Seminari 10

Matematika za ekonomiste 2

Damir Horvat

FOI, Varaždin

Sadržaj

prvi zadatak

Zadatak 1

Julio štedi za novi auto. Uplaćuje 79 € *početkom svakog mjeseca kroz* 27 *mjeseci počevši od 1.6.2011.*

a) *Kolikim iznosom raspolaže 1.2.2016.?*

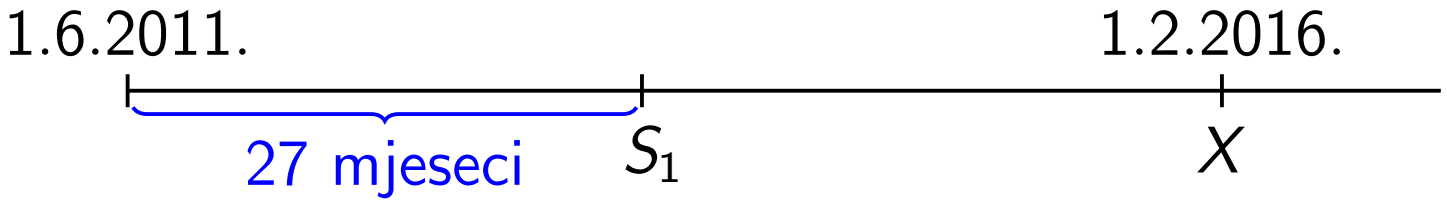
b) *Koliko novaca Julio mora ulagati kvartalno tijekom iduće tri godine kako bi tada, zajedno s uštedevinom, mogao kupiti auto vrijedan* 9 000 €*?*

Na raniju uštedevinu također se obračunavaju kamate. Godišnja kamatna stopa iznosi 6.3%*.*

Rješenje

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$

a)



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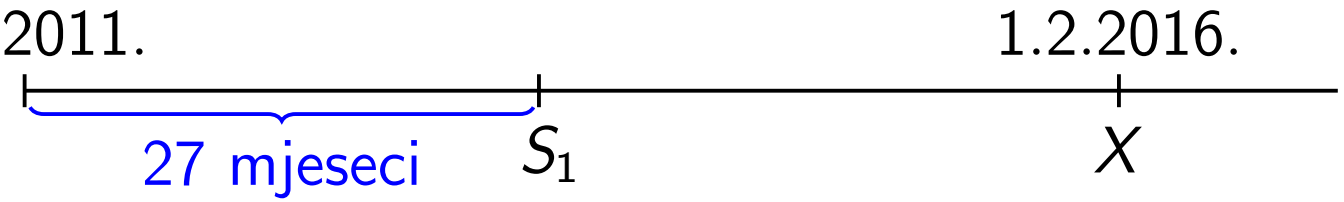

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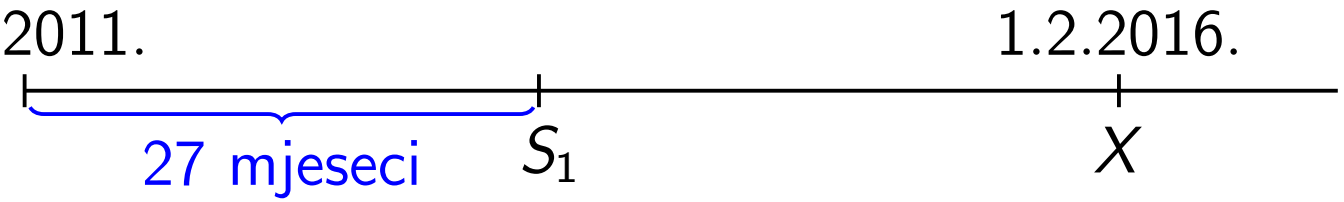

$$S_1 = R_1 \cdot r_1 \cdot \frac{r_1^{n_1} - 1}{r_1 - 1} = 79 \cdot \sqrt[12]{1.063} \cdot \frac{\sqrt[12]{1.063}^{27} - 1}{\sqrt[12]{1.063} - 1}$$

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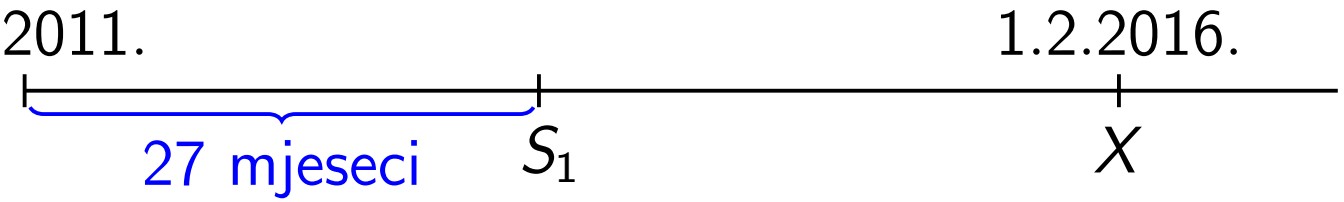

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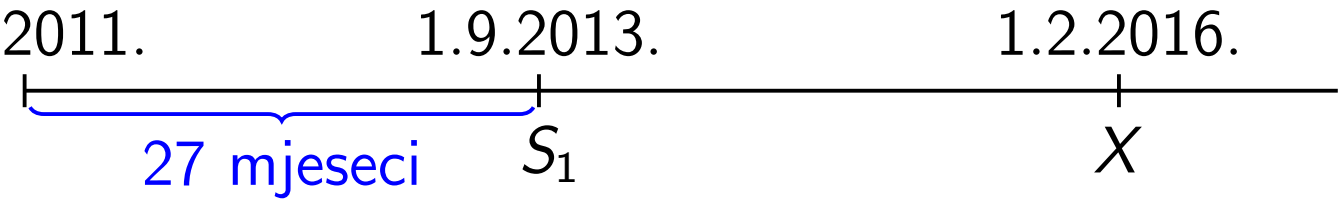

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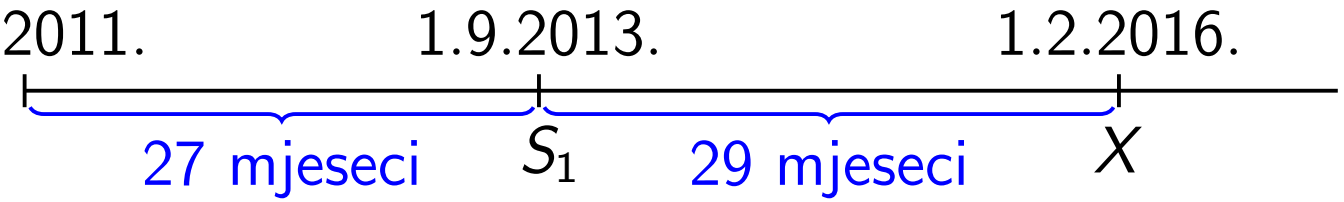

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$$X = S_1 \cdot r_1^{29}$$

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$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$

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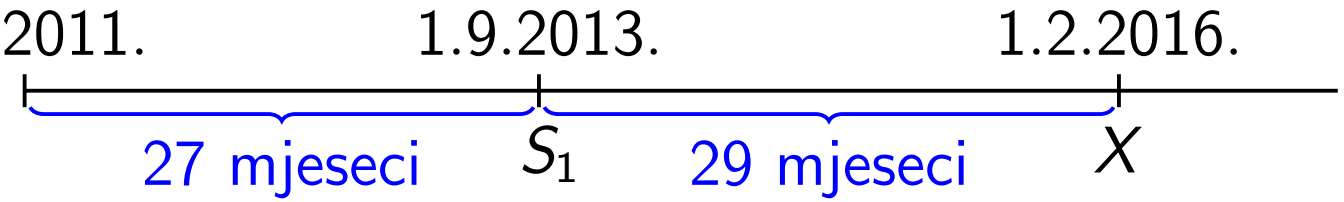

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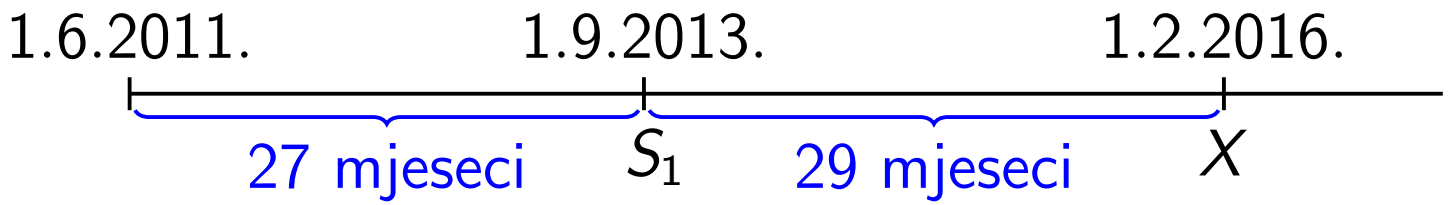

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$$X = S_1 \cdot r_1^{29} = 2292.39 \cdot \sqrt[12]{1.063}^{29} = 2657.11$$

Rješenje

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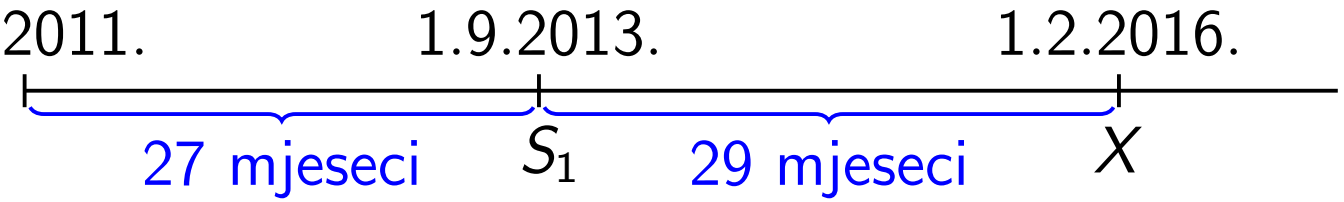

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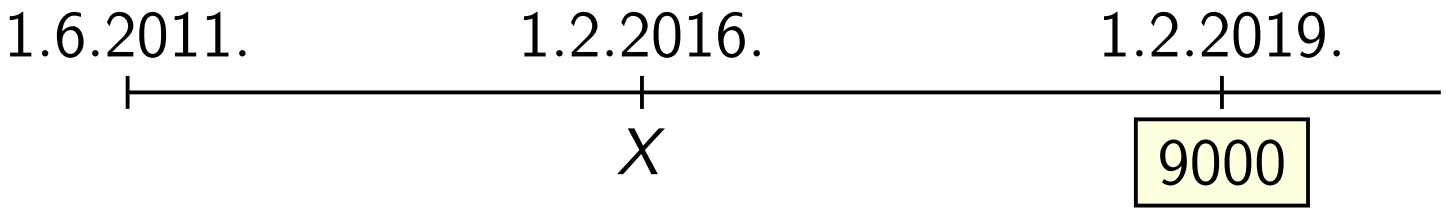
Julio 1.2.2016. raspolaže s 2657.11 €.

b)



$$S_2 = 9000 - X \cdot r_2^{12}$$

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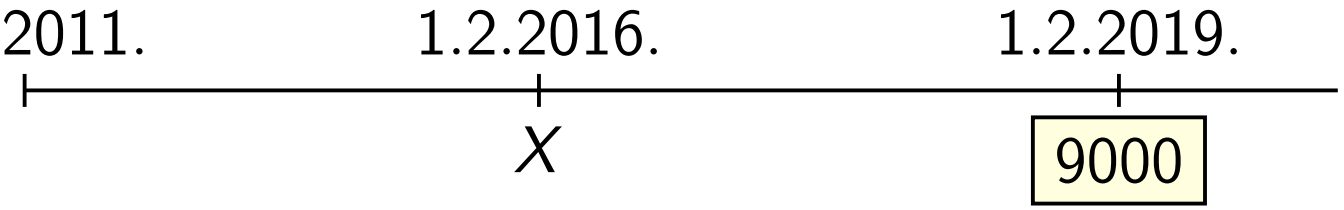

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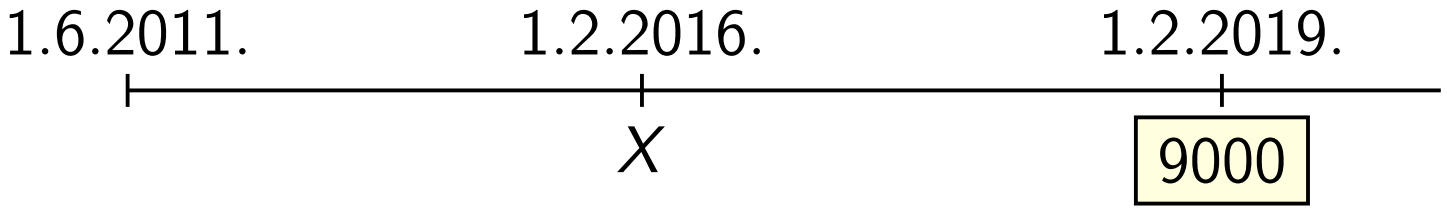
novac koji mu nedostaje

toliko bi imao 1.2.2019. da ništa ne uplaćuje

b)



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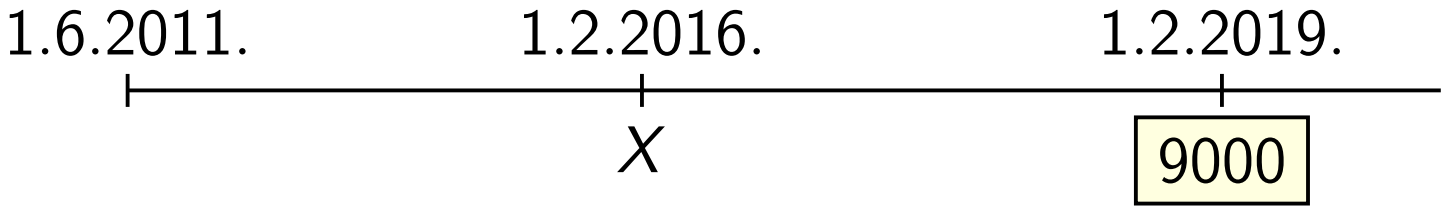

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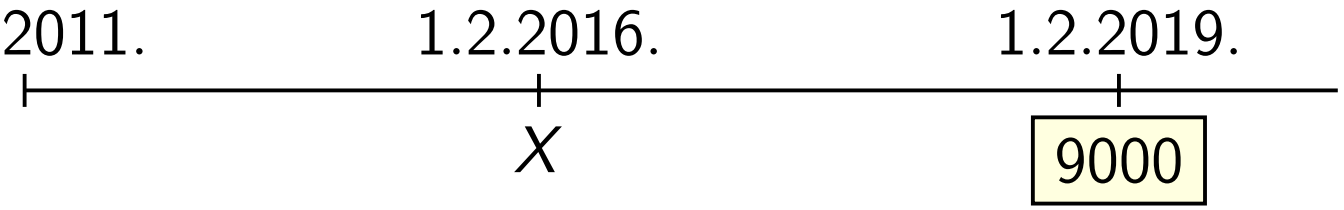

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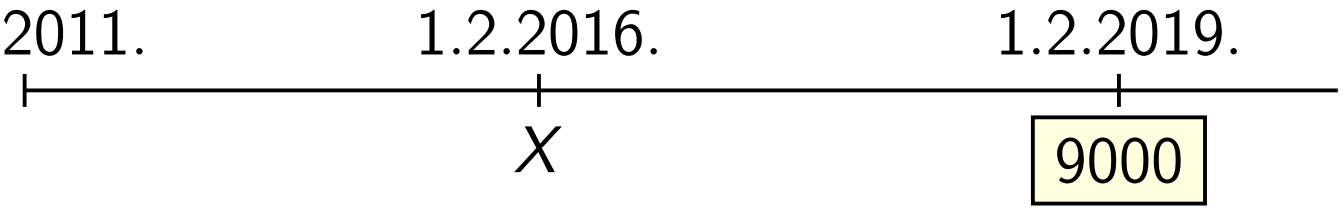

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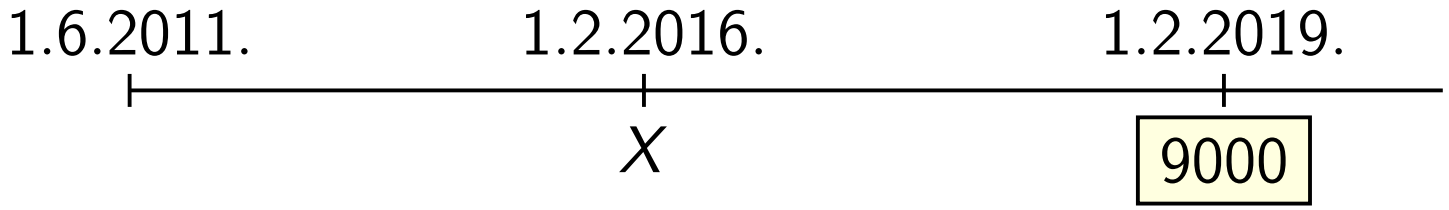

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$- 1$

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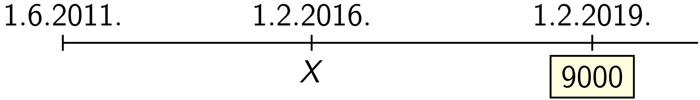

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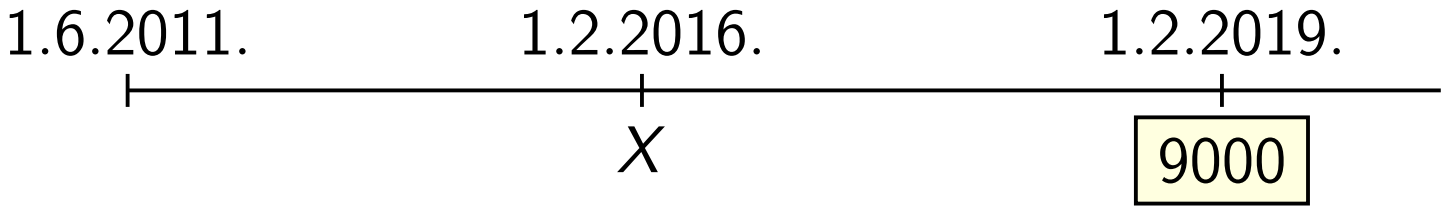

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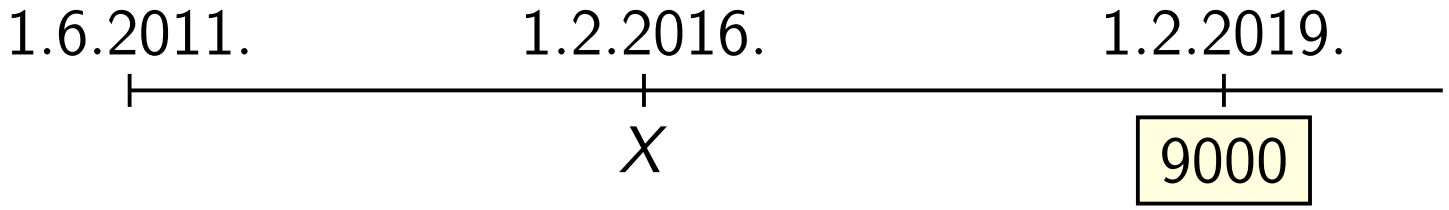

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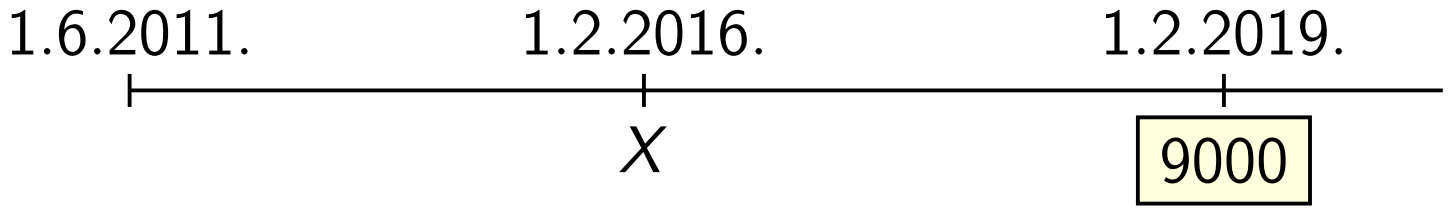

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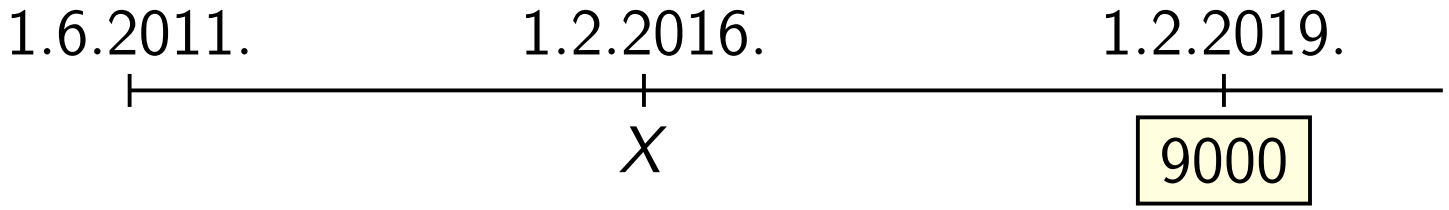
$$R_2$$

-1

-1

b)



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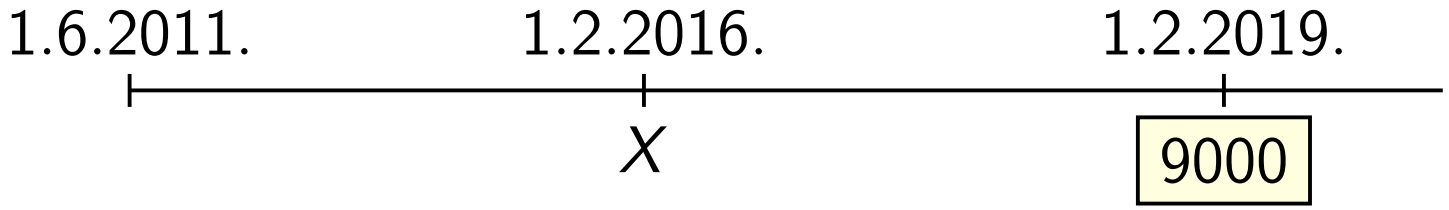

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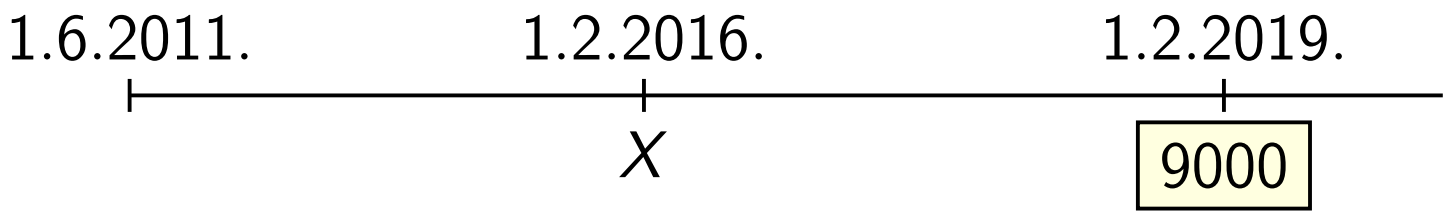

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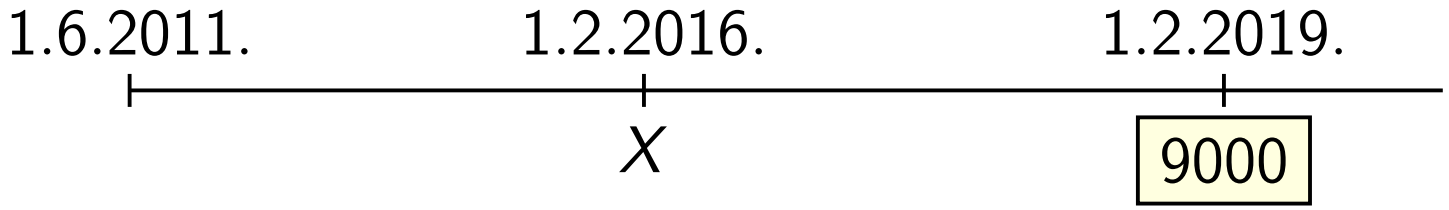
$$\frac{}{} \quad \begin{matrix} -1 \\ \hline -1 \end{matrix}$$

b)



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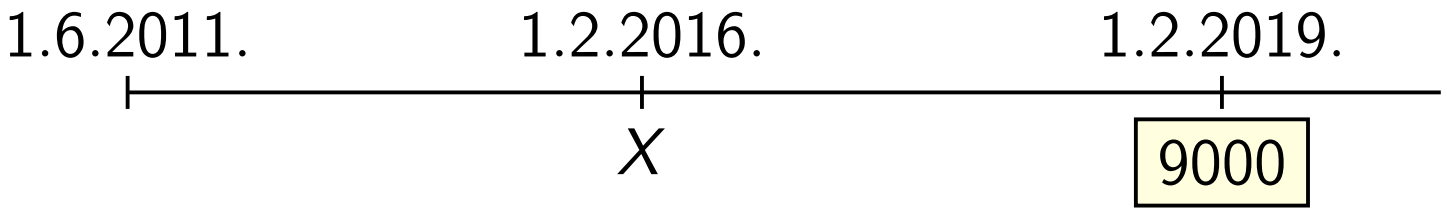

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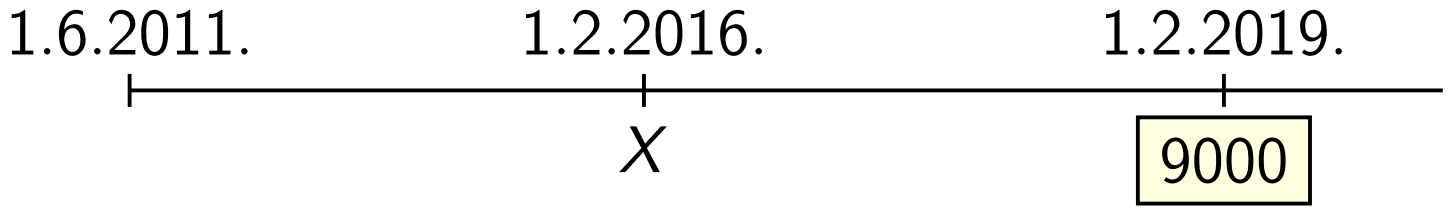

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$n_2 = 12\,\text{kvartala}$

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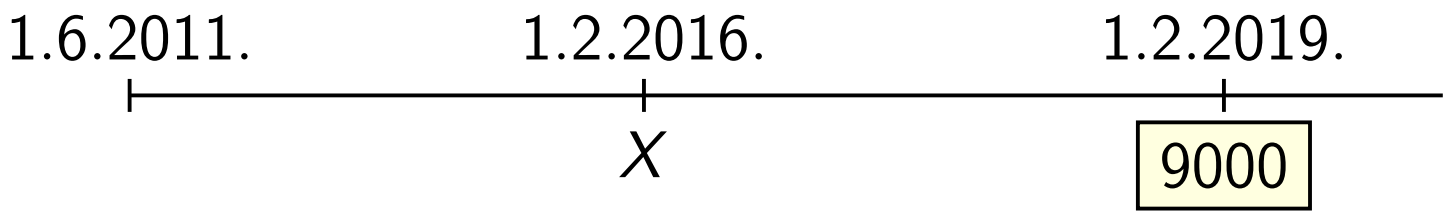

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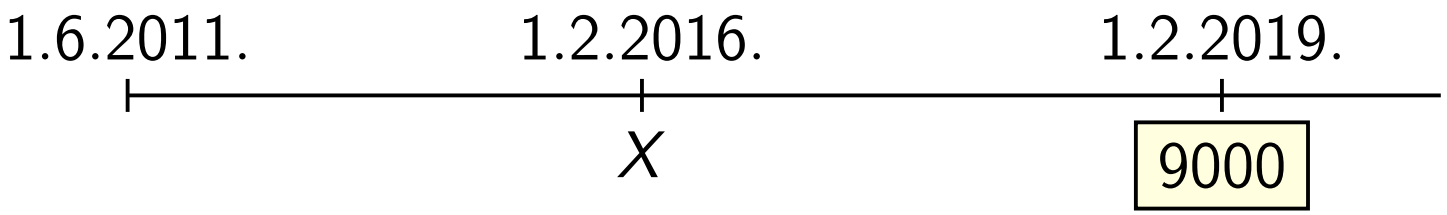
$$R_2 = \frac{5808.39 \cdot \left(\sqrt[4]{1.063} - 1\right)}{\sqrt[4]{1.063} \cdot \left(\sqrt[4]{1.063}^{12} - 1\right)} = 437.68$$

b)



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$$R_2 = \frac{5808.39 \cdot \left(\sqrt[4]{1.063} - 1\right)}{\sqrt[4]{1.063} \cdot \left(\sqrt[4]{1.063}^{12} - 1\right)} = 437.68$$

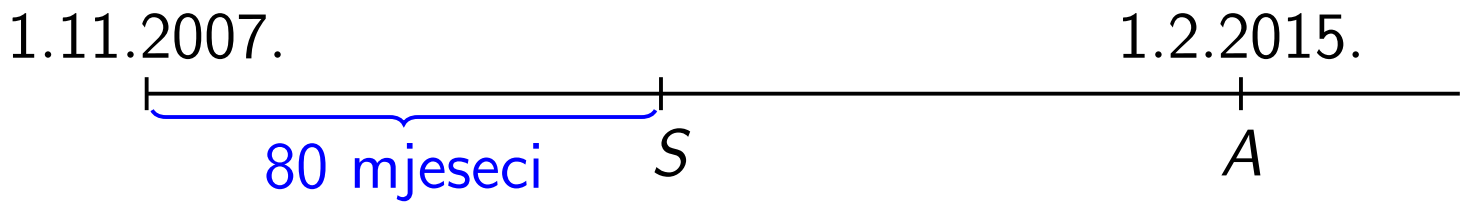
Tijekom iduće tri godine Julio kvartalno mora ulagati 437.68 €.

drugi zadatak

Zadatak 2

Roditelji su štedjeli za studij svoje djece. Od 1.11.2007. na račun su uplaćivali 420 € *početkom svakog mjeseca tijekom* 80 *mjeseci. Od 1.2.2015. svako od dvoje djece dobiva mjesečnu rentu tijekom iduće četiri godine. O kojem se iznosu radi ako je godišnja kamatna stopa* 4.5%*?*

Rješenje



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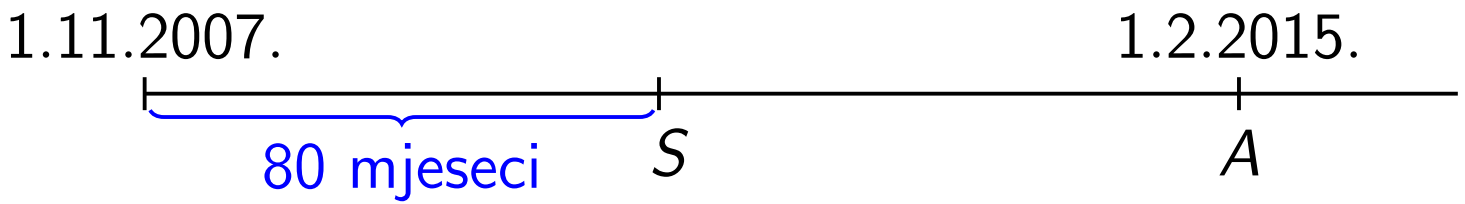

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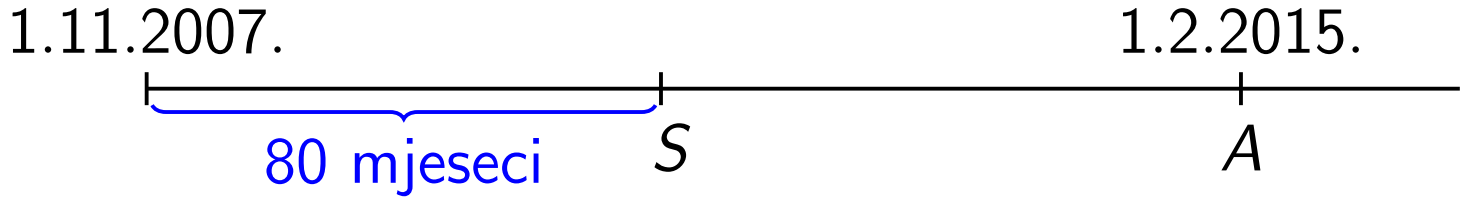
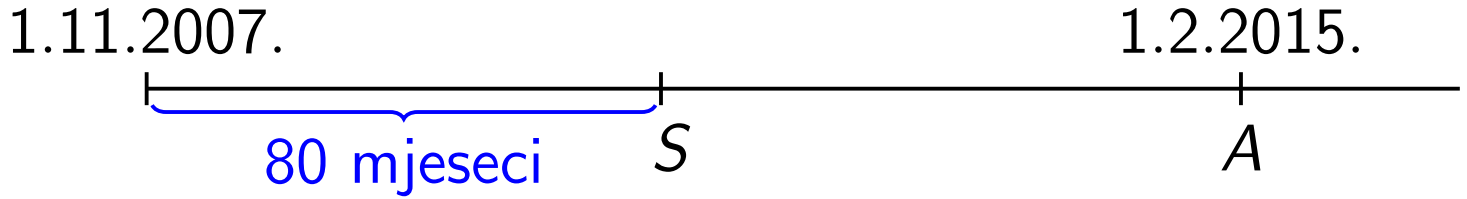
Rješenje



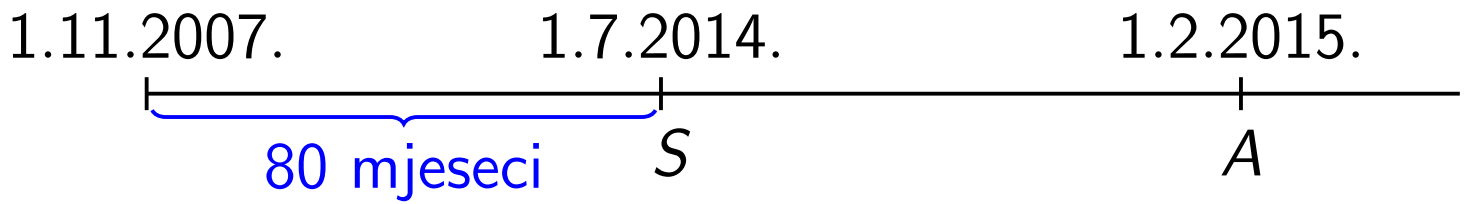
$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1}$$

Rješenje



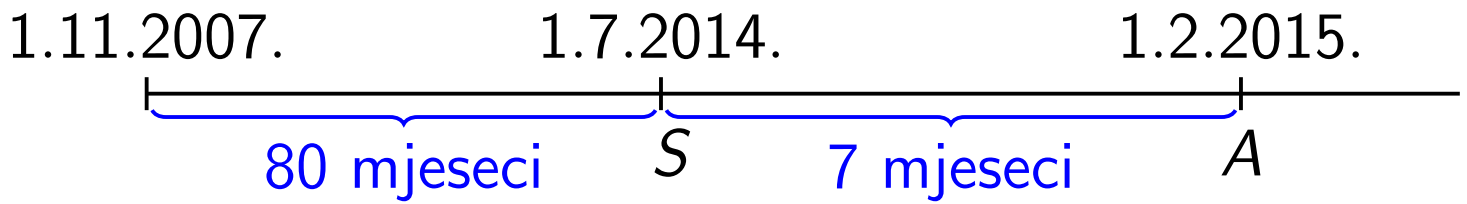
$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

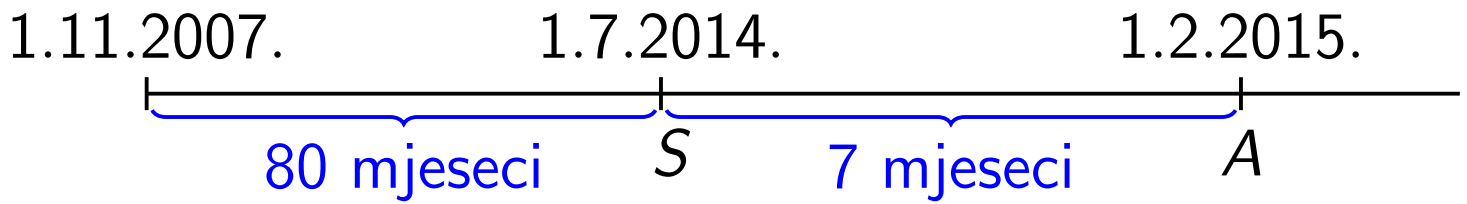
Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

$$A = S \cdot r^7$$

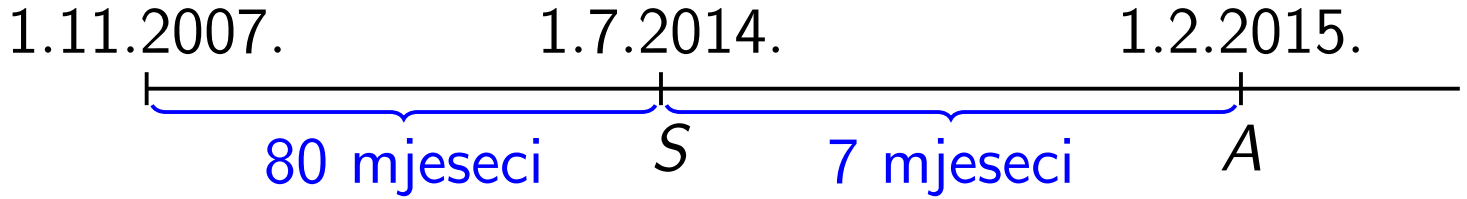
Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

$$A = S \cdot r^7 = 39\,121.33 \cdot \sqrt[12]{1.045}^{7}$$

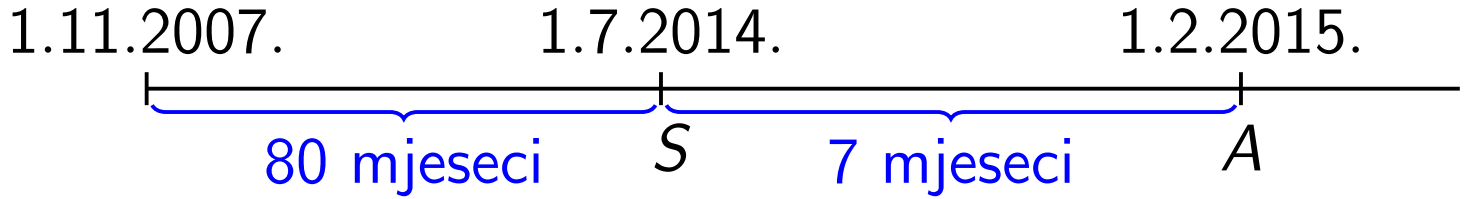
Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

$$A = S \cdot r^7 = 39\,121.33 \cdot \sqrt[12]{1.045}^{7} = 40\,138.84$$

Rješenje

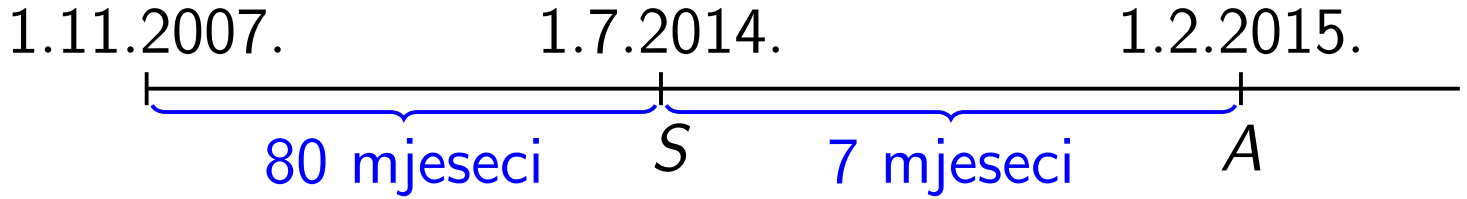


$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

$$A = S \cdot r^7 = 39\,121.33 \cdot \sqrt[12]{1.045}^{7} = 40\,138.84$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

Rješenje

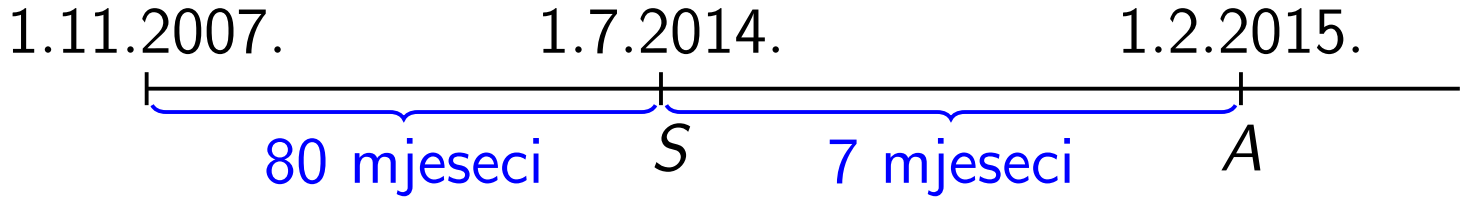


$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

$$A = S \cdot r^7 = 39\,121.33 \cdot \sqrt[12]{1.045}^{7} = 40\,138.84$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)} \qquad R = \frac{A \cdot r^{n-1} \cdot (r - 1)}{r^n - 1}$$

Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

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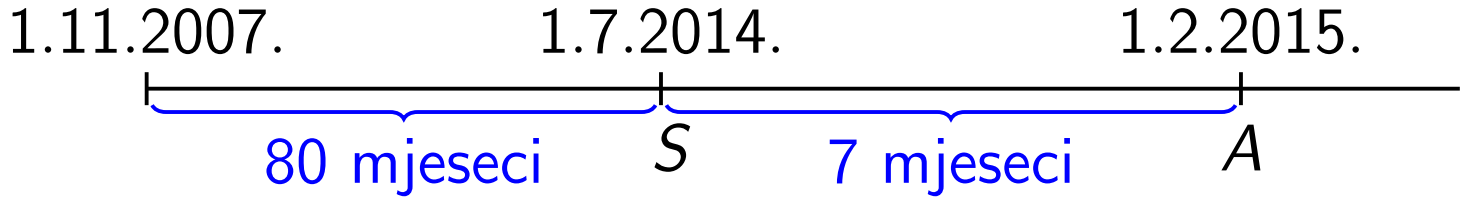
$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)} \qquad R = \frac{A \cdot r^{n-1} \cdot (r - 1)}{r^n - 1}$$

$$R = \frac{40\,138.84 \cdot \sqrt[12]{1.045}^{47} \cdot (\sqrt[12]{1.045} - 1)}{\sqrt[12]{1.045}^{48} - 1}$$

Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

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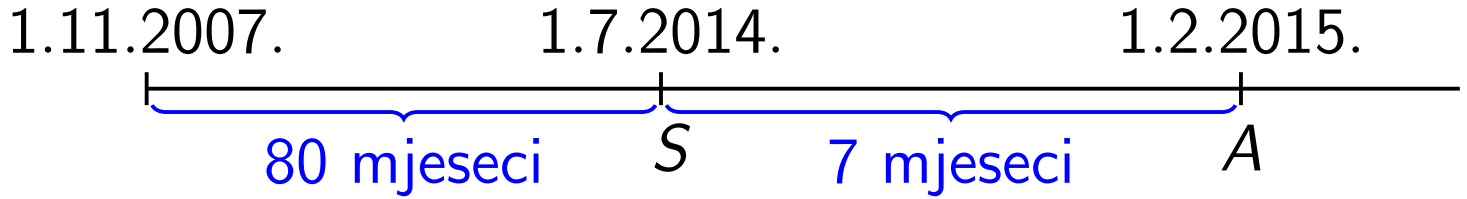
$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)} \qquad R = \frac{A \cdot r^{n-1} \cdot (r - 1)}{r^n - 1}$$

$$R = \frac{40\,138.84 \cdot \sqrt[12]{1.045}^{47} \cdot (\sqrt[12]{1.045} - 1)}{\sqrt[12]{1.045}^{48} - 1} = 910.33$$

Rješenje



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1} = 420 \cdot \sqrt[12]{1.045} \cdot \frac{\sqrt[12]{1.045}^{80} - 1}{\sqrt[12]{1.045} - 1} = 39\,121.33$$

$$A = S \cdot r^7 = 39\,121.33 \cdot \sqrt[12]{1.045}^{7} = 40\,138.84$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)} \qquad R = \frac{A \cdot r^{n-1} \cdot (r - 1)}{r^n - 1}$$

$$R = \frac{40\,138.84 \cdot \sqrt[12]{1.045}^{47} \cdot (\sqrt[12]{1.045} - 1)}{\sqrt[12]{1.045}^{48} - 1} = 910.33$$

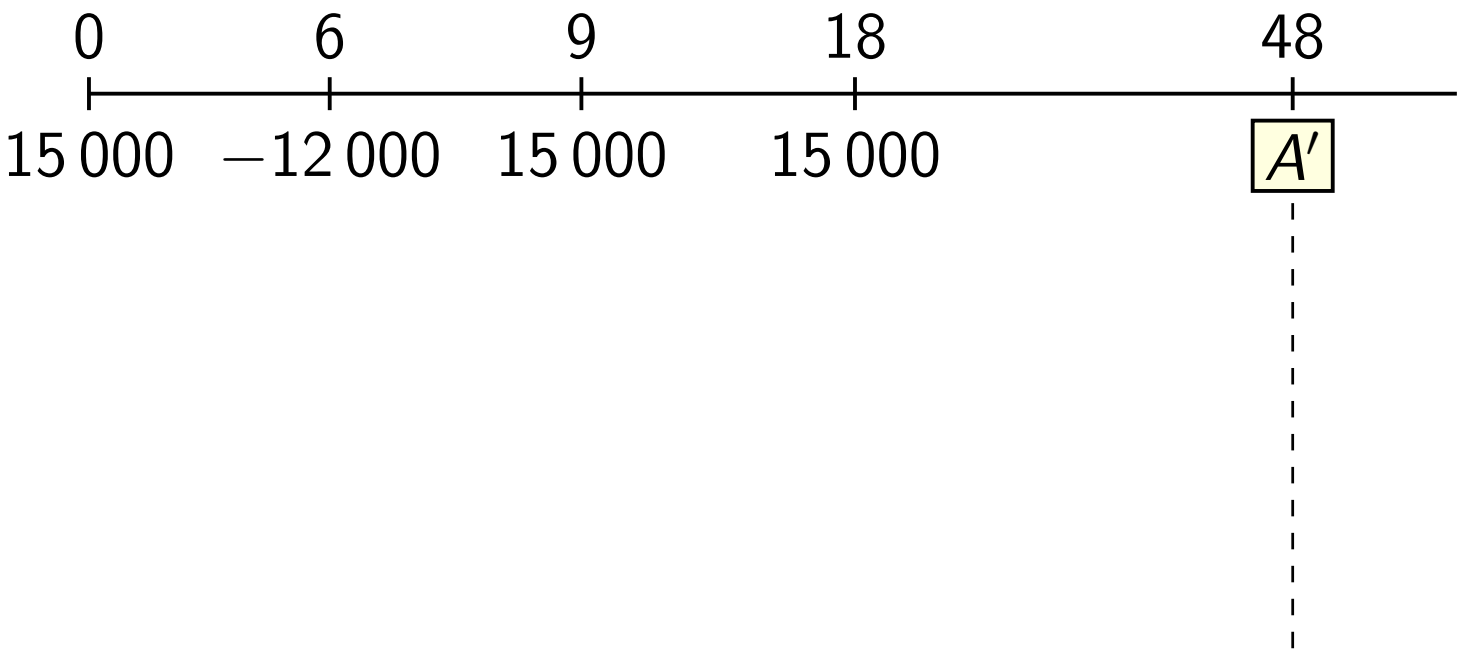
Svako dijete dobiva mjesečno 455.16 €.

treći zadatak

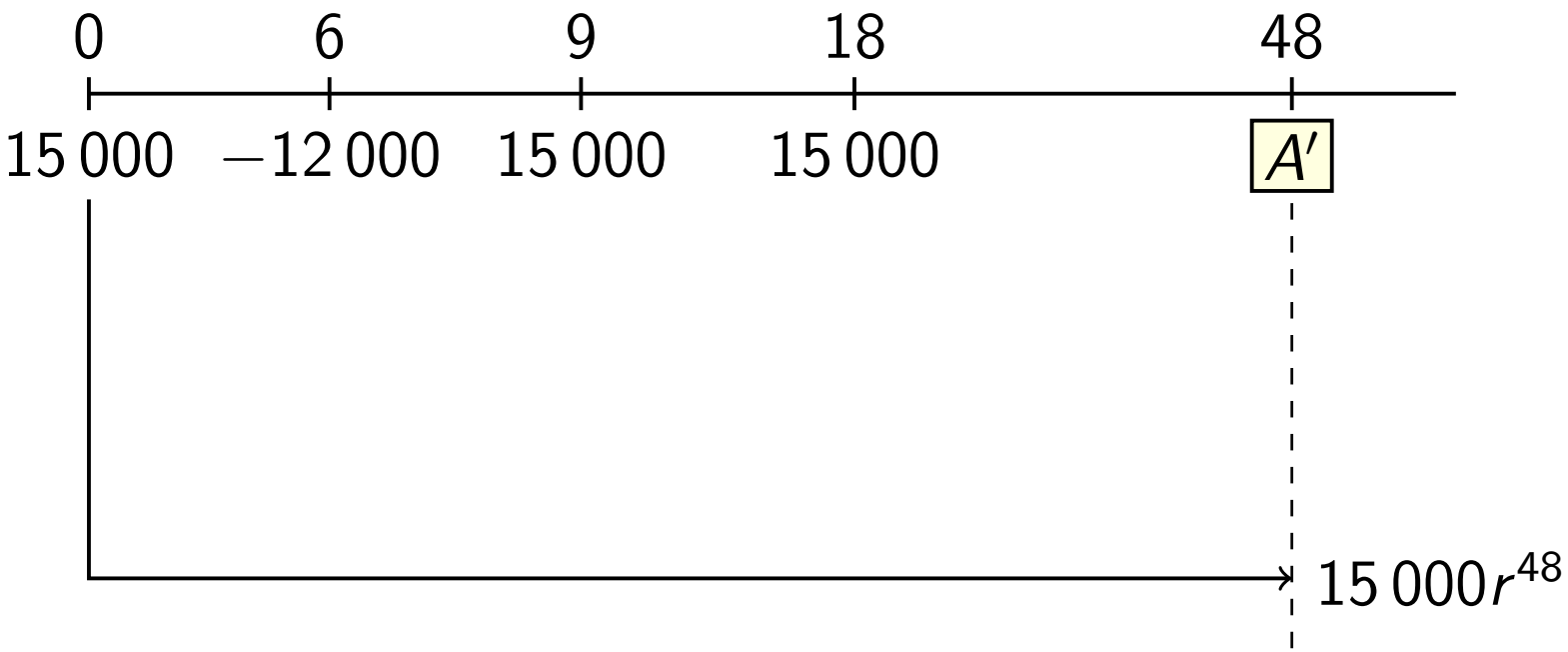
Zadatak 3

Netko uplati tri puta u razmacima od 9 *mjeseci* 15 000 *kn uz godišnju kamatnu stopu* 5.5%. *Šest mjeseci nakon prve uplate podigne* 12 000 *kn. Na osnovu svote s kojom raspolaže četiri godine nakon prve uplate želi primati postnumerando mjesečnu rentu visine* 3500 *kn. Koliko će takvih renti primiti i kolika je krnja isplata?*

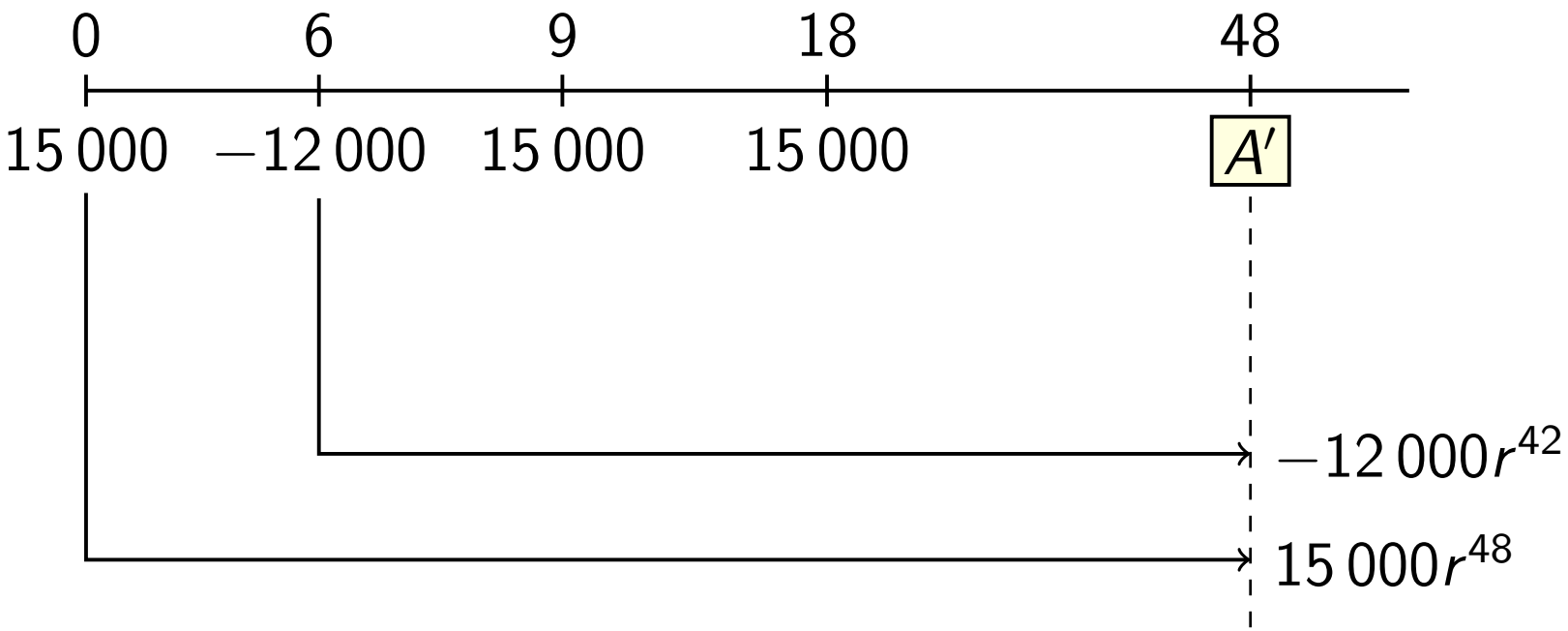
Rješenje



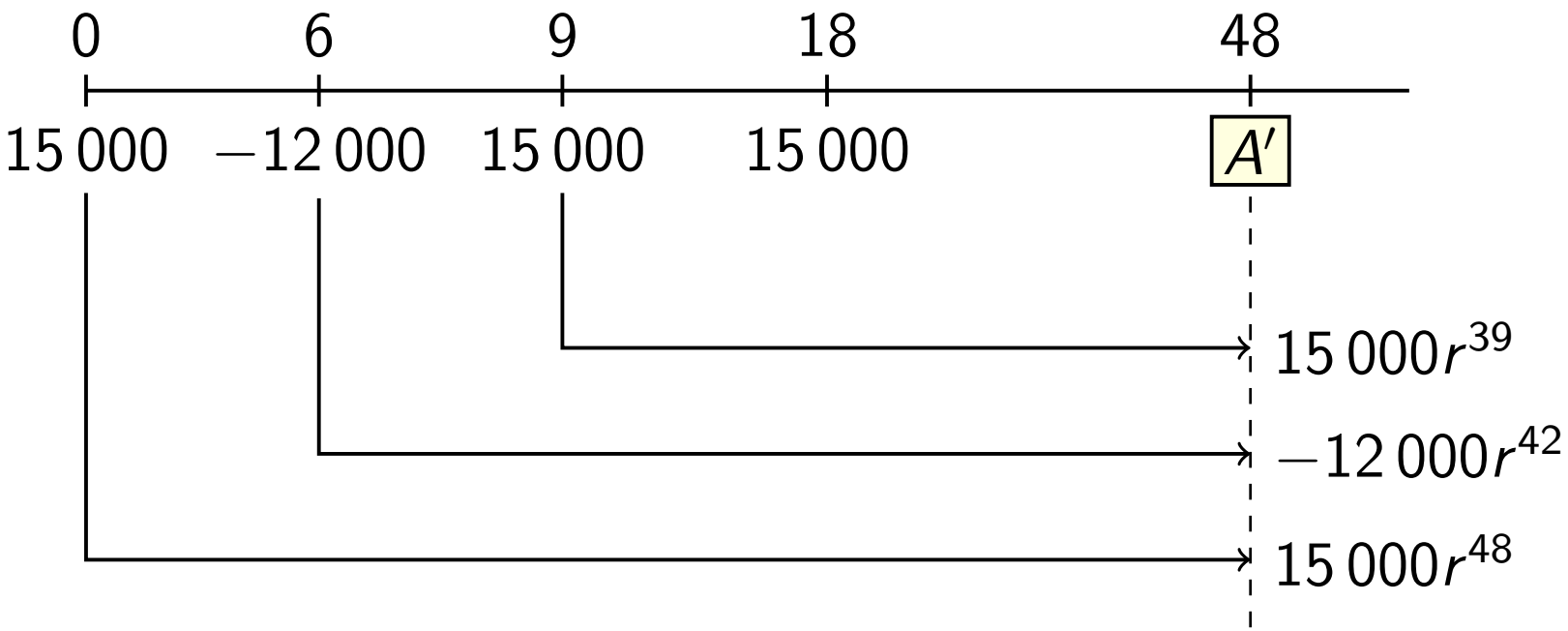
Rješenje



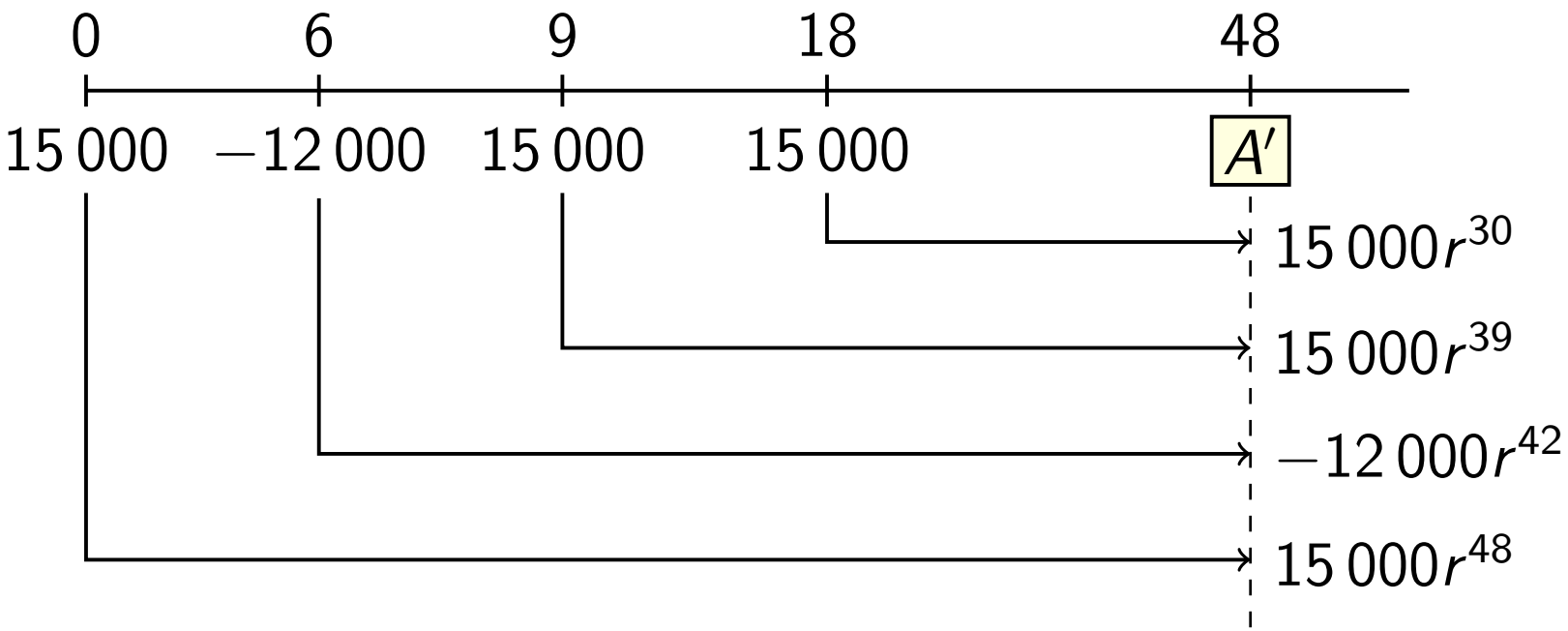
Rješenje



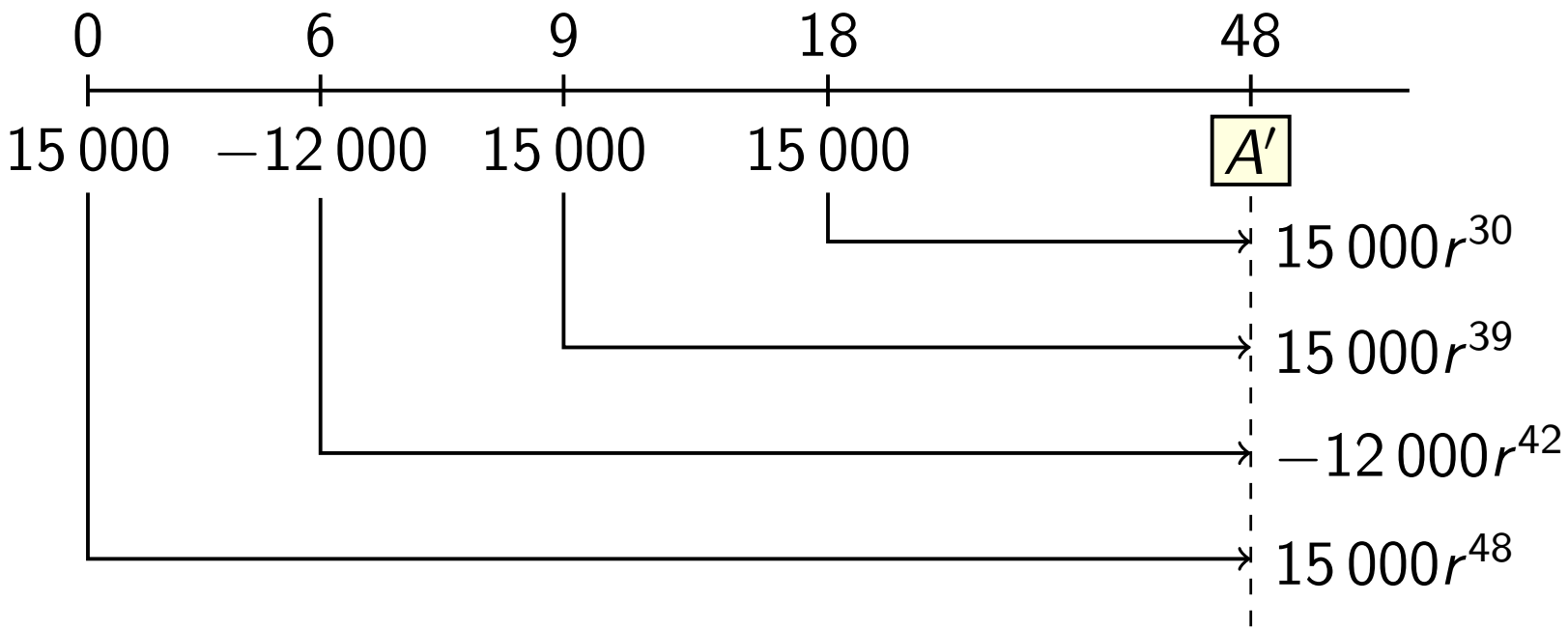
Rješenje


0
6
9
18
48
15 000
−12 000
15 000
15 000
A′
15 000r^39
−12 000r^42
15 000r^48

Rješenje

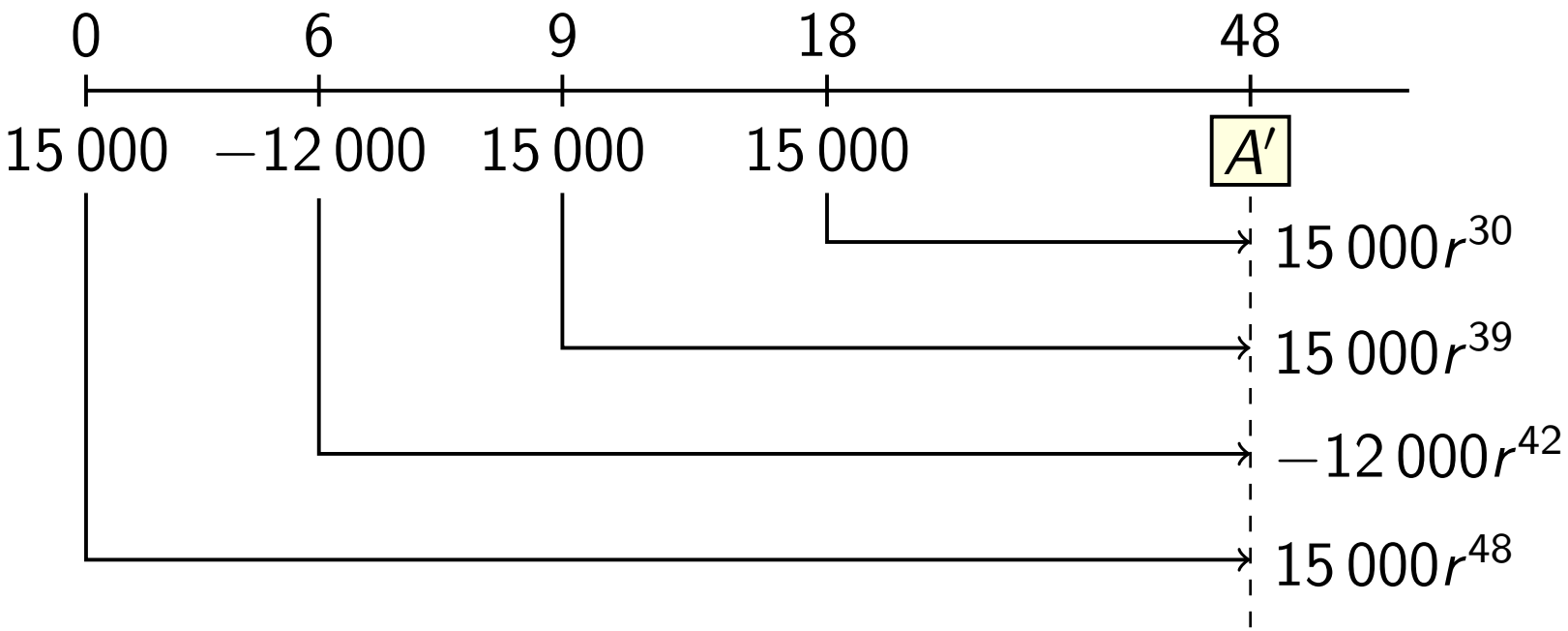

0
6
9
18
48
15 000
−12 000
15 000
15 000
A′
15 000r^30
15 000r^39
−12 000r^42
15 000r^48

Rješenje



$$A' = 15\,000r^{48} - 12\,000r^{42} + 15\,000r^{39} + 15\,000r^{30}$$

Rješenje

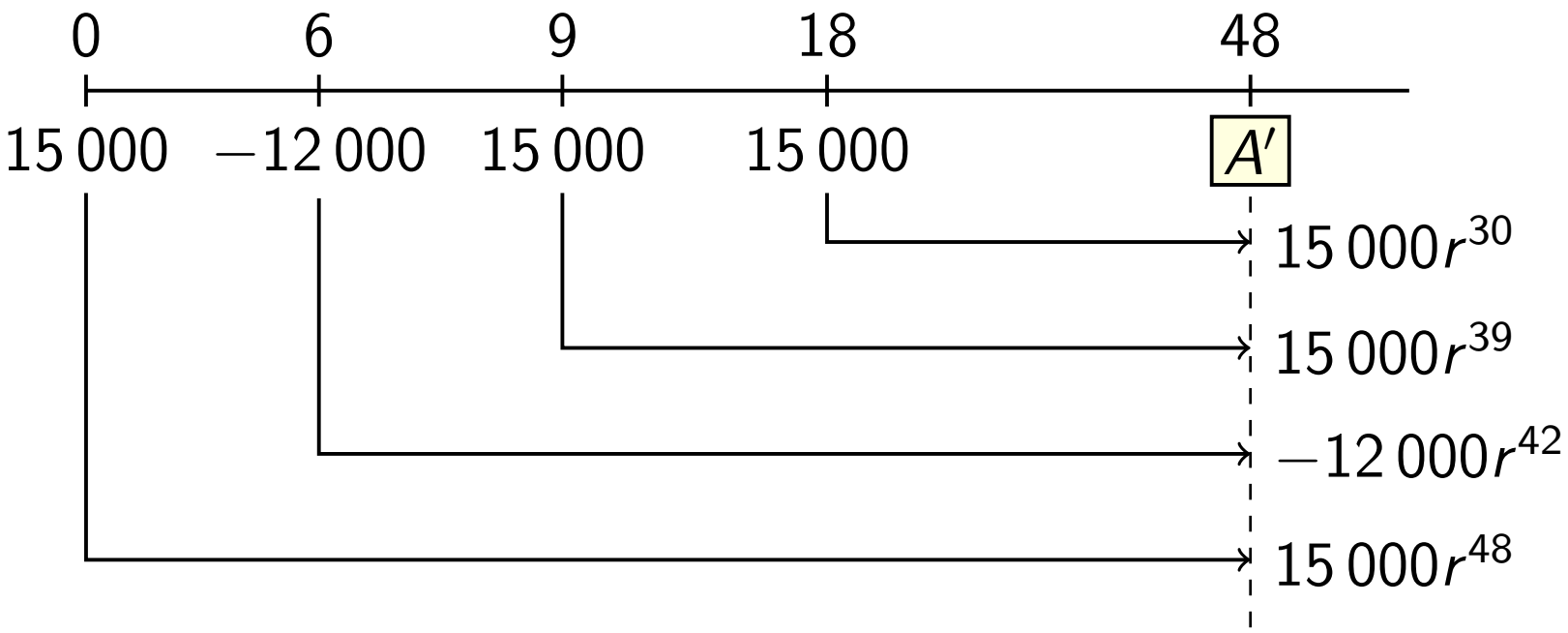


$$A' = 15\,000r^{48} - 12\,000r^{42} + 15\,000r^{39} + 15\,000r^{30}$$

$$A' = 15\,000 \cdot \sqrt[12]{1.055}^{48} - 12\,000 \cdot \sqrt[12]{1.055}^{42} +$$

$$+ 15\,000 \cdot \sqrt[12]{1.055}^{39} + 15\,000 \cdot \sqrt[12]{1.055}^{30}$$

Rješenje



$$A' = 15\,000r^{48} - 12\,000r^{42} + 15\,000r^{39} + 15\,000r^{30}$$

$$A' = 15\,000 \cdot \sqrt[12]{1.055}^{48} - 12\,000 \cdot \sqrt[12]{1.055}^{42} +$$

$$+ 15\,000 \cdot \sqrt[12]{1.055}^{39} + 15\,000 \cdot \sqrt[12]{1.055}^{30}$$

$$A' = 39\,108.48$$

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r - 1)} \qquad n = \frac{\log \frac{R}{R - A' \cdot (r - 1)}}{\log r}$$

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r - 1)} \qquad n = \frac{\log \frac{R}{R - A' \cdot (r - 1)}}{\log r}$$

$$n = \frac{\log \frac{3500}{3500 - 39\,108.48 \cdot \left(\sqrt[12]{1.055} - 1\right)}}{\log \sqrt[12]{1.055}}$$

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \qquad n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \frac{3500}{3500 - 39\,108.48 \cdot \left(\sqrt[12]{1.055} - 1\right)}}{\log \sqrt[12]{1.055}} = 11.48$$

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \qquad n = \frac{\log \dfrac{R}{R - A' \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{3500}{3500 - 39\,108.48 \cdot \left(\sqrt[12]{1.055} - 1\right)}}{\log \sqrt[12]{1.055}} = 11.48$$

Dakle, $n = 11$, tj. može se primiti 11 isplata visine 3500 kn.

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \qquad n = \frac{\log \dfrac{R}{R - A' \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{3500}{3500 - 39\,108.48 \cdot \left(\sqrt[12]{1.055} - 1\right)}}{\log \sqrt[12]{1.055}} = 11.48$$

Dakle, $n = 11$, tj. može se primiti 11 isplata visine 3500 kn.

Krnja isplata

$$R' = \left(A' - R \cdot \frac{r^n - 1}{r^n \cdot (r-1)}\right) \cdot r^{n+1}$$

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \qquad n = \frac{\log \dfrac{R}{R - A' \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{3500}{3500 - 39\,108.48 \cdot \left(\sqrt[12]{1.055} - 1\right)}}{\log \sqrt[12]{1.055}} = 11.48$$

Dakle, $n = 11$, tj. može se primiti 11 isplata visine 3500 kn.

Krnja isplata

$$R' = \left(A' - R \cdot \frac{r^n - 1}{r^n \cdot (r-1)}\right) \cdot r^{n+1}$$

$$R' = \left(39\,108.48 - 3500 \cdot \frac{\sqrt[12]{1.055}^{11} - 1}{\sqrt[12]{1.055}^{11} \cdot \left(\sqrt[12]{1.055} - 1\right)}\right) \cdot \sqrt[12]{1.055}^{11+1}$$

$$A' = R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \qquad n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \frac{3500}{3500 - 39\,108.48 \cdot \left(\sqrt[12]{1.055} - 1\right)}}{\log \sqrt[12]{1.055}} = 11.48$$

Dakle, $n = 11$, tj. može se primiti 11 isplata visine 3500 kn.

Krnja isplata

$$R' = \left(A' - R \cdot \frac{r^n - 1}{r^n \cdot (r-1)}\right) \cdot r^{n+1} \qquad R' = 1710.93$$

$$R' = \left(39\,108.48 - 3500 \cdot \frac{\sqrt[12]{1.055}^{11} - 1}{\sqrt[12]{1.055}^{11} \cdot \left(\sqrt[12]{1.055} - 1\right)}\right) \cdot \sqrt[12]{1.055}^{11+1}$$

četvrti zadatak

Zadatak 4

Ivan ima brižne roditelje i strica Stjepana. Kako bi Ivanu osigurali bezbrižno studiranje, njegovi roditelji zajedno sa stricom Stjepanom su otvorili račun u banci. Krajem svakog mjeseca tijekom četiri godine, počevši od ožujka 2007., roditelji izdvajaju 500 *kn, a stric Stjepan* 700 *kn i ukupni iznos uplaćuju na Ivanov račun. Zbog troškova upisa na fakultet, Ivan je sa svojeg računa 1. rujna 2011. godine podigao* 2000 *kn.*

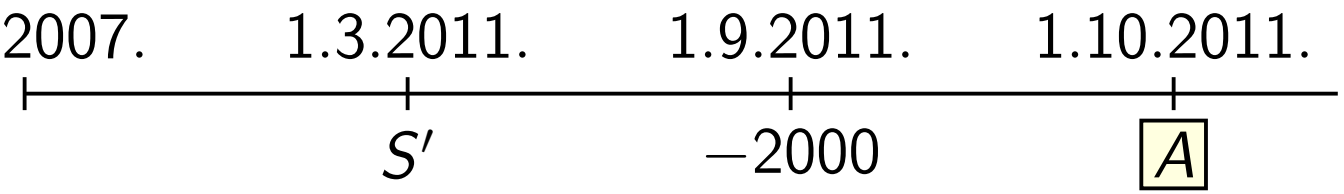
a) *Od 1. listopada 2011. Ivan želi početkom svakog mjeseca primati rentu u iznosu od* 1200 *kn. Koliko takvih isplata Ivan može primiti temeljem iznosa s kojim raspolaže na svojem računu?*

b) *Kolika je krnja isplata?*

Godišnja kamatna stopa je 6.5%*.*

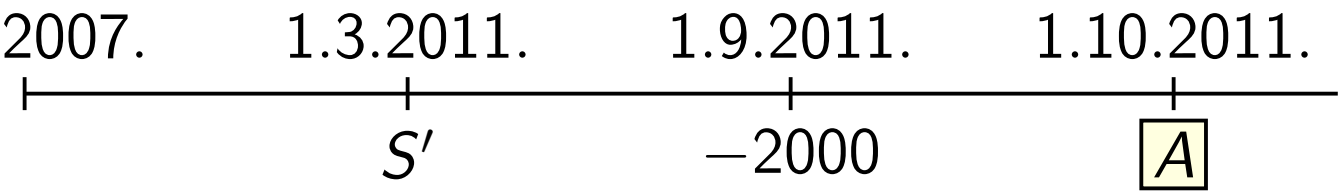
Rješenje

a)



Rješenje

a)

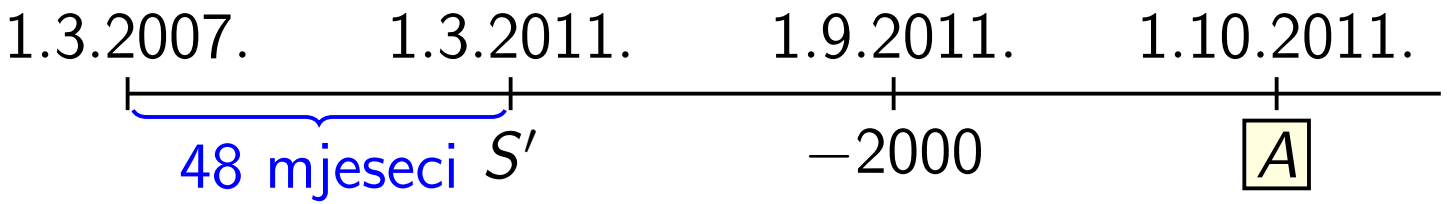


$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

Rješenje

a)



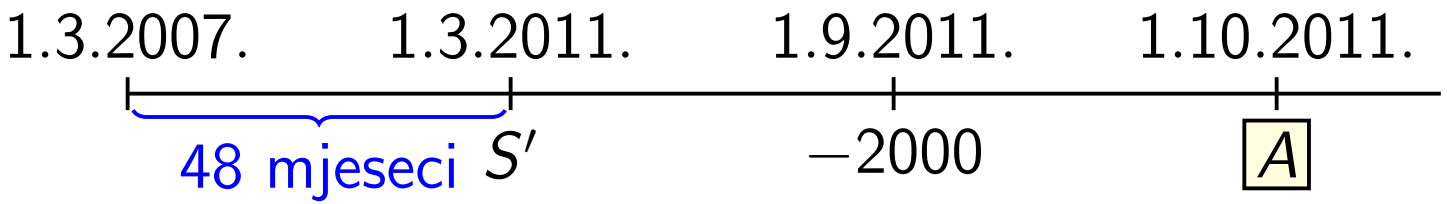
$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1}$$

Rješenje

a)



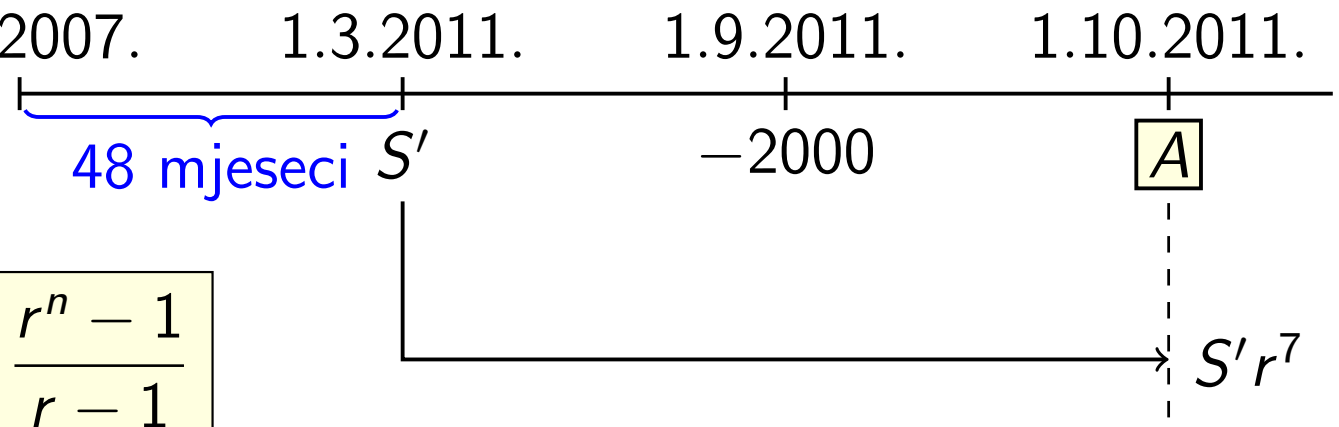
$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1} = 65\,332.50$$

Rješenje

a)



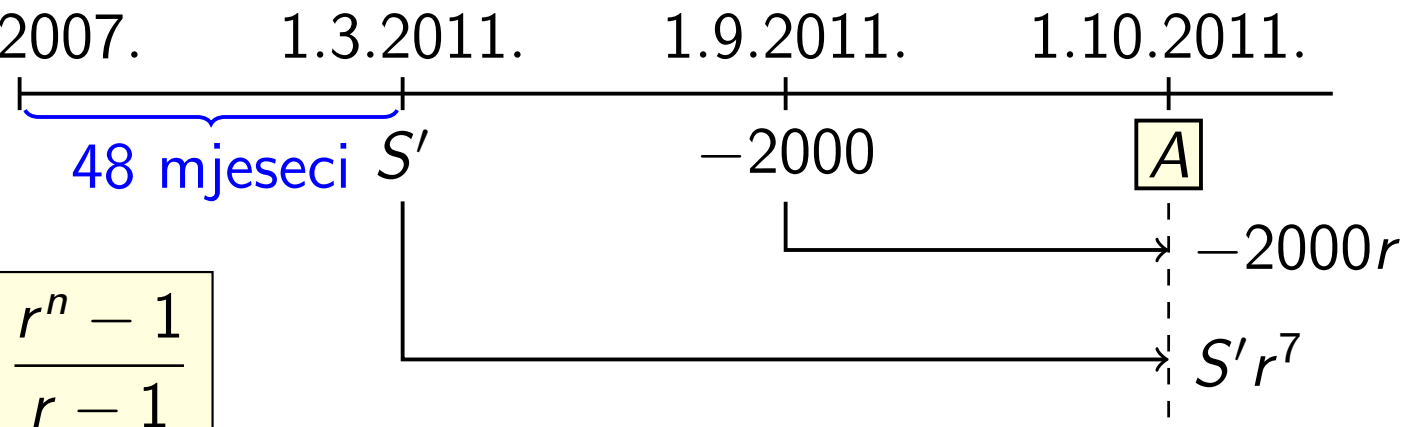
$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1} = 65\,332.50$$

Rješenje

a)



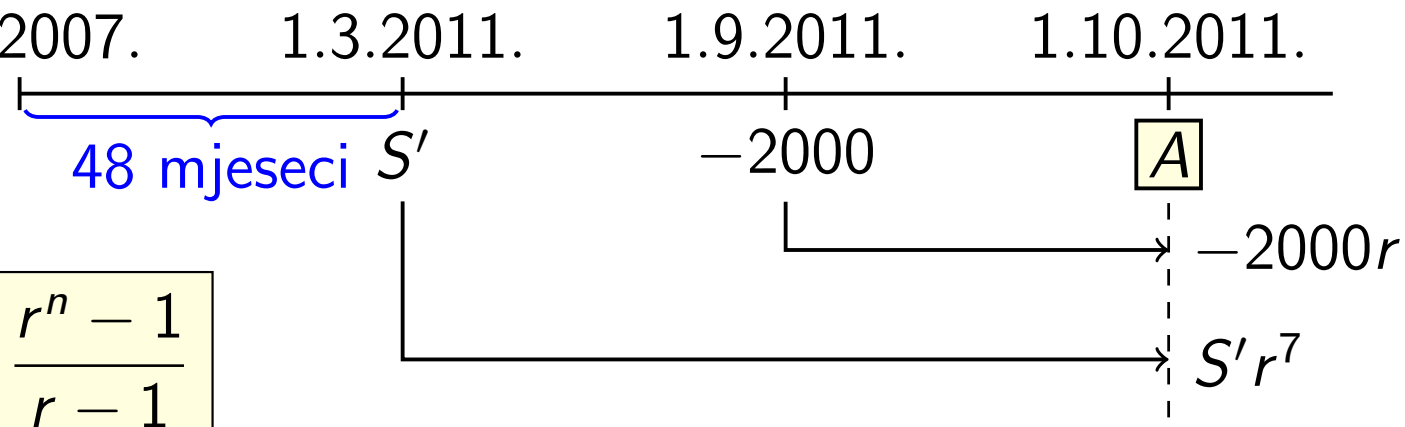
$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1} = 65\,332.50$$

Rješenje

a)



$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

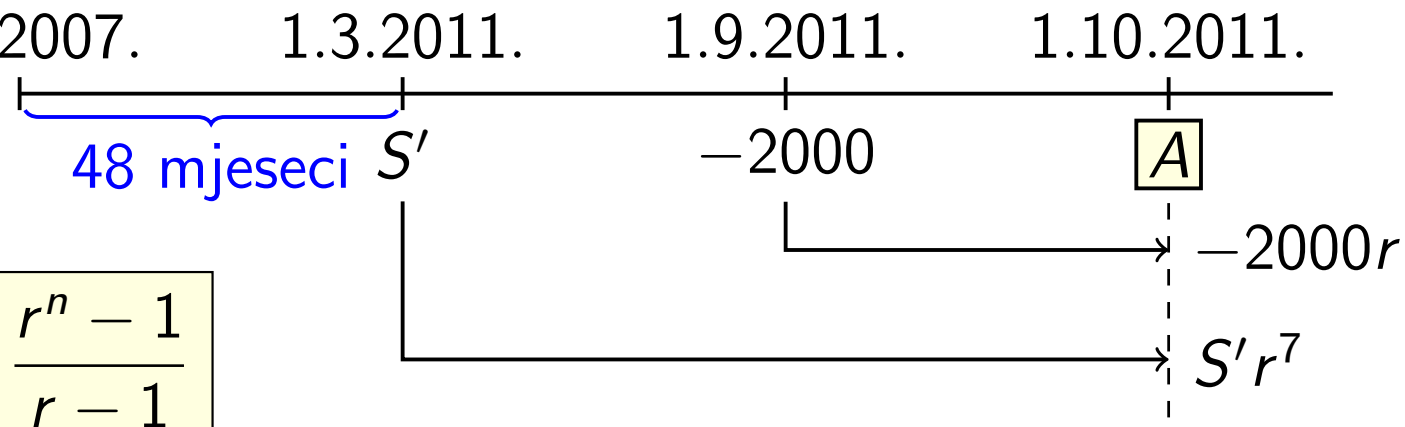
$$R = 500 + 700 = 1200$$

$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1} = 65\,332.50$$

$$A = S'r^7 - 2000r$$

Rješenje

a)



$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

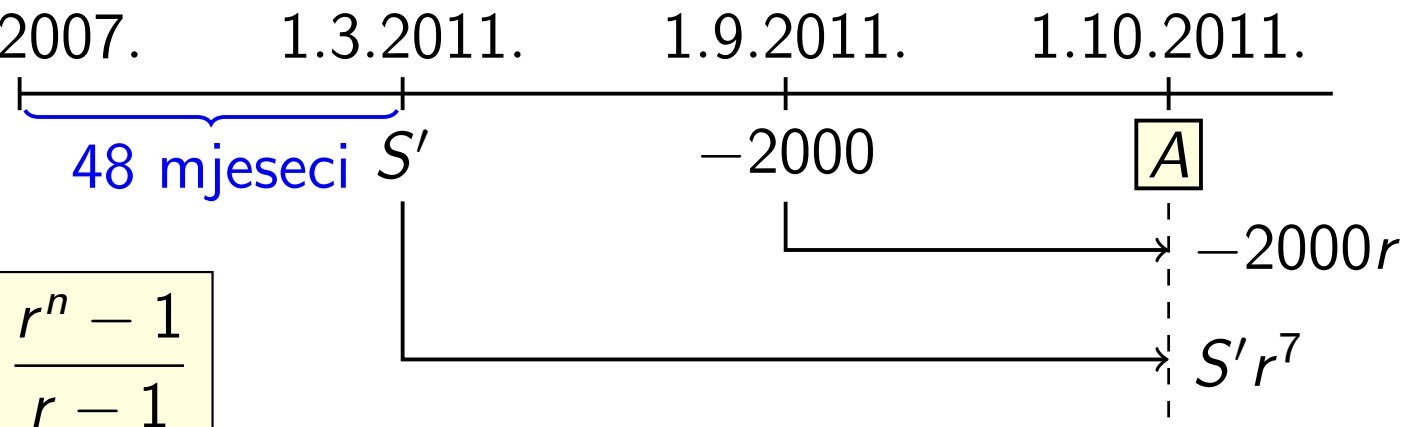
$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1} = 65\,332.50$$

$$A = S'r^7 - 2000r$$

$$A = 65\,332.50 \cdot \sqrt[12]{1.065}^{7} - 2000 \cdot \sqrt[12]{1.065}$$

Rješenje

a)



$$S' = R \cdot \frac{r^n - 1}{r - 1}$$

$$R = 500 + 700 = 1200$$

$$S' = 1200 \cdot \frac{\sqrt[12]{1.065}^{48} - 1}{\sqrt[12]{1.065} - 1} = 65\,332.50$$

$$A = S'r^7 - 2000r$$

$$A = 65\,332.50 \cdot \sqrt[12]{1.065}^{7} - 2000 \cdot \sqrt[12]{1.065}$$

$$A = 65\,766.61$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \frac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \dfrac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{1200 \cdot \sqrt[12]{1.065}}{1200 \cdot \sqrt[12]{1.065} - 65\,766.61 \cdot \left(\sqrt[12]{1.065} - 1\right)}}{\log \sqrt[12]{1.065}}$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \dfrac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{1200 \cdot \sqrt[12]{1.065}}{1200 \cdot \sqrt[12]{1.065} - 65\,766.61 \cdot \left(\sqrt[12]{1.065} - 1\right)}}{\log \sqrt[12]{1.065}} = 64.42$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \dfrac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{1200 \cdot \sqrt[12]{1.065}}{1200 \cdot \sqrt[12]{1.065} - 65\,766.61 \cdot \left(\sqrt[12]{1.065} - 1\right)}}{\log \sqrt[12]{1.065}} = 64.42$$

Dakle, $n = 64$, tj. Ivan može primiti 64 isplate visine 1200 kn.

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \dfrac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{1200 \cdot \sqrt[12]{1.065}}{1200 \cdot \sqrt[12]{1.065} - 65\,766.61 \cdot \left(\sqrt[12]{1.065} - 1\right)}}{\log \sqrt[12]{1.065}} = 64.42$$

Dakle, $n = 64$, tj. Ivan može primiti 64 isplate visine 1200 kn.

b) **Krnja isplata**

$$R' = \left(A - R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)}\right) \cdot r^n$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \frac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \frac{1200 \cdot \sqrt[12]{1.065}}{1200 \cdot \sqrt[12]{1.065} - 65\,766.61 \cdot \left(\sqrt[12]{1.065} - 1\right)}}{\log \sqrt[12]{1.065}} = 64.42$$

Dakle, $n = 64$, tj. Ivan može primiti 64 isplate visine 1200 kn.

b) **Krnja isplata**

$$R' = \left(A - R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)}\right) \cdot r^n$$

$$R' = \left(65\,766.61 - 1200 \cdot \frac{\sqrt[12]{1.065}^{64} - 1}{\sqrt[12]{1.065}^{64-1} \cdot \left(\sqrt[12]{1.065} - 1\right)}\right) \cdot \sqrt[12]{1.065}^{64}$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \dfrac{R \cdot r}{R \cdot r - A \cdot (r-1)}}{\log r}$$

$$n = \frac{\log \dfrac{1200 \cdot \sqrt[12]{1.065}}{1200 \cdot \sqrt[12]{1.065} - 65\,766.61 \cdot \left(\sqrt[12]{1.065} - 1\right)}}{\log \sqrt[12]{1.065}} = 64.42$$

Dakle, $n = 64$, tj. Ivan može primiti 64 isplate visine 1200 kn.

b) **Krnja isplata**

$$R' = \left(A - R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)}\right) \cdot r^n \qquad R' = 506.70$$

$$R' = \left(65\,766.61 - 1200 \cdot \frac{\sqrt[12]{1.065}^{64} - 1}{\sqrt[12]{1.065}^{64-1} \cdot \left(\sqrt[12]{1.065} - 1\right)}\right) \cdot \sqrt[12]{1.065}^{64}$$

peti zadatak

Zadatak 5

Tomislav je uložio u banku 4000 €.

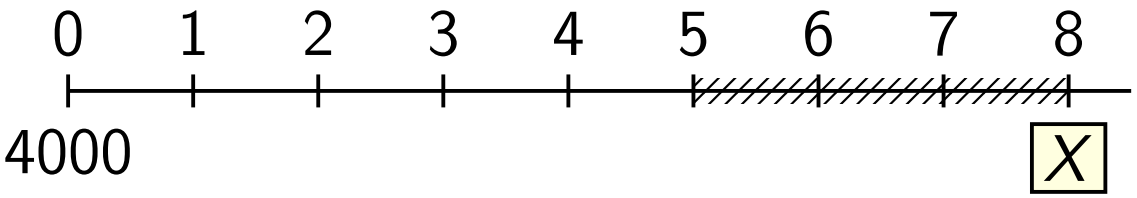
a) *Kolikim će iznosom raspolagati na kraju osme godine ako u posljednje tri godine bude podizao iz banke po* 500 € *početkom svakog kvartala?*

b) *Ukoliko od kraja desete godine od raspoloživog iznosa Tomislav želi primati prenumerando mjesečne isplate visine* 300 €, *koliko takvih isplata može primiti i kolika je krnja isplata?*

Polugodišnja kamatna stopa je 5.1%.

Rješenje

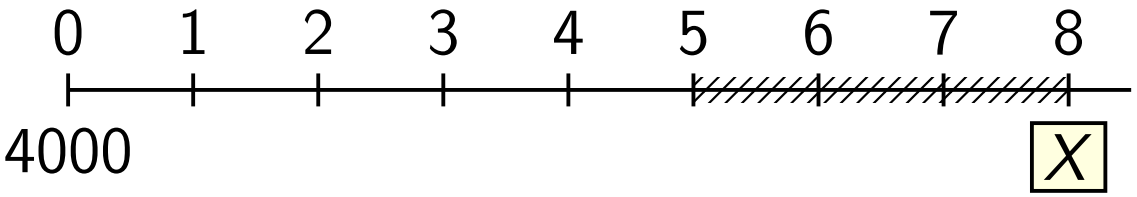
a) Prvi način: preko prenumerando isplata



Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

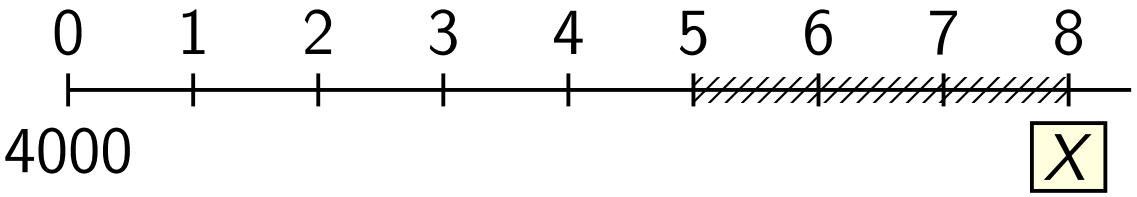
$$r_1 = \sqrt{1.051}$$

$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

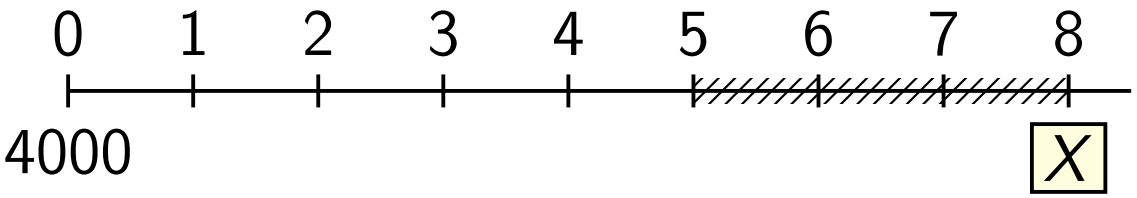
$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)}$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

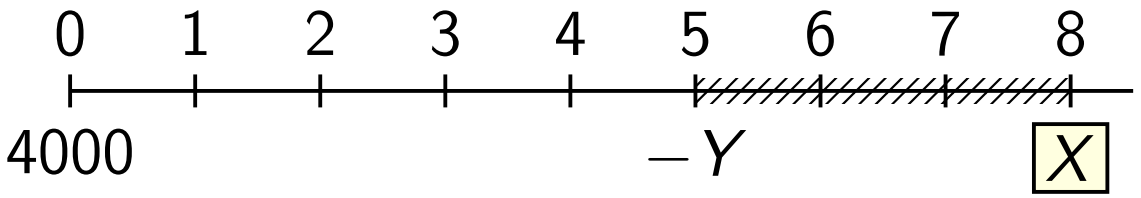
$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

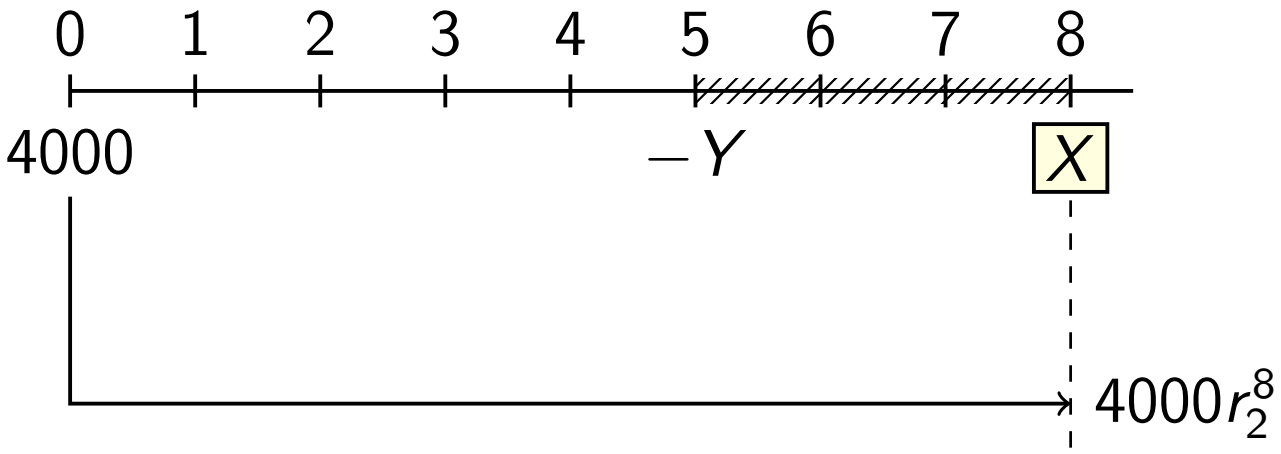
$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

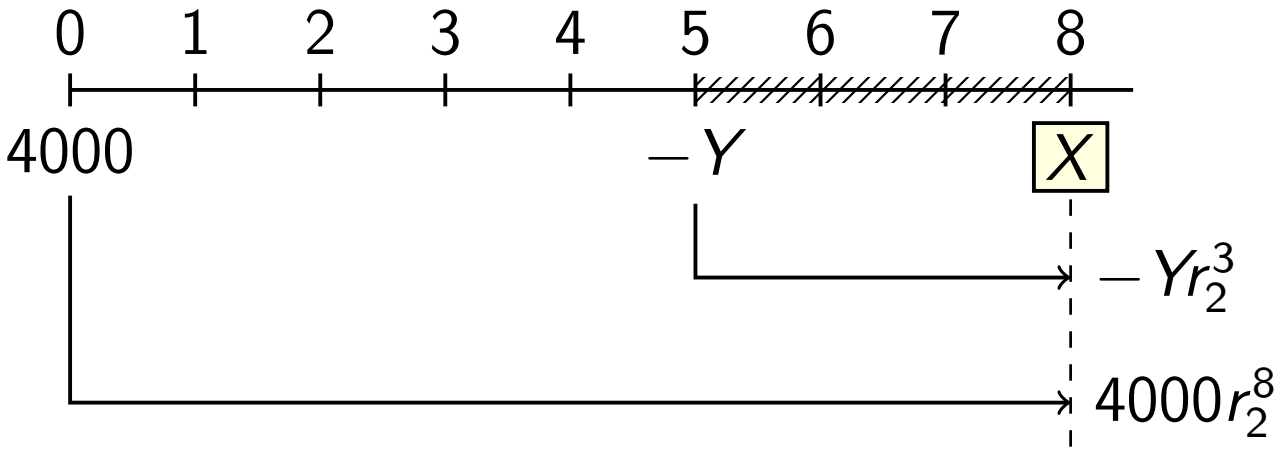
$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

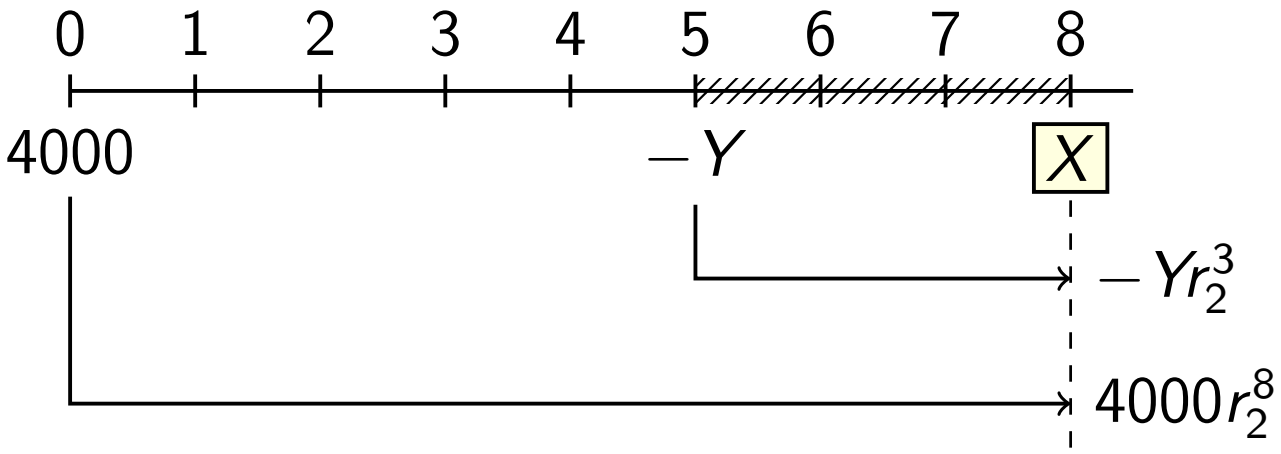
$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

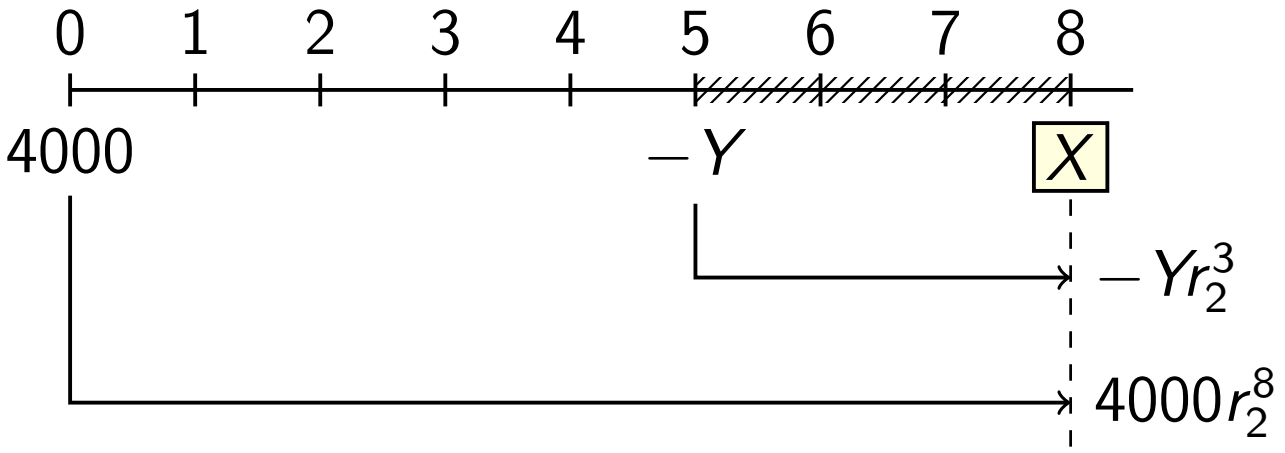
$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

$$X = 4000 r_2^8 - Y r_2^3$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

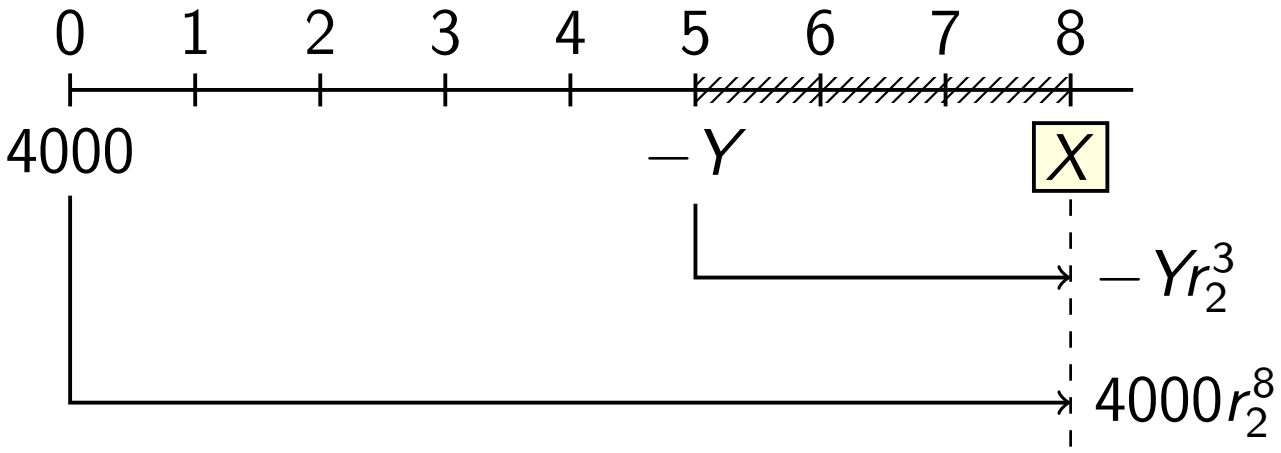
$$X = 4000 r_2^8 - Y r_2^3$$

$$X = 4000 \cdot \left(1.051^2\right)^8 - 5252.22 \cdot \left(1.051^2\right)^3$$

Rješenje

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r - 1)}$$

a) Prvi način: preko prenumerando isplata



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

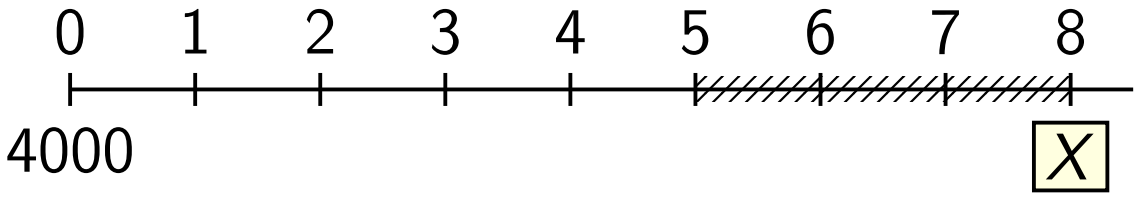
$$Y = R \cdot \frac{r_1^n - 1}{r_1^{n-1} \cdot (r_1 - 1)}$$

$$Y = 500 \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051}^{12-1} \cdot \left(\sqrt{1.051} - 1\right)} = 5252.22$$

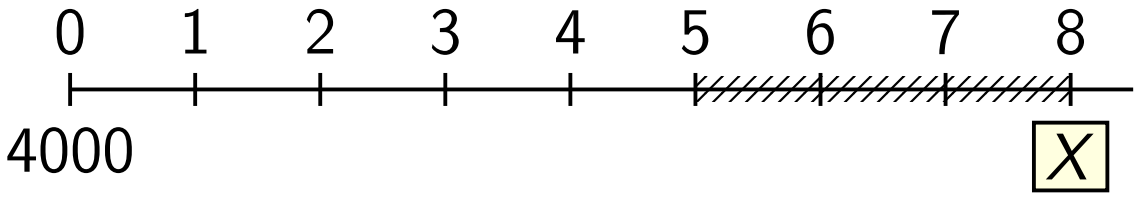
$$X = 4000 r_2^8 - Y r_2^3$$

$$X = 4000 \cdot \left(1.051^2\right)^8 - 5252.22 \cdot \left(1.051^2\right)^3 = 1786.71$$

a) Drugi način: preko prenumerando uplata



a) Drugi način: preko prenumerando uplata



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$

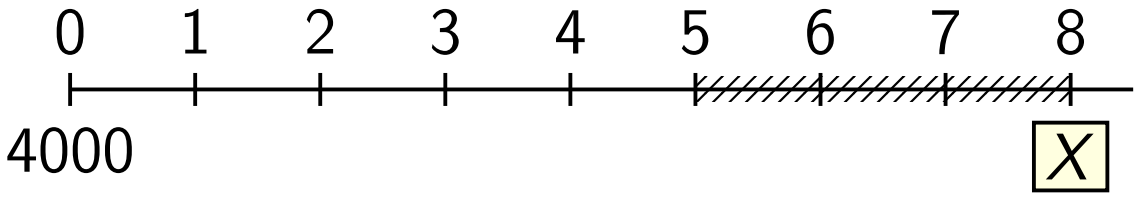
kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

a) Drugi način: preko prenumerando uplata

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$



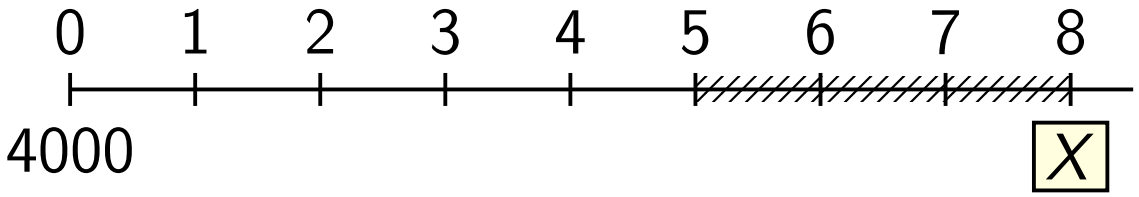
kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1}$$

a) Drugi način: preko prenumerando uplata



$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$

kvartalni dekurzivni kamatni faktor

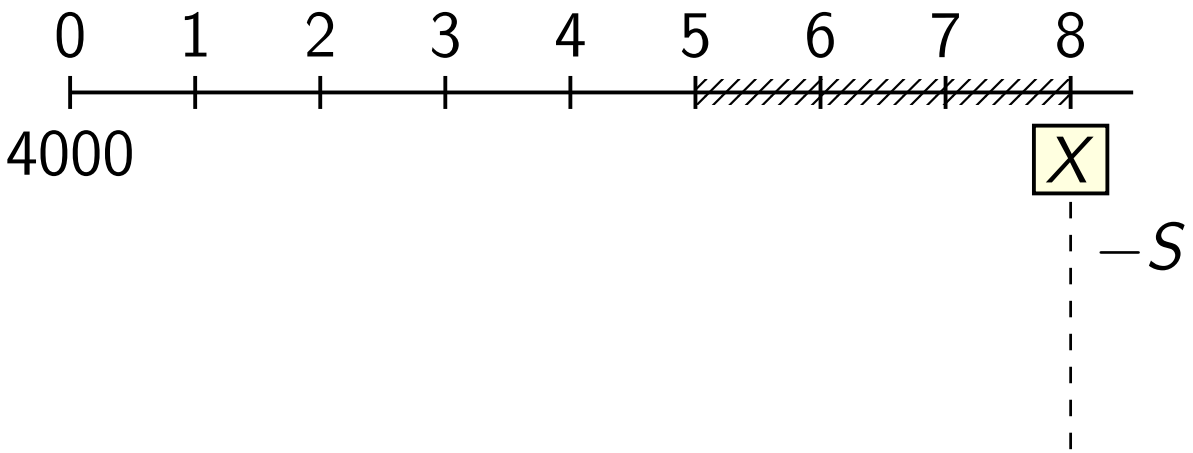
$$r_1 = \sqrt{1.051}$$

$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1} = 7078.798$$

a) Drugi način: preko prenumerando uplata

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$



kvartalni dekurzivni kamatni faktor

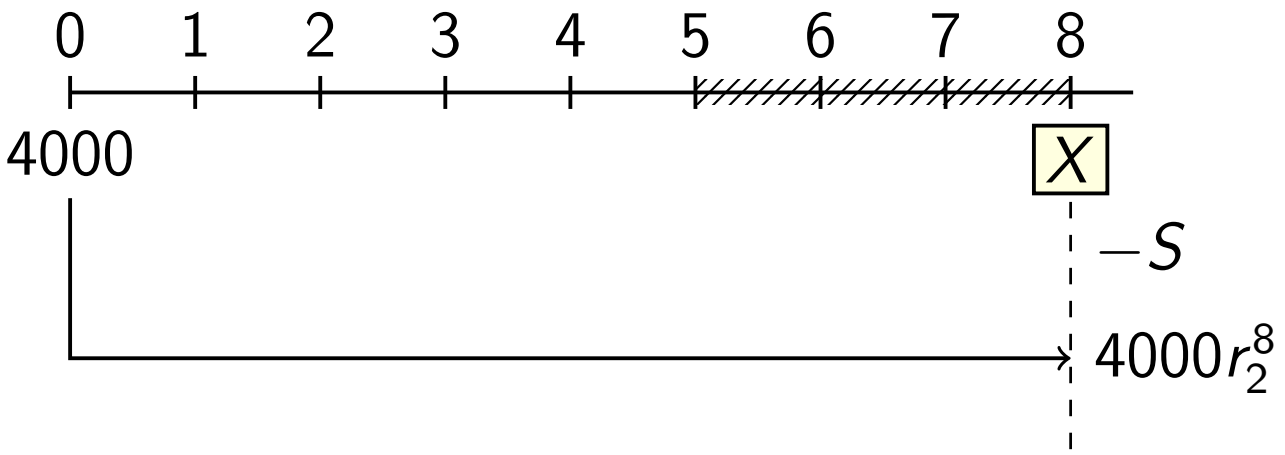
$$r_1 = \sqrt{1.051}$$

$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1} = 7078.798$$

a) Drugi način: preko prenumerando uplata

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

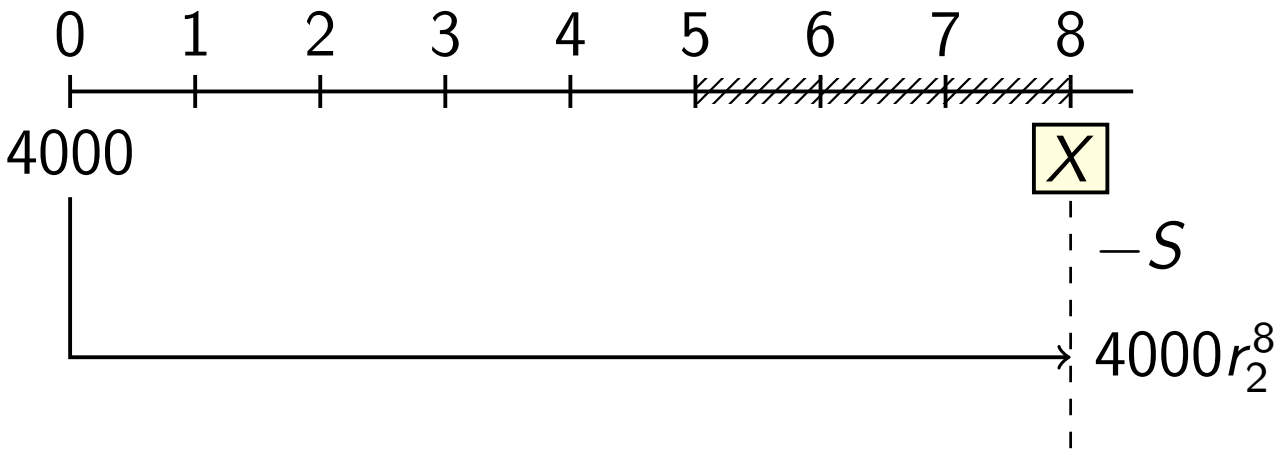
$$r_2 = 1.051^2$$

$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1} = 7078.798$$

a) Drugi način: preko prenumerando uplata

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

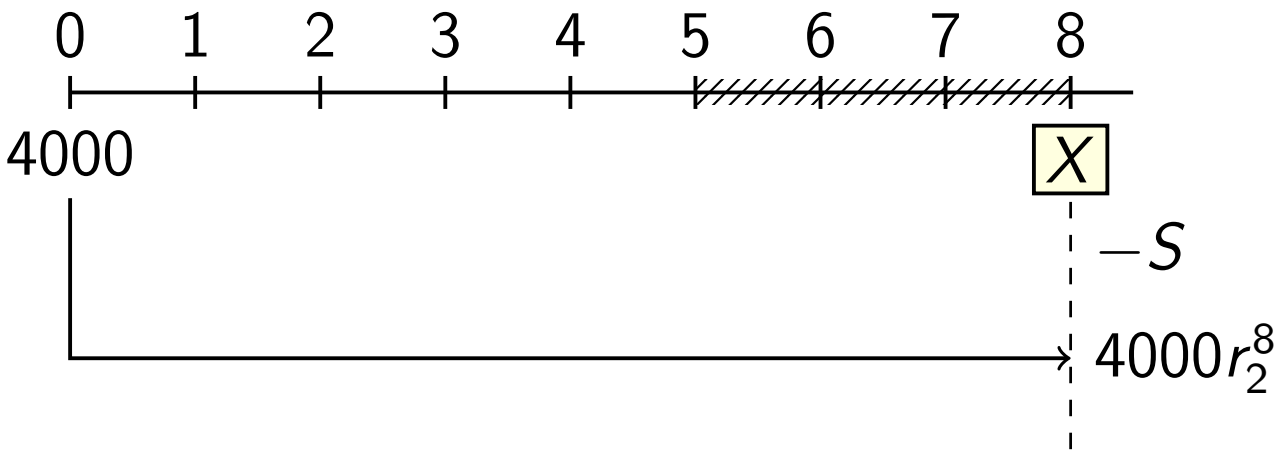
$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1} = 7078.798$$

$$X = 4000 r_2^8 - S$$

a) Drugi način: preko prenumerando uplata

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

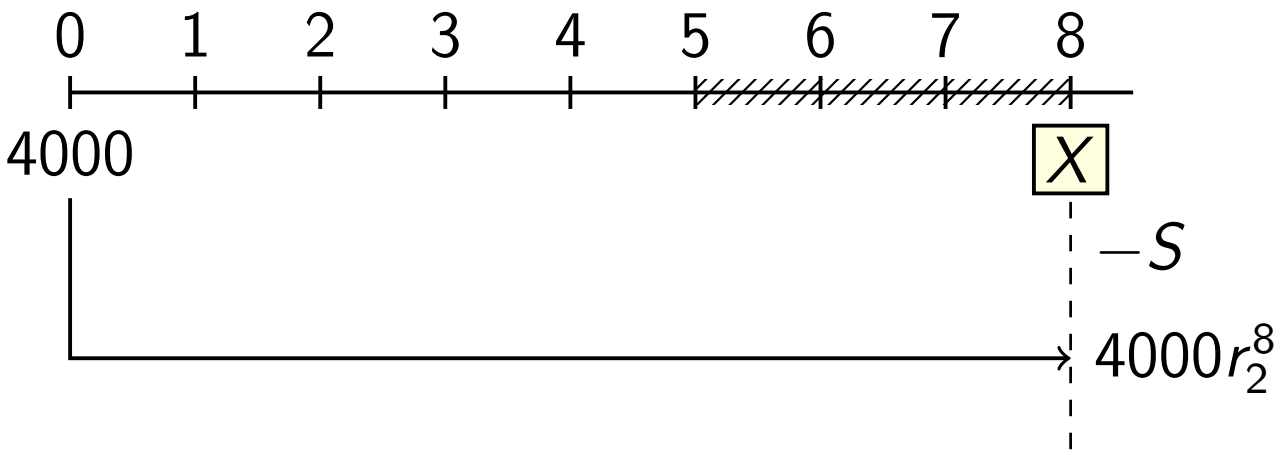
$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1} = 7078.798$$

$$X = 4000 r_2^8 - S$$

$$X = 4000 \cdot \left(1.051^2\right)^8 - 7078.798$$

a) Drugi način: preko prenumerando uplata

$$S = R \cdot r \cdot \frac{r^n - 1}{r - 1}$$



kvartalni dekurzivni kamatni faktor

$$r_1 = \sqrt{1.051}$$

godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

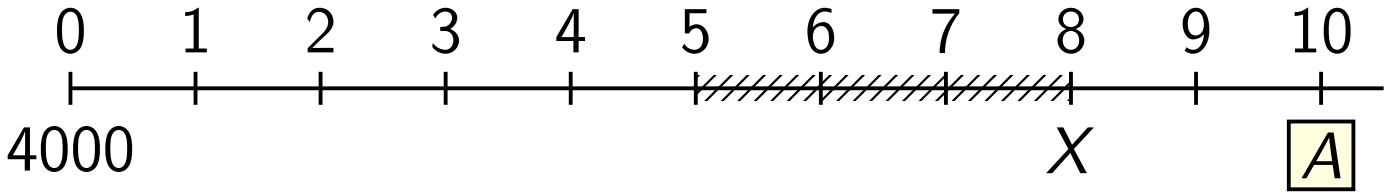
$$S = R \cdot r_1 \cdot \frac{r_1^n - 1}{r_1 - 1}$$

$$S = 500 \cdot \sqrt{1.051} \cdot \frac{\sqrt{1.051}^{12} - 1}{\sqrt{1.051} - 1} = 7078.798$$

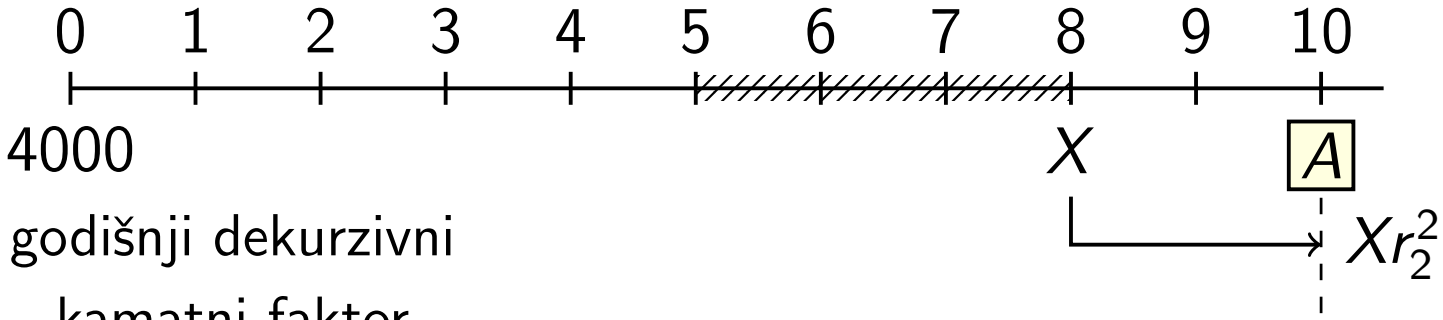
$$X = 4000 r_2^8 - S$$

$$X = 4000 \cdot \left(1.051^2\right)^8 - 7078.798 = 1786.71$$

b)



b)

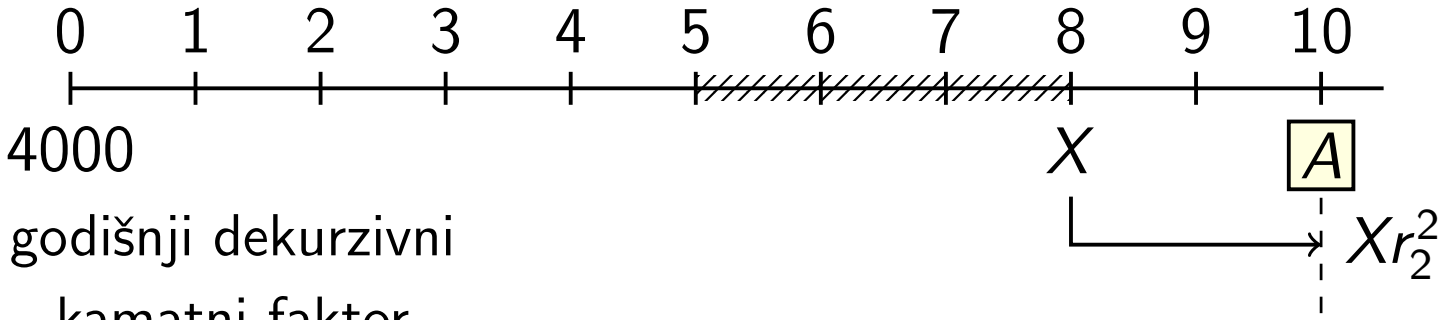


godišnji dekurzivni kamatni faktor

$$\boxed{r_2 = 1.051^2}$$

$$A = Xr_2^2$$

b)

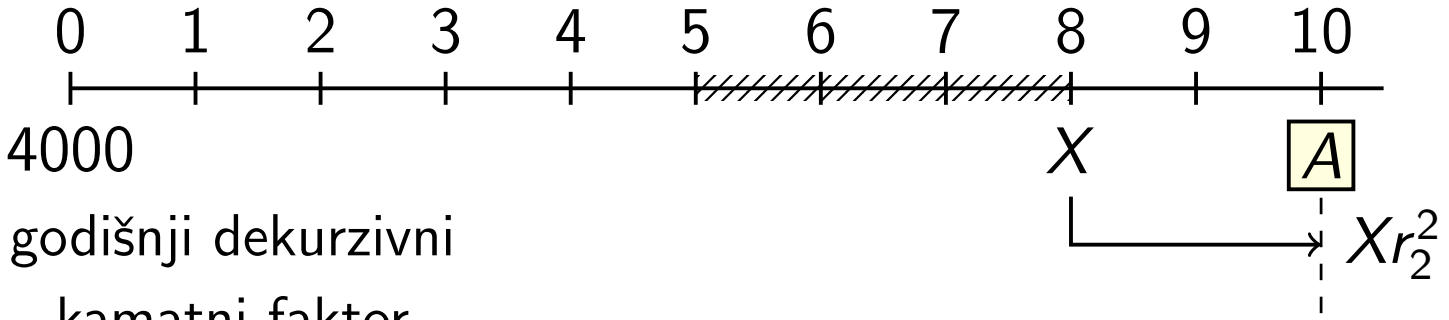


godišnji dekurzivni kamatni faktor

$$\boxed{r_2 = 1.051^2}$$

$$A = Xr_2^2 = 1786.71 \cdot \left(1.051^2\right)^2$$

b)

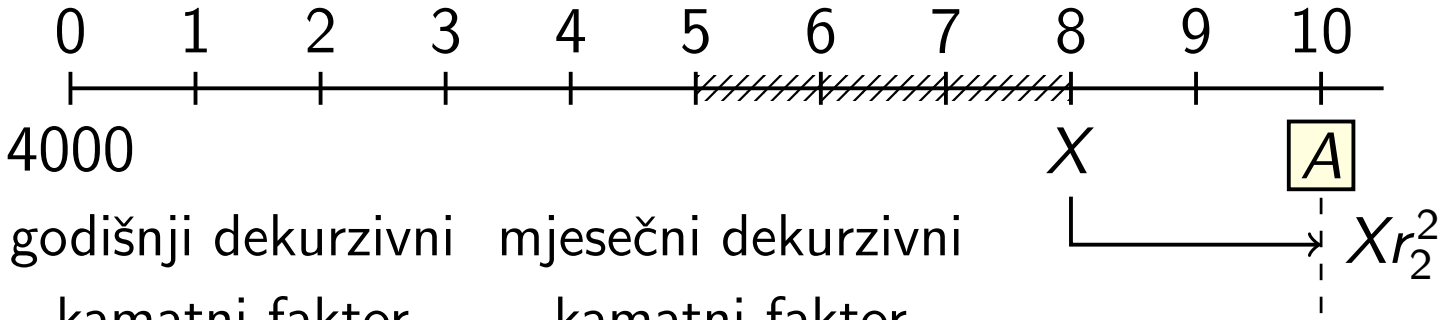


godišnji dekurzivni kamatni faktor

$$\boxed{r_2 = 1.051^2}$$

$$A = Xr_2^2 = 1786.71 \cdot \left(1.051^2\right)^2 = 2180.04$$

b)



godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

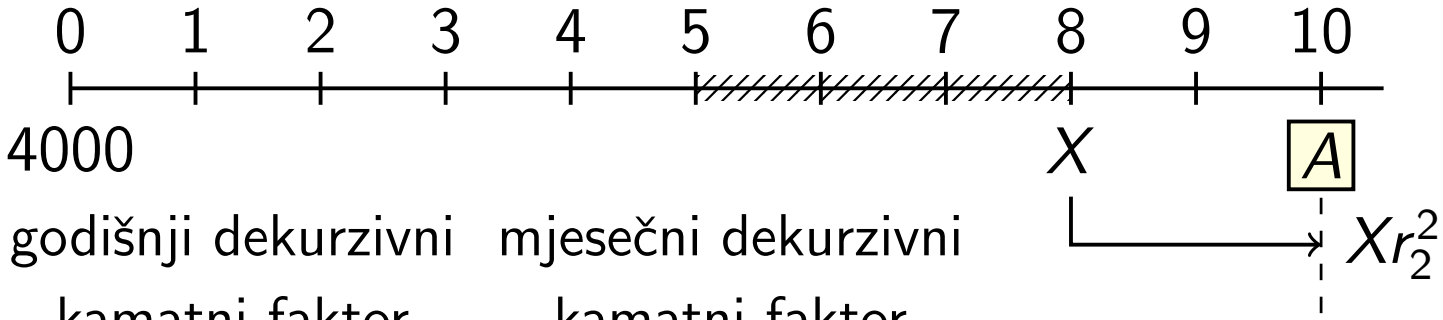
mjesečni dekurzivni kamatni faktor

$$r_3 = \sqrt[6]{1.051}$$

$$A = Xr_2^2 = 1786.71 \cdot \left(1.051^2\right)^2 = 2180.04$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \frac{R \cdot r_3}{R \cdot r_3 - A \cdot (r_3 - 1)}}{\log r_3}$$

b)



godišnji dekurzivni kamatni faktor

$$r_2 = 1.051^2$$

mjesečni dekurzivni kamatni faktor

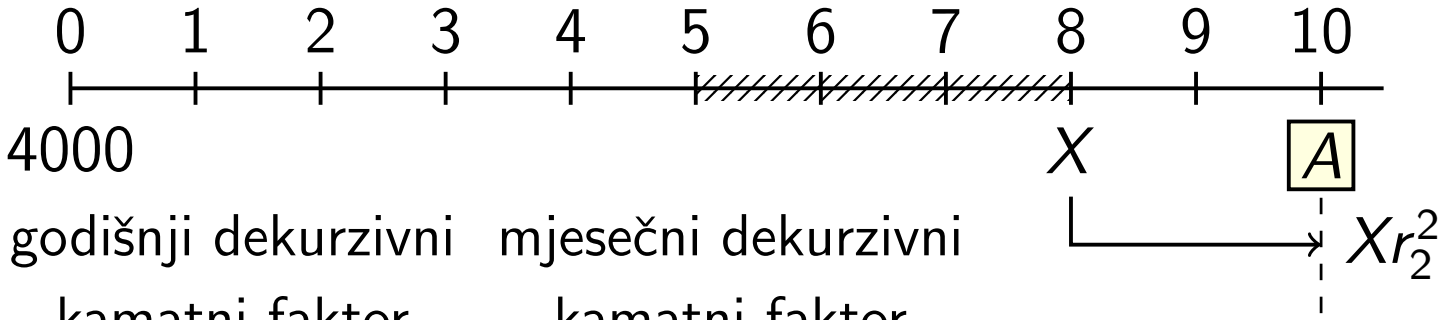
$$r_3 = \sqrt[6]{1.051}$$

$$A = Xr_2^2 = 1786.71 \cdot \left(1.051^2\right)^2 = 2180.04$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \frac{R \cdot r_3}{R \cdot r_3 - A \cdot (r_3 - 1)}}{\log r_3}$$

$$n = \frac{\log \frac{300 \cdot \sqrt[6]{1.051}}{300 \cdot \sqrt[6]{1.051} - 2180.04 \cdot \left(\sqrt[6]{1.051} - 1\right)}}{\log \sqrt[6]{1.051}}$$

b)



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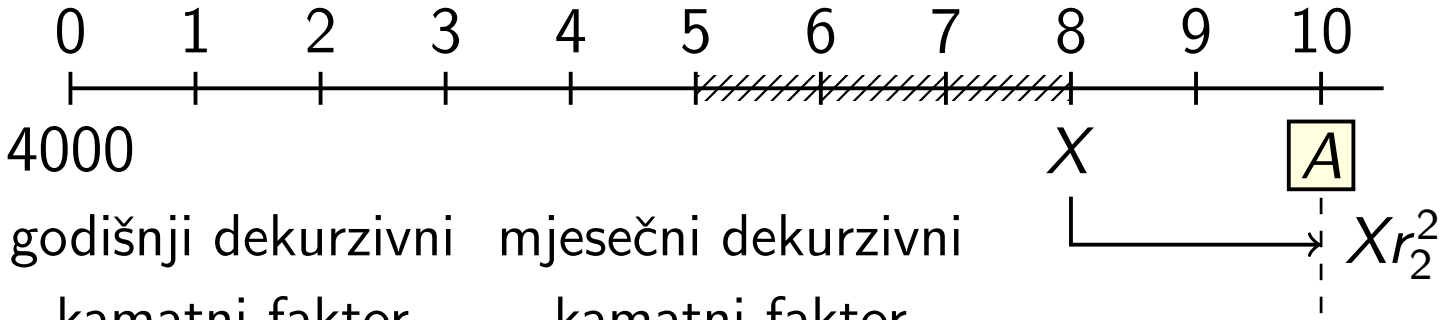
$$n = \frac{\log \frac{300 \cdot \sqrt[6]{1.051}}{300 \cdot \sqrt[6]{1.051} - 2180.04 \cdot \left(\sqrt[6]{1.051} - 1\right)}}{\log \sqrt[6]{1.051}} = 7.46$$

b)



godišnji dekurzivni kamatni faktor

$$\boxed{r_2 = 1.051^2}$$

mjesečni dekurzivni kamatni faktor

$$\boxed{r_3 = \sqrt[6]{1.051}}$$

$$A = Xr_2^2 = 1786.71 \cdot \left(1.051^2\right)^2 = 2180.04$$

$$A = R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)} \qquad n = \frac{\log \dfrac{R \cdot r_3}{R \cdot r_3 - A \cdot (r_3 - 1)}}{\log r_3}$$

$$n = \frac{\log \dfrac{300 \cdot \sqrt[6]{1.051}}{300 \cdot \sqrt[6]{1.051} - 2180.04 \cdot \left(\sqrt[6]{1.051} - 1\right)}}{\log \sqrt[6]{1.051}} = 7.46 \qquad \boxed{n = 7}$$

Tomislav može primiti ukupno 7 mjesečnih isplata visine 300 €.

Krnja isplata

$$R' = \left(A - R \cdot \frac{r_3^n - 1}{r_3^{n-1} \cdot (r_3 - 1)} \right) \cdot r_3^n$$

Krnja isplata

$$R' = \left(A - R \cdot \frac{r_3^n - 1}{r_3^{n-1} \cdot (r_3 - 1)} \right) \cdot r_3^n$$

$$R' = \left(2180.04 - 300 \cdot \frac{\sqrt[6]{1.051}^{7} - 1}{\sqrt[6]{1.051}^{7-1} \cdot \left(\sqrt[6]{1.051} - 1 \right)} \right) \cdot \sqrt[6]{1.051}^{7}$$

Krnja isplata

$$R' = \left(A - R \cdot \frac{r_3^n - 1}{r_3^{n-1} \cdot (r_3 - 1)} \right) \cdot r_3^n$$

$$R' = \left(2180.04 - 300 \cdot \frac{\sqrt[6]{1.051}^{7} - 1}{\sqrt[6]{1.051}^{7-1} \cdot \left(\sqrt[6]{1.051} - 1 \right)} \right) \cdot \sqrt[6]{1.051}^{7}$$

$$R' = 139.19$$

Krnja isplata iznosi 139.19 €.

šesti zadatak

Vječna renta

- Postnumerando slučaj

$$\left(A'_\infty - \frac{R}{r}\right) \cdot r = A'_\infty \quad \Longrightarrow \quad A'_\infty = \frac{R}{r-1}$$

$$A'_\infty = \lim_{n\to\infty} A' = \lim_{n\to\infty} R \cdot \frac{r^n - 1}{r^n \cdot (r-1)}$$

- Prenumerando slučaj

$$(A_\infty - R) \cdot r = A_\infty \quad \Longrightarrow \quad A_\infty = \frac{Rr}{r-1}$$

$$A_\infty = \lim_{n\to\infty} A = \lim_{n\to\infty} R \cdot \frac{r^n - 1}{r^{n-1} \cdot (r-1)}$$

- Veza između postnumerando i prenumerando vječne rente

$$A_\infty = A'_\infty + R$$

Zadatak 6

Godine 2000. *ustanovljena je znanstvena zaklada za školarine s fondom od* 300 000 € *tako da kamate omogućuju vječne rente visine* 40 000 € *krajem svake godine.*

a) *Uz koju je godišnju kamatnu stopu uložen polazni iznos?*

b) *Na kraju* 2010. *godine kamatna stopa je promijenjena na* 9.5%. *Kolika je u tom slučaju visina godišnjih, a kolika mjesečnih isplata krajem razdoblja?*

c) *Umjesto da se nastavi s vječnim isplatama, od* 2015. *godine zaklada se odlučila za konačni broj isplata visine* 40 000 € *krajem svake godine. Koliko je takvih renti moguće isplatiti i kolika je krnja isplata?*

Rješenje

a)

$$A'_{\infty} = \frac{R}{r-1}$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty (r-1) = R$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty (r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty (r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty (r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty (r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

$$r = \frac{17}{15} \approx 1.1333$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1} \qquad r = 1 + \frac{p}{100}$$

$$A'_\infty (r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

$$r = \frac{17}{15} \approx 1.1333$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty (r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

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Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty(r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

$$r = \frac{17}{15} \approx 1.1333$$

$$r = 1 + \frac{p}{100}$$

$$r - 1 = \frac{p}{100}$$

$$p = 100(r-1)$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty(r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

$$r = \frac{17}{15} \approx 1.1333$$

$$r = 1 + \frac{p}{100}$$

$$r - 1 = \frac{p}{100}$$

$$p = 100(r-1)$$

$$p = 100 \cdot (1.1333 - 1)$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty(r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

$$r = \frac{17}{15} \approx 1.1333$$

$$r = 1 + \frac{p}{100}$$

$$r - 1 = \frac{p}{100}$$

$$p = 100(r-1)$$

$$p = 100 \cdot (1.1333 - 1)$$

$$p = 13.33$$

Rješenje

a) $A'_\infty = 300\,000, \quad R = 40\,000$

$$A'_\infty = \frac{R}{r-1}$$

$$A'_\infty(r-1) = R$$

$$r - 1 = \frac{R}{A'_\infty}$$

$$r = \frac{R}{A'_\infty} + 1$$

$$r = \frac{40\,000}{300\,000} + 1$$

$$r = \frac{17}{15} \approx 1.1333$$

$$r = 1 + \frac{p}{100}$$

$$r - 1 = \frac{p}{100}$$

$$p = 100(r-1)$$

$$p = 100 \cdot (1.1333 - 1)$$

$$p = 13.33$$

Godišnja kamatna stopa iznosi 13.33%.

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty (r_1 - 1)$$

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1)$$

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1) = 28\,500$$

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1) = 28\,500$$

Visina godišnjih isplata jednaka je 28 500 €.

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1) = 28\,500$$

Visina godišnjih isplata jednaka je 28 500 €.

- mjesečni kamatni faktor: $r_2 = \sqrt[12]{1.095}$

$$R_{\text{mj}} = A'_\infty(r_2 - 1)$$

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1) = 28\,500$$

Visina godišnjih isplata jednaka je 28 500 €.

- mjesečni kamatni faktor: $r_2 = \sqrt[12]{1.095}$

$$R_{\text{mj}} = A'_\infty(r_2 - 1) = 300\,000 \cdot \left(\sqrt[12]{1.095} - 1\right)$$

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1) = 28\,500$$

Visina godišnjih isplata jednaka je 28 500 €.

- mjesečni kamatni faktor: $r_2 = \sqrt[12]{1.095}$

$$R_{\text{mj}} = A'_\infty(r_2 - 1) = 300\,000 \cdot \left(\sqrt[12]{1.095} - 1\right) = 2277.46$$

b) $A'_\infty = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

$$A'_\infty = \frac{R}{r-1}$$

- godišnji kamatni faktor: $r_1 = 1.095$

$$R_{\text{god}} = A'_\infty(r_1 - 1) = 300\,000 \cdot (1.095 - 1) = 28\,500$$

Visina godišnjih isplata jednaka je 28 500 €.

- mjesečni kamatni faktor: $r_2 = \sqrt[12]{1.095}$

$$R_{\text{mj}} = A'_\infty(r_2 - 1) = 300\,000 \cdot \left(\sqrt[12]{1.095} - 1\right) = 2277.46$$

Visina mjesečnih isplata jednaka je 2277.46 €.

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r}$$

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r} = \frac{\log \frac{40\,000}{40\,000 - 300\,000 \cdot (1.095 - 1)}}{\log 1.095}$$

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r} = \frac{\log \frac{40\,000}{40\,000 - 300\,000 \cdot (1.095 - 1)}}{\log 1.095}$$

$$n = 13.74$$

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r} = \frac{\log \frac{40\,000}{40\,000 - 300\,000 \cdot (1.095 - 1)}}{\log 1.095}$$

$n = 13.74 \qquad \boxed{n = 13}$

Moguće je isplatiti ukupno 13 takvih renti visine 40 000 €.

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r} = \frac{\log \frac{40\,000}{40\,000 - 300\,000 \cdot (1.095 - 1)}}{\log 1.095}$$

$$n = 13.74 \qquad \boxed{n = 13}$$

Moguće je isplatiti ukupno 13 takvih renti visine 40 000 €.

- krnja isplata

$$R' = \left(A' - R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \right) \cdot r^{n+1}$$

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r} = \frac{\log \frac{40\,000}{40\,000 - 300\,000 \cdot (1.095 - 1)}}{\log 1.095}$$

$$n = 13.74 \qquad \boxed{n = 13}$$

Moguće je isplatiti ukupno 13 takvih renti visine 40 000 €.

- krnja isplata

$$R' = \left(A' - R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \right) \cdot r^{n+1}$$

$$R' = \left(300\,000 - 40\,000 \cdot \frac{1.095^{13} - 1}{1.095^{13} \cdot (1.095 - 1)} \right) \cdot 1.095^{13+1}$$

c) $A' = 300\,000, \quad R = 40\,000, \quad p = 9.5\%$

- broj isplata

$$n = \frac{\log \frac{R}{R - A' \cdot (r-1)}}{\log r} = \frac{\log \frac{40\,000}{40\,000 - 300\,000 \cdot (1.095 - 1)}}{\log 1.095}$$

$$n = 13.74 \qquad \boxed{n = 13}$$

Moguće je isplatiti ukupno 13 takvih renti visine 40 000 €.

- krnja isplata

$$R' = \left(A' - R \cdot \frac{r^n - 1}{r^n \cdot (r-1)} \right) \cdot r^{n+1}$$

$$R' = \left(300\,000 - 40\,000 \cdot \frac{1.095^{13} - 1}{1.095^{13} \cdot (1.095 - 1)} \right) \cdot 1.095^{13+1}$$

$$R' = 29\,760.13$$

sedmi zadatak

Zadatak 7

Poslodavci su odlučili osnovati fond za stipendiranje trideset studenata informatike. Po završetku studija stipendiju dobiva novi student. Stipendije počinju s isplatom 1. rujna, a po studentu iznose 1200 *kn mjesečno.*

a) *Koliki iznos trebaju poslodavci uplatiti u fond 1. rujna da bi fond imao dostatna sredstva?*

b) *Ukoliko odluče stipendije isplaćivati kvartalno u iznosu od* 3600 *kn, hoće li isti iznos biti dostatan?*

Godišnja kamatna stopa iznosi 8%*.*

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 =$ $\qquad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1}$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1}$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1}$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Poslodavci trebaju uplatiti 5 631 248.92 kn.

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Poslodavci trebaju uplatiti 5 631 248.92 kn.

b) $R_2 =$ $\qquad r_2 = \sqrt[4]{1.08}$

$$A_\infty = \frac{R_2 r_2}{r_2 - 1}$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Poslodavci trebaju uplatiti 5 631 248.92 kn.

b) $R_2 = 30 \cdot 3600 = 108\,000, \quad r_2 = \sqrt[4]{1.08}$

$$A_\infty = \frac{R_2 r_2}{r_2 - 1}$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Poslodavci trebaju uplatiti 5 631 248.92 kn.

b) $R_2 = 30 \cdot 3600 = 108\,000, \quad r_2 = \sqrt[4]{1.08}$

$$A_\infty = \frac{R_2 r_2}{r_2 - 1} = \frac{108\,000 \cdot \sqrt[4]{1.08}}{\sqrt[4]{1.08} - 1}$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Poslodavci trebaju uplatiti 5 631 248.92 kn.

b) $R_2 = 30 \cdot 3600 = 108\,000, \quad r_2 = \sqrt[4]{1.08}$

$$A_\infty = \frac{R_2 r_2}{r_2 - 1} = \frac{108\,000 \cdot \sqrt[4]{1.08}}{\sqrt[4]{1.08} - 1} = 5\,667\,402.84$$

Rješenje

$$A_\infty = \frac{Rr}{r-1}$$

a) $R_1 = 30 \cdot 1200 = 36\,000, \quad r_1 = \sqrt[12]{1.08}$

$$A_\infty = \frac{R_1 r_1}{r_1 - 1} = \frac{36\,000 \cdot \sqrt[12]{1.08}}{\sqrt[12]{1.08} - 1} = 5\,631\,248.92$$

Poslodavci trebaju uplatiti 5 631 248.92 kn.

b) $R_2 = 30 \cdot 3600 = 108\,000, \quad r_2 = \sqrt[4]{1.08}$

$$A_\infty = \frac{R_2 r_2}{r_2 - 1} = \frac{108\,000 \cdot \sqrt[4]{1.08}}{\sqrt[4]{1.08} - 1} = 5\,667\,402.84$$

U ovom slučaju poslodavci bi trebali uplatiti veći iznos.

osmi zadatak

Zadatak 8

Rajka je odlučila štedjeti na način da krajem svakog mjeseca idućih 10 *godina uplaćuje na posebni bankovni račun* 10% *svoje plaće. Trenutno zarađuje* 3500 *kn mjesečno, a godišnja stopa rasta plaće je* 2.1%*. Banka svoju imovinu ukamaćuje uz godišnju kamatnu stopu od* 4.3%*. Nakon isteka* 10 *godina, dvije godine nije trošila ušteđevinu, a zatim je odlučila da krajem svakog mjeseca idućih* 5 *godina podigne određeni iznos tako da svaki mjesec podigne* 20 *kn više nego prethodni mjesec. Koliko je Rajka uštedjela tijekom deset godina? Kolika je visina prve i posljednje isplate?*

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- Iznos potreban za povećanje isplata (20 kn)

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- Posljednja isplata:

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- Posljednja isplata: $557.76 + 60 \cdot 20 = 1757.76$