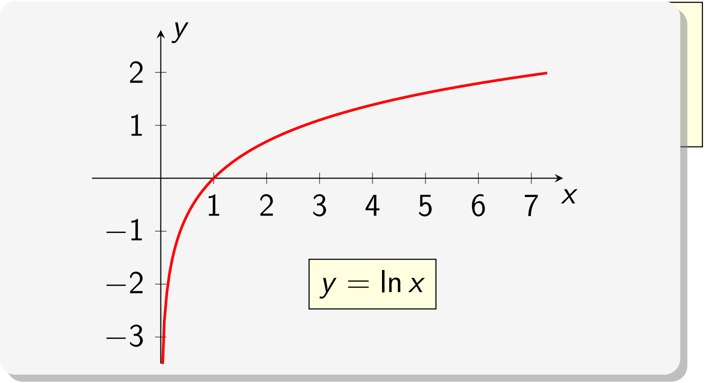
Rješenje

$$E_{q,p} = -$$

$$\frac{p}{q} \cdot q' = -$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -$$

$$\frac{\mathrm{d}q}{q} = -$$



$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ *za koju je* $E_{q,p} = -2p$ *i* $q(0) = 2$.

Rješenje

$$y = y(x)$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

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$$\frac{p}{q} \cdot q' = -2p$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$y = y(x)$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C}$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$E_{q,p} = -2p$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$y = y(x)$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

$$y = y(x)$$

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Rješenje

$$E_{q,p} = -2p$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln \frac{|q|}{C} = -2p$$

$$\frac{|q|}{C}$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

$$y = y(x)$$
$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

Rješenje

$$E_{q,p} = -2p$$
$$\frac{p}{q} \cdot q' = -2p$$
$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$
$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$
$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$
$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$
$$\ln\frac{|q|}{C} = -2p$$
$$\frac{|q|}{C} =$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

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Rješenje

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

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$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e^{-2p}$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ *za koju je* $E_{q,p} = -2p$ *i* $q(0) = 2$.

Rješenje

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$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e^{-2p}$$

$$|q| = Ce^{-2p}$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

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$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e^{-2p}$$

$$|q| = Ce^{-2p}, \quad C > 0$$

Zadatak 5

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$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$
$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$
$$\ln\frac{|q|}{C} = -2p$$
$$\frac{|q|}{C} = e^{-2p}$$
$$|q| = Ce^{-2p}, \quad C > 0$$
$$q = Ce^{-2p}, \quad C \in \mathbb{R} \setminus \{0\}$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

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$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

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$$|q| = Ce^{-2p}, \quad C > 0$$

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$$q(0) = 2$$

Zadatak 5

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$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e^{-2p}$$

$$|q| = Ce^{-2p}, \quad C > 0$$

$$q = Ce^{-2p}, \quad C \in \mathbb{R} \setminus \{0\}$$

$$q(0) = 2$$

$$C \cdot e^{-2\cdot 0} = 2$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

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$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e^{-2p}$$

$$|q| = Ce^{-2p}, \quad C > 0$$

$$q = Ce^{-2p}, \quad C \in \mathbb{R} \setminus \{0\}$$

$$q(0) = 2$$

$$C \cdot e^{-2\cdot 0} = 2$$

$$C = 2$$

Zadatak 5

Odredite funkciju potražnje $q(p)$ za koju je $E_{q,p} = -2p$ i $q(0) = 2$.

Rješenje

$$\boxed{\begin{aligned} y &= y(x) \\ y' &= \frac{\mathrm{d}y}{\mathrm{d}x} \end{aligned}}$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p + \ln C, \quad C > 0$$

$$\ln|q| - \ln C = -2p$$

$$\ln\frac{|q|}{C} = -2p$$

$$\frac{|q|}{C} = e^{-2p}$$

$$|q| = Ce^{-2p}, \quad C > 0$$

$$q = Ce^{-2p}, \quad C \in \mathbb{R} \setminus \{0\}$$

$$q(0) = 2$$

$$C \cdot e^{-2\cdot 0} = 2$$

$$C = 2$$

$$\boxed{q(p) = 2e^{-2p}}$$

drugi način

$$E_{q,p} = -2p$$

$\boxed{\text{drugi način}}$

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q'$$

$\boxed{\text{drugi način}}$

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p}$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q}$$

drugi način

$$
\begin{aligned}
E_{q,p} &= -2p \\
\frac{p}{q} \cdot q' &= -2p \\
\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} &= -2p \\
\frac{\mathrm{d}q}{q} &=
\end{aligned}
$$

drugi način

$$
\begin{aligned}
E_{q,p} &= -2p \\
\frac{p}{q} \cdot q' &= -2p \\
\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} &= -2p \\
\frac{\mathrm{d}q}{q} &= -2\,\mathrm{d}p
\end{aligned}
$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q}$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} =$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

drugi način

$$E_{q,p} = -2p \qquad\qquad \ln|q|$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| =$$

drugi način

$$E_{q,p} = -2p \qquad\qquad \ln|q| = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

drugi način

$$
\begin{aligned}
E_{q,p} &= -2p \\
\frac{p}{q} \cdot q' &= -2p \\
\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} &= -2p \\
\frac{\mathrm{d}q}{q} &= -2\,\mathrm{d}p \\
\int \frac{\mathrm{d}q}{q} &= -2 \int \mathrm{d}p
\end{aligned}
$$

$$\ln|q| = -2p +$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p + C$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p + C, \quad C \in \mathbb{R}$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

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$$\int \frac{\mathrm{d}q}{q} = -2\int \mathrm{d}p$$

$$\ln|q| = -2p + C, \quad C \in \mathbb{R}$$

$$|q|$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

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$$\ln|q| = -2p + C, \quad C \in \mathbb{R}$$

$$|q| =$$

drugi način

$$E_{q,p} = -2p$$

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$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + C, \quad C \in \mathbb{R}$$

$$|q| = e$$

drugi način

$$E_{q,p} = -2p$$

$$\frac{p}{q} \cdot q' = -2p$$

$$\frac{p}{q} \cdot \frac{\mathrm{d}q}{\mathrm{d}p} = -2p$$

$$\frac{\mathrm{d}q}{q} = -2\,\mathrm{d}p$$

$$\int \frac{\mathrm{d}q}{q} = -2 \int \mathrm{d}p$$

$$\ln|q| = -2p + C, \quad C \in \mathbb{R}$$

$$|q| = e^{-2p+C}$$

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$$q = \pm e^{-2p+C}$$

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$$|q| = e^{-2p+C}$$

$$q = \pm e^{-2p+C}, \quad C \in \mathbb{R}$$

$$q(0) = 2$$

drugi način

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$$q = \pm e^{-2p+C}, \quad C \in \mathbb{R}$$

$$q(0) = 2$$

$$e^{-2\cdot 0+C} = 2$$

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$$e^{C} = 2$$

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$$C = \ln 2$$

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$$q =$$

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$$|q| = e^{-2p+C}$$

$$q = \pm e^{-2p+C}, \quad C \in \mathbb{R}$$

$$q(0) = 2$$

$$e^{-2\cdot 0+C} = 2$$

$$e^{C} = 2$$

$$C = \ln 2$$

$$q = e^{-2p+\ln 2}$$

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$$e^{C} = 2$$

$$C = \ln 2$$

$$q = e^{-2p+\ln 2} = e^{-2p} \cdot e^{\ln 2}$$

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$$q(0) = 2$$

$$e^{-2\cdot 0 + C} = 2$$

$$e^{C} = 2$$

$$C = \ln 2$$

$$q = e^{-2p+\ln 2} = e^{-2p} \cdot e^{\ln 2} = 2e^{-2p}$$