

Pokazatelji isplativosti ulaganja

FINANCIJSKA MATEMATIKA, IPS

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Zadatak 1

Investitoru su prezentirana dva projekta A i B s dobicima krajem godina danima u tablici. Pomoću NPV metode odaberite isplativiji projekt ako znamo da je cijena kapitala 8%.

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

Rješenje

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4}$$

Rješenje

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4}$$

$$NPV(A) = -600\,000 + \frac{180\,000}{1.08} + \frac{160\,000}{1.08^2} + \frac{170\,000}{1.08^3} + \frac{210\,000}{1.08^4}$$

Rješenje

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4}$$

$$NPV(A) = -600\,000 + \frac{180\,000}{1.08} + \frac{160\,000}{1.08^2} + \frac{170\,000}{1.08^3} + \frac{210\,000}{1.08^4}$$

$$NPV(A) = -6851.37$$

Rješenje

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4}$$

$$NPV(A) = -600\,000 + \frac{180\,000}{1.08} + \frac{160\,000}{1.08^2} + \frac{170\,000}{1.08^3} + \frac{210\,000}{1.08^4}$$

$$NPV(A) = -6851.37$$

$$NPV(B) = -600\,000 + \frac{100\,000}{1.08} + \frac{140\,000}{1.08^2} + \frac{215\,000}{1.08^3} + \frac{300\,000}{1.08^4}$$

Rješenje

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4}$$

$$NPV(A) = -600\,000 + \frac{180\,000}{1.08} + \frac{160\,000}{1.08^2} + \frac{170\,000}{1.08^3} + \frac{210\,000}{1.08^4}$$

$$NPV(A) = -6851.37$$

$$NPV(B) = -600\,000 + \frac{100\,000}{1.08} + \frac{140\,000}{1.08^2} + \frac{215\,000}{1.08^3} + \frac{300\,000}{1.08^4}$$

$$NPV(B) = 3802.92$$

Rješenje

projekt	0	1	2	3	4
A	−600 000	180 000	160 000	170 000	210 000
B	−600 000	100 000	140 000	215 000	300 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4}$$

$$NPV(A) = -600\,000 + \frac{180\,000}{1.08} + \frac{160\,000}{1.08^2} + \frac{170\,000}{1.08^3} + \frac{210\,000}{1.08^4}$$

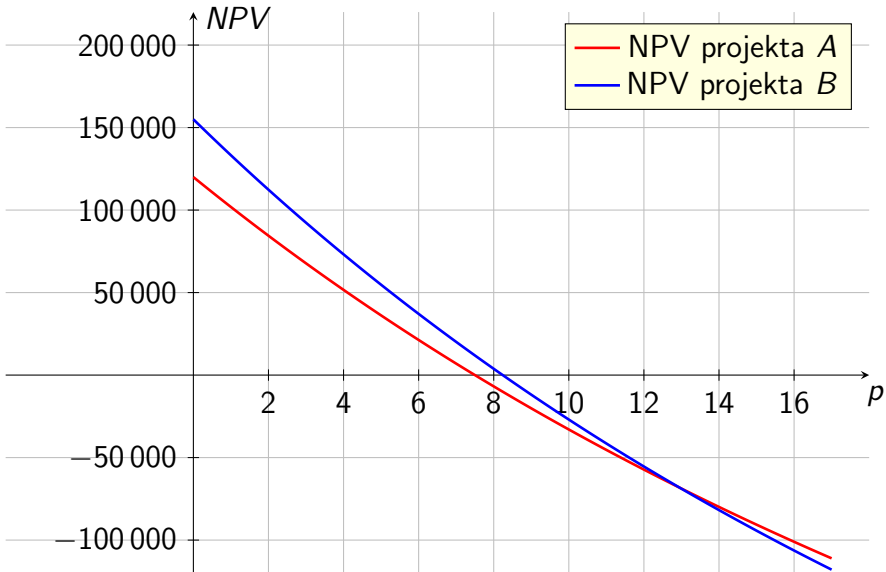
$$NPV(A) = -6851.37$$

$$NPV(B) = -600\,000 + \frac{100\,000}{1.08} + \frac{140\,000}{1.08^2} + \frac{215\,000}{1.08^3} + \frac{300\,000}{1.08^4}$$

$$NPV(B) = 3802.92$$

Isplativiji je projekt *B*. Projekt *A* nije isplativ jer je $NPV(A) < 0$.

NPV projekata A i B kao funkcija cijene kapitala



drugi zadatak

Zadatak 2

Investitoru je ponuđen projekt u koji bi trebalo uložiti 1800 kn odmah i 10 500 kn za dvije godine. Očekivani dobiti na kraju prve tri godine su dani u tablici.

godina	1.	2.	3.
očekivani dobiti	6050 kn	3750 kn	2500 kn

- a) Radi li se o isplativom projektu ako je cijena kapitala 8%?
- b) Radi li se o isplativom projektu ako je cijena kapitala 12%?

Rješenje

a)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo *NPV* danog projekta ako je cijena kapitala 8%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

Rješenje

a)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo *NPV* danog projekta ako je cijena kapitala 8%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -1800 + \frac{6050}{1.08} + \frac{3750 - 10\,500}{1.08^2} + \frac{2500}{1.08^3}$$

Rješenje

a)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo NPV danog projekta ako je cijena kapitala 8%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -1800 + \frac{6050}{1.08} + \frac{3750 - 10\,500}{1.08^2} + \frac{2500}{1.08^3}$$

$$NPV = -0.6046$$

Rješenje

a)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo NPV danog projekta ako je cijena kapitala 8%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -1800 + \frac{6050}{1.08} + \frac{3750 - 10\,500}{1.08^2} + \frac{2500}{1.08^3}$$

$$NPV = -0.6046$$

Kako je $NPV < 0$, projekt nije isplativ uz cijenu kapitala od 8%.

b)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo *NPV* danog projekta ako je cijena kapitala 12%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

b)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo *NPV* danog projekta ako je cijena kapitala 12%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -1800 + \frac{6050}{1.12} + \frac{3750 - 10\,500}{1.12^2} + \frac{2500}{1.12^3}$$

b)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo *NPV* danog projekta ako je cijena kapitala 12%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -1800 + \frac{6050}{1.12} + \frac{3750 - 10\,500}{1.12^2} + \frac{2500}{1.12^3}$$

$$NPV = 0.18$$

b)

godina	1.	2.	3.
očekivani dobici	6050 kn	3750 kn	2500 kn

Izračunamo NPV danog projekta ako je cijena kapitala 12%.

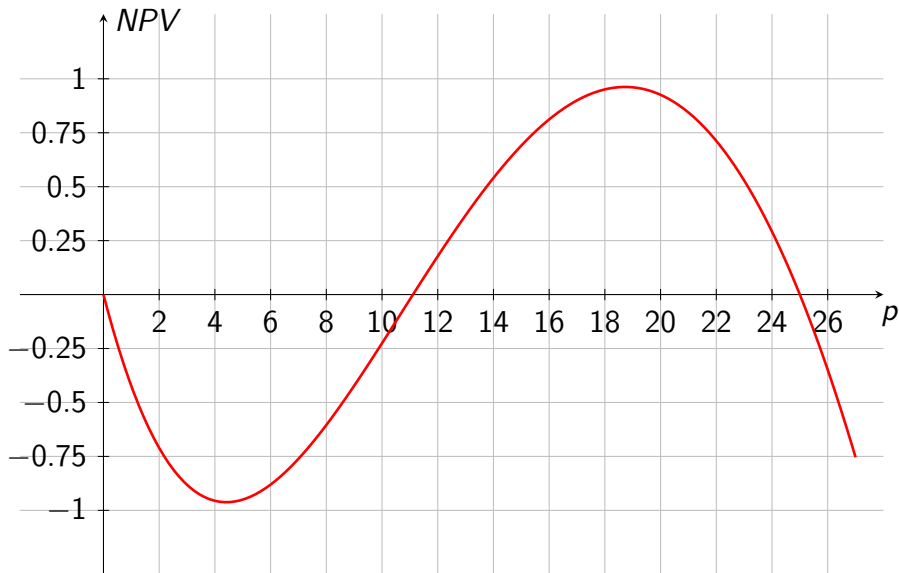
$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -1800 + \frac{6050}{1.12} + \frac{3750 - 10\,500}{1.12^2} + \frac{2500}{1.12^3}$$

$$NPV = 0.18$$

Kako je $NPV > 0$, projekt je isplativ uz cijenu kapitala od 12%.

NPV projekta kao funkcija cijene kapitala



treći zadatak

Zadatak 3

U projekt treba uložiti 9 000 €. Očekivani dobici na kraju prve tri godine su dani u tablici.

<i>godina</i>	<i>1.</i>	<i>2.</i>	<i>3.</i>
<i>očekivani dobici</i>	3000 €	3500 €	4000 €

- a) *Ispitajte koja je od ponuđenih kamatnih stopa 4.5445% i 7.7285% bolja aproksimacija za IRR zadanog projekta.*
- b) *Uz koju cijenu kapitala je promatrani projekt isplativ?*
- c) *Ako je cijena kapitala 5%, do kojeg uloženog iznosa je investitoru projekt isplativ?*

Rješenje

a)

godina	1.	2.	3.
očekivani dobiti	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 4.5445%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

Rješenje

a)

godina	1.	2.	3.
očekivani dobiti	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 4.5445%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -9000 + \frac{3000}{1.045445} + \frac{3500}{1.045445^2} + \frac{4000}{1.045445^3}$$

Rješenje

a)

godina	1.	2.	3.
očekivani dobiti	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 4.5445%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -9000 + \frac{3000}{1.045445} + \frac{3500}{1.045445^2} + \frac{4000}{1.045445^3}$$

$$NPV = 572.63064$$

godina	1.	2.	3.
očekivani dobici	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 7.7285%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

godina	1.	2.	3.
očekivani dobici	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 7.7285%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -9000 + \frac{3000}{1.077285} + \frac{3500}{1.077285^2} + \frac{4000}{1.077285^3}$$

godina	1.	2.	3.
očekivani dobici	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 7.7285%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -9000 + \frac{3000}{1.077285} + \frac{3500}{1.077285^2} + \frac{4000}{1.077285^3}$$

$$NPV = 0.00528$$

godina	1.	2.	3.
očekivani dobici	3000 €	3500 €	4000 €

Izračunamo *NPV* danog projekta za cijenu kapitala 7.7285%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$NPV = -9000 + \frac{3000}{1.077285} + \frac{3500}{1.077285^2} + \frac{4000}{1.077285^3}$$

$$NPV = 0.00528$$

Kako je za cijenu kapitala 7.7285% *NPV* projekta bliže nuli, zaključujemo da je kamatna stopa 7.7285% bolja aproksimacija za *IRR* danog projekta.

b) Projekt je isplativ po onoj cijeni kapitala p za koju vrijedi

$$p < IRR.$$

Iz prethodnog dijela zadatka smo dobili da je 7.7285% jako dobra aproksimacija za IRR promatranog projekta jer je po toj kamatnoj stopi NPV bio jako blizu nule. Stoga možemo reći da je projekt isplativ ako je cijena kapitala manja od 7.7285%.

c)

godina	1.	2.	3.
očekivani dobici	3000 €	3500 €	4000 €

Investitoru je projekt isplativ do onog iznosa za koji je $NPV = 0$ uz cijenu kapitala 5%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

c)

godina	1.	2.	3.
očekivani dobici	3000 €	3500 €	4000 €

Investitoru je projekt isplativ do onog iznosa za koji je $NPV = 0$ uz cijenu kapitala 5%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$0 = F_0 + \frac{3000}{1.05} + \frac{3500}{1.05^2} + \frac{4000}{1.05^3}$$

c)

godina	1.	2.	3.
očekivani dobiti	3000 €	3500 €	4000 €

Investitoru je projekt isplativ do onog iznosa za koji je $NPV = 0$ uz cijenu kapitala 5%.

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$0 = F_0 + \frac{3000}{1.05} + \frac{3500}{1.05^2} + \frac{4000}{1.05^3}$$

$$F_0 = -9487.10$$

c)

godina	1.	2.	3.
očekivani dobiti	3000 €	3500 €	4000 €

Investitoru je projekt isplativ do onog iznosa za koji je $NPV = 0$ uz cijenu kapitala 5%.

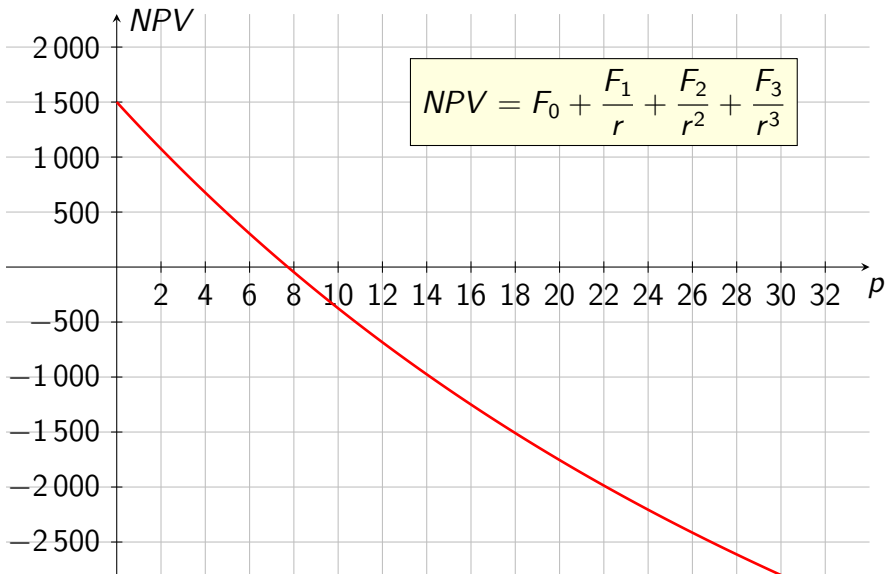
$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$0 = F_0 + \frac{3000}{1.05} + \frac{3500}{1.05^2} + \frac{4000}{1.05^3}$$

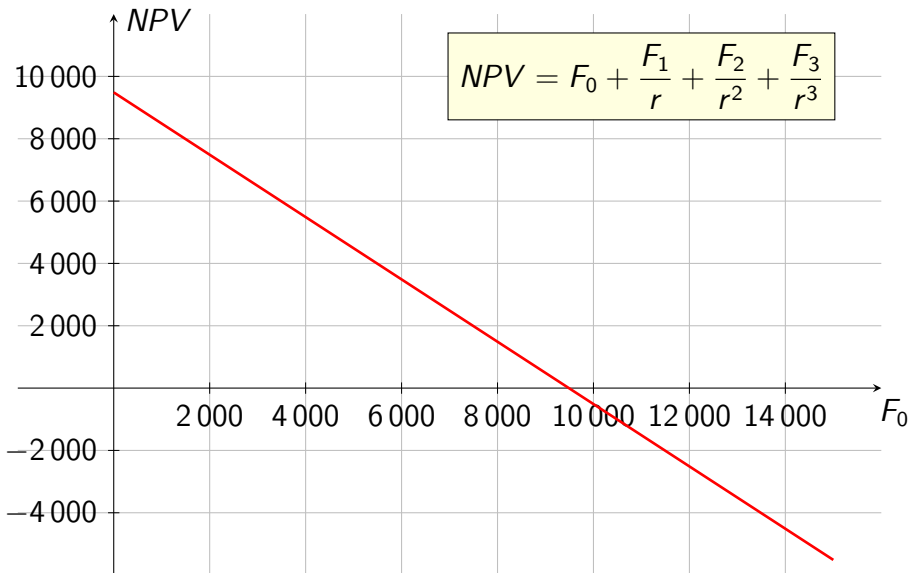
$$F_0 = -9487.10$$

Investitoru se isplati uložiti u projekt maksimalno 9487.10 €.

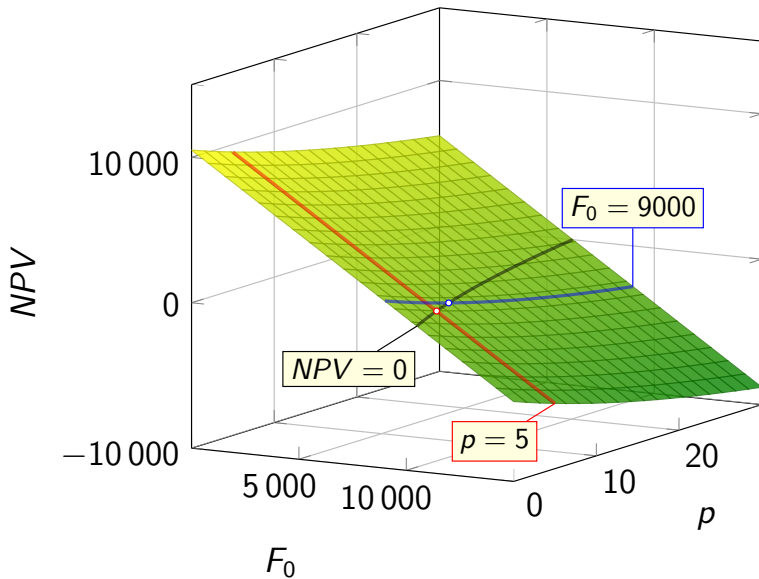
NPV projekta kao funkcija cijene kapitala



NPV projekta kao funkcija ulaganja uz $p = 5\%$



NPV kao funkcija ulaganja i cijene kapitala



čtvrti zadatak

Zadatak 4

Koliki prihod je na kraju druge godine ostvarila investicija visine 200 000 kn uz cijenu kapitala 7% ako je NPV jednak 14 559.82 kn, a prihodi na kraju prve i treće godine su redom 50 000 kn i 120 000 kn?

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$\frac{F_2}{r^2} = NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3}$$

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$\frac{F_2}{r^2} = NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3}$$

$$F_2 = r^2 \cdot \left(NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3} \right)$$

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$\frac{F_2}{r^2} = NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3}$$

$$F_2 = r^2 \cdot \left(NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3} \right)$$

$$F_2 = 1.07^2 \cdot \left(14\,559.82 + 200\,000 - \frac{50\,000}{1.07} - \frac{120\,000}{1.07^3} \right)$$

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$\frac{F_2}{r^2} = NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3}$$

$$F_2 = r^2 \cdot \left(NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3} \right)$$

$$F_2 = 1.07^2 \cdot \left(14\,559.82 + 200\,000 - \frac{50\,000}{1.07} - \frac{120\,000}{1.07^3} \right)$$

$$F_2 = 80\,000.01$$

Rješenje

- $r = 1.07$, $NPV = 14\,559.82$,
 $F_0 = -200\,000$, $F_1 = 50\,000$, $F_3 = 120\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3}$$

$$\frac{F_2}{r^2} = NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3}$$

$$F_2 = r^2 \cdot \left(NPV - F_0 - \frac{F_1}{r} - \frac{F_3}{r^3} \right)$$

$$F_2 = 1.07^2 \cdot \left(14\,559.82 + 200\,000 - \frac{50\,000}{1.07} - \frac{120\,000}{1.07^3} \right)$$

$$F_2 = 80\,000.01$$

Na kraju druge godine ostvaren je prihod od 80 000.01 kn.

peti zadatak

Zadatak 5

NPV i IRR metodom ocijenite je li isplativ projekt u koji treba uložiti 200 000 kn, a koji na kraju prve godine donosi 50 000 kn, a na kraju druge godine 180 000 kn dobiti. Investitor sredstva za ulaganje u projekt posuđuje po godišnjoj kamatnoj stopi od 7%.

Rješenje

- $r = 1.07$, $F_0 = -200\,000$, $F_1 = 50\,000$, $F_2 = 180\,000$

Rješenje

- $r = 1.07$, $F_0 = -200\,000$, $F_1 = 50\,000$, $F_2 = 180\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2}$$

Rješenje

- $r = 1.07$, $F_0 = -200\,000$, $F_1 = 50\,000$, $F_2 = 180\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2}$$

$$NPV = -200\,000 + \frac{50\,000}{1.07} + \frac{180\,000}{1.07^2}$$

Rješenje

- $r = 1.07$, $F_0 = -200\,000$, $F_1 = 50\,000$, $F_2 = 180\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2}$$

$$NPV = -200\,000 + \frac{50\,000}{1.07} + \frac{180\,000}{1.07^2}$$

$$NPV = 3947.94$$

Rješenje

- $r = 1.07$, $F_0 = -200\,000$, $F_1 = 50\,000$, $F_2 = 180\,000$

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2}$$

$$NPV = -200\,000 + \frac{50\,000}{1.07} + \frac{180\,000}{1.07^2}$$

$$NPV = 3947.94$$

Kako je $NPV > 0$, radi se o isplativom projektu.

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0$$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \Bigg/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \Big/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

$$r_1 = \frac{-5 \pm \sqrt{25 - 4 \cdot (-20) \cdot 18}}{2 \cdot (-20)}$$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \Bigg/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

$$r_1 = \frac{-5 \pm \sqrt{25 - 4 \cdot (-20) \cdot 18}}{2 \cdot (-20)}$$

$$r_1^{(1)} = -0.83188, \quad r_1^{(2)} = 1.08188$$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \bigg/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

$$r_1 = \frac{-5 \pm \sqrt{25 - 4 \cdot (-20) \cdot 18}}{2 \cdot (-20)}$$

~~$$r_1^{(1)} = 0.83188, \quad r_1^{(2)} = 1.08188$$~~

- $r_1 = 1.08188,$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \bigg/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

$$r_1 = \frac{-5 \pm \sqrt{25 - 4 \cdot (-20) \cdot 18}}{2 \cdot (-20)}$$

~~$$r_1^{(1)} = 0.83188, \quad r_1^{(2)} = 1.08188$$~~

- $r_1 = 1.08188, \quad IRR = 100(r_1 - 1),$

- $r_1 = 1 + \frac{IRR}{100}, \quad F_0 = -200\,000, \quad F_1 = 50\,000, \quad F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \bigg/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

$$r_1 = \frac{-5 \pm \sqrt{25 - 4 \cdot (-20) \cdot 18}}{2 \cdot (-20)}$$

~~$$r_1^{(1)} = 0.83188, \quad r_1^{(2)} = 1.08188$$~~

- $r_1 = 1.08188, \quad IRR = 100(r_1 - 1), \quad IRR = 8.188\%$

- $r_1 = 1 + \frac{IRR}{100}$, $F_0 = -200\,000$, $F_1 = 50\,000$, $F_2 = 180\,000$

$$NPV = 0$$

$$F_0 + \frac{F_1}{r_1} + \frac{F_2}{r_1^2} = 0$$

$$-200\,000 + \frac{50\,000}{r_1} + \frac{180\,000}{r_1^2} = 0 \quad \bigg/ \cdot \frac{r_1^2}{10\,000}$$

$$-20r_1^2 + 5r_1 + 18 = 0$$

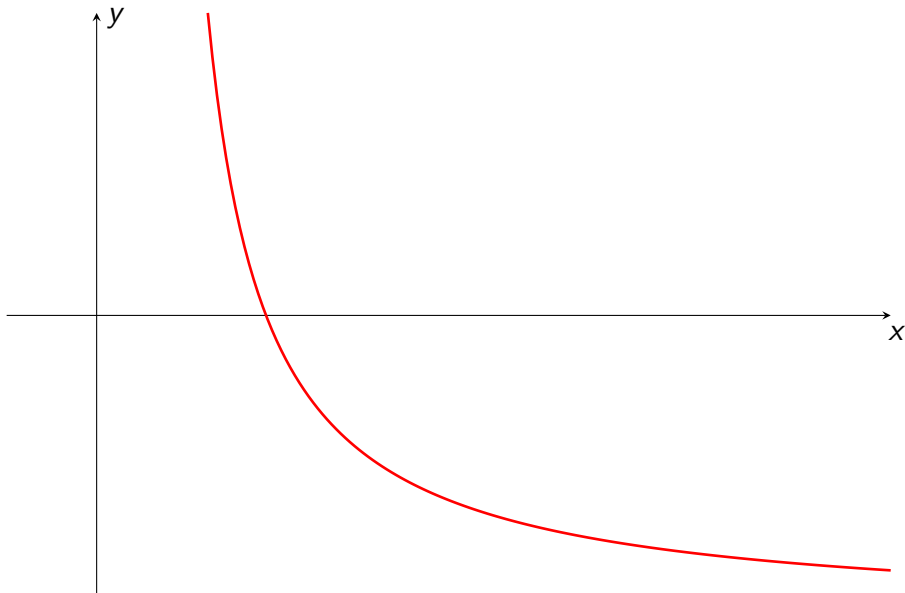
$$r_1 = \frac{-5 \pm \sqrt{25 - 4 \cdot (-20) \cdot 18}}{2 \cdot (-20)}$$

~~$$r_1^{(1)} = 0.83188, \quad r_1^{(2)} = 1.08188$$~~

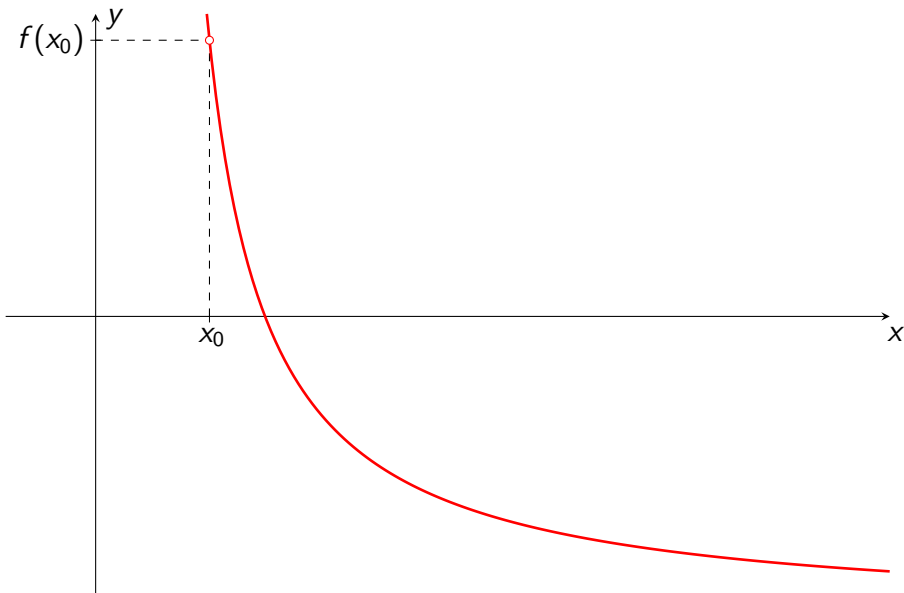
- $r_1 = 1.08188$, $IRR = 100(r_1 - 1)$, $IRR = 8.188\%$
- IRR je veći od zadane cijene kapitala pa je projekt isplativ.

Metoda Regula falsi

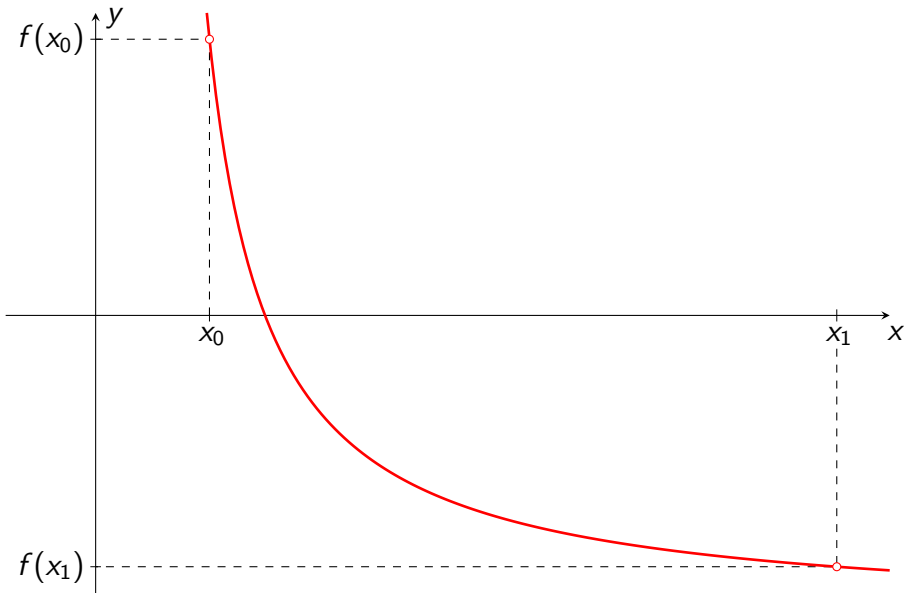
Metoda Regula falsi



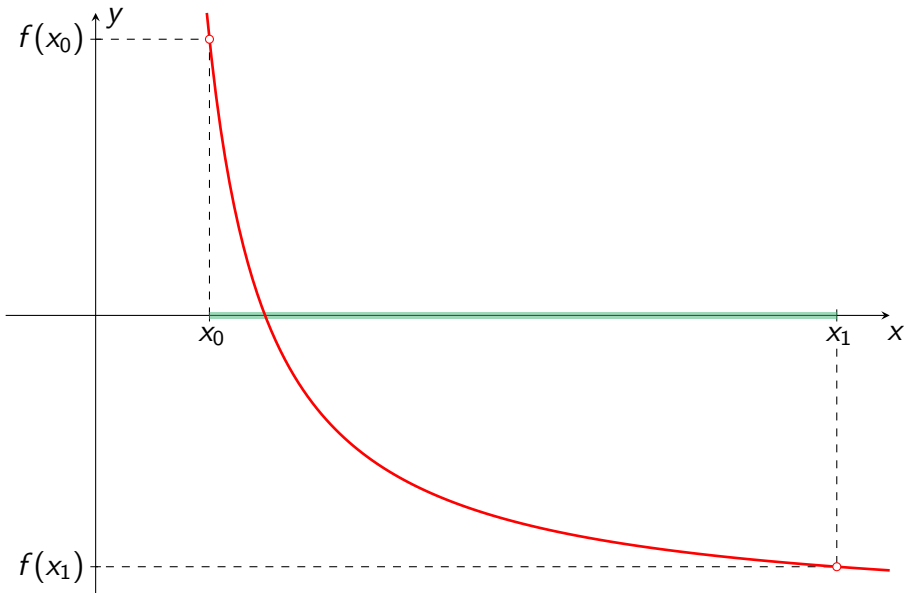
Metoda Regula falsi



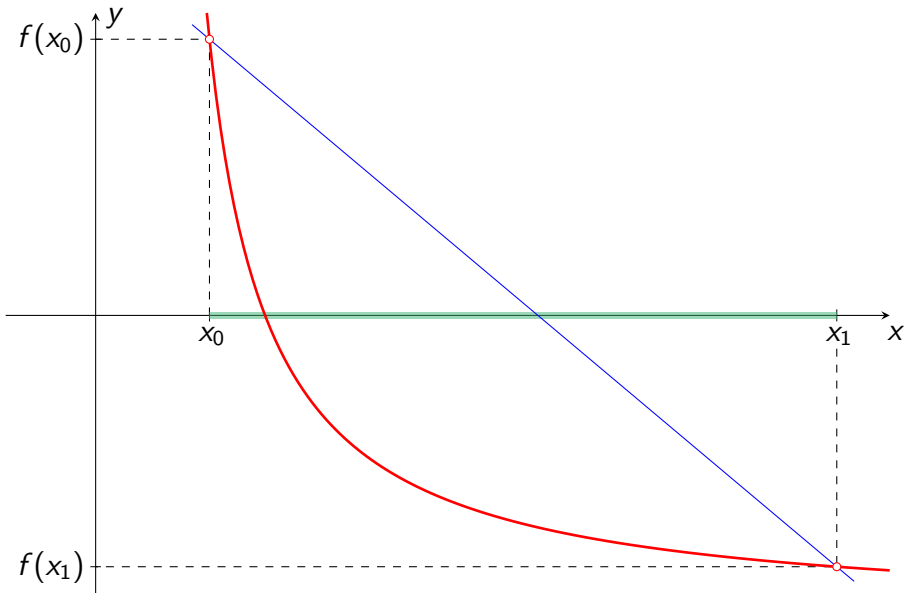
Metoda Regula falsi



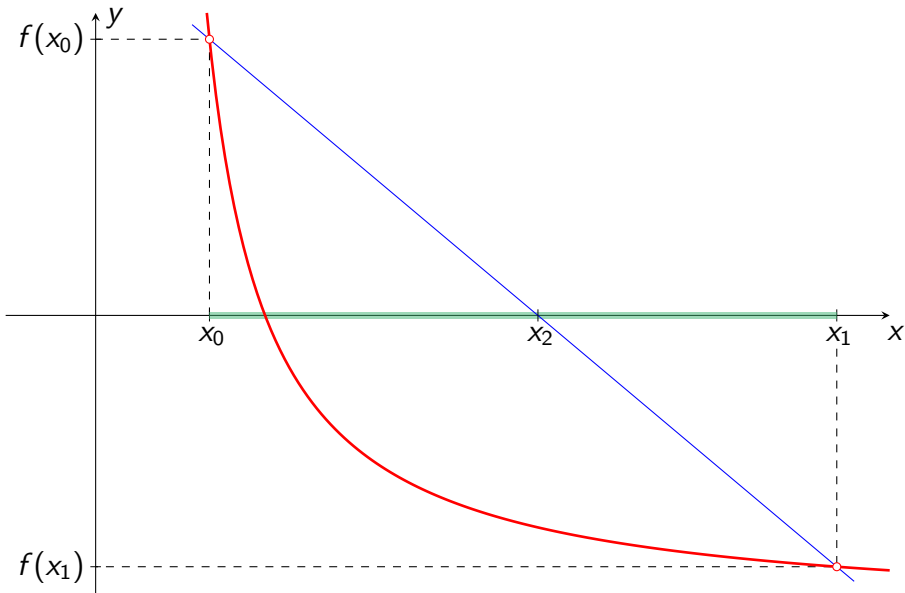
Metoda Regula falsi



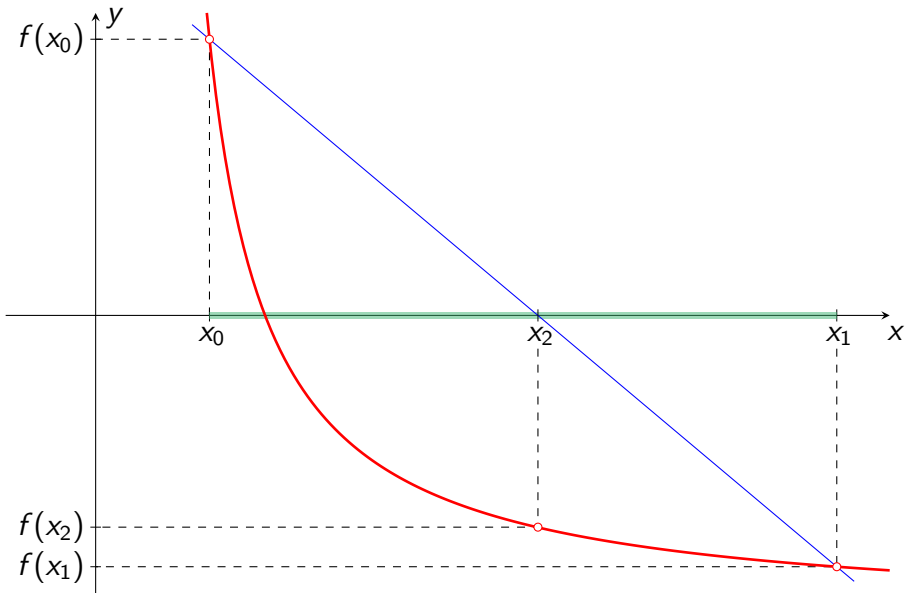
Metoda Regula falsi



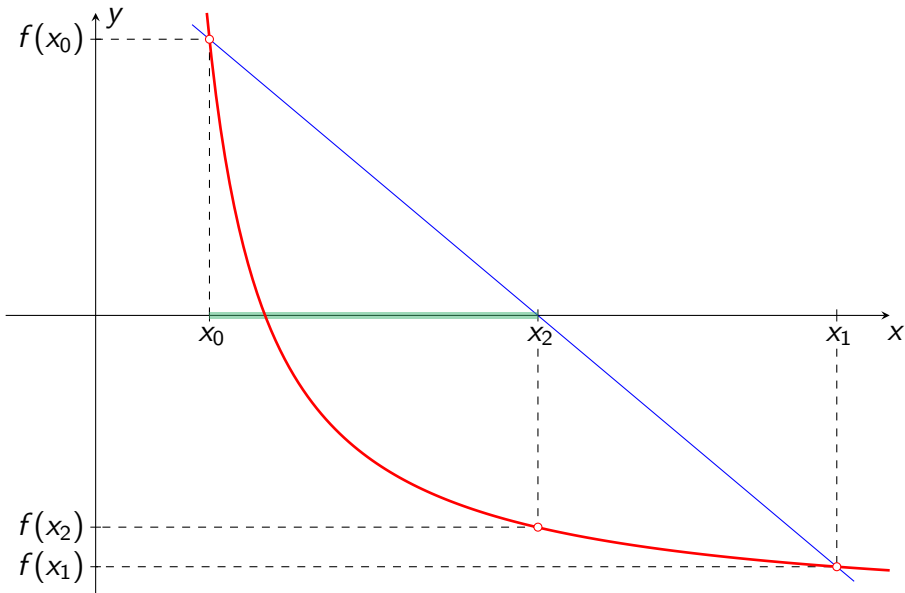
Metoda Regula falsi



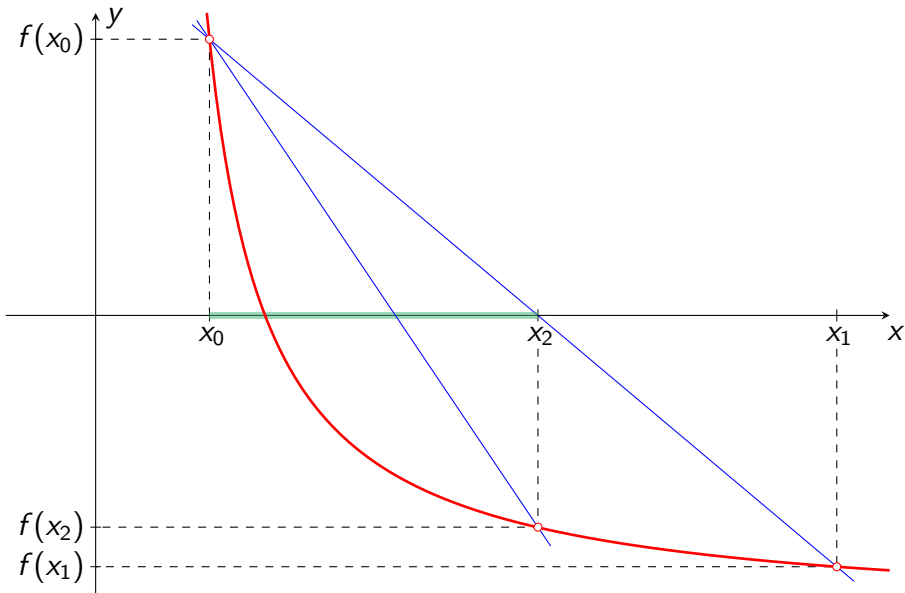
Metoda Regula falsi



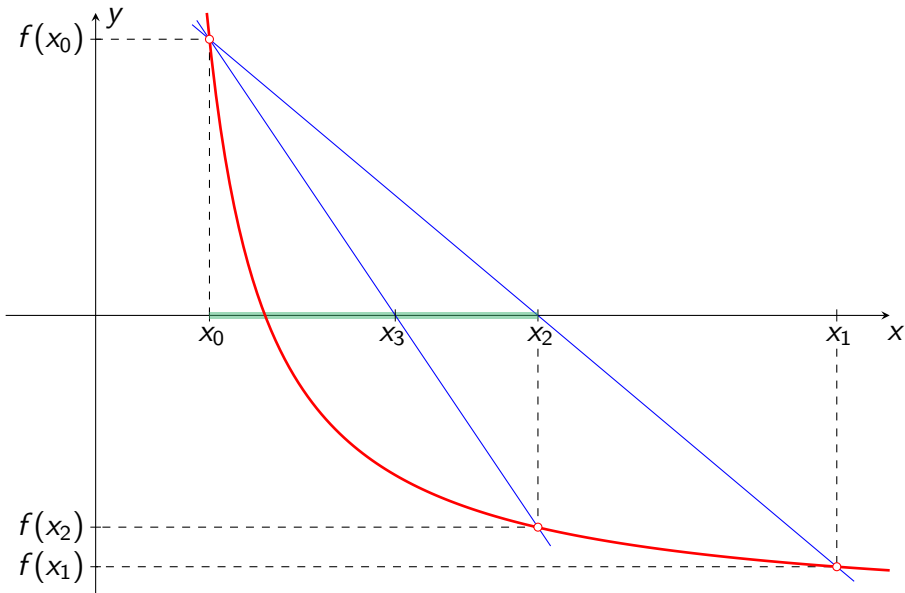
Metoda Regula falsi



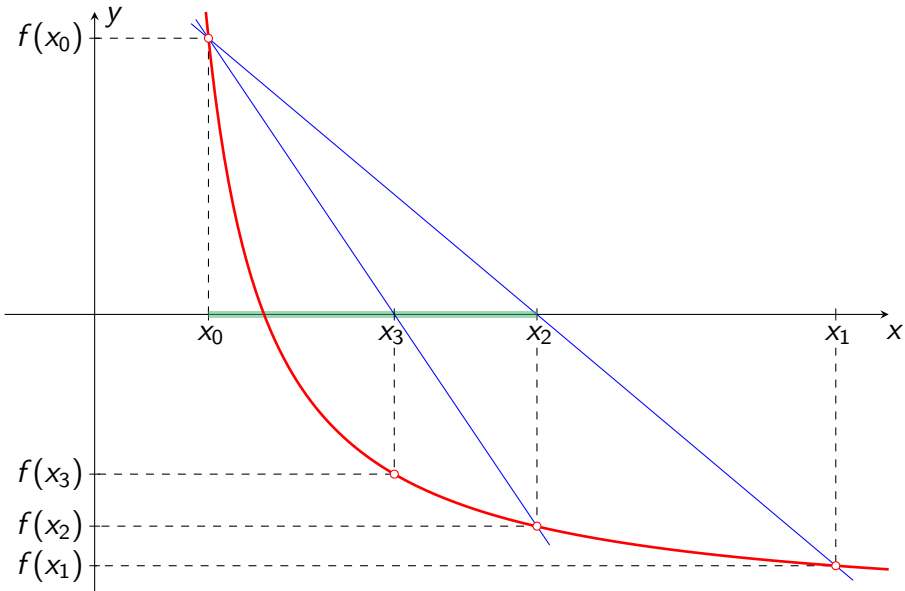
Metoda Regula falsi



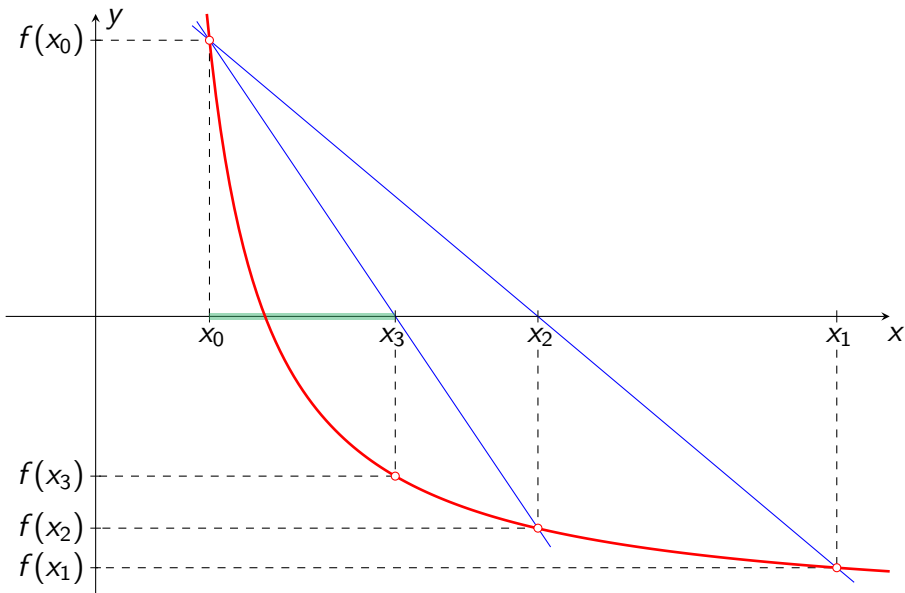
Metoda Regula falsi



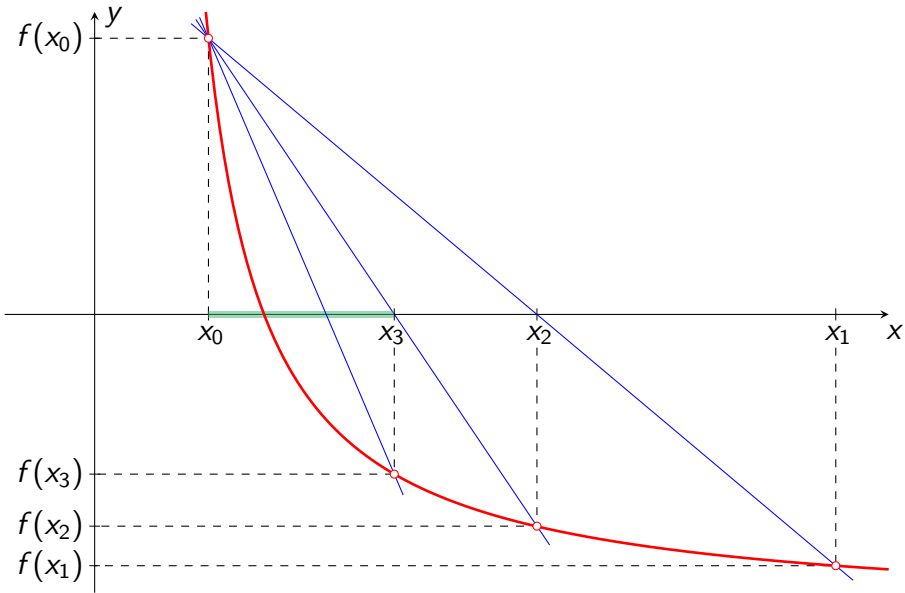
Metoda Regula falsi



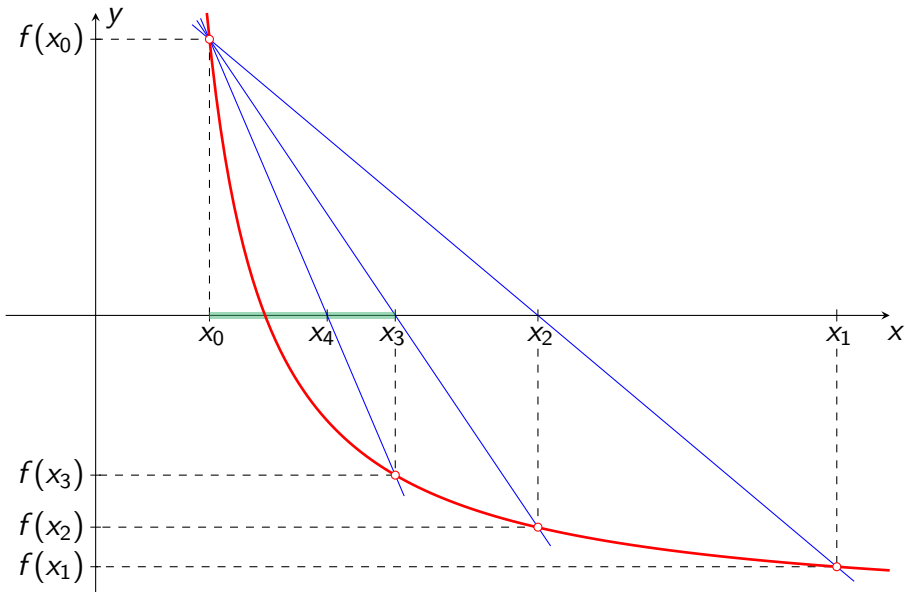
Metoda Regula falsi



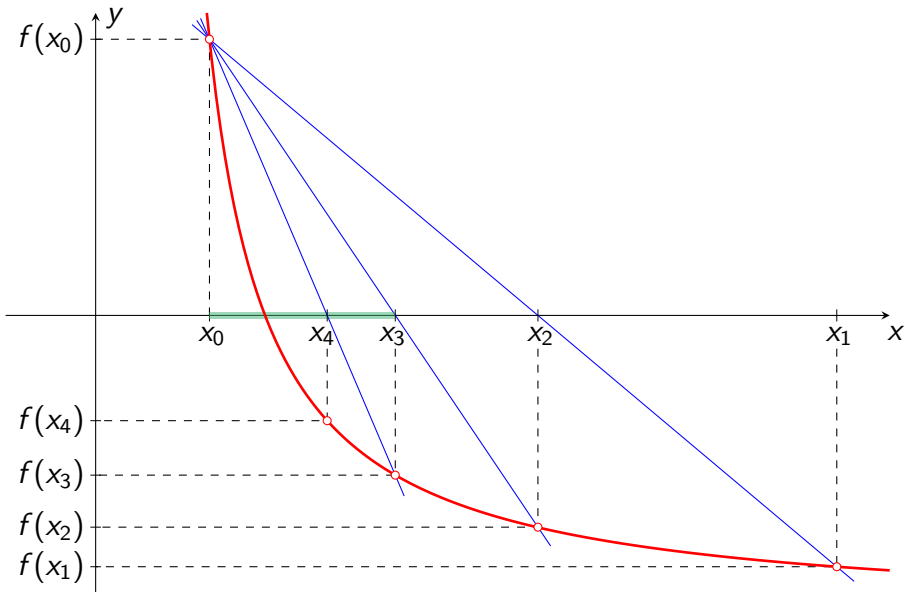
Metoda Regula falsi



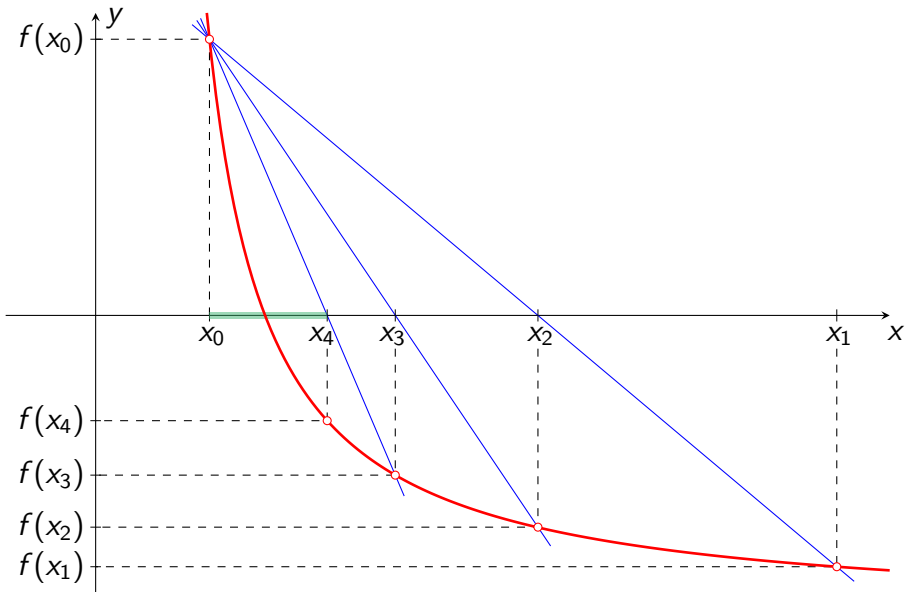
Metoda Regula falsi



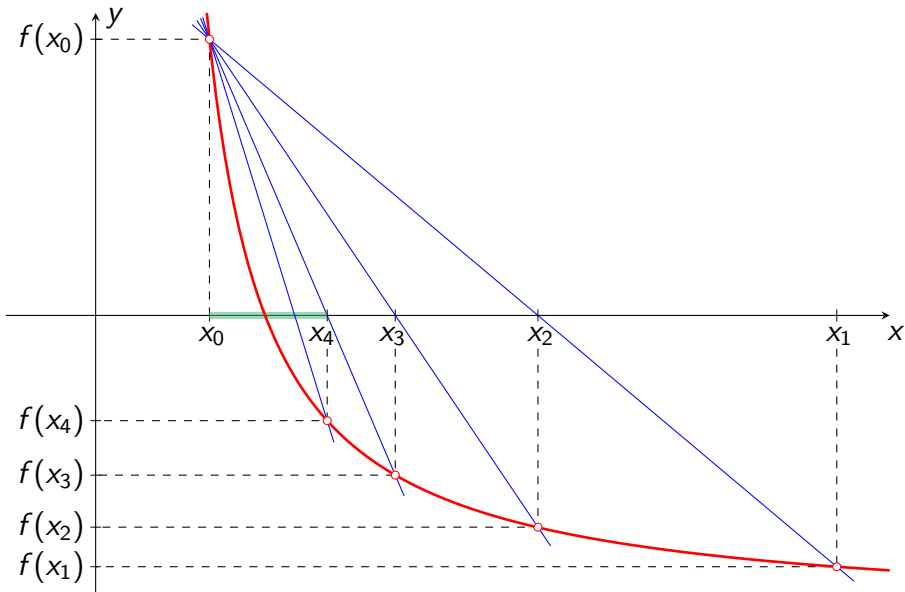
Metoda Regula falsi



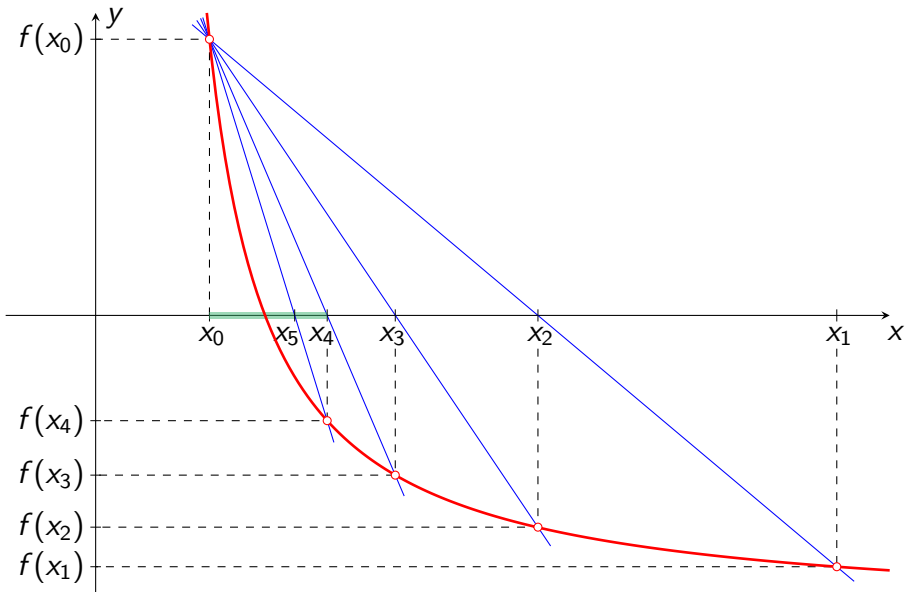
Metoda Regula falsi



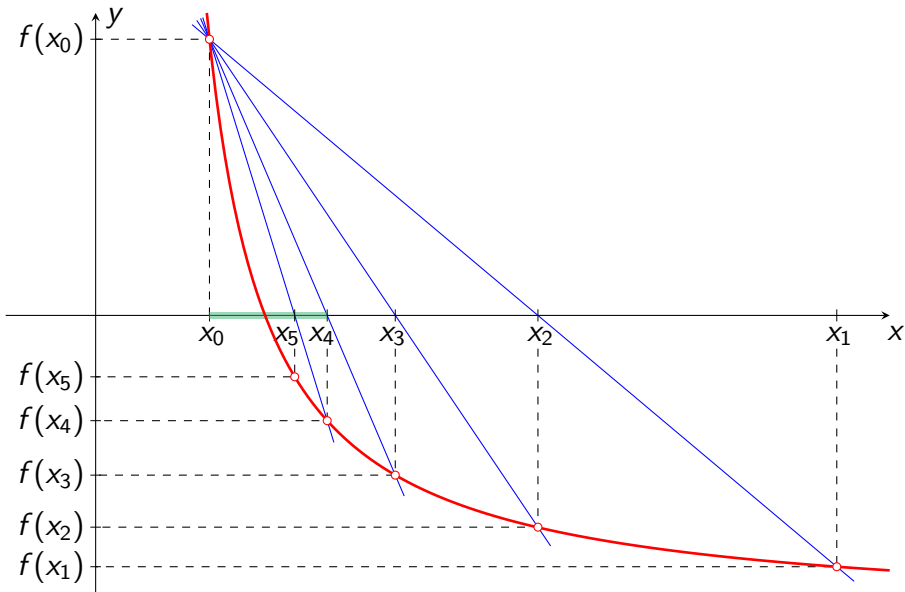
Metoda Regula falsi



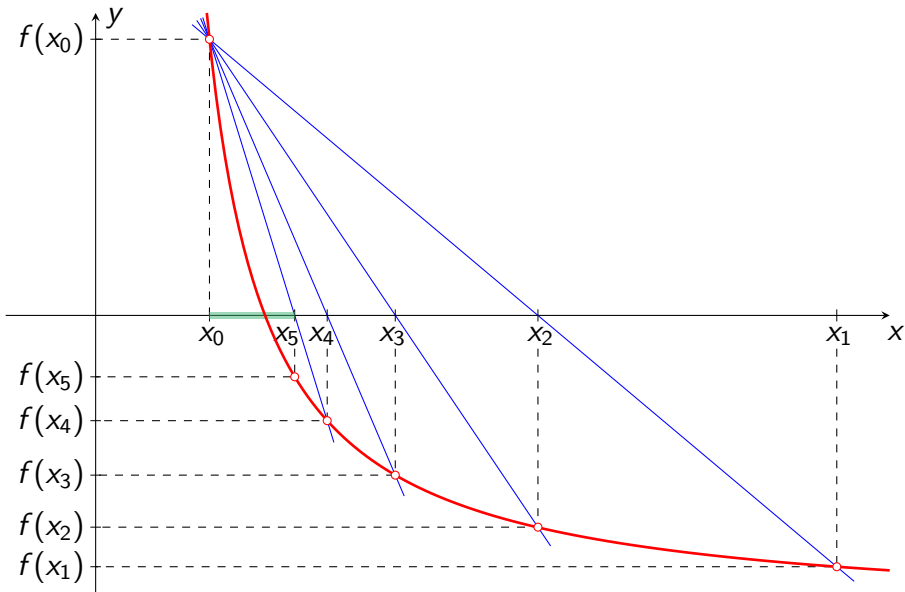
Metoda Regula falsi



Metoda Regula falsi



Metoda Regula falsi



Metoda Regula falsi

- Zadane točky: $(a, f(a))$, $(b, f(b))$

Metoda Regula falsi

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

- Zadane točke: $(a, f(a))$, $(b, f(b))$
- Jednadžba pravca kroz zadane točke

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

Metoda Regula falsi

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

- Zadane točke: $(a, f(a))$, $(b, f(b))$
- Jednadžba pravca kroz zadane točke

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

- Tražimo presjek pravca s x-osi

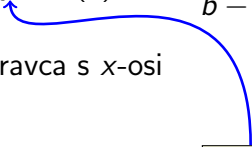
Metoda Regula falsi

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

- Zadane točke: $(a, f(a))$, $(b, f(b))$
- Jednadžba pravca kroz zadane točke

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

- Tražimo presjek pravca s x-osi

$$y = 0$$


Metoda Regula falsi

$$-f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

Metoda Regula falsi

$$-f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

$$\frac{f(b) - f(a)}{b - a} \cdot x = -f(a) + a \cdot \frac{f(b) - f(a)}{b - a}$$

Metoda Regula falsi

$$-f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

$$\frac{f(b) - f(a)}{b - a} \cdot x = -f(a) + a \cdot \frac{f(b) - f(a)}{b - a} \Bigg/ \cdot \frac{b - a}{f(b) - f(a)}$$

$$x = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)} + a$$

Metoda Regula falsi

$$-f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

$$\frac{f(b) - f(a)}{b - a} \cdot x = -f(a) + a \cdot \frac{f(b) - f(a)}{b - a} \Bigg/ \cdot \frac{b - a}{f(b) - f(a)}$$

$$x = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)} + a$$

$$x = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

Algoritam: Regula falsi

Ulaz: Nепrekidna funkcija na intervalu $\langle a, b \rangle$ takva da je $f(a) \cdot f(b) < 0$.

Izlaz: Nultočka funkcije f na intervalu $\langle a, b \rangle$ na zadanu točnost.

while *uvjet konvergencije nije istinit* **do**

$$c := \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)};$$

if $f(a) \cdot f(c) < 0$ **then**

$b := c$;

else

$a := c$;

end

end

return c

Uvjeti konvergencije

Neka je c_i vrijednost broja c dobivena u i -toj iteraciji.

- Petlju možemo prekinuti u trenutku kada je

$$|c_i - c_{i-1}| < \delta$$

za neki unaprijed zadani mali broj $\delta > 0$.

- Ili, petlju možemo prekinuti u trenutku kada je

$$|f(c_i)| < \epsilon$$

za neki unaprijed zadani mali broj $\epsilon > 0$.

- Također, iz sigurnosnih razloga u implementaciji ograničimo maksimalni broj iteracija u slučaju da niti jedan od gornja dva uvjeta ne bi mogao biti zadovoljen nakon razumnog broja iteracija.

šesti zadatak

Zadatak 6

Banka mora odlučiti između dviju investicija u iznosu od 200 000 € čiji novčani tokovi su procijenjeni u sljedećoj tablici.

<i>godina</i>	0.	1.	2.	3.	4.	5.
<i>A</i>	−200 000	80 000	60 000	70 000	100 000	100 000
<i>B</i>	−200 000	30 000	85 000	95 000	90 000	85 000

- a) *Odredite NPV oba projekta uz cijenu kapitala 7%.*
- b) *Približno odredite IRR oba projekta pomoću metode Regula falsi.*

Rješenje

a)

godina	0.	1.	2.	3.	4.	5.
A	−200 000	80 000	60 000	70 000	100 000	100 000
B	−200 000	30 000	85 000	95 000	90 000	85 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4} + \frac{F_5}{r^5}$$

Rješenje

a)

godina	0.	1.	2.	3.	4.	5.
A	-200 000	80 000	60 000	70 000	100 000	100 000
B	-200 000	30 000	85 000	95 000	90 000	85 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4} + \frac{F_5}{r^5}$$

$$NPV(A) = -200\,000 + \frac{80\,000}{1.07} + \frac{60\,000}{1.07^2} + \frac{70\,000}{1.07^3} + \frac{100\,000}{1.07^4} + \frac{100\,000}{1.07^5}$$

Rješenje

a)

godina	0.	1.	2.	3.	4.	5.
A	-200 000	80 000	60 000	70 000	100 000	100 000
B	-200 000	30 000	85 000	95 000	90 000	85 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4} + \frac{F_5}{r^5}$$

$$NPV(A) = -200\,000 + \frac{80\,000}{1.07} + \frac{60\,000}{1.07^2} + \frac{70\,000}{1.07^3} + \frac{100\,000}{1.07^4} + \frac{100\,000}{1.07^5}$$

$$NPV(A) = 131\,901.67$$

Rješenje

a)

godina	0.	1.	2.	3.	4.	5.
A	-200 000	80 000	60 000	70 000	100 000	100 000
B	-200 000	30 000	85 000	95 000	90 000	85 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4} + \frac{F_5}{r^5}$$

$$NPV(A) = -200\,000 + \frac{80\,000}{1.07} + \frac{60\,000}{1.07^2} + \frac{70\,000}{1.07^3} + \frac{100\,000}{1.07^4} + \frac{100\,000}{1.07^5}$$

$$NPV(A) = 131\,901.67$$

$$NPV(B) = -200\,000 + \frac{30\,000}{1.07} + \frac{85\,000}{1.07^2} + \frac{95\,000}{1.07^3} + \frac{90\,000}{1.07^4} + \frac{85\,000}{1.07^5}$$

Rješenje

a)

godina	0.	1.	2.	3.	4.	5.
A	-200 000	80 000	60 000	70 000	100 000	100 000
B	-200 000	30 000	85 000	95 000	90 000	85 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4} + \frac{F_5}{r^5}$$

$$NPV(A) = -200\,000 + \frac{80\,000}{1.07} + \frac{60\,000}{1.07^2} + \frac{70\,000}{1.07^3} + \frac{100\,000}{1.07^4} + \frac{100\,000}{1.07^5}$$

$$NPV(A) = 131\,901.67$$

$$NPV(B) = -200\,000 + \frac{30\,000}{1.07} + \frac{85\,000}{1.07^2} + \frac{95\,000}{1.07^3} + \frac{90\,000}{1.07^4} + \frac{85\,000}{1.07^5}$$

$$NPV(B) = 109\,092.37$$

Rješenje

a) Projekt A je isplativiji jer je $NPV(A) > 0$ i $NPV(A) > NPV(B)$.

godina	0.	1.	2.	3.	4.	5.
A	-200 000	80 000	60 000	70 000	100 000	100 000
B	-200 000	30 000	85 000	95 000	90 000	85 000

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \frac{F_3}{r^3} + \frac{F_4}{r^4} + \frac{F_5}{r^5}$$

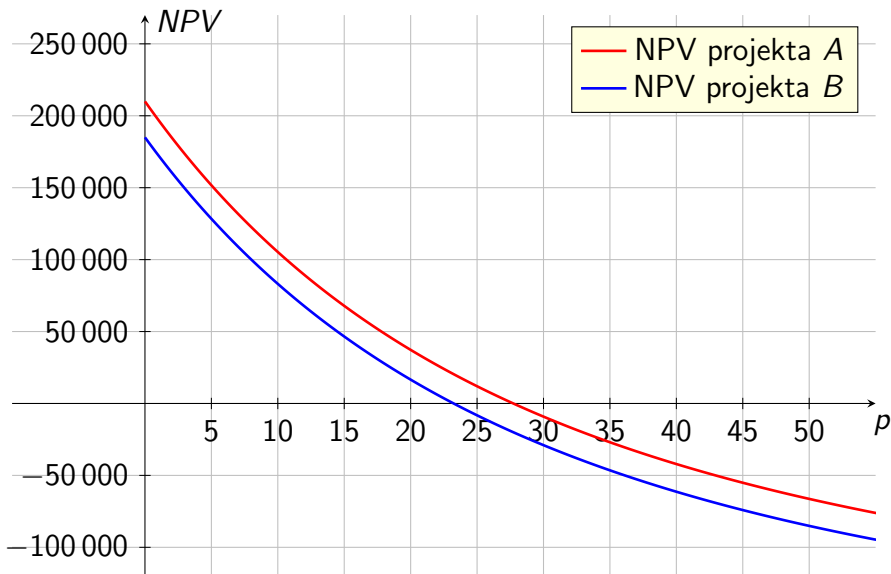
$$NPV(A) = -200\,000 + \frac{80\,000}{1.07} + \frac{60\,000}{1.07^2} + \frac{70\,000}{1.07^3} + \frac{100\,000}{1.07^4} + \frac{100\,000}{1.07^5}$$

$$NPV(A) = 131\,901.67$$

$$NPV(B) = -200\,000 + \frac{30\,000}{1.07} + \frac{85\,000}{1.07^2} + \frac{95\,000}{1.07^3} + \frac{90\,000}{1.07^4} + \frac{85\,000}{1.07^5}$$

$$NPV(B) = 109\,092.37$$

NPV projekata A i B kao funkcija cijene kapitala



b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	
2			
3			
4			

	NPV_A
25%	
30%	

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	
2			
3			
4			

	NPV_A
25%	
30%	

$$\begin{aligned}
 NPV_A(25) = & -200\,000 + \frac{80\,000}{1.25} + \frac{60\,000}{1.25^2} + \\
 & + \frac{70\,000}{1.25^3} + \frac{100\,000}{1.25^4} + \frac{100\,000}{1.25^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	
2			
3			
4			

	NPV_A
25%	11968
30%	

$$\begin{aligned}
 NPV_A(25) = & -200\,000 + \frac{80\,000}{1.25} + \frac{60\,000}{1.25^2} + \\
 & + \frac{70\,000}{1.25^3} + \frac{100\,000}{1.25^4} + \frac{100\,000}{1.25^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	
2			
3			
4			

	NPV_A
25%	11968
30%	

$$\begin{aligned}
 NPV_A(30) = & -200\,000 + \frac{80\,000}{1.3} + \frac{60\,000}{1.3^2} + \\
 & + \frac{70\,000}{1.3^3} + \frac{100\,000}{1.3^4} + \frac{100\,000}{1.3^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	
2			
3			
4			

	NPV_A
25%	11968
30%	- 9151.26

$$NPV_A(30) = -200\,000 + \frac{80\,000}{1.3} + \frac{60\,000}{1.3^2} +$$

$$+ \frac{70\,000}{1.3^3} + \frac{100\,000}{1.3^4} + \frac{100\,000}{1.3^5}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	
2			
3			
4			

	NPV_A
25%	11968
30%	- 9151.26

$$c_1 = \frac{25 \cdot NPV_A(30) - 30 \cdot NPV_A(25)}{NPV_A(30) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2			
3			
4			

	NPV_A
25%	11968
30%	- 9151.26

$$c_1 = \frac{25 \cdot NPV_A(30) - 30 \cdot NPV_A(25)}{NPV_A(30) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2			
3			
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	

$$\begin{aligned}
 NPV_A(27.83343) = & -200\,000 + \frac{80\,000}{1.2783343} + \frac{60\,000}{1.2783343^2} + \\
 & + \frac{70\,000}{1.2783343^3} + \frac{100\,000}{1.2783343^4} + \frac{100\,000}{1.2783343^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2			
3			
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33

$$\begin{aligned}
 NPV_A(27.83343) = & -200\,000 + \frac{80\,000}{1.2783343} + \frac{60\,000}{1.2783343^2} + \\
 & + \frac{70\,000}{1.2783343^3} + \frac{100\,000}{1.2783343^4} + \frac{100\,000}{1.2783343^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	
3			
4			

	NPV_A
25%	11968
30%	− 9151.26
27.83343%	− 451.33

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	
3			
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33

$$c_2 = \frac{25 \cdot NPV_A(27.83343) - 27.83343 \cdot NPV_A(25)}{NPV_A(27.83343) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3			
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33

$$c_2 = \frac{25 \cdot NPV_A(27.83343) - 27.83343 \cdot NPV_A(25)}{NPV_A(27.83343) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3			
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	

$$\begin{aligned}
 NPV_A(27.73046) = & -200\,000 + \frac{80\,000}{1.2773046} + \frac{60\,000}{1.2773046^2} + \\
 & + \frac{70\,000}{1.2773046^3} + \frac{100\,000}{1.2773046^4} + \frac{100\,000}{1.2773046^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3			
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39

$$\begin{aligned}
 NPV_A(27.73046) = & -200\,000 + \frac{80\,000}{1.2773046} + \frac{60\,000}{1.2773046^2} + \\
 & + \frac{70\,000}{1.2773046^3} + \frac{100\,000}{1.2773046^4} + \frac{100\,000}{1.2773046^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	
4			

	NPV_A
25%	11968
30%	− 9151.26
27.83343%	− 451.33
27.73046%	− 21.39

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39

$$c_3 = \frac{25 \cdot NPV_A(27.73046) - 27.73046 \cdot NPV_A(25)}{NPV_A(27.73046) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39

$$c_3 = \frac{25 \cdot NPV_A(27.73046) - 27.73046 \cdot NPV_A(25)}{NPV_A(27.73046) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	

$$\begin{aligned}
 NPV_A(27.72559) = & -200\,000 + \frac{80\,000}{1.2772559} + \frac{60\,000}{1.2772559^2} + \\
 & + \frac{70\,000}{1.2772559^3} + \frac{100\,000}{1.2772559^4} + \frac{100\,000}{1.2772559^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4			

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	- 1.01

$$\begin{aligned}
 NPV_A(27.72559) = & -200\,000 + \frac{80\,000}{1.2772559} + \frac{60\,000}{1.2772559^2} + \\
 & + \frac{70\,000}{1.2772559^3} + \frac{100\,000}{1.2772559^4} + \frac{100\,000}{1.2772559^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4	25	27.72559	

	NPV_A
25%	11968
30%	− 9151.26
27.83343%	− 451.33
27.73046%	− 21.39
27.72559%	− 1.01

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4	25	27.72559	

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	- 1.01

$$c_4 = \frac{25 \cdot NPV_A(27.72559) - 27.72559 \cdot NPV_A(25)}{NPV_A(27.72559) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4	25	27.72559	27.72536

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	- 1.01

$$c_4 = \frac{25 \cdot NPV_A(27.72559) - 27.72559 \cdot NPV_A(25)}{NPV_A(27.72559) - NPV_A(25)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4	25	27.72559	27.72536

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	- 1.01
27.72536%	

$$\begin{aligned}
 NPV_A(27.72536) = & -200\,000 + \frac{80\,000}{1.2772536} + \frac{60\,000}{1.2772536^2} + \\
 & + \frac{70\,000}{1.2772536^3} + \frac{100\,000}{1.2772536^4} + \frac{100\,000}{1.2772536^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4	25	27.72559	27.72536

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	- 1.01
27.72536%	- 0.05

$$\begin{aligned}
 NPV_A(27.72536) = & -200\,000 + \frac{80\,000}{1.2772536} + \frac{60\,000}{1.2772536^2} + \\
 & + \frac{70\,000}{1.2772536^3} + \frac{100\,000}{1.2772536^4} + \frac{100\,000}{1.2772536^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	25	30	27.83343
2	25	27.83343	27.73046
3	25	27.73046	27.72559
4	25	27.72559	27.72536

	NPV_A
25%	11968
30%	- 9151.26
27.83343%	- 451.33
27.73046%	- 21.39
27.72559%	- 1.01
27.72536%	- 0.05

$$IRR(A) = 27.72536\%$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	
2			
3			
4			

	NPV_B
20%	
25%	

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	
2			
3			
4			

	NPV_B
20%	
25%	

$$\begin{aligned}
 NPV_B(20) = & -200\,000 + \frac{30\,000}{1.2} + \frac{85\,000}{1.2^2} + \\
 & + \frac{95\,000}{1.2^3} + \frac{90\,000}{1.2^4} + \frac{85\,000}{1.2^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	
2			
3			
4			

	NPV_B
20%	16567
25%	

$$\begin{aligned}
 NPV_B(20) = & -200\,000 + \frac{30\,000}{1.2} + \frac{85\,000}{1.2^2} + \\
 & + \frac{95\,000}{1.2^3} + \frac{90\,000}{1.2^4} + \frac{85\,000}{1.2^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	
2			
3			
4			

	NPV_B
20%	16567
25%	

$$\begin{aligned}
 NPV_B(25) = & -200\,000 + \frac{30\,000}{1.25} + \frac{85\,000}{1.25^2} + \\
 & + \frac{95\,000}{1.25^3} + \frac{90\,000}{1.25^4} + \frac{85\,000}{1.25^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	
2			
3			
4			

	NPV_B
20%	16567
25%	- 8243.2

$$\begin{aligned}
 NPV_B(25) = & -200\,000 + \frac{30\,000}{1.25} + \frac{85\,000}{1.25^2} + \\
 & + \frac{95\,000}{1.25^3} + \frac{90\,000}{1.25^4} + \frac{85\,000}{1.25^5} +
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	
2			
3			
4			

	NPV_B
20%	16567
25%	- 8243.2

$$c_1 = \frac{20 \cdot NPV_B(25) - 25 \cdot NPV_B(20)}{NPV_B(25) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2			
3			
4			

	NPV_B
20%	16567
25%	- 8243.2

$$c_1 = \frac{20 \cdot NPV_B(25) - 25 \cdot NPV_B(20)}{NPV_B(25) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2			
3			
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	

$$\begin{aligned}
 NPV_B(23.33875) = & -200\,000 + \frac{30\,000}{1.2333875} + \frac{85\,000}{1.2333875^2} + \\
 & + \frac{95\,000}{1.2333875^3} + \frac{90\,000}{1.2333875^4} + \frac{85\,000}{1.2333875^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2			
3			
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13

$$\begin{aligned}
 NPV_B(23.33875) = & -200\,000 + \frac{30\,000}{1.2333875} + \frac{85\,000}{1.2333875^2} + \\
 & + \frac{95\,000}{1.2333875^3} + \frac{90\,000}{1.2333875^4} + \frac{85\,000}{1.2333875^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	
3			
4			

	NPV_B
20%	16567
25%	− 8243.2
23.33875%	− 499.13

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	
3			
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13

$$c_2 = \frac{20 \cdot NPV_B(23.33875) - 23.33875 \cdot NPV_B(20)}{NPV_B(23.33875) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3			
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13

$$c_2 = \frac{20 \cdot NPV_B(23.33875) - 23.33875 \cdot NPV_B(20)}{NPV_B(23.33875) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3			
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	

$$\begin{aligned}
 NPV_B(23.2411) = & -200\,000 + \frac{30\,000}{1.232411} + \frac{85\,000}{1.232411^2} + \\
 & + \frac{95\,000}{1.232411^3} + \frac{90\,000}{1.232411^4} + \frac{85\,000}{1.232411^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3			
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25

$$\begin{aligned}
 NPV_B(23.2411) = & -200\,000 + \frac{30\,000}{1.232411} + \frac{85\,000}{1.232411^2} + \\
 & + \frac{95\,000}{1.232411^3} + \frac{90\,000}{1.232411^4} + \frac{85\,000}{1.232411^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	
4			

	NPV_B
20%	16567
25%	− 8243.2
23.33875%	− 499.13
23.2411%	− 29.25

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25

$$c_3 = \frac{20 \cdot NPV_B(23.2411) - 23.2411 \cdot NPV_B(20)}{NPV_B(23.2411) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25

$$c_3 = \frac{20 \cdot NPV_B(23.2411) - 23.2411 \cdot NPV_B(20)}{NPV_B(23.2411) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	

$$\begin{aligned}
 NPV_B(23.23539) = & -200\,000 + \frac{30\,000}{1.2323539} + \frac{85\,000}{1.2323539^2} + \\
 & + \frac{95\,000}{1.2323539^3} + \frac{90\,000}{1.2323539^4} + \frac{85\,000}{1.2323539^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4			

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72

$$\begin{aligned}
 NPV_B(23.23539) = & -200\,000 + \frac{30\,000}{1.2323539} + \frac{85\,000}{1.2323539^2} + \\
 & + \frac{95\,000}{1.2323539^3} + \frac{90\,000}{1.2323539^4} + \frac{85\,000}{1.2323539^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4	20	23.23539	

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4	20	23.23539	

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72

$$c_4 = \frac{20 \cdot NPV_B(23.23539) - 23.23539 \cdot NPV_B(20)}{NPV_B(23.23539) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4	20	23.23539	23.23505

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72

$$c_4 = \frac{20 \cdot NPV_B(23.23539) - 23.23539 \cdot NPV_B(20)}{NPV_B(23.23539) - NPV_B(20)}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4	20	23.23539	23.23505

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72
23.23505%	

$$\begin{aligned}
 NPV_B(23.23505) = & -200\,000 + \frac{30\,000}{1.2323505} + \frac{85\,000}{1.2323505^2} + \\
 & + \frac{95\,000}{1.2323505^3} + \frac{90\,000}{1.2323505^4} + \frac{85\,000}{1.2323505^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4	20	23.23539	23.23505

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72
23.23505%	- 0.08

$$\begin{aligned}
 NPV_B(23.23505) = & -200\,000 + \frac{30\,000}{1.2323505} + \frac{85\,000}{1.2323505^2} + \\
 & + \frac{95\,000}{1.2323505^3} + \frac{90\,000}{1.2323505^4} + \frac{85\,000}{1.2323505^5}
 \end{aligned}$$

b)

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

i	a	b	c_i
1	20	25	23.33875
2	20	23.33875	23.2411
3	20	23.2411	23.23539
4	20	23.23539	23.23505

	NPV_B
20%	16567
25%	- 8243.2
23.33875%	- 499.13
23.2411%	- 29.25
23.23539%	- 1.72
23.23505%	- 0.08

$$IRR(B) = 23.23505\%$$

Efektivna kamatna stopa kod kredita

Odluka o efektivnoj kamatnoj stopi kreditnih institucija i kreditnih unija te ugovaranju usluga s potrošačima (HNB 2009)

- Odlukom o jedinstvenom iskazivanju efektivne kamatne stope na kredite kreditne unije Hrvatska narodna banka propisuje način iskazivanja efektivne kamatne stope.
- Ovom odlukom izračunavanje efektivne kamatne stope i predočavanje javnosti i klijentu postaje **dužnost kreditora**.
- Efektivna kamatna stopa iskazuje se dvjema decimalama, uz zaokruživanje na dvije decimale.
- Ugovor o kreditu mora sadržavati odgovarajuću odredbu iz koje je razvidno da je **klijent upoznat s kreditnim uvjetima i efektivnom kamatnom stopom te da mu je uručena otplatna tablica**.

Efektivna kamatna stopa

- Osnovu jedinstvenog načina izračunavanja efektivne kamatne stope na kredite i depozite čine **složeni kamatni račun** i **dekurzivni obračun kamata**.
- Jedinstveni način izračunavanja efektivne kamatne stope temelji se na **metodi neto sadašnje vrijednosti**.

Efektivna kamatna stopa

- **Efektivna kamatna stopa** je kamatna stopa primjenom koje se diskontirani novčani primici izjednačuju s diskontiranim novčanim izdacima koji se odnose na odobrene kredite, odnosno primljene depozite, tj. ona kamatna stopa primjenom koje se diskontirana serija neto novčanih tokova izjednačuje s nulom.
- **Novčani tokovi** uključuju svaki novčani transfer između korisnika kredita i davatelja kredita koji je izravno vezan uz odobrenje kredita, odnosno koji je dio uvjeta korištenja kredita (npr. isplata glavnice, uplata anuiteta, jednokratna provizija za odobrenje kredita, periodična naknada za servisiranje kredita i sl.) ili koji uvjetuje odobravanje kredita (npr. naknada za obradu kreditnog zahtjeva).

Matematička podloga za izračunavanje EKS

Efektivna kamatna stopa je razlika između zbroja konačnih vrijednosti uplata davatelju kredita, odnosno primatelju depozita, i zbroja početnih vrijednosti isplata korisniku kredita, tj. deponentu, iskazana kao postotak od zbroja početnih vrijednosti isplata korisniku kredita, tj. deponentu, izražena na godišnjoj razini.

EKS je definiran sljedećim matematičkim izrazom

$$e = 100 \left[\left(1 + \frac{e_n}{100} \right)^{\frac{t}{n}} - 1 \right]$$

gdje je

$$\begin{aligned} e_n &= 100 \cdot \frac{\sum_k BV_n(NT_k+) - \sum_k SV_0(NT_k-)}{\sum_k SV_0(NT_k-)} = \\ &= 100 \cdot \left(\frac{\sum_k BV_n(NT_k+)}{\sum_k SV_0(NT_k-)} - 1 \right) = \\ &= 100 \cdot \left(\frac{\sum_k (NT_k+)(1 + \frac{e_1}{100})^{n-d_k}}{\sum_k (NT_k-)(1 + \frac{e_1}{100})^{-d_k}} - 1 \right) \\ e_1 &= 100 \left[\left(1 + \frac{e_n}{100} \right)^{\frac{1}{n}} - 1 \right] \end{aligned}$$

- e – EKS
- e_n – EKS izražena na razini n dana trajanja kredita
- e_1 – EKS izražena na razini jednog dana
- t – broj dana u godini (ovisno o metodi računanja dana)
- n – ukupni broj dana trajanja kredita
- (NT_k-) – zbroj isplata korisniku kredita tijekom k -tog dana
(negativni novčani tok)
- (NT_k+) – zbroj uplata davatelju kredita tijekom k -tog dana
(pozitivni novčani tok)
- $BV_n(x)$ – buduća vrijednost iznosa x na kraju n -tog dana
- $SV_0(x)$ – sadašnja vrijednost iznosa x na kraju nultog dana
- d_k – broj proteklih dana od 0-tog dana do novčanog toka na k -ti promatrani dan

Vrijedi

$$1 + \frac{e_1}{100} = \left(1 + \frac{e_n}{100}\right)^{\frac{1}{n}} = \left(1 + \frac{e}{100}\right)^{\frac{1}{t}}.$$

Može se pokazati da vrijedi

$$\sum_k NNT_k \cdot \left(1 + \frac{e}{100}\right)^{-\frac{d_k}{t}} = 0$$

gdje je

$$NNT_k = (NT_{k+}) - (NT_{k-}).$$

EKS kod kredita

- Napraviti otplatnu tablicu: izračunati anuitet, kamate, otplatne kvote i ostatke duga.
- Odrediti neto novčane tokove NNT_k i diskontirane neto novčane tokove $DNNT_k$.

NNT_k je zbroj anuiteta i drugih uplata (pozitivni novčani tok) umanjen za zbroj isplata kredita i drugih isplata (negativni novčani tok) u k -tom razdoblju.

$$DNNT_k = NNT_k \cdot \left(1 + \frac{PGS}{100}\right)^{-\frac{d_k}{t}}$$

PGS – postotna godišnja stopa

EKS kod kredita

- Izračunamo postotnu godišnju kamatnu stopu tako da sumu diskontiranih neto novčanih tokova izjednačimo s nulom i pronađemo rješenje pripadne jednačbe.

$$\sum_k NNT_k \cdot \left(1 + \frac{PGS}{100}\right)^{-\frac{d_k}{t}} = 0$$

- Odredimo efektivnu kamatnu stopu prema formuli

$$EKS = PGS \cdot \frac{UDIK}{UDIK - UDTSP}.$$

EKS kod kredita

- Diskontirane isplate kredita

$$DIK_k = IK_k \cdot \left(1 + \frac{PGS}{100}\right)^{-\frac{d_k}{t}}$$

$$UDIK = \sum_k DIK_k$$

- Diskontirani tokovi sigurnosnog pologa

$$DTSP_k = TSP_k \cdot \left(1 + \frac{PGS}{100}\right)^{-\frac{d_k}{t}}$$

$$UDTSP = \sum_k DTSP_k$$