Složeni dekurzivni kamatni račun

Financijska matematika, IPS

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Sadržaj

prvi zadatak

Zadatak 1

Uz koju je mjesečnu kamatnu stopu posuđeno 8000 *kn ako nakon* 32 *mjeseca dužnik treba vratiti* 9500 *kn? Kolika je ekvivalentna godišnja kamatna stopa? Obračun kamata je složeni i dekurzivni.*

Zadatak 1

Uz koju je mjesečnu kamatnu stopu posuđeno 8000 kn ako nakon 32 mjeseca dužnik treba vratiti 9500 kn? Kolika je ekvivalentna godišnja kamatna stopa? Obračun kamata je složeni i dekurzivni.

Rješenje

- $C_0 = 8000, \quad n = 32, \quad C_{32} = 9500$
- Koristimo formulu $C_n = C_0 r^n$.

$$C_{32} = C_0 \cdot r_{\mathrm{mj}}^{32}$$

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$$r_{\mathrm{mj}}^{32} = \frac{C_{32}}{C_0}$$

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$$r_{\mathrm{mj}} = \sqrt[32]{\frac{9500}{8000}}$$

$$r_{\mathrm{mj}} = 1.0053847665\cdots$$

$$r_{\mathrm{mj}} = 1 + \frac{p_{\mathrm{mj}}}{100}$$

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$$p_{\mathrm{mj}} = 0.5384767\%$$

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$$r_{\mathrm{god}} = r_{\mathrm{mj}}^{12}$$

$$C_{32} = C_0 \cdot r_{\mathrm{mj}}^{32}$$

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$$r_{\mathrm{god}} = r_{\mathrm{mj}}^{12}$$

$$r_{\mathrm{god}} = 1.066565685\cdots$$

$$r_{\mathrm{god}} = 1 + \frac{p_{\mathrm{god}}}{100}$$

$$p_{\mathrm{god}} = 100(r_{\mathrm{god}} - 1)$$

$$p_{\mathrm{god}} = 6.65657\%$$

Nekoliko napomena

- kvartalni dekurzivni kamatni faktor

$$r_{\mathrm{kv}} = r_{\mathrm{mj}}^3 \quad \text{ili} \quad r_{\mathrm{kv}} = \sqrt[4]{r_{\mathrm{god}}} \quad \text{ili} \quad r_{\mathrm{kv}} = \sqrt{r_{\mathrm{pgod}}}$$

- polugodišnji dekurzivni kamatni faktor

$$r_{\mathrm{pgod}} = r_{\mathrm{mj}}^6 \quad \text{ili} \quad r_{\mathrm{pgod}} = \sqrt{r_{\mathrm{god}}} \quad \text{ili} \quad r_{\mathrm{pgod}} = r_{\mathrm{kv}}^2$$

- mjesečni dekurzivni kamatni faktor

$$r_{\mathrm{mj}} = \sqrt[12]{r_{\mathrm{god}}} \quad \text{ili} \quad r_{\mathrm{mj}} = \sqrt[3]{r_{\mathrm{kv}}} \quad \text{ili} \quad r_{\mathrm{mj}} = \sqrt[6]{r_{\mathrm{pgod}}}$$

- godišnji dekurzivni kamatni faktor

$$r_{\mathrm{god}} = r_{\mathrm{mj}}^{12} \quad \text{ili} \quad r_{\mathrm{god}} = r_{\mathrm{pgod}}^2 \quad \text{ili} \quad r_{\mathrm{god}} = r_{\mathrm{kv}}^4$$

drugi zadatak

Zadatak 2

Zadana je glavnica od 1200 *kn i godišnja kamatna stopa* 5%.

a) *Odredite vrijednost glavnice nakon* 8 *mjeseci uz konformno ukamaćivanje.*
b) *Odredite vrijednost glavnice nakon* 8 *mjeseci uz relativno mjesečno ukamaćivanje.*

Obračuna kamata je složeni i dekurzivni.

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

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b) Relativna mjesečna kamatna stopa: $p_r = \dfrac{5}{12}$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \frac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100} = 1 + \frac{5}{1200} = \frac{241}{240}$$

Rješenje

a) Mjesečni dekurzivni kamatni faktor je $r = \sqrt[12]{1.05}$ pa korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \sqrt[12]{1.05}^{\,8} = 1239.67$$

b) Relativna mjesečna kamatna stopa: $p_r = \frac{5}{12}$

Mjesečni dekurzivni kamatni faktor:

$$r = 1 + \frac{p_r}{100} = 1 + \frac{\frac{5}{12}}{100} = 1 + \frac{5}{1200} = \frac{241}{240}$$

Korištenjem formule $C_n = C_0 r^n$ dobivamo

$$C_8 = 1200 \cdot \left(\frac{241}{240}\right)^8 = 1240.59$$

treći zadatak

Zadatak 3

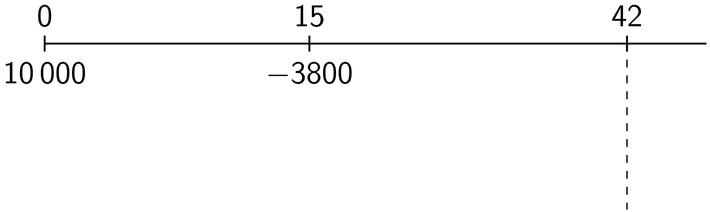
Stipe uplati 10 000 *kn, a nakon* 15 *mjeseci podigne* 3800 *kn.*

a) *Koliko novaca ima Stipe tri i pol godine nakon prve uplate?*
b) *Nakon koliko će mjeseci, u odnosu na zadnje stanje, Stipe ponovo raspolagati s* 10 000 *kn?*
c) *Koliko bi novaca morao podići četiri godine nakon prve uplate tako da bi pet godina nakon prve uplate imao polovicu iznosa kojeg je uplatio?*

Godišnja kamatna stopa je 11.1%.

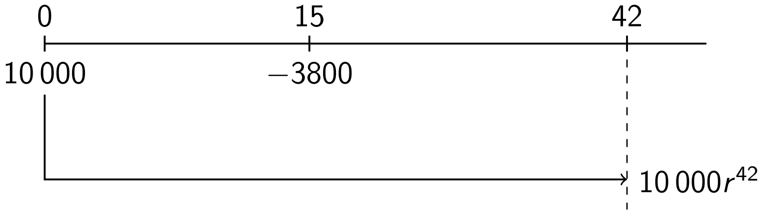
Rješenje

a)



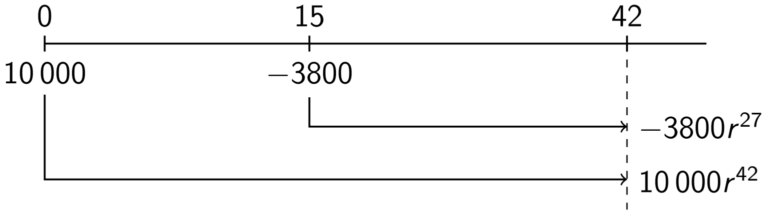
Rješenje

a)



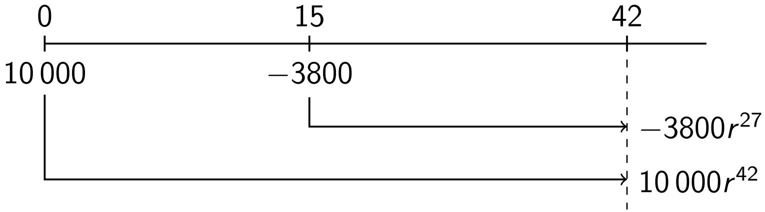
Rješenje

a)



Rješenje

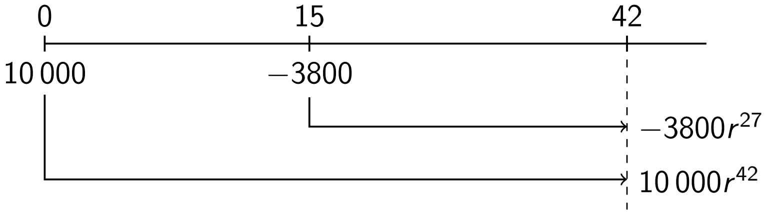
a)



$$C_{42} = 10\,000r^{42} - 3800r^{27}$$

Rješenje

a)

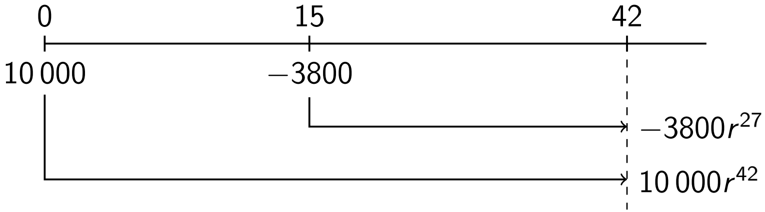


$$\boxed{r = \sqrt[12]{1.111}} \quad C_{42} = 10\,000 r^{42} - 3800 r^{27}$$

$$C_{42} = 10\,000 \cdot \sqrt[12]{1.111}^{\,42} - 3800 \cdot \sqrt[12]{1.111}^{\,27}$$

Rješenje

a)



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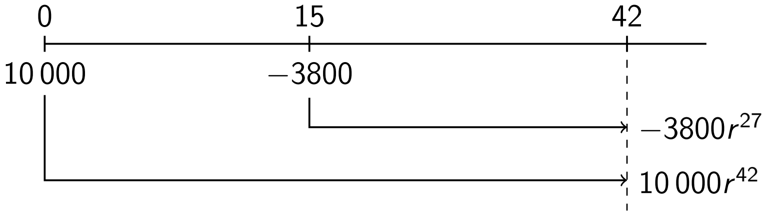
$$C_{42} = 9638.88$$

Rješenje

a)



$$\boxed{r = \sqrt[12]{1.111}}$$

$$C_{42} = 10\,000r^{42} - 3800r^{27}$$

$$C_{42} = 10\,000 \cdot \sqrt[12]{1.111}^{\,42} - 3800 \cdot \sqrt[12]{1.111}^{\,27}$$

$$C_{42} = 9638.88$$

Tri i pol godine nakon prve uplate Stipe ima 9638.88 kn.

b)

$$C_{42} r^n = 10\,000$$

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$$r^n = \frac{10\,000}{C_{42}}$$

b)

$$C_{42} r^n = 10\,000$$

$$r^n = \frac{10\,000}{C_{42}} \Bigg/ \log$$

$$n \log r = \log \frac{10\,000}{C_{42}}$$

b)

$$C_{42} r^n = 10\,000$$

$$r^n = \frac{10\,000}{C_{42}} \;\Big/\; \log$$

$$n \log r = \log \frac{10\,000}{C_{42}}$$

$$n = \frac{\log \frac{10\,000}{C_{42}}}{\log r}$$

b)

$$\begin{aligned}
C_{42} r^n &= 10\,000 \\
r^n &= \frac{10\,000}{C_{42}} \quad \Bigg/ \log \\
n \log r &= \log \frac{10\,000}{C_{42}} \\
n &= \frac{\log \dfrac{10\,000}{C_{42}}}{\log r} \\
n &= \frac{\log \dfrac{10\,000}{9638.88}}{\log \sqrt[12]{1.111}}
\end{aligned}$$

b)

$$C_{42} r^n = 10\,000$$

$$r^n = \frac{10\,000}{C_{42}} \Big/ \log$$

$$n \log r = \log \frac{10\,000}{C_{42}}$$

$$n = \frac{\log \dfrac{10\,000}{C_{42}}}{\log r}$$

$$n = \frac{\log \dfrac{10\,000}{9638.88}}{\log \sqrt[12]{1.111}}$$

$$n = 4.19$$

b)

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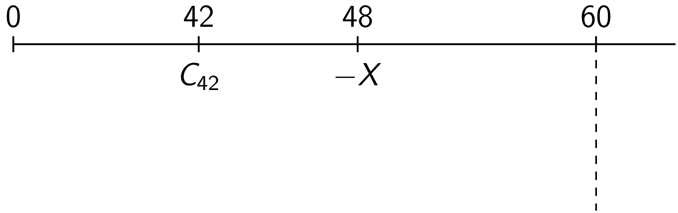
$$n = \frac{\log \frac{10\,000}{C_{42}}}{\log r}$$

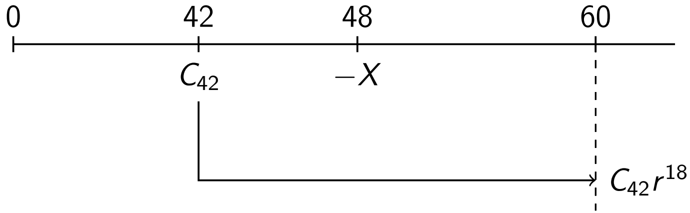
$$n = \frac{\log \frac{10\,000}{9638.88}}{\log \sqrt[12]{1.111}}$$

$$n = 4.19$$

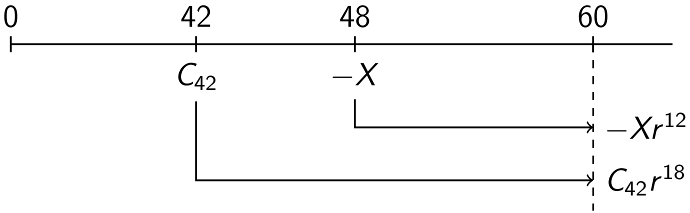
Stipe će ponovo raspolagati s 10 000 kn nakon 5 mjeseci od zadnjeg stanja.

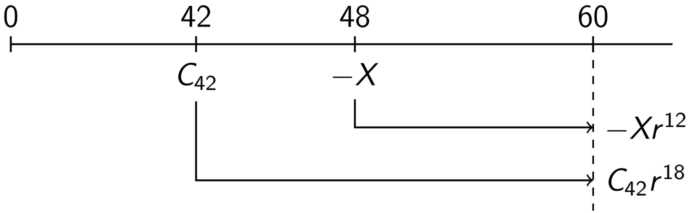
c) 

c)



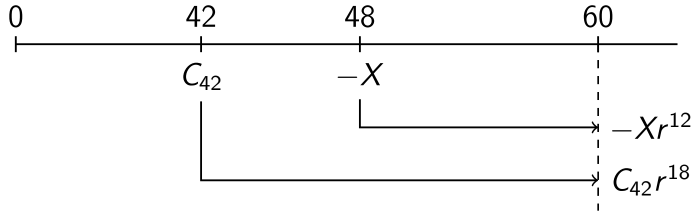
c)



c) 

$$C_{42}r^{18} - Xr^{12} = 5000$$

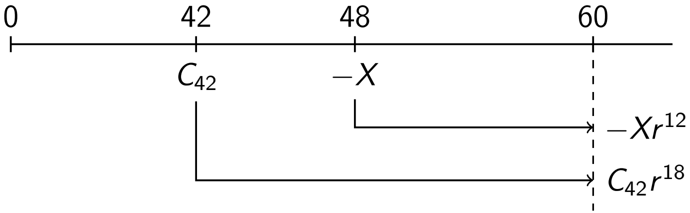
c)



$$C_{42}r^{18} - Xr^{12} = 5000$$

$$Xr^{12} = C_{42}r^{18} - 5000$$

c)

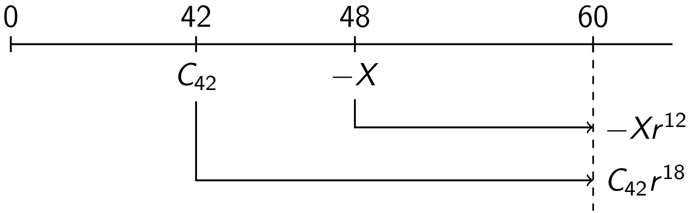


$$C_{42}r^{18} - Xr^{12} = 5000$$

$$Xr^{12} = C_{42}r^{18} - 5000$$

$$X = \frac{C_{42}r^{18} - 5000}{r^{12}}$$

c)



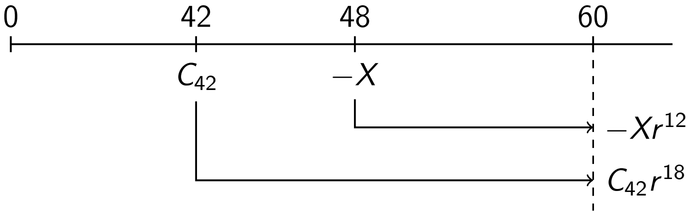
$$C_{42}r^{18} - Xr^{12} = 5000$$

$$Xr^{12} = C_{42}r^{18} - 5000$$

$$X = \frac{C_{42}r^{18} - 5000}{r^{12}}$$

$$X = \frac{9638.88 \cdot \sqrt[12]{1.111}^{\,18} - 5000}{\sqrt[12]{1.111}^{\,12}}$$

c)



$$C_{42}r^{18} - Xr^{12} = 5000$$

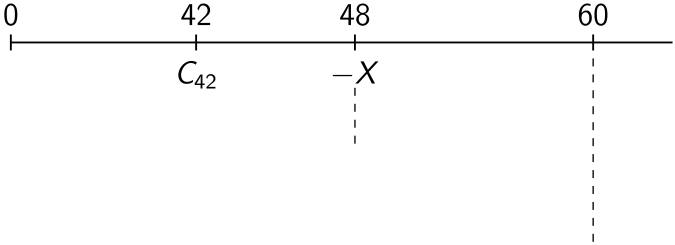
$$Xr^{12} = C_{42}r^{18} - 5000$$

$$X = \frac{C_{42}r^{18} - 5000}{r^{12}}$$

$$X = \frac{9638.88 \cdot \sqrt[12]{1.111}^{18} - 5000}{\sqrt[12]{1.111}^{12}}$$

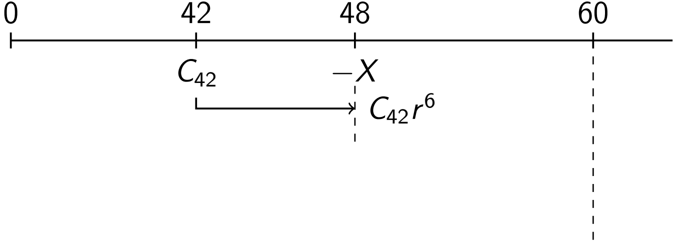
$$X = 5659.32$$

Drugi način razmišljanja



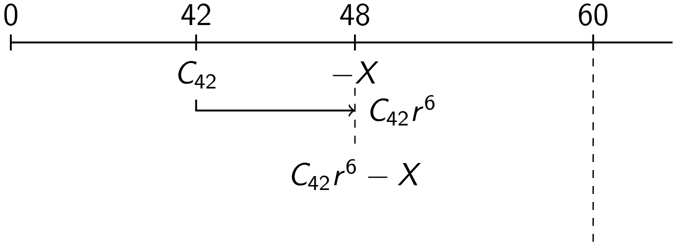
C_{42}

Drugi način razmišljanja



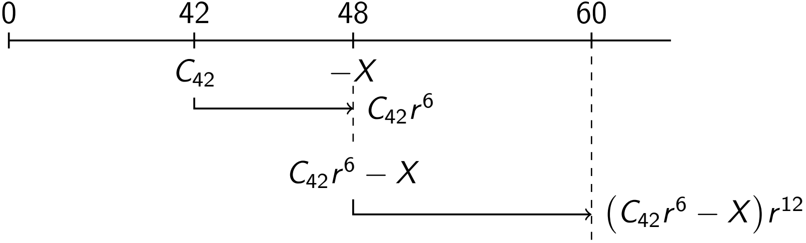
$$C_{42}r^6$$

Drugi način razmišljanja



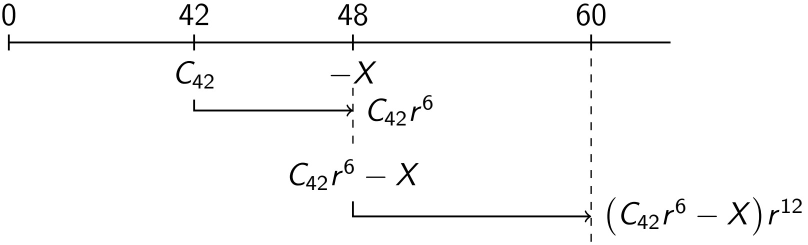
$$C_{42}r^6 - X$$

Drugi način razmišljanja



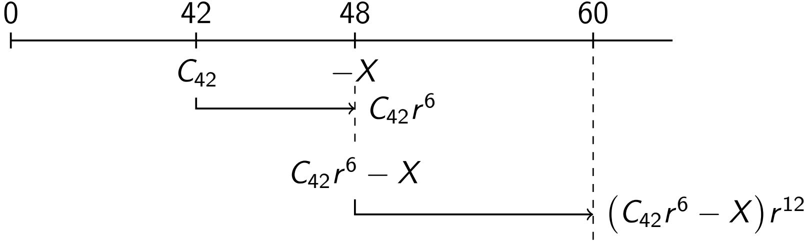
$$\left(C_{42}r^6 - X\right)r^{12}$$

Drugi način razmišljanja



$$\left(C_{42}r^6 - X\right)r^{12} = 5000$$

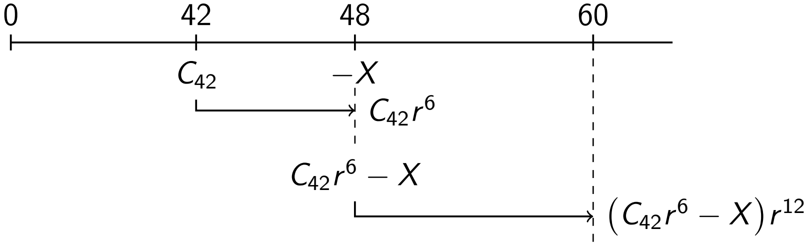
Drugi način razmišljanja



$$\left(C_{42}r^6 - X\right)r^{12} = 5000$$

$$C_{42}r^{18} - Xr^{12} = 5000$$

Drugi način razmišljanja



$$\big(C_{42}r^6 - X\big)r^{12} = 5000$$

$$C_{42}r^{18} - Xr^{12} = 5000$$

$$\vdots$$

$$X = 5659.32$$

četvrti zadatak

Zadatak 4

Martina uplati nepoznati iznos. Nakon 8 *mjeseci podigne polovinu tog iznosa, a* 5 *kvartala nakon toga uplati još* $\frac{1}{5}$ *tog nepoznatog iznosa.*

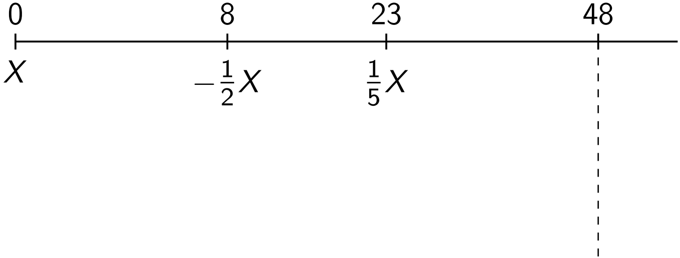
a) *Koliki je iznos uplaćen ako četiri godine nakon prve uplate Martina ima* 8000 *kn? Skicirajte tijek novca!*

b) *Nakon koliko će kvartala u odnosu na prvu uplatu Martina raspolagati s dvostruko većim iznosom od prve uplate?*

Godišnja dekurzivna kamatna stopa je 7.25%.

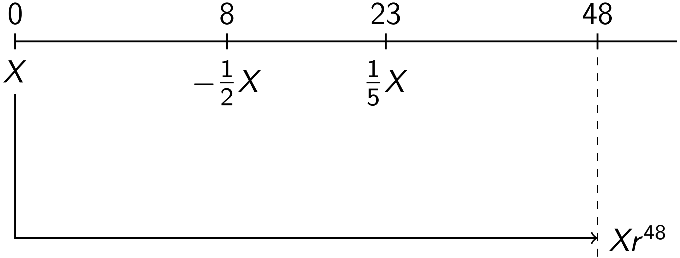
Rješenje

a)



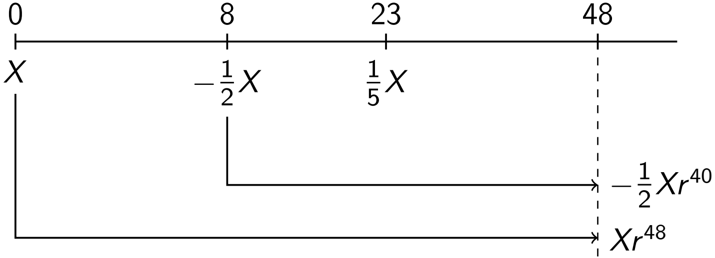
Rješenje

a)



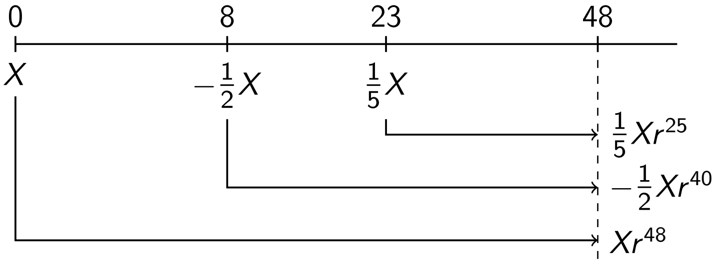
Rješenje

a)



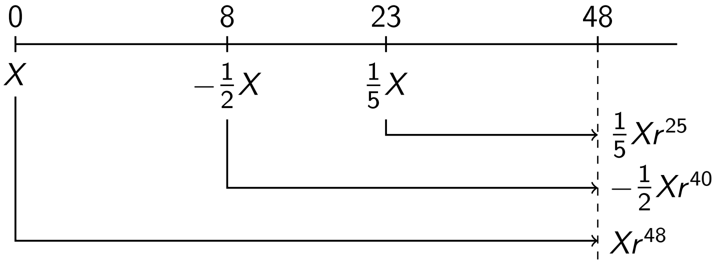
Rješenje

a)



Rješenje

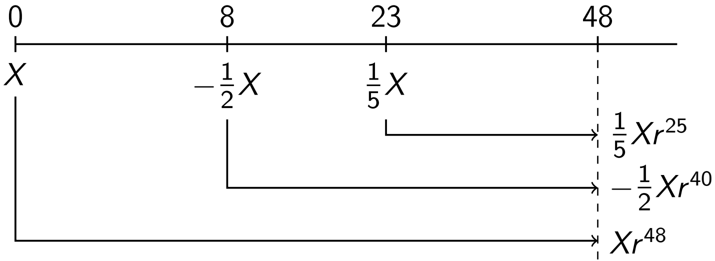
a)



$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

Rješenje

a)

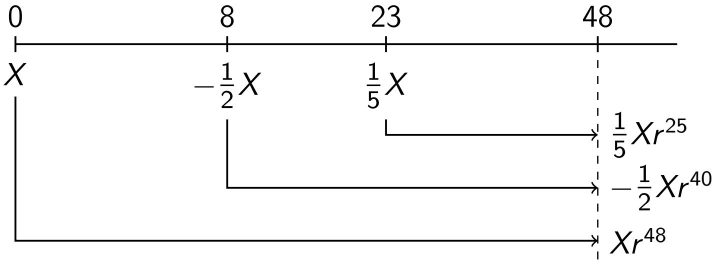


$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

Rješenje

a)



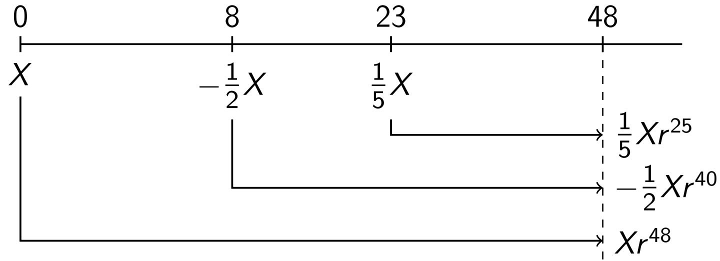
$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

$$X = \frac{8000}{r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}}$$

Rješenje

a)



$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

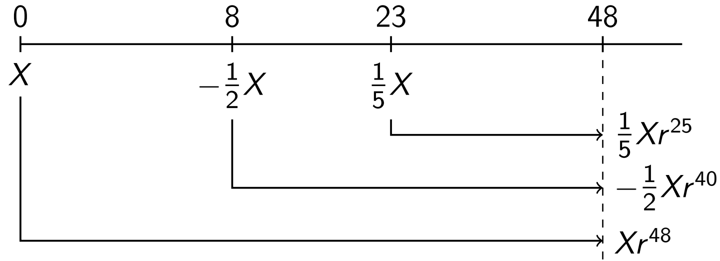
$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

$$X = \frac{8000}{r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}}$$

$$\boxed{r = \sqrt[12]{1.0725}}$$

Rješenje

a)



$$Xr^{48} - \frac{1}{2}Xr^{40} + \frac{1}{5}Xr^{25} = 8000$$

$$X\left(r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}\right) = 8000$$

$$X = \frac{8000}{r^{48} - \frac{1}{2}r^{40} + \frac{1}{5}r^{25}}$$

$$\boxed{r = \sqrt[12]{1.0725}}$$

$$X = 8666.44$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^{\,8} = 9080.41, \qquad 2X = 17332.88$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^n = 2X$$

$$\sqrt[4]{1.0725}^n = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^{\,8} = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^{\,n} = 2X$$

$$\sqrt[4]{1.0725}^{\,n} = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^{\,8} = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^{\,n} = 2X$$

$$\sqrt[4]{1.0725}^{\,n} = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = \frac{\log \frac{8666.44}{4000}}{\log \sqrt[4]{1.0725}}$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^{\,8} = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^{\,n} = 2X$$

$$\sqrt[4]{1.0725}^{\,n} = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = \frac{\log \frac{8666.44}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = 44.19$$

b) U prvih 8 mjeseci uplaćeni iznos se neće udvostručiti.

$$Xr^8 = 8666.44 \cdot \sqrt[12]{1.0725}^8 = 9080.41, \qquad 2X = 17332.88$$

Stoga gledamo od zadnjeg stanja.

$$8000 \cdot \sqrt[4]{1.0725}^{\,n} = 2X$$

$$\sqrt[4]{1.0725}^{\,n} = \frac{2X}{8000} \Big/ \log$$

$$n \log \sqrt[4]{1.0725} = \log \frac{X}{4000}$$

$$n = \frac{\log \frac{X}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = \frac{\log \frac{8666.44}{4000}}{\log \sqrt[4]{1.0725}}$$

$$n = 44.19$$

Martina će raspolagati s dvostruko većim iznosom od uplaćenog nakon $45 + 16 = 61$ kvartala od prve uplate.

peti zadatak

Zadatak 5

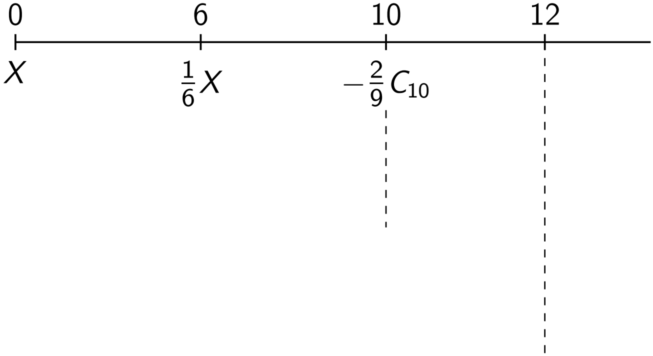
Viktorija uplati nepoznati iznos. Nakon pola godine uloži još šestinu tog iznosa, a četiri mjeseca nakon toga podigne dvije devetine svote s kojom raspolaže u tom trenutku.

a) *Koliki je početni ulog ako na kraju godine Viktorija na računu ima* $2950\,\mathrm{kn}$*, a godišnja kamatna stopa je* 8.25%*?*

b) *Nakon koliko će polugodišta, u odnosu na zadnje stanje, Viktorija raspolagati s* $4500\,\mathrm{kn}$*?*

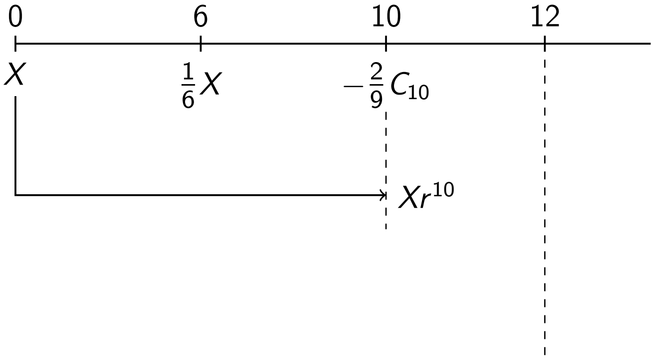
Rješenje

a)



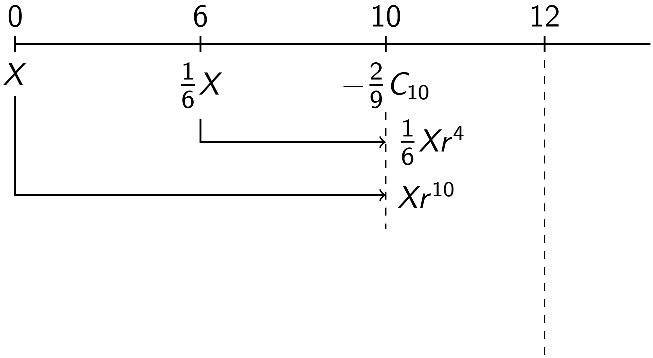
Rješenje

a)



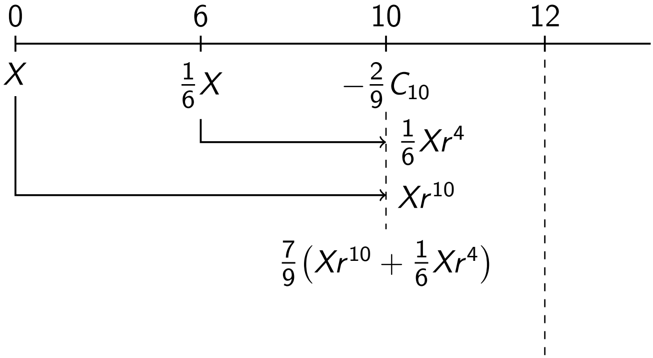
Rješenje

a)



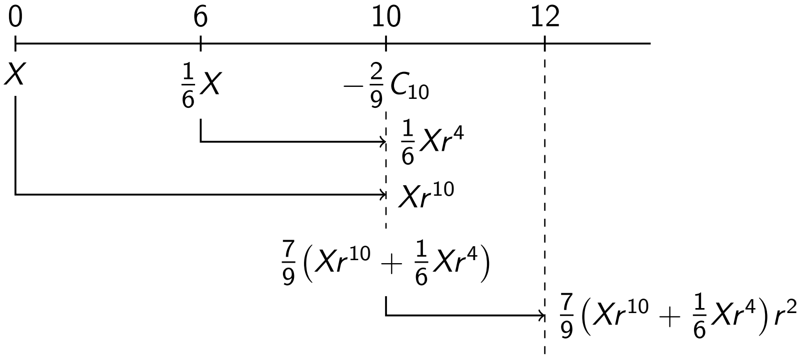
Rješenje

a)


0
6
10
12
X
$\frac{1}{6}X$
$-\frac{2}{9}C_{10}$
$\frac{1}{6}Xr^4$
Xr^{10}
$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^4\right)$

Rješenje

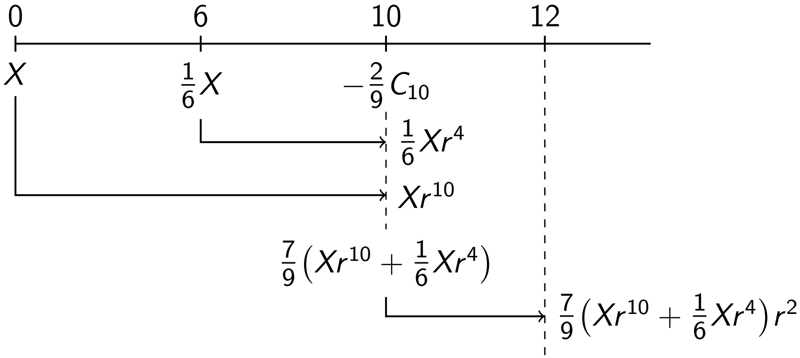
a)



0
6
10
12
X
$\frac{1}{6}X$
$-\frac{2}{9}C_{10}$
$\frac{1}{6}Xr^4$
Xr^{10}
$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)$
$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2$

Rješenje

a)



$$\frac{7}{9}\left(Xr^{10}+\frac{1}{6}Xr^4\right)r^2=2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$X\left(\frac{7}{9}r^{12} + \frac{7}{54}r^{6}\right) = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$X\left(\frac{7}{9}r^{12} + \frac{7}{54}r^{6}\right) = 2950$$

$$X = \frac{2950}{\frac{7}{9}r^{12} + \frac{7}{54}r^{6}}$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

$$X\left(\frac{7}{9}r^{12} + \frac{7}{54}r^{6}\right) = 2950$$

$$\boxed{r = \sqrt[12]{1.0825}}$$

$$X = \frac{2950}{\frac{7}{9}r^{12} + \frac{7}{54}r^{6}}$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^{4}\right)r^{2} = 2950$$

$$\frac{7}{9}Xr^{12} + \frac{7}{54}Xr^{6} = 2950$$

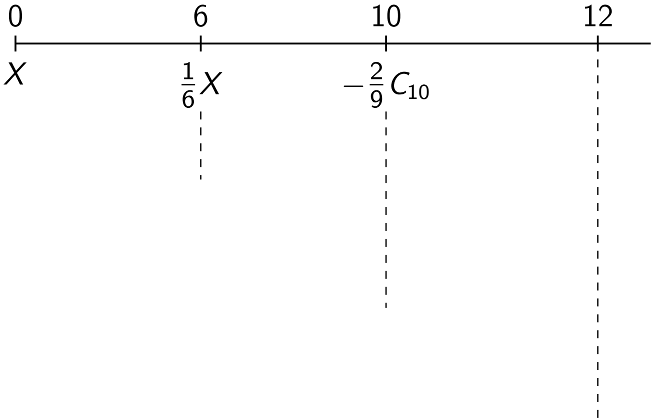
$$X\left(\frac{7}{9}r^{12} + \frac{7}{54}r^{6}\right) = 2950$$

$$\boxed{r = \sqrt[12]{1.0825}}$$

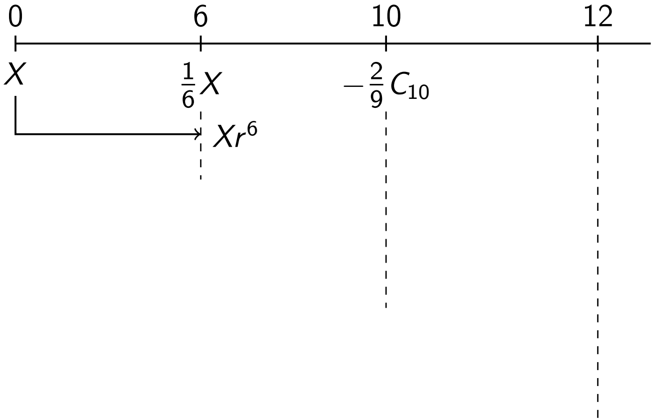
$$X = \frac{2950}{\frac{7}{9}r^{12} + \frac{7}{54}r^{6}}$$

$$X = 3020.02$$

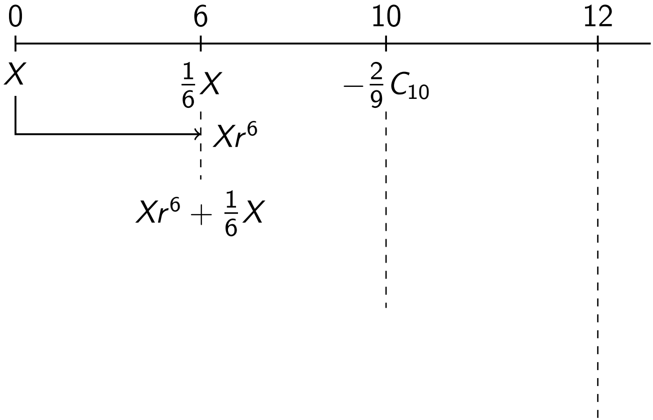
Drugi način razmišljanja



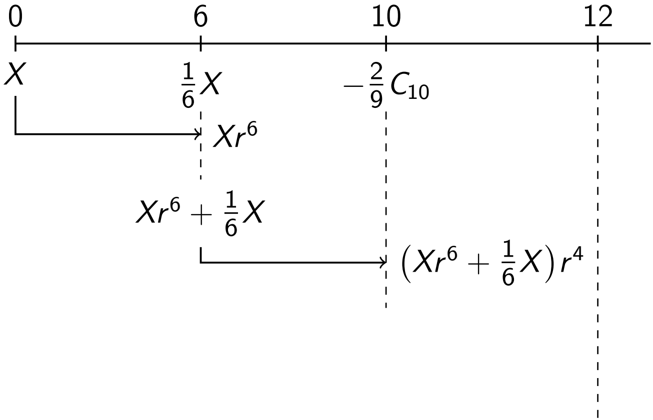
Drugi način razmišljanja



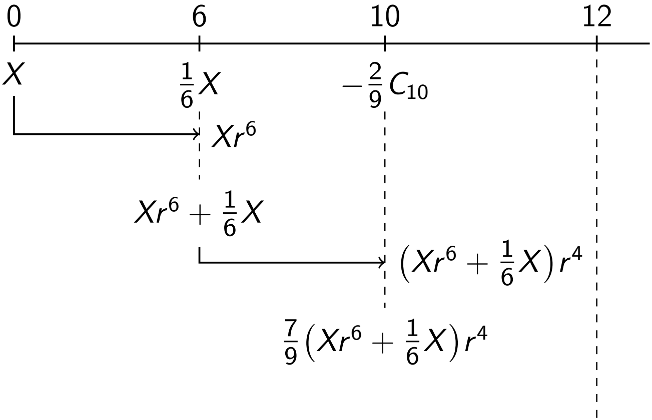
Drugi način razmišljanja



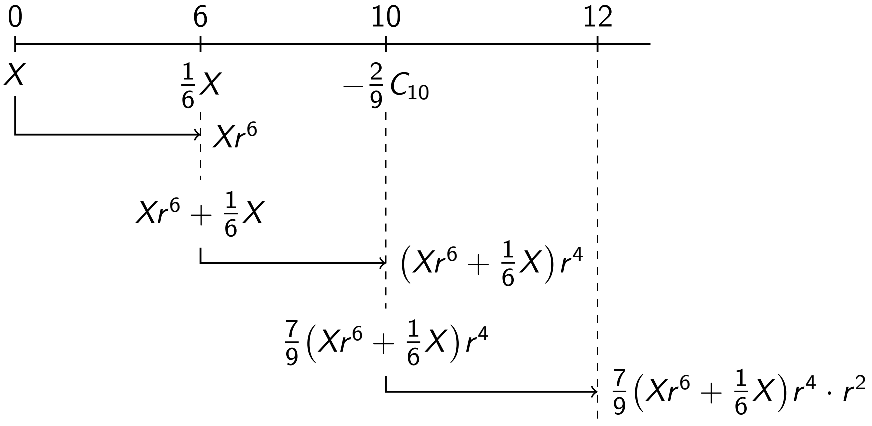
Drugi način razmišljanja



Drugi način razmišljanja

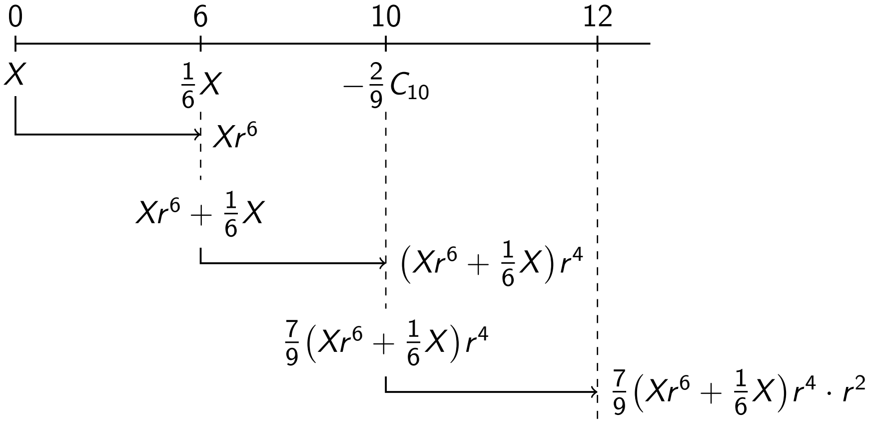


Drugi način razmišljanja



$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

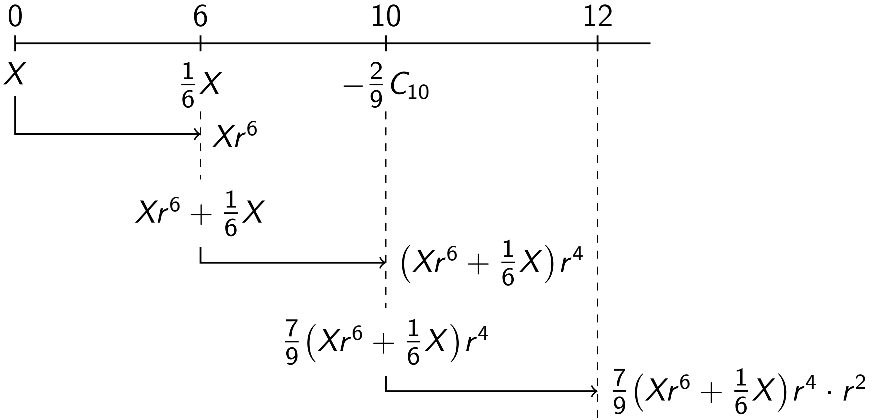
Drugi način razmišljanja



$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2 = 2950$$

Drugi način razmišljanja

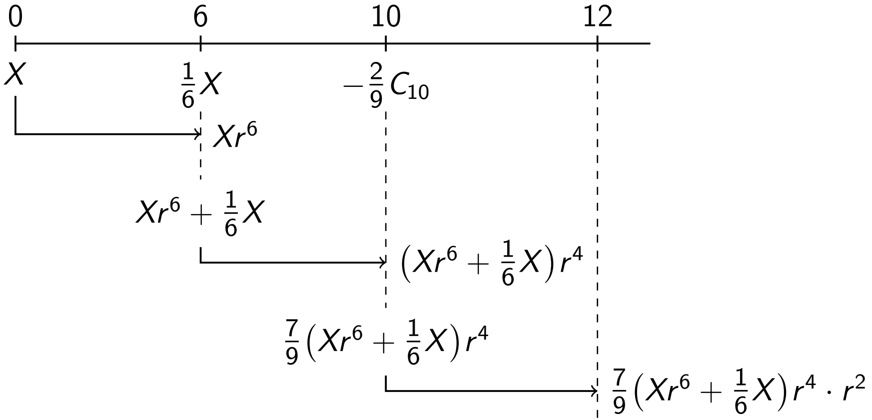


$$\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 = 2950$$

$$\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2 = 2950$$

$$\vdots$$

Drugi način razmišljanja



$$
\begin{aligned}
\frac{7}{9}\left(Xr^6 + \frac{1}{6}X\right)r^4 \cdot r^2 &= 2950 \\
\frac{7}{9}\left(Xr^{10} + \frac{1}{6}Xr^4\right)r^2 &= 2950 \\
&\vdots \\
X &= 3020.02
\end{aligned}
$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950}$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950} \;\Big/\; \log$$

$$n \log \sqrt{1.0825} = \log \frac{90}{59}$$

b)

$$
\begin{aligned}
2950 \cdot \sqrt{1.0825}^{\,n} &= 4500 \\
\sqrt{1.0825}^{\,n} &= \frac{4500}{2950} \Bigg/ \log \\
n \log \sqrt{1.0825} &= \log \frac{90}{59} \\
n &= \frac{\log \frac{90}{59}}{\log \sqrt{1.0825}}
\end{aligned}
$$

b)

$$
\begin{aligned}
2950 \cdot \sqrt{1.0825}^{\,n} &= 4500 \\
\sqrt{1.0825}^{\,n} &= \frac{4500}{2950} \;\Big/\; \log \\
n \log \sqrt{1.0825} &= \log \frac{90}{59} \\
n &= \frac{\log \frac{90}{59}}{\log \sqrt{1.0825}} \\
n &= 10.65
\end{aligned}
$$

b)

$$2950 \cdot \sqrt{1.0825}^{\,n} = 4500$$

$$\sqrt{1.0825}^{\,n} = \frac{4500}{2950} \Big/ \log$$

$$n \log \sqrt{1.0825} = \log \frac{90}{59}$$

$$n = \frac{\log \frac{90}{59}}{\log \sqrt{1.0825}}$$

$$n = 10.65$$

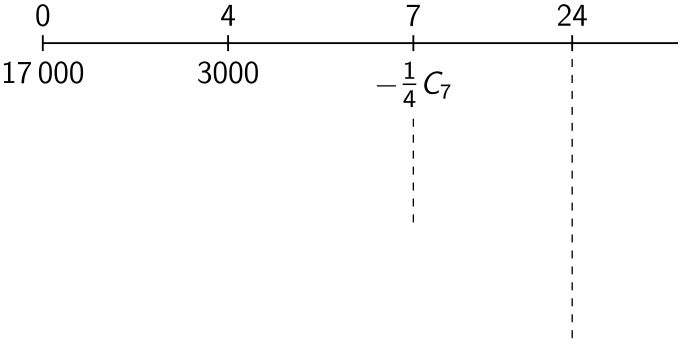
Viktorija će raspolagati s 4500 kn nakon 11 polugodišta od zadnjeg stanja.

šesti zadatak

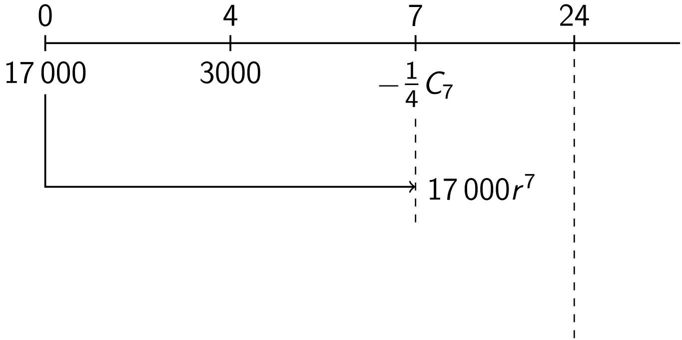
Zadatak 6

Netko uloži 17 000 *kn uz mjesečnu kamatnu stopu* 1.02%*. Nakon četiri mjeseca uloži još* 3000 *kn, a tri mjeseca poslije podigne četvrtinu iznosa s kojim raspolaže u tom trenutku. S kojom svotom raspolaže dvije godine nakon prve uplate?*

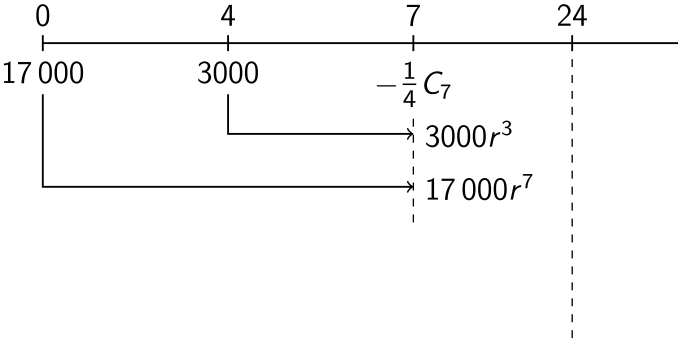
Rješenje



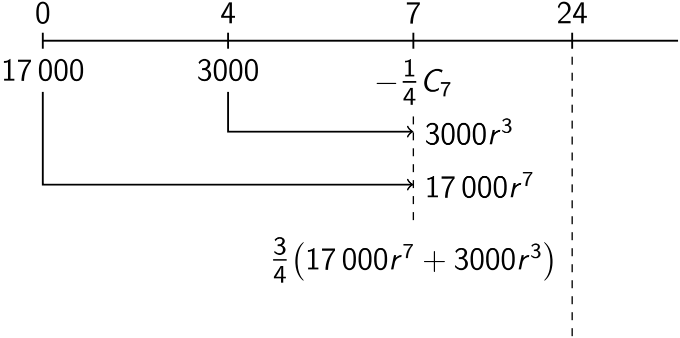
Rješenje



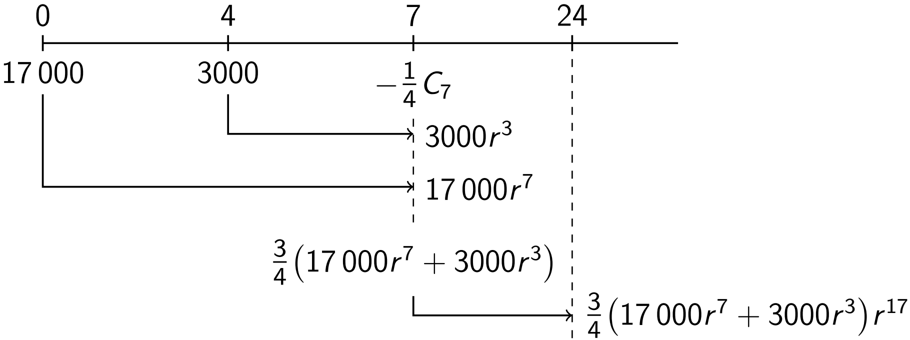
Rješenje



Rješenje

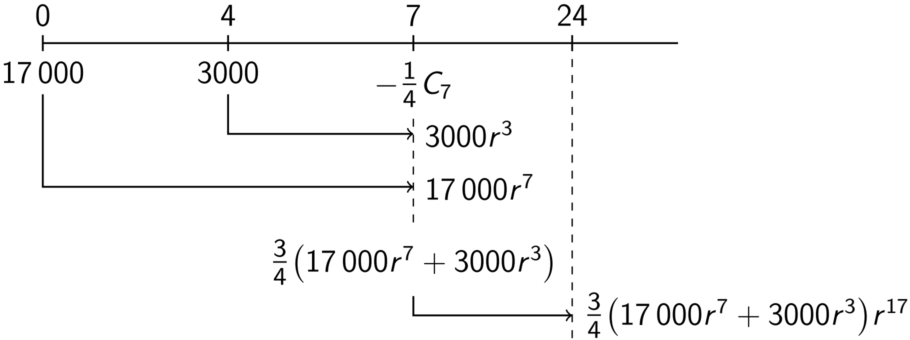

0
4
7
24
17 000
3000
$-\frac{1}{4}C_7$
$3000r^3$
$17\,000r^7$
$\frac{3}{4}\left(17\,000r^7 + 3000r^3\right)$

Rješenje



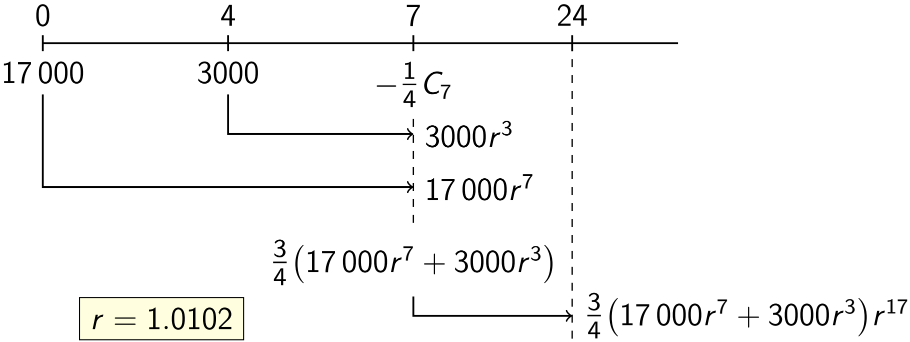
0
4
7
24
17 000
3000
$-\frac{1}{4}C_7$
$3000r^3$
$17\,000r^7$
$\frac{3}{4}\left(17\,000r^7 + 3000r^3\right)$
$\frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$

Rješenje



$$C_{24} = \frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$$

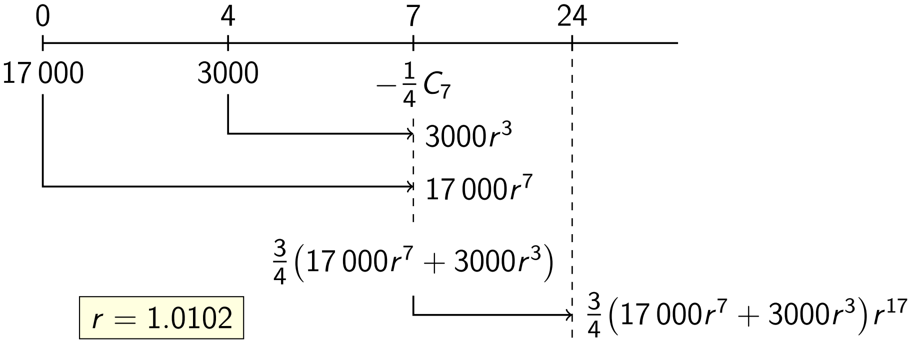
Rješenje



$$C_{24} = \frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$$

$$C_{24} = \frac{3}{4}\left(17\,000 \cdot 1.0102^7 + 3000 \cdot 1.0102^3\right) \cdot 1.0102^{17}$$

Rješenje



$$\boxed{r = 1.0102}$$

$$C_{24} = \frac{3}{4}\left(17\,000r^7 + 3000r^3\right)r^{17}$$

$$C_{24} = \frac{3}{4}\left(17\,000 \cdot 1.0102^7 + 3000 \cdot 1.0102^3\right) \cdot 1.0102^{17}$$

$$C_{24} = 19\,022.55$$

sedmi zadatak

Zadatak 7

Količina pastrva u ribogojilištu za 3 *mjeseca se poveća za* 70%. *Odredite prosječnu stopu godišnjeg prirasta pastrva.*

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

- 3 mjeseca = 0.25 godina

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{100}}$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_{0.25} = C_0 e^{\frac{0.25p}{100}}$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_{0.25} = C_0 e^{\frac{0.25p}{100}}$$

$$1.7 C_0 = C_0 e^{\frac{p}{400}}$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{100}} \qquad 1.7 = e^{\frac{p}{400}}$$

$$C_{0.25} = C_0 e^{\frac{0.25p}{100}}$$

$$1.7 C_0 = C_0 e^{\frac{p}{400}}$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

$$\boxed{a^x = b \quad \Leftrightarrow \quad x = \log_a b}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \dfrac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$\begin{aligned} C_n &= C_0 e^{\frac{np}{100}} \\ C_{0.25} &= C_0 e^{\frac{0.25p}{100}} \\ 1.7 C_0 &= C_0 e^{\frac{p}{400}} \end{aligned} \qquad \begin{aligned} 1.7 &= e^{\frac{p}{400}} \\ \frac{p}{400} &= \ln 1.7 \end{aligned}$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

$$\boxed{a^x = b \;\Leftrightarrow\; x = \log_a b}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_{0.25} = C_0 e^{\frac{0.25p}{100}}$$

$$1.7 C_0 = C_0 e^{\frac{p}{400}}$$

$$1.7 = e^{\frac{p}{400}}$$

$$\frac{p}{400} = \ln 1.7$$

$$p = 400 \cdot \ln 1.7$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

$$\boxed{a^x = b \iff x = \log_a b}$$

- 3 mjeseca $= 0.25$ godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$\begin{aligned} C_n &= C_0 e^{\frac{np}{100}} \\ C_{0.25} &= C_0 e^{\frac{0.25p}{100}} \\ 1.7 C_0 &= C_0 e^{\frac{p}{400}} \end{aligned}$$

$$\begin{aligned} 1.7 &= e^{\frac{p}{400}} \\ \frac{p}{400} &= \ln 1.7 \\ p &= 400 \cdot \ln 1.7 \end{aligned}$$

$$p \approx 212.25$$

Zadatak 7

Količina pastrva u ribogojilištu za 3 mjeseca se poveća za 70%. Odredite prosječnu stopu godišnjeg prirasta pastrva.

Rješenje

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

$$\boxed{a^x = b \;\Leftrightarrow\; x = \log_a b}$$

- 3 mjeseca = 0.25 godina
- $C_{0.25} = C_0 + \frac{70}{100} C_0 = 1.7 C_0$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_{0.25} = C_0 e^{\frac{0.25p}{100}}$$

$$1.7 C_0 = C_0 e^{\frac{p}{400}}$$

$$1.7 = e^{\frac{p}{400}}$$

$$\frac{p}{400} = \ln 1.7$$

$$p = 400 \cdot \ln 1.7$$

$$p \approx 212.25$$

Prosječna stopa godišnjeg prirasta pastrva jednaka je 212.25%.

osmi zadatak

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\,m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\,m^3$ *drvne mase?*

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\,m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\,m^3$ *drvne mase?*

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\,m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\,m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\, m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\, m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}}$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\, m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\, m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}}$$

$$180\,000 = 160\,000 e^{\frac{p}{100}}$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\, m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\, m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}}$$

$$180\,000 = 160\,000 e^{\frac{p}{100}}$$

$$1.125 = e^{\frac{p}{100}}$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\, m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\, m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}}$$

$$180\,000 = 160\,000 e^{\frac{p}{100}}$$

$$1.125 = e^{\frac{p}{100}}$$

$$\frac{p}{100} = \ln 1.125$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\, m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\, m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

$$\boxed{a^x = b \;\Leftrightarrow\; x = \log_a b}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}} \qquad 1.125 = e^{\frac{p}{100}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}} \qquad \frac{p}{100} = \ln 1.125$$

$$180\,000 = 160\,000 e^{\frac{p}{100}} \qquad p = 100 \cdot \ln 1.125$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\,m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\,m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

$$\boxed{a^x = b \;\Leftrightarrow\; x = \log_a b}$$

Rješenje

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}}$$

$$180\,000 = 160\,000 e^{\frac{p}{100}}$$

$$1.125 = e^{\frac{p}{100}}$$

$$\frac{p}{100} = \ln 1.125$$

$$p = 100 \cdot \ln 1.125$$

$$p \approx 11.778$$

Zadatak 8

U nekoj šumi prije deset godina bilo je $160\,000\, m^3$ *drvne mase. Koliki je prirodni prirast u promilima ako u međuvremenu nije bilo sječe i sada u šumi ima* $180\,000\, m^3$ *drvne mase?*

$$\boxed{C_n = C_0 e^{\frac{np}{100}}}$$

Rješenje

$$\boxed{a^x = b \;\Leftrightarrow\; x = \log_a b}$$

- $C_0 = 160\,000, \quad C_{10} = 180\,000$
- Neprekidno ukamaćivanje

$$C_n = C_0 e^{\frac{np}{1000}}$$

$$C_{10} = C_0 e^{\frac{10p}{1000}}$$

$$180\,000 = 160\,000 e^{\frac{p}{100}}$$

$$1.125 = e^{\frac{p}{100}}$$

$$\frac{p}{100} = \ln 1.125$$

$$p = 100 \cdot \ln 1.125$$

$$p \approx 11.778$$

Prirodni godišnji prirast jednak je $11.778\,‰$.

Domaća zadaća

Domaća zadaća

Zadatak 9

Kolika će glavnica uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 10% *donijeti* 64 000 *kn kamata ako je novac oročen na* 13 *kvartala?*

Domaća zadaća

Zadatak 9

Kolika će glavnica uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 10% *donijeti* 64 000 *kn kamata ako je novac oročen na* 13 *kvartala?*

Rješenje

- $C_n = C_0 r^n, \quad r = \sqrt[4]{1.1}, \quad n = 13$

Domaća zadaća

Zadatak 9

Kolika će glavnica uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 10% *donijeti* 64 000 *kn kamata ako je novac oročen na* 13 *kvartala?*

Rješenje

- $C_n = C_0 r^n, \quad r = \sqrt[4]{1.1}, \quad n = 13$
- Kamate: $I = 64\,000$

$$I = C_n - C_0 = C_0 r^n - C_0 = C_0\left(r^n - 1\right)$$

Domaća zadaća

Zadatak 9

Kolika će glavnica uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 10% *donijeti* 64 000 *kn kamata ako je novac oročen na* 13 *kvartala?*

Rješenje

- $C_n = C_0 r^n, \quad r = \sqrt[4]{1.1}, \quad n = 13$
- Kamate: $I = 64\,000$

$$I = C_n - C_0 = C_0 r^n - C_0 = C_0\left(r^n - 1\right)$$

$$C_0 = \frac{I}{r^n - 1} = \frac{64\,000}{\sqrt[4]{1.1}^{\,13} - 1} = 176\,262.26$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$C_n = C_0 r^n$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$\begin{aligned} C_n &= C_0 r^n \\ 2C_0 &= C_0 r^n \end{aligned}$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$\begin{aligned} C_n &= C_0 r^n \\ 2C_0 &= C_0 r^n \\ r^n &= 2 \end{aligned}$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$
\begin{aligned}
C_n &= C_0 r^n \\
2C_0 &= C_0 r^n \\
r^n &= 2 \\
n &= \log_r 2
\end{aligned}
$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$\begin{aligned} C_n &= C_0 r^n \\ 2C_0 &= C_0 r^n \\ r^n &= 2 \\ n &= \log_r 2 \end{aligned}$$

$$n = \frac{\log 2}{\log r}$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$\begin{aligned} C_n &= C_0 r^n \\ 2C_0 &= C_0 r^n \\ r^n &= 2 \\ n &= \log_r 2 \end{aligned}$$

$$n = \frac{\log 2}{\log r}$$

$$n = \frac{\log 2}{\log \sqrt[12]{1.12}}$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$\begin{aligned} C_n &= C_0 r^n \\ 2C_0 &= C_0 r^n \\ r^n &= 2 \\ n &= \log_r 2 \end{aligned}$$

$$n = \frac{\log 2}{\log r}$$

$$n = \frac{\log 2}{\log \sqrt[12]{1.12}}$$

$$n = 73.395$$

Domaća zadaća

Zadatak 10

Nakon koliko će mjeseci glavnica udvostručiti svoju vrijednost uz složeni dekurzivni obračun kamata i godišnju kamatnu stopu 12%?

Rješenje

- $C_n = 2C_0, \quad r = \sqrt[12]{1.12}$

$$\begin{aligned} C_n &= C_0 r^n \\ 2C_0 &= C_0 r^n \\ r^n &= 2 \\ n &= \log_r 2 \end{aligned}$$

$$n = \frac{\log 2}{\log r}$$

$$n = \frac{\log 2}{\log \sqrt[12]{1.12}}$$

$$n = 73.395$$

Glavnica će udvostručiti svoju vrijednost nakon 74 mjeseca.

Zadatak 11

Neki grad imao je prije pet godina 19 000 *stanovnika, a danas ima* 24 000 *stanovnika. Koliko će taj grad imati stanovnika za sedam godina ako godišnja stopa rasta ostane ista?*

Domaća zadaća

Zadatak 11

Neki grad imao je prije pet godina 19 000 *stanovnika, a danas ima* 24 000 *stanovnika. Koliko će taj grad imati stanovnika za sedam godina ako godišnja stopa rasta ostane ista?*

Rješenje

- $C_0 = 19\,000, \quad C_5 = 24\,000$
- Neprekidno ukamaćivanje: $C_n = C_0 e^{\frac{np}{100}}$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$
\begin{aligned}
C_n &= C_0 e^{\frac{np}{100}} \\
C_5 &= C_0 e^{\frac{5p}{100}} \\
24\,000 &= 19\,000 e^{\frac{p}{20}} \\
\frac{24}{19} &= e^{\frac{p}{20}}
\end{aligned}
$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln \frac{24}{19}$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln \frac{24}{19}$$

$$p = 20 \cdot \ln \frac{24}{19}$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln \frac{24}{19}$$

$$p = 20 \cdot \ln \frac{24}{19}$$

$$p = 4.6723$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln \frac{24}{19}$$

$$p = 20 \cdot \ln \frac{24}{19}$$

$$p = 4.6723$$

Godišnja stopa rasta stanovništva jednaka je 4.6723%.

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln\frac{24}{19}$$

$$p = 20 \cdot \ln\frac{24}{19}$$

$$p = 4.6723$$

Godišnja stopa rasta stanovništva jednaka je 4.6723%.

$$C_{12} = C_5 e^{\frac{7p}{100}}$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln\frac{24}{19}$$

$$p = 20 \cdot \ln\frac{24}{19}$$

$$p = 4.6723$$

Godišnja stopa rasta stanovništva jednaka je 4.6723%.

$$C_{12} = C_5 e^{\frac{7p}{100}}$$

$$C_{12} = 24\,000 \cdot e^{\frac{7 \cdot 4.6723}{100}}$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln \frac{24}{19}$$

$$p = 20 \cdot \ln \frac{24}{19}$$

$$p = 4.6723$$

Godišnja stopa rasta stanovništva jednaka je 4.6723%.

$$C_{12} = C_5 e^{\frac{7p}{100}}$$

$$C_{12} = 24\,000 \cdot e^{\frac{7 \cdot 4.6723}{100}}$$

$$C_{12} = 33\,285.27$$

$$C_n = C_0 e^{\frac{np}{100}}$$

$$C_5 = C_0 e^{\frac{5p}{100}}$$

$$24\,000 = 19\,000 e^{\frac{p}{20}}$$

$$\frac{24}{19} = e^{\frac{p}{20}}$$

$$\frac{p}{20} = \ln \frac{24}{19}$$

$$p = 20 \cdot \ln \frac{24}{19}$$

$$p = 4.6723$$

Godišnja stopa rasta stanovništva jednaka je 4.6723%.

$$C_{12} = C_5 e^{\frac{7p}{100}}$$

$$C_{12} = 24\,000 \cdot e^{\frac{7 \cdot 4.6723}{100}}$$

$$C_{12} = 33\,285.27$$

Grad će imati 33 285 stanovnika.