Seminari 2

Matematika za ekonomiste 2

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FOI, Varaždin

Sadržaj

prvi zadatak

Zadatak 1

Zadana je funkcija troškova $T = 0.001x^3 + 10x + 2000$.

a) *Odredite funkcije prosječnih i graničnih troškova i nacrtajte njihove grafove na istoj slici na segmentu* $[0, 250]$.

b) *Odredite za koju količinu proizvodnje su prosječni troškovi minimalni. U kakvom su odnosu granični i prosječni troškovi za tu količinu proizvodnje? Kako se taj odnos može vidjeti na grafu?*

$$T = 0.001x^3 + 10x + 2000$$

Rješenje

- Funkcija prosječnih troškova

$$T_p(x) =$$

$$T = 0.001x^3 + 10x + 2000$$

Rješenje

- Funkcija prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

$$T = 0.001x^3 + 10x + 2000$$

Rješenje

- Funkcija prosječnih troškova

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$

$$T = 0.001x^3 + 10x + 2000$$

Rješenje

- Funkcija prosječnih troškova

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

$$T = 0.001x^3 + 10x + 2000$$

Rješenje

- Funkcija prosječnih troškova

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- Funkcija graničnih troškova

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Rješenje

- Funkcija prosječnih troškova

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- Funkcija graničnih troškova

$$T_g(x) = T'(x)$$

$$T = 0.001x^3 + 10x + 2000$$

Rješenje

- Funkcija prosječnih troškova

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- Funkcija graničnih troškova

$$T_g(x) = T'(x)$$

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$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

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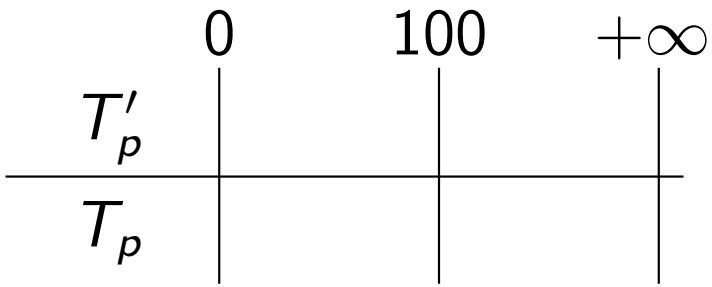

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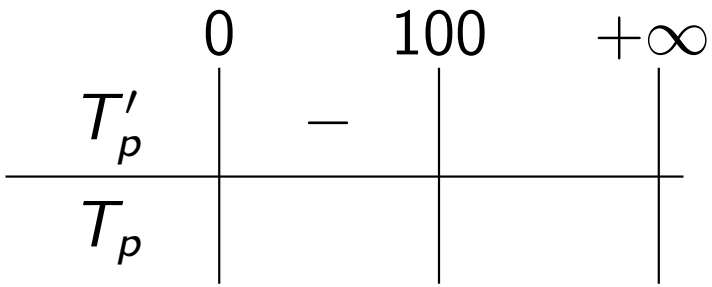

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	0		100		$+\infty$
T_p'		$-$		$+$	
T_p					

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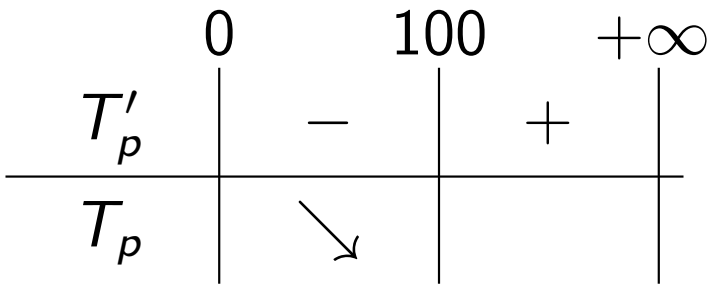

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T_p		$\searrow$		$\nearrow$	

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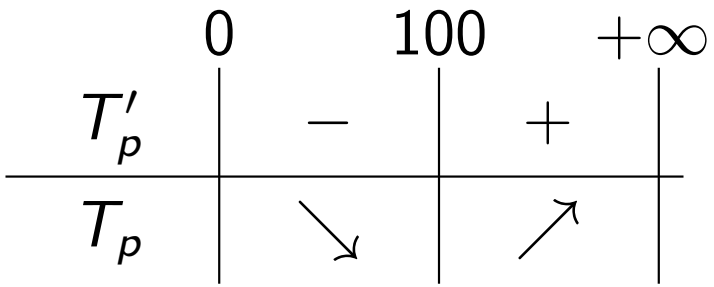

minimum

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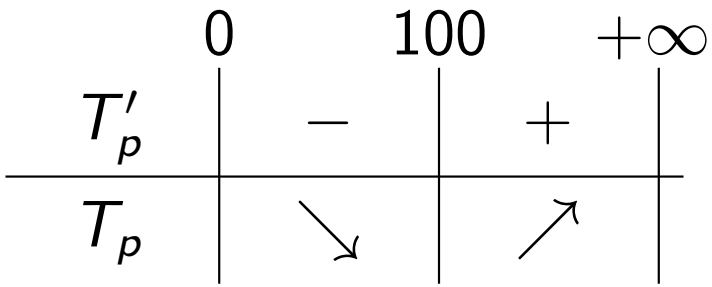

minimum

Prosječni troškovi su minimalni za 100 proizvoda.

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

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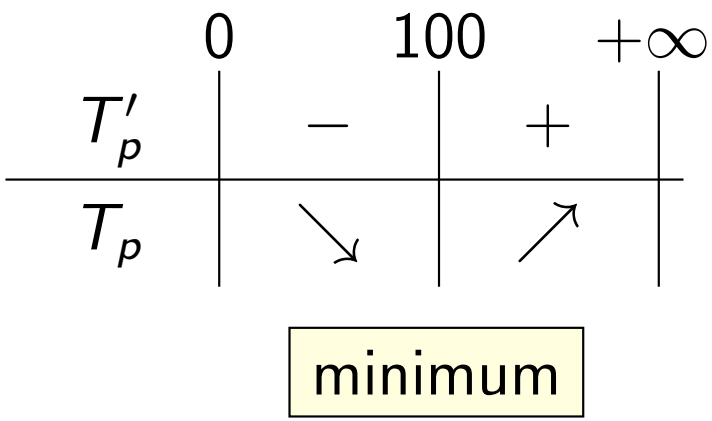

minimum

$T_p(100) =$

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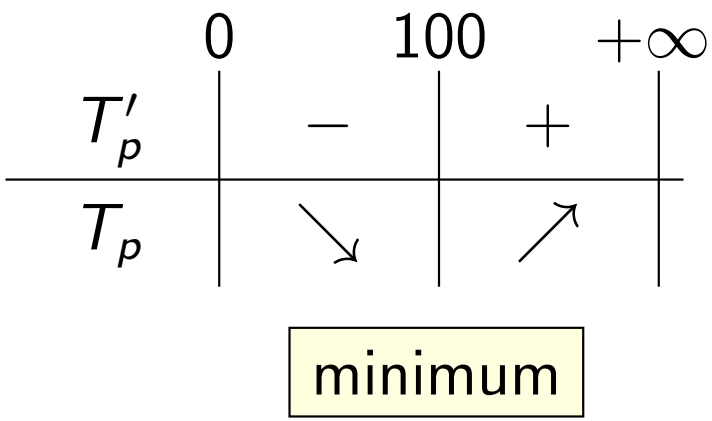

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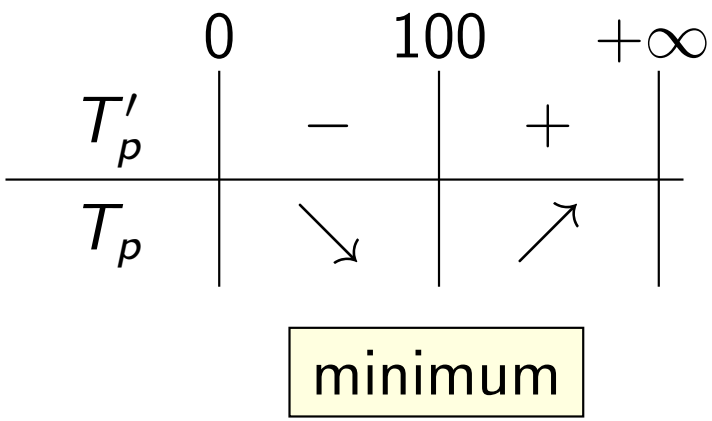

minimum

$$T_p(100) = 0.001 \cdot 100^2 + 10 + \frac{2000}{100}$$

$$T_p(100) = 40$$

Prosječni troškovi su minimalni za 100 proizvoda.

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$$\boxed{T_g(x) = 0.003x^2 + 10}$$

	0		100		$+\infty$
T_p'		$-$		$+$	
T_p		$\searrow$		$\nearrow$	

minimum

$$T_p(100) = 0.001 \cdot 100^2 + 10 + \frac{2000}{100}$$

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$$\boxed{T_g(x) = 0.003x^2 + 10}$$

	0		100		$+\infty$
T_p'		$-$		$+$	
T_p		$\searrow$		$\nearrow$	

minimum

$$T_p(100) = 0.001 \cdot 100^2 + 10 + \frac{2000}{100}$$

$$T_p(100) = 40$$

$$T_g(100) = 0.003 \cdot 100^2 + 10$$

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$$T_g(x) = 0.003x^2 + 10$$

	0		100		$+\infty$
T_p'		$-$		$+$	
T_p		$\searrow$		$\nearrow$	

minimum

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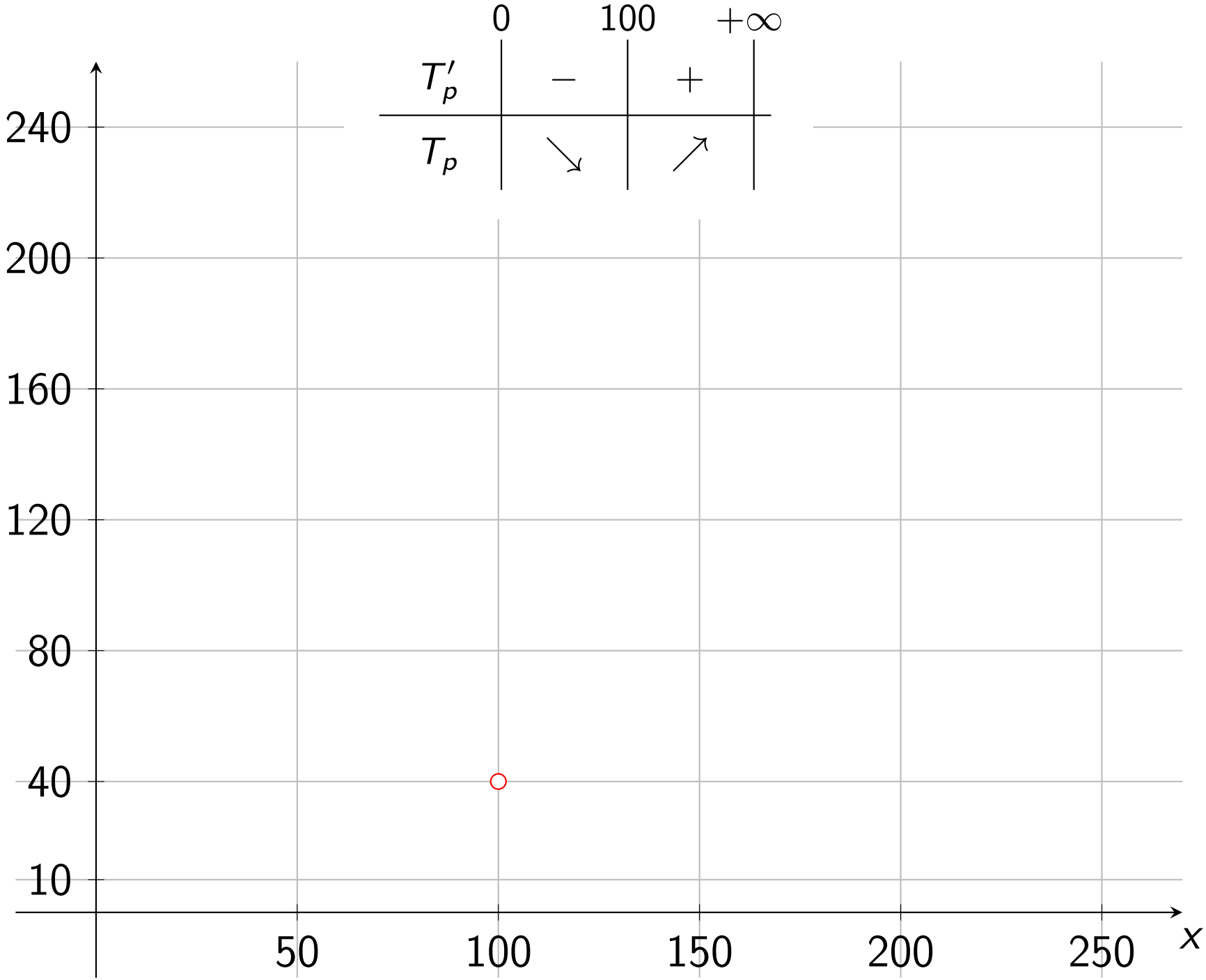
$$T_p(100) = 40$$

$$T_g(100) = 0.003 \cdot 100^2 + 10$$

$$T_g(100) = 40$$

Prosječni troškovi su minimalni za 100 proizvoda. Za ovu količinu proizvodnje granični troškovi su jednaki prosječnim troškovima i iznose 40 novčanih jedinica.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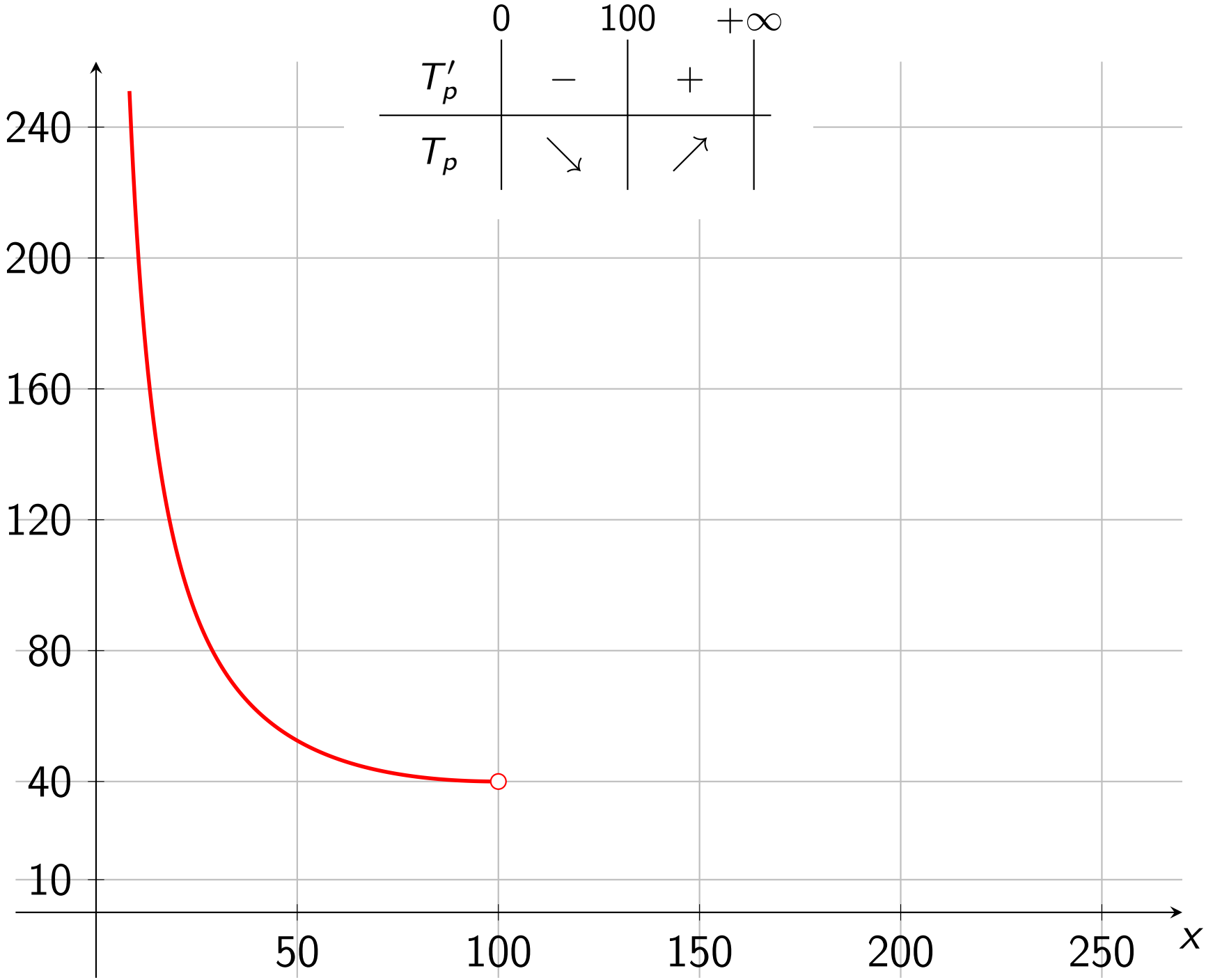
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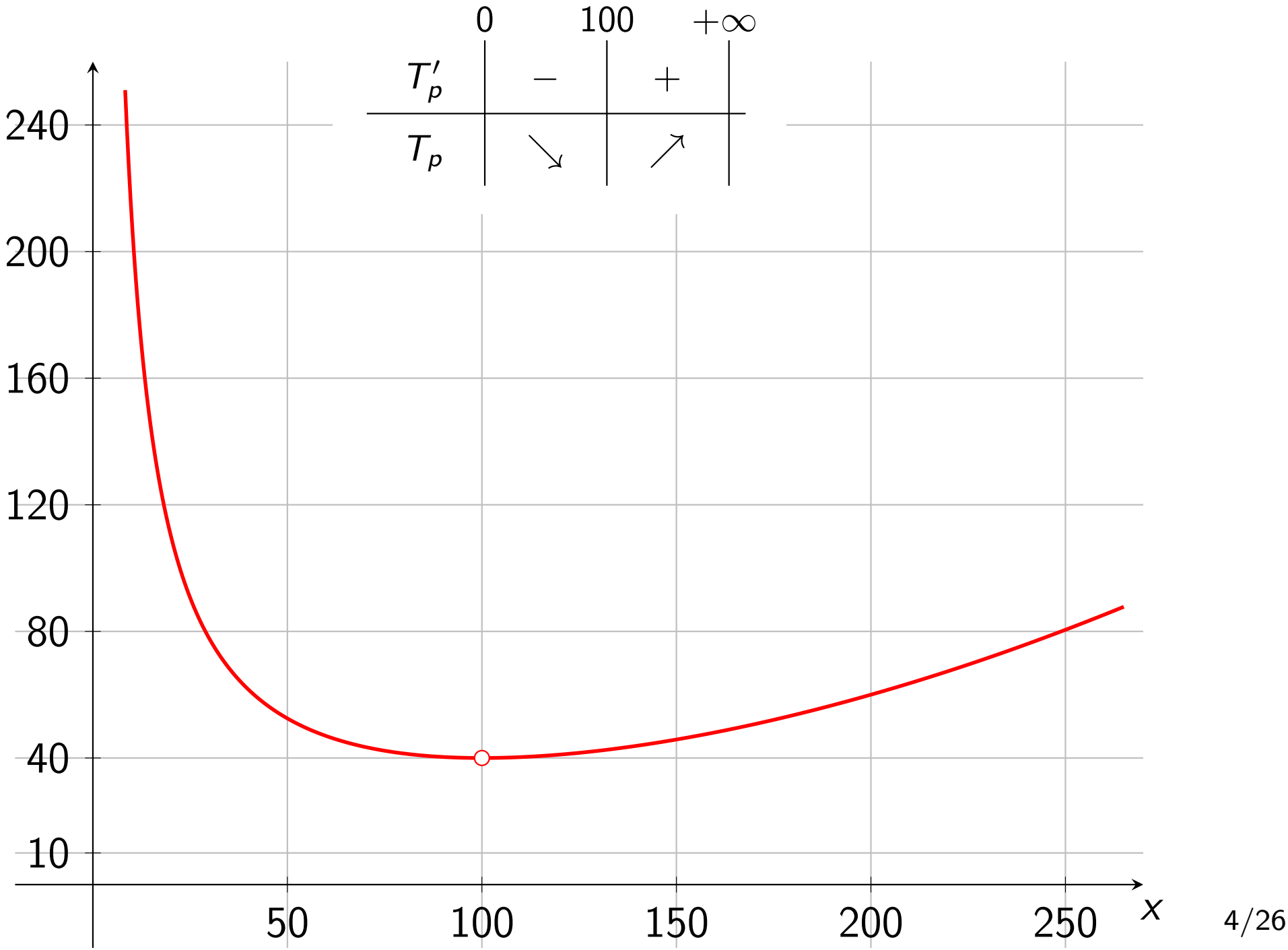
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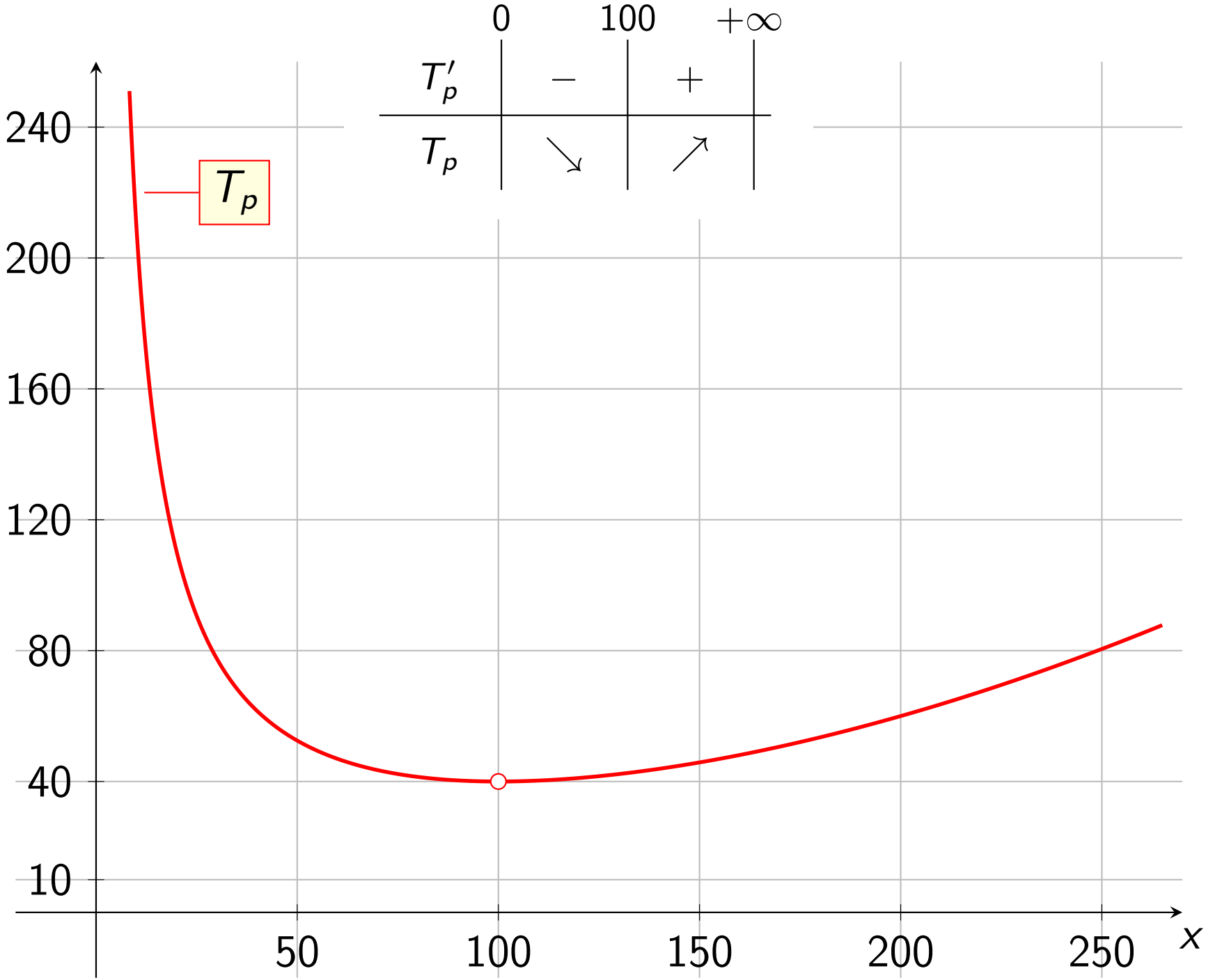
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$+\infty$
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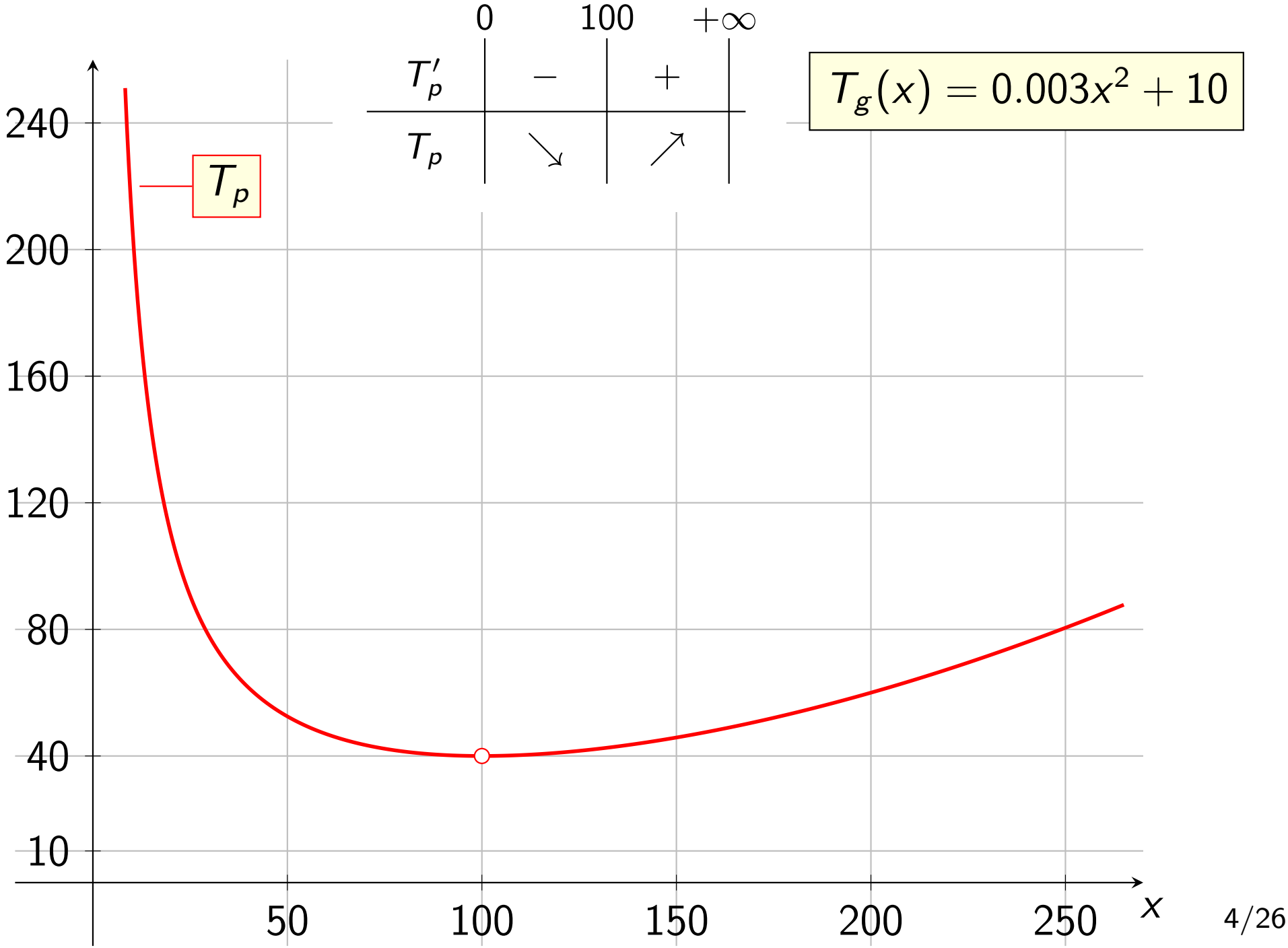
0
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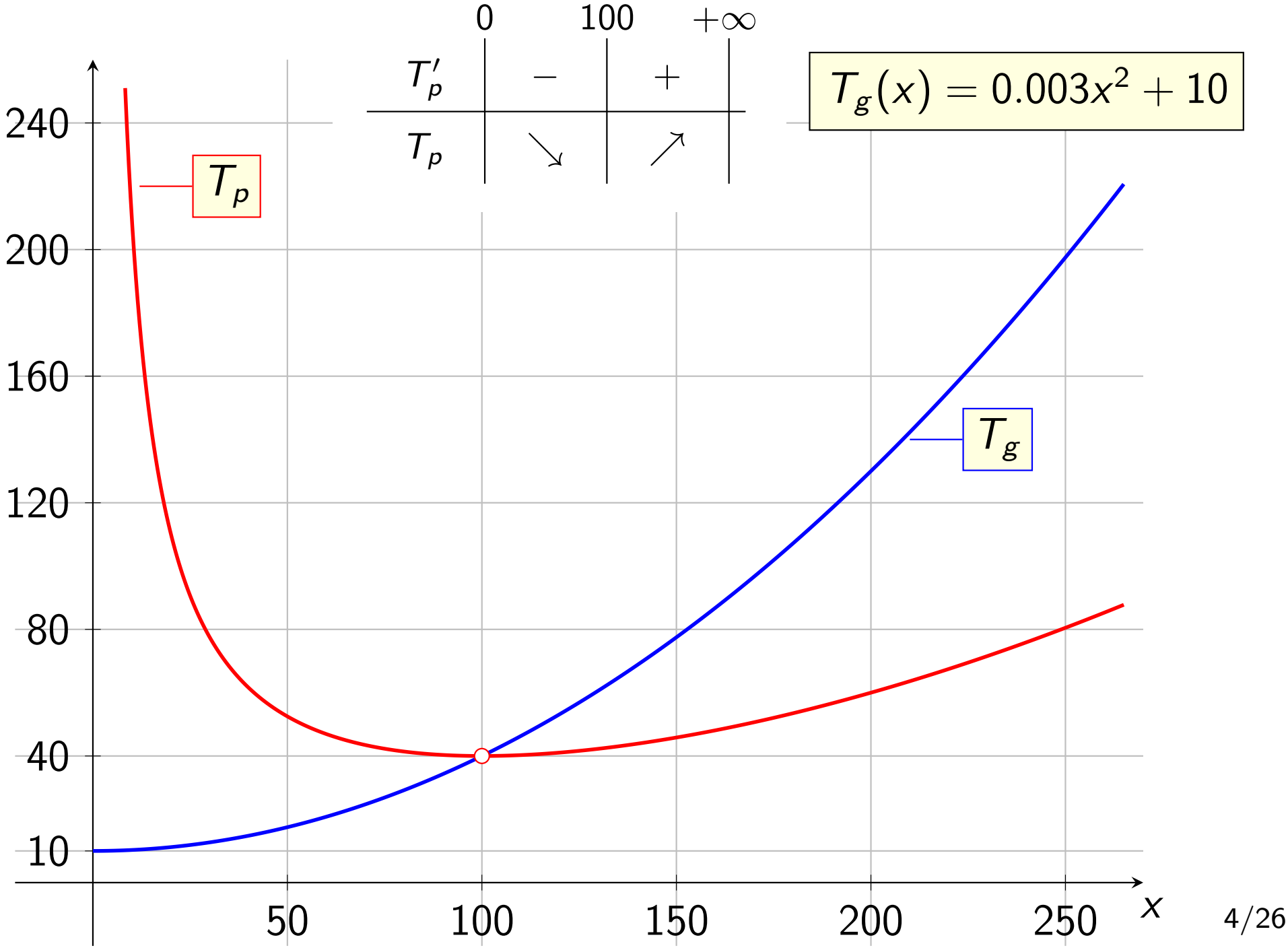
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Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

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Općeniti odnos graničnih i prosječnih troškova

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Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

$$T_p'(x) = \frac{\qquad\qquad\qquad\qquad}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

$$T_p'(x) = \frac{T'(x)}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

$$T_p'(x) = \frac{T'(x) \cdot}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

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$$T_p'(x) = \frac{T'(x) \cdot x}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

$$T_p'(x) = \frac{T'(x) \cdot x -}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

$$T_p'(x) = \frac{T'(x) \cdot x - T(x)}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

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$$T_p'(x) = \frac{x \cdot \left(T'(x) - \qquad \right)}{x^2}$$

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$$T_p(x) = \frac{T(x)}{x}$$

$$T_p'(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^2}$$

$$T_p'(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x} \right)}{x^2}$$

Općeniti odnos graničnih i prosječnih troškova

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$$T_p'(x) = \text{———————}$$

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Općeniti odnos graničnih i prosječnih troškova

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$$T_p'(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x} \right)}{x^2}$$

$$T_p'(x) = \frac{T_g(x) -}{x}$$

Općeniti odnos graničnih i prosječnih troškova

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Općeniti odnos graničnih i prosječnih troškova

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- Ako su granični troškovi veći od prosječnih troškova, tada prosječni troškovi rastu.

Općeniti odnos graničnih i prosječnih troškova

$$T_p(x) = \frac{T(x)}{x}$$

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$$T_p'(x) = \frac{T_g(x) - T_p(x)}{x}$$

- Ako su granični troškovi veći od prosječnih troškova, tada prosječni troškovi rastu.
- Ako su granični troškovi manji od prosječnih troškova, tada prosječni troškovi padaju.

- Kako na grafu funkcije troškova uočiti, ako postoji, točku proizvodnje x_0 u kojoj su prosječni troškovi minimalni?

- Kako na grafu funkcije troškova uočiti, ako postoji, točku proizvodnje x_0 u kojoj su prosječni troškovi minimalni?

$$T_p'(x) = \frac{T'(x) \cdot x - T(x)}{x^2} = \frac{T_g(x) - T_p(x)}{x}$$

- Kako na grafu funkcije troškova uočiti, ako postoji, točku proizvodnje x_0 u kojoj su prosječni troškovi minimalni?

$$T_p'(x) = \frac{T'(x) \cdot x - T(x)}{x^2} = \frac{T_g(x) - T_p(x)}{x}$$

- Iz $T_p'(x_0) = 0$ slijedi

$$T'(x_0) = \frac{T(x_0)}{x_0}.$$

- Kako na grafu funkcije troškova uočiti, ako postoji, točku proizvodnje x_0 u kojoj su prosječni troškovi minimalni?

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$$T'(x_0) = \frac{T(x_0)}{x_0}.$$

- Jednadžba tangente na graf funkcije troškova u točki x_0

$$t \ldots y - T(x_0) = T'(x_0) \cdot (x - x_0)$$

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$$t \ldots y - T(x_0) = T'(x_0) \cdot (x - x_0)$$

$$t \ldots y = \frac{T(x_0)}{x_0} \cdot x$$

- Kako na grafu funkcije troškova uočiti, ako postoji, točku proizvodnje x_0 u kojoj su prosječni troškovi minimalni?

$$T_p'(x) = \frac{T'(x) \cdot x - T(x)}{x^2} = \frac{T_g(x) - T_p(x)}{x}$$

- Iz $T_p'(x_0) = 0$ slijedi

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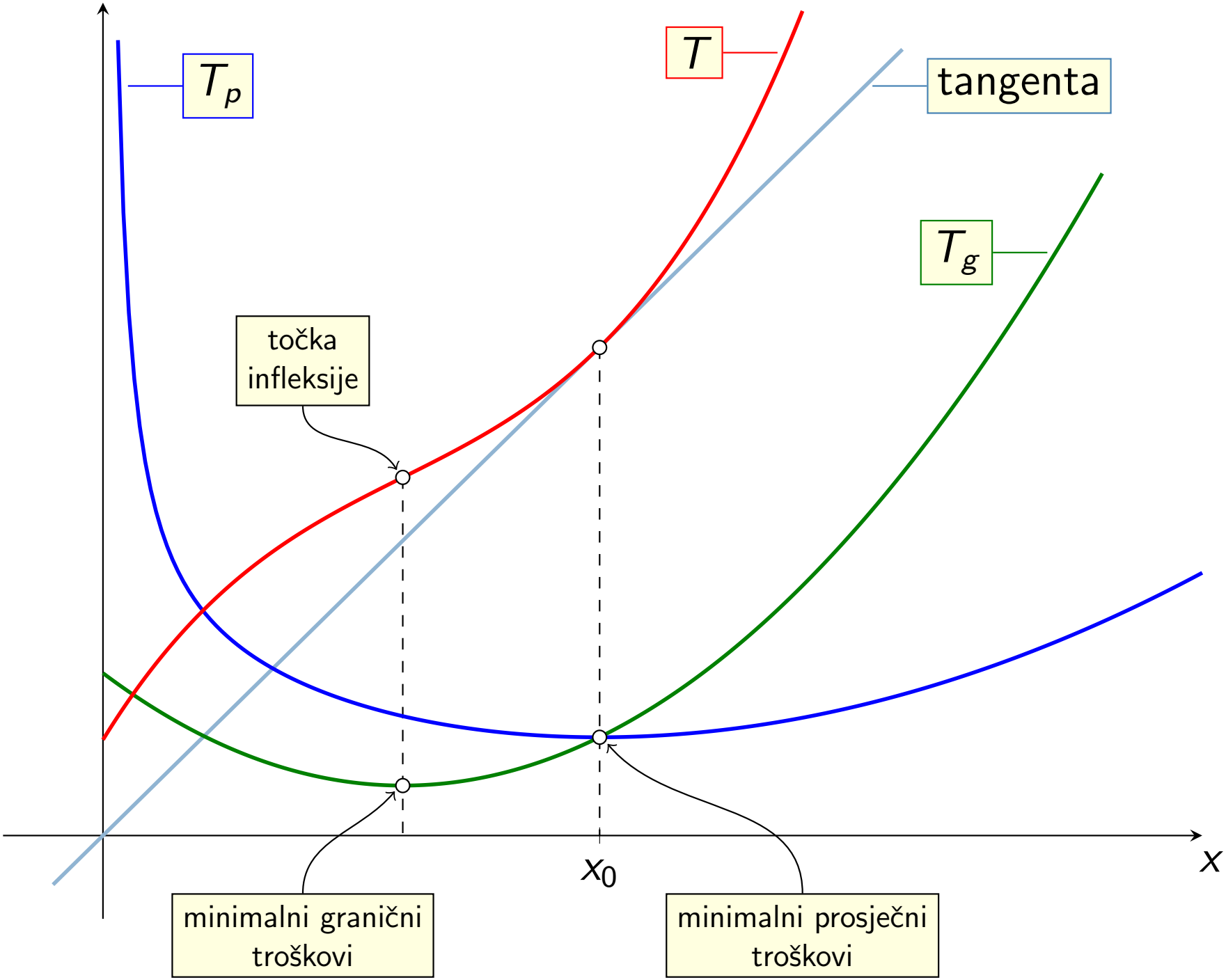
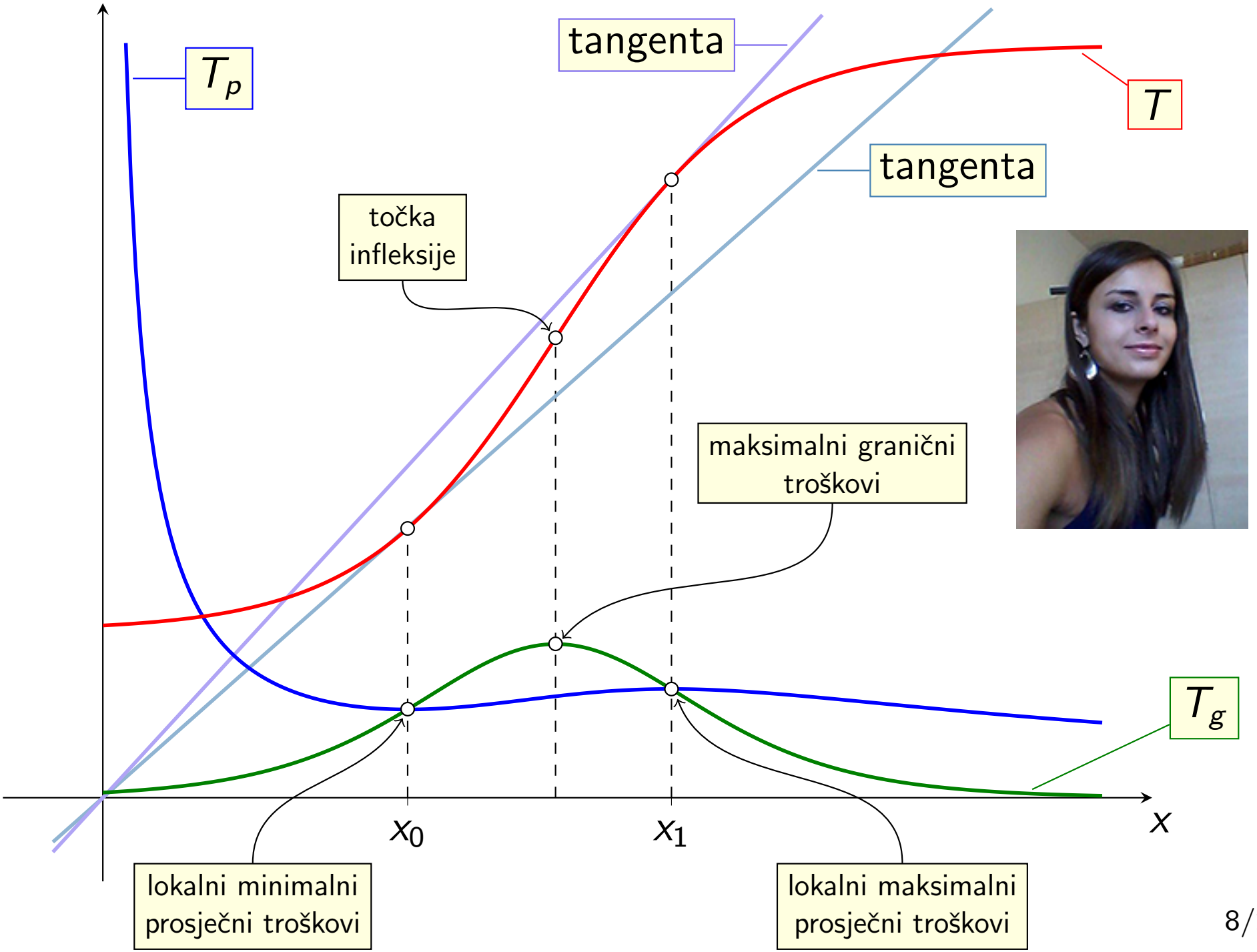
- Jednadžba tangente na graf funkcije troškova u točki x_0

$$t \ldots y - T(x_0) = T'(x_0) \cdot (x - x_0)$$

$$t \ldots y = \frac{T(x_0)}{x_0} \cdot x$$

tangenta prolazi kroz ishodište koordinatnog sustava


T_p
T
tangenta
T_g
točka infleksije
x_0
x
minimalni granični troškovi
minimalni prosječni troškovi


T_p
tangenta
T
tangenta
točka infleksije
maksimalni granični troškovi
T_g
x_0
x_1
x
lokalni minimalni prosječni troškovi
lokalni maksimalni prosječni troškovi

Neka svojstva funkcije troškova

- Funkcija troškova je u pravilu rastuća funkcija na $[0, +\infty\rangle$.
- Ako je funkcija troškova konveksna, tada se granični troškovi povećavaju s porastom proizvodnje.
- Ako je funkcija troškova konkavna, tada se granični troškovi smanjuju s porastom proizvodnje.
- Tangenta na graf funkcije troškova u točki u kojoj su prosječni troškovi minimalni prolazi kroz ishodište koordinatnog sustava, a funkcija troškova je konveksna u toj točki.
- Razina proizvodnje za koju su prosječni troškovi minimalni, prosječni troškovi su jednaki graničnim troškovima.
- Prosječni troškovi padaju ako su veći od graničnih troškova. Prosječni troškovi rastu ako su manji od graničnih troškova.

Zadatak (Domaća zadaća)

Zadana je linearna funkcija troškova $T = 2x + 5$.

a) *Odredite funkcije prosječnih i graničnih troškova.*

b) *Nacrtajte grafove funkcije troškova, prosječnih troškova i graničnih troškova na istoj slici.*

c) *Za koju razinu proizvodnje su prosječni troškovi minimalni? Objasnite što se događa u ovom slučaju.*

d) *Kako se odmah mogao donijeti zaključak o minimalnim prosječnim troškovima na temelju zadane funkcije troškova? Koristite svojstvo tangente koje smo ranije spomenuli.*

drugi zadatak

Zadatak 2

Zadana je funkcija troškova $T = 45\sqrt{3Q + 2209}$.

a) *Odredite fiksne, varijabilne i prosječne troškove.*

b) *Koliko je proizvoda proizvedeno ako su troškovi jednaki* 5000*?*

c) *Odredite granične troškove za* 10 *proizvoda i interpretirajte rezultat.*

d) *Odredite funkciju elastičnosti troškova.*

e) *Odredite elastičnost troškova za* 10 *proizvoda i interpretirajte rezultat.*

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F =$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0)$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209}$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$\boxed{T = 45\sqrt{3Q + 2209}}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

Prosječni troškovi

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

Prosječni troškovi

$$T_p = \frac{T}{Q}$$

$$T = 45\sqrt{3Q + 2209}$$

Rješenje

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

Prosječni troškovi

$$T_p = \frac{T}{Q} = \frac{45\sqrt{3Q + 2209}}{Q}$$

b)

$$T = 5000$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000 \;\Big/^{2}$$

$$81 \cdot (3Q + 2209) = 1\,000\,000$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000 \;\Big/^{2}$$

$$81 \cdot (3Q + 2209) = 1\,000\,000$$

$$243Q + 178\,929 = 1\,000\,000$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000 \;\Big/^{2}$$

$$81 \cdot (3Q + 2209) = 1\,000\,000$$

$$243Q + 178\,929 = 1\,000\,000$$

$$243Q = 821\,071$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000 \;\Big/^{2}$$

$$81 \cdot (3Q + 2209) = 1\,000\,000$$

$$243Q + 178\,929 = 1\,000\,000$$

$$243Q = 821\,071$$

$$Q = \frac{821\,071}{243}$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000 \;\Big/^{2}$$

$$81 \cdot (3Q + 2209) = 1\,000\,000$$

$$243Q + 178\,929 = 1\,000\,000$$

$$243Q = 821\,071$$

$$Q = \frac{821\,071}{243}$$

$$Q \approx 3378.89$$

b)

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 \;\Big/ : 5$$

$$9\sqrt{3Q + 2209} = 1000 \;\Big/^{2}$$

$$81 \cdot (3Q + 2209) = 1\,000\,000$$

$$243Q + 178\,929 = 1\,000\,000$$

$$243Q = 821\,071$$

$$Q = \frac{821\,071}{243}$$

$$Q \approx 3378.89$$

Proizvedeno je približno 3379 proizvoda.

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' =$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}}$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)'$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}}$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) =$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}}$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}} = \frac{135}{2\sqrt{2239}}$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x}\,)' = \frac{1}{2\sqrt{x}}$$

$$T = 45\sqrt{3Q + 2209}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}} = \frac{135}{2\sqrt{2239}} \approx 1.43$$

$$\left(\sqrt{\text{nešto}}\,\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$\left(\sqrt{x}\,\right)' = \frac{1}{2\sqrt{x}}$$

$$\boxed{T = 45\sqrt{3Q + 2209}}$$

c)

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}} = \frac{135}{2\sqrt{2239}} \approx 1.43$$

Ako na razini proizvodnje od 10 proizvoda proizvodnju povećamo za jedan proizvod, troškovi će se povećati za 1.43 novčane jedinice.

$$\boxed{\left(\sqrt{\text{nešto}}\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'} \qquad \boxed{\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T = 45\sqrt{3Q + 2209}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T'$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T = 45\sqrt{3Q + 2209}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \text{———————}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}}$$

$$\boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{\qquad\qquad}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}}$$

$$\boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}}$$

$$\boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \text{\_\_\_\_\_}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{3Q}{\qquad\qquad}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{3Q}{2(3Q + 2209)}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

e)

$$E_{T,Q}(10) = \frac{3 \cdot 10}{6 \cdot 10 + 4418}$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

e)

$$E_{T,Q}(10) = \frac{3 \cdot 10}{6 \cdot 10 + 4418} = \frac{15}{2239} \approx 0.0067$$

$$\boxed{T' = \frac{135}{2\sqrt{3Q + 2209}}} \qquad \boxed{T = 45\sqrt{3Q + 2209}}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

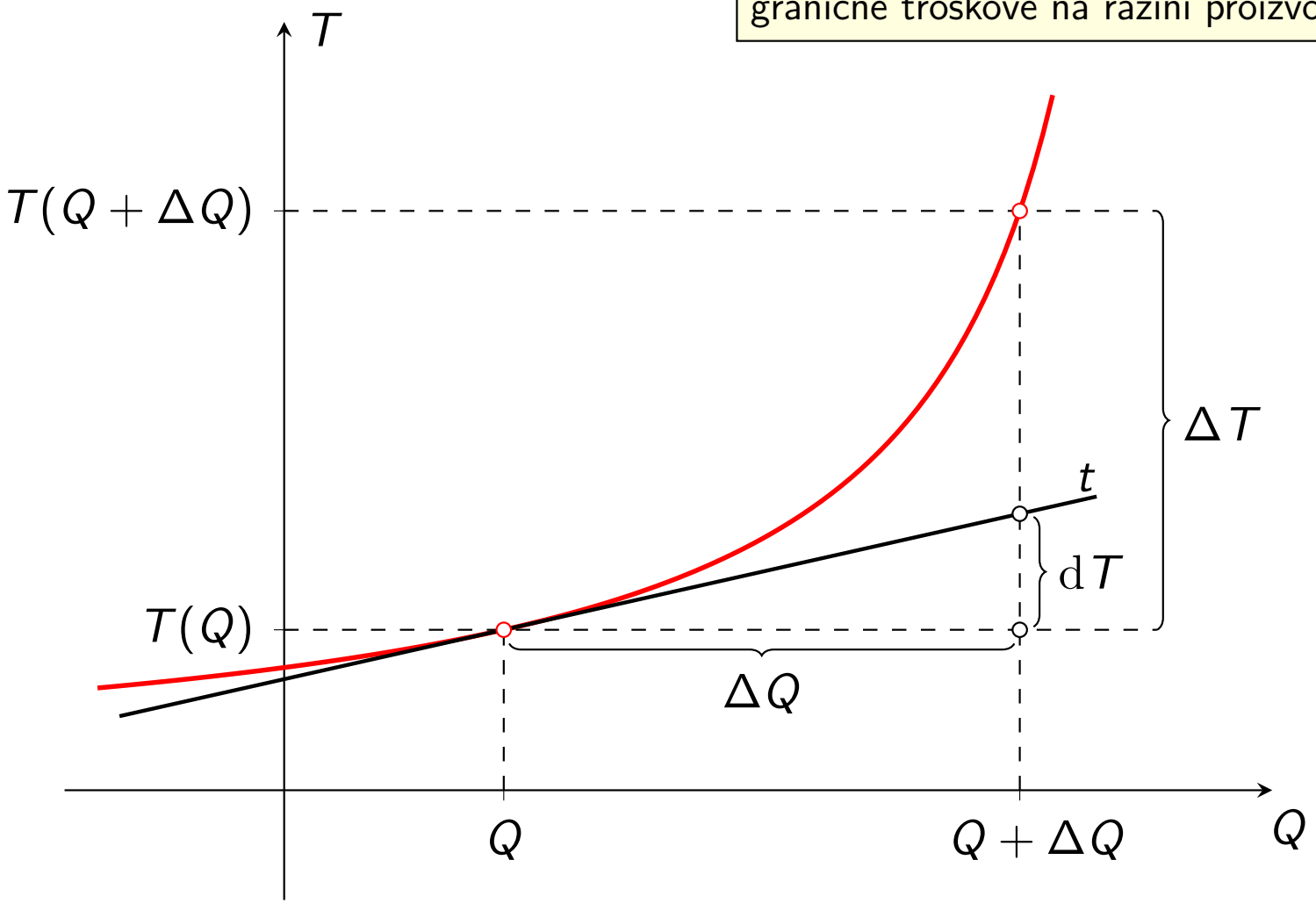
e)

$$E_{T,Q}(10) = \frac{3 \cdot 10}{6 \cdot 10 + 4418} = \frac{15}{2239} \approx 0.0067$$

Ako na razini proizvodnje od 10 proizvoda proizvodnju povećamo za 1%, troškovi će se povećati za 0.0067%.

Vizualizacija graničnih troškova

Kada je $\Delta Q = 1$, tada $\mathrm{d}T$ predstavlja granične troškove na razini proizvodnje Q.



treći zadatak

Zadatak 3

Zadana je funkcija ponude $q = 2.43\sqrt[4]{18 + 0.2p^2}$.

a) *Koliko se proizvoda nudi ako je cijena jednog proizvoda* 100 *kn.*

b) *Odredite funkciju elastičnosti ponude.*

c) *Odredite elastičnost ponude na razini cijene od* 30 *kn i interpretirajte rezultat.*

d) *Odredite cijenu za koju je elastičnost ponude jednaka* $\frac{1}{22}$.

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Rješenje

a) $q(100) = 2.43\sqrt[4]{18 + 0.2 \cdot 100^2} \approx 16.29$

Ako je cijena jednog proizvoda 100 kn, nudi se približno 16 proizvoda.

b) $q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$

$$q' =$$

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$$q' = 2.43 \cdot$$

$$(x^n)' = nx^{n-1}$$

$$\big((\text{nešto})^n\big)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$

b) $q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$

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$$= \frac{p^2}{2p^2 + 180}$$

$$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$

c)

$$E_{q,p} = \frac{p^2}{2p^2 + 180}$$

c)

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$$E_{q,p}(30) = \frac{30^2}{2 \cdot 30^2 + 180}$$

$$E_{q,p}(30) = \frac{5}{11} \approx 0.45$$

Ako na razini cijene od 30 kn cijenu povećamo za 1%, ponuda će porasti za 0.45%.

d)

$$E_{q,p} = \frac{1}{22}$$

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$$E_{q,p} = \frac{1}{22}$$

$$\frac{p^2}{2p^2 + 180} = \frac{1}{22}$$

$$22p^2 = 2p^2 + 180$$

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$$p = 3$$

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$$\frac{p^2}{2p^2 + 180} = \frac{1}{22}$$

$$22p^2 = 2p^2 + 180$$

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$$p^2 = 9$$

$$p = 3$$

Na razini cijene $p = 3$ elastičnost ponude jednaka je $\frac{1}{22}$.

četvrti zadatak

Zadatak 4

Zadana je funkcija ponude $q = (2p + 1)\log_4(5p)$.

a) *Izračunajte elastičnost ponude za cijenu* $p = 10$ *i interpretirajte rezultat.*

b) *Odredite koliko se proizvoda nudi po cijeni* $p = 18$. *Da li je za tu cijenu ponuda elastična ili neelastična?*

Rješenje

$$q = (2p + 1)\log_4(5p)$$

a)

$$q' =$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p + 1)\log_4(5p)$$

a)

$$q' = (2p + 1)'$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p + 1)\log_4(5p)$$

a)

$$q' = (2p + 1)' \cdot$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$\boxed{q = (2p+1)\log_4(5p)}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p)$$

$$\boxed{(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)}$$

Rješenje

$$\boxed{q = (2p+1)\log_4(5p)}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) +$$

$$\boxed{(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)}$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1)$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$\boxed{q = (2p+1)\log_4(5p)}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot$$

$$\boxed{(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)}$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)'$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p)$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot$$

$$\left(\log_a x\right)' = \frac{1}{x \ln a}$$

$$\left(\log_a(\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4}$$

$$\left(\log_a x\right)' = \frac{1}{x \ln a}$$

$$\left(\log_a(\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5$$

$$\left(\log_a x\right)' = \frac{1}{x \ln a}$$

$$\left(\log_a(\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$\left(\log_a x\right)' = \frac{1}{x \ln a}$$

$$\left(\log_a(\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Rješenje

$$\boxed{q = (2p+1)\log_4(5p)}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$\boxed{E_{q,p} = \frac{p}{q} \cdot q'}$$

Rješenje

$$\boxed{q = (2p+1)\log_4(5p)}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$\boxed{E_{q,p} = \frac{p}{q} \cdot q'}$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10)$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4}$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4}$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) = \frac{10}{59.26} \cdot 7.16$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) = \frac{10}{59.26} \cdot 7.16 \approx 1.21$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$$

Rješenje

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

a)

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot \left(\log_4(5p)\right)' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) = \frac{10}{59.26} \cdot 7.16 \approx 1.21$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4(5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$$

Ako na razini cijene $p = 10$ cijenu povećamo za 1%, ponuda će se povećati za 1.21%.

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) =$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) =$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) =$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18+1}{18\ln 4}$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) =$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18+1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4}$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) =$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18+1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) = \frac{18}{120.099}\cdot 7.97$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18+1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) = \frac{18}{120.099}\cdot 7.97 \approx 1.195$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18+1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) $q(18) = (2\cdot 18+1)\log_4(5\cdot 18) = 37\log_4 90 \approx 120.099$

Po cijeni $p = 18$ nudi se oko 120 proizvoda.

$$E_{q,p} = \frac{p}{q}\cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)}\cdot q'(18) = \frac{18}{120.099}\cdot 7.97 \approx 1.195$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18+1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

Kako je $|E_{q,p}(18)| = |1.195| = 1.195 > 1$, zaključujemo da je ponuda elastična za cijenu $p = 18$.

peti zadatak

Zadatak 5

Funkcija potražnje zadana je s $Q = 2005 + 25 \cdot 0.7^p$.

a) *Odredite elastičnost potražnje za* $p = 4$ *i interpretirajte rezultat.*

b) *Ako se cijena na razini* $p = 4$ *poveća za* 1%, *da li će prihod porasti ili će se smanjiti? Objasnite!*

c) *Odredite cijenu za koju je potražnja jednaka* 2010.

Rješenje

$$Q = 2005 + 25 \cdot 0.7^p$$

$$E_{Q,p} = \frac{p}{Q} \cdot Q'$$

a)

$$Q' =$$

Rješenje

$$Q = 2005 + 25 \cdot 0.7^p$$

$$E_{Q,p} = \frac{p}{Q} \cdot Q'$$

a)

$$Q' = 0 +$$

Rješenje

$$Q = 2005 + 25 \cdot 0.7^p$$

$$E_{Q,p} = \frac{p}{Q} \cdot Q'$$

a)

$$Q' = 0 + 25 \cdot$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

Rješenje

$$(a^x)' = a^x \ln a$$

$$Q = 2005 + 25 \cdot 0.7^p$$

$$E_{Q,p} = \frac{p}{Q} \cdot Q'$$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = -2.14094$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = -2.14094$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$E_{Q,p}(4) = \frac{4}{\qquad}$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = -2.14094$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$E_{Q,p}(4) = \frac{4}{2011.0025}$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = -2.14094$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$E_{Q,p}(4) = \frac{4}{2011.0025} \cdot (-2.14094)$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

a)

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = -2.14094$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$E_{Q,p}(4) = \frac{4}{2011.0025} \cdot (-2.14094)$$

$$E_{Q,p}(4) \approx -0.0043$$

Rješenje

$(a^x)' = a^x \ln a$

$Q = 2005 + 25 \cdot 0.7^p$

$E_{Q,p} = \frac{p}{Q} \cdot Q'$

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Ako na razini cijene $p = 4$ cijenu povećamo za 1%, potražnja će se smanjiti za 0.0043%.

b) Ako se cijena na razini $p = 4$ poveća za 1%, prihod će se povećati jer je potražnja na toj razini cijene neelastična, tj. $|E_{q,p}(4)| < 1$.

$Q = 2005 + 25 \cdot 0.7^p$

c)

prvi način

$2005 + 25 \cdot 0.7^p = 2010$

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prvi način

$$2005 + 25 \cdot 0.7^p = 2010$$

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c)

prvi način

$$2005 + 25 \cdot 0.7^p = 2010$$

$$25 \cdot 0.7^p = 5 \,/ : 25$$

$$0.7^p = 0.2$$

$Q = 2005 + 25 \cdot 0.7^p$

c)

prvi način

$$2005 + 25 \cdot 0.7^p = 2010$$

$$25 \cdot 0.7^p = 5 \,/ : 25$$

$$0.7^p = 0.2$$

$$p = \log_{0.7} 0.2$$

$Q = 2005 + 25 \cdot 0.7^p$

c)

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$$2005 + 25 \cdot 0.7^p = 2010$$

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$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.7}$$

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$$p = 4.51$$

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drugi način

$$2005 + 25 \cdot 0.7^p = 2010$$

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$$2005 + 25 \cdot 0.7^p = 2010$$

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drugi način

$$2005 + 25 \cdot 0.7^p = 2010$$
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$$0.7^p = 0.2 \,/ \log$$
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prvi način

$$\begin{aligned}
2005 + 25 \cdot 0.7^p &= 2010 \\
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\end{aligned}$$

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2005 + 25 \cdot 0.7^p &= 2010 \\
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\log 0.7^p &= \log 0.2 \\
p \cdot \log 0.7 &= \log 0.2 \,/ : \log 0.7 \\
p &= \frac{\log 0.2}{\log 0.7}
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