Matrice

MATEMATIKA ZA EKONOMISTE 1

Damir Horvat

FOI, Varaždin

Matrice

Zadatak 1

Napišite matricu $A = [a_{ij}]$ *tipa* $(3,4)$ *ako je*

$$a_{ij} = \begin{cases} \cos \dfrac{i\pi}{2}, & \text{ako je } i > j \\ \log_2(i+j), & \text{ako je } i \leqslant j \end{cases}$$

Rješenje $\log_2 4 = \log_2 2^2 = 2$

$\boxed{\log_a a^x = x}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & \log_2 3 & 2 & \log_2 5 \\ -1 & 2 & \log_2 5 & \log_2 6 \\ 0 & 0 & \log_2 6 & \log_2 7 \end{bmatrix}$$

$$a_{11} = \log_2(1+1) = \log_2 2 = 1$$

$$a_{12} = \log_2(1+2) = \log_2 3$$

$$a_{13} = \log_2(1+3) = \log_2 4 = 2$$

$$a_{14} = \log_2(1+4) = \log_2 5$$

$$a_{21} = \cos\frac{2\pi}{2} = \cos\pi = -1$$

$$a_{22} = \log_2(2+2) = \log_2 4 = 2$$

$$a_{23} = \log_2(2+3) = \log_2 5$$

$$a_{24} = \log_2(2+4) = \log_2 6$$

$$a_{31} = \cos\frac{3\pi}{2} = 0$$

$$a_{32} = \cos\frac{3\pi}{2} = 0$$

$$a_{33} = \log_2(3+3) = \log_2 6$$

$$a_{34} = \log_2(3+4) = \log_2 7$$

Zadatak 2

Dopunite matricu

$$\begin{bmatrix} \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot \\ e & 3 & \cdot \end{bmatrix}$$

tako da bude

a) *simetrična,*

b) *antisimetrična.*

Rješenje

a) $a_{ij} = a_{ji} \xrightarrow{i=j} a_{ii} = a_{ii}$

$$\begin{bmatrix} a & -1 & e \\ -1 & b & 3 \\ e & 3 & c \end{bmatrix} \qquad a, b, c \in \mathbb{R}$$

b) $a_{ij} = -a_{ji} \xrightarrow{i=j} a_{ii} = -a_{ii}$

$$2a_{ii} = 0$$

$$a_{ii} = 0$$

$$\begin{bmatrix} 0 & -1 & -e \\ 1 & 0 & -3 \\ e & 3 & 0 \end{bmatrix}$$

Zadatak 3

Odredite $a, b \in \mathbb{R}$ tako da matrica

$$A = \begin{bmatrix} a^2 + 4b^2 & a & b \\ a - b & a^2 & a + b \\ a - 1 & b - 1 & b^2 \end{bmatrix}$$

bude

a) *gornje trokutasta,*

b) *donje trokutasta.*

a) gornje trokutasta matrica

$$A = \begin{bmatrix} a^2 + 4b^2 & a & b \\ a - b & a^2 & a + b \\ a - 1 & b - 1 & b^2 \end{bmatrix}$$

$$\left.\begin{aligned} a - b &= 0 \\ a - 1 &= 0 \\ b - 1 &= 0 \end{aligned}\right\} \quad \begin{aligned} & 1 - 1 = 0 \\ & \rightsquigarrow a = 1 \\ & \rightsquigarrow b = 1 \end{aligned}$$

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

b) donje trokutasta matrica

$$A = \begin{bmatrix} a^2 + 4b^2 & a & b \\ a - b & a^2 & a + b \\ a - 1 & b - 1 & b^2 \end{bmatrix}$$

$$\left.\begin{aligned} a &= 0 \\ b &= 0 \\ a + b &= 0 \end{aligned}\right\} \quad \begin{aligned} & \rightsquigarrow a = 0 \\ & \rightsquigarrow b = 0 \\ & 0 + 0 = 0 \end{aligned}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Zadatak 4

Odredite $a, b \in \mathbb{R}$ tako da matrica

$$B = \begin{bmatrix} 1 & a & b \\ a^2 - 9 & 3 & a \\ a^2 + b & b + 9 & 2 \end{bmatrix}$$

bude

a) *gornje trokutasta,*

b) *simetrična.*

Rješenje

a) gornje trokutasta matrica

$$B = \begin{bmatrix} 1 & a & b \\ a^2 - 9 & 3 & a \\ a^2 + b & b + 9 & 2 \end{bmatrix}$$

$$\left.\begin{aligned} a^2 - 9 &= 0 \\ a^2 + b &= 0 \\ b + 9 &= 0 \end{aligned}\right\} \quad \begin{aligned} & \rightsquigarrow a^2 - 9 = 0 \\ & \rightsquigarrow a^2 - 9 = 0 \\ & \rightsquigarrow b = -9 \end{aligned} \quad \rightsquigarrow \begin{aligned} a^2 &= 9 \\ a_1 &= 3 \\ a_2 &= -3 \end{aligned}$$

$a = 3$

$b = -9$

$$B_1 = \begin{bmatrix} 1 & 3 & -9 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

$a = -3$

$b = -9$

$$B_2 = \begin{bmatrix} 1 & -3 & -9 \\ 0 & 3 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

b) simetrična matrica

$$B=\begin{bmatrix}1 & a & b\\ a^2-9 & 3 & a\\ a^2+b & b+9 & 2\end{bmatrix}$$

$$\left.\begin{aligned} a^2-9&=a\\ a^2+b&=b\\ b+9&=a\end{aligned}\right\} \rightsquigarrow \begin{aligned} &a^2=0\\ &\boxed{a=0}\end{aligned} \qquad -9=0$$

nema rješenja

Ne postoje $a, b \in \mathbb{R}$ za koje bi matrica B bila simetrična matrica.

Zadatak 5

Zadane su matrice

$$A=\begin{bmatrix}1 & 2\\ 0 & -3\\ 5 & 4\end{bmatrix} \quad i \quad B=\begin{bmatrix}1 & 0 & -2 & 5\\ 8 & 4 & -1 & 3\end{bmatrix}.$$

Odredite:

a) A^T,

b) BA,

c) AB.

Rješenje

a)

$$A^T=\begin{bmatrix}1 & 0 & 5\\ 2 & -3 & 4\end{bmatrix}$$

b) $BA \longleftarrow$ nije definirano

$$(2,\boxed{4)\ (3},2)$$
$$\neq$$

c)

$$AB=\begin{bmatrix}1 & 2\\ 0 & -3\\ 5 & 4\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & -2 & 5\\ 8 & 4 & -1 & 3\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & c_{14}\\ c_{21} & c_{22} & c_{23} & c_{24}\\ c_{31} & c_{32} & c_{33} & c_{34}\end{bmatrix}$$

$$AB=\begin{bmatrix}17 & 8 & -4 & 11\\ 24 & -12 & 3 & -9\\ 7 & 16 & -14 & 37\end{bmatrix}$$

$$\begin{aligned}
c_{11}&=(1,2)\cdot(1,8)=1\cdot 1+2\cdot 8=17\\
c_{12}&=(1,2)\cdot(0,4)=1\cdot 0+2\cdot 4=8\\
c_{13}&=(1,2)\cdot(-2,-1)=1\cdot(-2)+2\cdot(-1)=-4\\
c_{14}&=(1,2)\cdot(5,3)=1\cdot 5+2\cdot 3=11\\
c_{21}&=(0,-3)\cdot(1,8)=0\cdot 1+(-3)\cdot 8=-24\\
c_{22}&=(0,-3)\cdot(0,4)=0\cdot 0+(-3)\cdot 4=-12\\
c_{23}&=(0,-3)\cdot(-2,-1)=0\cdot(-2)+(-3)\cdot(-1)=3\\
c_{24}&=(0,-3)\cdot(5,3)=0\cdot 5+(-3)\cdot 3=-9\\
c_{31}&=(5,4)\cdot(1,8)=5\cdot 1+4\cdot 8=37\\
c_{32}&=(5,4)\cdot(0,4)=5\cdot 0+4\cdot 4=16\\
c_{33}&=(5,4)\cdot(-2,-1)=5\cdot(-2)+4\cdot(-1)=-14\\
c_{34}&=(5,4)\cdot(5,3)=5\cdot 5+4\cdot 3=37
\end{aligned}$$

Zadatak 6

Odredite matricu $3AB - 7BA$ ako je

$$A = \begin{bmatrix} 3 & 1 & -4 \\ -4 & 6 & -2 \\ 5 & 8 & 5 \end{bmatrix} \quad i \quad B = \begin{bmatrix} 3 & 7 & -4 \\ 2 & 1 & 0 \\ -5 & 3 & 2 \end{bmatrix}.$$

Rješenje

$$3AB - 7BA \neq -4AB$$
$$3AB - 7BA \neq -4BA$$

$$AB = \begin{bmatrix} 3 & 1 & -4 \\ -4 & 6 & -2 \\ 5 & 8 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 7 & -4 \\ 2 & 1 & 0 \\ -5 & 3 & 2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 7 & -4 \\ 2 & 1 & 0 \\ -5 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 & -4 \\ -4 & 6 & -2 \\ 5 & 8 & 5 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

$$AB = \begin{bmatrix} 31 & 10 & -20 \\ 10 & -28 & 12 \\ 6 & 58 & -10 \end{bmatrix} \qquad BA = \begin{bmatrix} -39 & 13 & -46 \\ 2 & 8 & -10 \\ -17 & 29 & 24 \end{bmatrix}$$

$$\begin{aligned}
c_{11} &= (3,1,-4) \cdot (3,2,-5) = 3 \cdot 3 + 1 \cdot 2 + (-4) \cdot (-5) = 31 \\
c_{12} &= (3,1,-4) \cdot (7,1,3) = 3 \cdot 7 + 1 \cdot 1 + (-4) \cdot 3 = 10 \\
c_{13} &= (3,1,-4) \cdot (-4,0,2) = 3 \cdot (-4) + 1 \cdot 0 + (-4) \cdot 2 = -20 \\
c_{21} &= (-4,6,-2) \cdot (3,2,-5) = -4 \cdot 3 + 6 \cdot 2 + (-2) \cdot (-5) = 10 \\
c_{22} &= (-4,6,-2) \cdot (7,1,3) = -4 \cdot 7 + 6 \cdot 1 + (-2) \cdot 3 = -28 \\
c_{23} &= (-4,6,-2) \cdot (-4,0,2) = -4 \cdot (-4) + 6 \cdot 0 + (-2) \cdot 2 = 12 \\
c_{31} &= (5,8,5) \cdot (3,2,-5) = 5 \cdot 3 + 8 \cdot 2 + 5 \cdot (-5) = 6 \\
c_{32} &= (5,8,5) \cdot (7,1,3) = 5 \cdot 7 + 8 \cdot 1 + 5 \cdot 3 = 58 \\
c_{33} &= (5,8,5) \cdot (-4,0,2) = 5 \cdot (-4) + 8 \cdot 0 + 5 \cdot 2 = -10
\end{aligned}$$

$$\begin{aligned}
d_{11} &= (3,7,-4) \cdot (3,-4,5) = 3 \cdot 3 + 7 \cdot (-4) + (-4) \cdot 5 = -39 \\
d_{12} &= (3,7,-4) \cdot (1,6,8) = 3 \cdot 1 + 7 \cdot 6 + (-4) \cdot 8 = 13 \\
d_{13} &= (3,7,-4) \cdot (-4,-2,5) = 3 \cdot (-4) + 7 \cdot (-2) + (-4) \cdot 5 = -46 \\
d_{21} &= (2,1,0) \cdot (3,-4,5) = 2 \cdot 3 + 1 \cdot (-4) + 0 \cdot 5 = 2 \\
d_{22} &= (2,1,0) \cdot (1,6,8) = 2 \cdot 1 + 1 \cdot 6 + 0 \cdot 8 = 8 \\
d_{23} &= (2,1,0) \cdot (-4,-2,5) = 2 \cdot (-4) + 1 \cdot (-2) + 0 \cdot 5 = -10 \\
d_{31} &= (-5,3,2) \cdot (3,-4,5) = -5 \cdot 3 + 3 \cdot (-4) + 2 \cdot 5 = -17 \\
d_{32} &= (-5,3,2) \cdot (1,6,8) = -5 \cdot 1 + 3 \cdot 6 + 2 \cdot 8 = 29 \\
d_{33} &= (-5,3,2) \cdot (-4,-2,5) = -5 \cdot (-4) + 3 \cdot (-2) + 2 \cdot 5 = 24
\end{aligned}$$

$$3AB-7BA = 3\cdot\begin{bmatrix}31 & 10 & -20\\ 10 & -28 & 12\\ 6 & 58 & -10\end{bmatrix} - 7\cdot\begin{bmatrix}-39 & 13 & -46\\ 2 & 8 & -10\\ -17 & 29 & 24\end{bmatrix} =$$

$$= \begin{bmatrix}93 & 30 & -60\\ 30 & -84 & 36\\ 18 & 174 & -30\end{bmatrix} - \begin{bmatrix}-273 & 91 & -322\\ 14 & 56 & -70\\ -119 & 203 & 168\end{bmatrix} =$$

$$= \begin{bmatrix}366 & -61 & 262\\ 16 & -140 & 106\\ 137 & -29 & -198\end{bmatrix}$$

Zadatak 7

Zadana je matrica $A = \begin{bmatrix}3 & -2\\ -1 & 5\end{bmatrix}$ *i polinom* $f(x) = x^3 + 2x^2 + 3$.

Odredite $f(A)$.

Rješenje

$$f(A) = A^3 + 2A^2 + 3I$$

$$\underset{\underset{=}{(2,\boxed{2)\,(2},2)}}{A^2} = A\cdot A = \begin{bmatrix}3 & -2\\ -1 & 5\end{bmatrix}\cdot\begin{bmatrix}3 & -2\\ -1 & 5\end{bmatrix} = \begin{bmatrix}11 & -16\\ -8 & 27\end{bmatrix}$$

$$\underset{\underset{=}{(2,\boxed{2)\,(2},2)}}{A^3} = A^2\cdot A = \begin{bmatrix}11 & -16\\ -8 & 27\end{bmatrix}\cdot\begin{bmatrix}3 & -2\\ -1 & 5\end{bmatrix} = \begin{bmatrix}49 & -102\\ -51 & 151\end{bmatrix}$$

$$f(A) = A^3 + 2A^2 + 3I$$

$$f(A) = \begin{bmatrix}49 & -102\\ -51 & 151\end{bmatrix} + 2\cdot\begin{bmatrix}11 & -16\\ -8 & 27\end{bmatrix} + 3\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$f(A) = \begin{bmatrix}49 & -102\\ -51 & 151\end{bmatrix} + \begin{bmatrix}22 & -32\\ -16 & 54\end{bmatrix} + \begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$f(A) = \begin{bmatrix}74 & -134\\ -67 & 208\end{bmatrix}$$

Može li se A^n izračunati za svaku matricu A pri čemu je $n \in \mathbb{N}$?

Potencirati se mogu samo kvadratne matrice.

$$\underset{(m,\boxed{n)\,(m},n)}{A\cdot A}$$

$$n = m$$

Zadatak 8

Izračunajte

$$\begin{bmatrix} 2 & -2 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

Rješenje

$$\begin{bmatrix} 2 & -2 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$(2,\boxed{3})(\boxed{3},1)$$

$$(2,-2,0)\cdot(2,1,3) = 2\cdot 2 + (-2)\cdot 1 + 0 \cdot 3 = 2$$

$$(1,-2,1)\cdot(2,1,3) = 1\cdot 2 + (-2)\cdot 1 + 1 \cdot 3 = 3$$

Zadatak 9

$k(AB) = (kA)B = A(kB)$

kvaziasocijativnost

Izračunajte AB ako je

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \quad i \quad B = \frac{9}{5}\begin{bmatrix} 1 & 0 & 4 \\ 3 & 6 & 8 \end{bmatrix}.$$

Rješenje

$$(2,\boxed{2})(\boxed{2},3)$$

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \cdot \frac{9}{5}\begin{bmatrix} 1 & 0 & 4 \\ 3 & 6 & 8 \end{bmatrix} = \frac{9}{5}\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 4 \\ 3 & 6 & 8 \end{bmatrix} = \frac{9}{5}\begin{bmatrix} 11 & 18 & 32 \\ 16 & 30 & 44 \end{bmatrix}$$

$$(2,3)\cdot(1,3) = 2\cdot 1 + 3\cdot 3 = 11 \qquad (1,5)\cdot(1,3) = 1\cdot 1 + 5\cdot 3 = 16$$

$$(2,3)\cdot(0,6) = 2\cdot 0 + 3\cdot 6 = 18 \qquad (1,5)\cdot(0,6) = 1\cdot 0 + 5\cdot 6 = 30$$

$$(2,3)\cdot(4,8) = 2\cdot 4 + 3\cdot 8 = 32 \qquad (1,5)\cdot(4,8) = 1\cdot 4 + 5\cdot 8 = 44$$

Zadatak 10

Izračunajte $XY + YX$ ako je

$$X = \begin{bmatrix} 1 & 2 & -5 \end{bmatrix} \quad i \quad Y = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}.$$

Rješenje

$$XY = \begin{bmatrix} 1 & 2 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -17 \end{bmatrix}$$

$$(1,\boxed{3})(\boxed{3},1)$$

$$(1,2,-5)\cdot(3,0,4) = 1\cdot 3 + 2\cdot 0 + (-5)\cdot 4 = -17$$

$$YX = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & -15 \\ 0 & 0 & 0 \\ 4 & 8 & -20 \end{bmatrix}$$

$$(3,\boxed{1})(\boxed{1},3)$$

- $XY + YX$ nije definirano jer matrice XY i YX nisu istog tipa.