Elementarna teorija brojeva

DISKRETNE STRUKTURE S TEORIJOM GRAFOVA

Damir Horvat

FOI, Varaždin

Sadržaj

Relacija *dijeli* na skupu cijelih brojeva

$$a \mid b \quad \overset{\text{def}}{\iff} \quad \exists k \in \mathbb{Z},\ \ b = ak$$

Važno svojstvo relacije *dijeli*

$$a, b, c \in \mathbb{Z},\ \ (c \mid a) \land (c \mid b) \quad \implies \quad c \mid k_1 a + k_2 b,\ \ \forall k_1, k_2 \in \mathbb{Z}$$

prvi zadatak

Zadatak 1

Dokažite da su prirodni brojevi n i $n+1$ relativno prosti.

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Rješenje

Tvrdimo

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Rješenje

Tvrdimo $M(n, n+1) = 1$

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$\boxed{\text{Tvrdimo}}$ $M(n, n+1) = 1$

Neka je $d = M(n, n+1)$.

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$$d = M(n, n+1) \;\Rightarrow\; d \mid n,$$

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Neka je $d = M(n, n+1)$.

$$d = M(n, n+1) \Rightarrow d \mid n, \; d \mid n+1 \Rightarrow$$

$$\Rightarrow d \mid 1 \cdot n + (-1) \cdot (n+1)$$

$$a, b, c \in \mathbb{Z}, \; (c \mid a) \wedge (c \mid b) \implies c \mid k_1 a + k_2 b, \; \forall k_1, k_2 \in \mathbb{Z}$$

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Neka je $d = M(n, n+1)$.

$$\boxed{d \in \mathbb{N}}$$

$$d = M(n, n+1) \;\Rightarrow\; d \mid n,\; d \mid n+1 \;\Rightarrow$$

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drugi zadatak

Zadatak 2

Odredite sve prirodne brojeve s kojima se može skratiti razlomak $\frac{5n+6}{8n+7}$ *pri čemu je* $n \in \mathbb{N}$.

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$$d = M(5n+6, 8n+7) \;\Rightarrow\; d \mid 5n+6,\; d \mid 8n+7 \;\Rightarrow$$

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \quad\Longrightarrow\quad c \mid k_1 a + k_2 b,\; \forall k_1, k_2 \in \mathbb{Z}$$

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$$d = M(5n+6, 8n+7) \;\Rightarrow\; d \mid 5n+6,\; d \mid 8n+7 \;\Rightarrow$$

$$\Rightarrow\; d \mid 8 \cdot (5n+6) - 5 \cdot (8n+7)$$

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \;\Longrightarrow\; c \mid k_1 a + k_2 b,\; \forall k_1, k_2 \in \mathbb{Z}$$

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$$\Rightarrow\; d \mid 8 \cdot (5n+6) - 5 \cdot (8n+7) \;\Rightarrow\; d \mid 13$$

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \;\Longrightarrow\; c \mid k_1 a + k_2 b,\; \forall k_1, k_2 \in \mathbb{Z}$$

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$$d = M(5n+6, 8n+7) \;\Rightarrow\; d \mid 5n+6,\; d \mid 8n+7 \;\Rightarrow$$

$$\Rightarrow\; d \mid 8\cdot(5n+6) - 5\cdot(8n+7) \;\Rightarrow\; d \mid 13 \;\Rightarrow\; d = 1 \text{ ili } d = 13$$

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \implies c \mid k_1 a + k_2 b,\; \forall k_1, k_2 \in \mathbb{Z}$$

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Dakle, razlomak $\frac{5n+6}{8n+7}$ se uopće ne može skratiti ili se može skratiti s brojem 13.

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \;\Longrightarrow\; c \mid k_1 a + k_2 b,\; \forall k_1, k_2 \in \mathbb{Z}$$

treći zadatak

$$a, b \in \mathbb{Z},\ \ b \neq 0 \quad \Longrightarrow \quad \exists! q, r \in \mathbb{Z},\ \ a = bq + r,\ \ 0 \leqslant r < |b|$$

$$a, b \in \mathbb{Z},\ b \neq 0 \quad \Longrightarrow \quad \exists! q, r \in \mathbb{Z},\ a = bq + r,\ 0 \leqslant r < |b|$$

$$q = \begin{cases} \left\lfloor \dfrac{a}{b} \right\rfloor, & \text{ako je } b > 0 \\ \left\lceil \dfrac{a}{b} \right\rceil, & \text{ako je } b < 0 \end{cases}$$

$$a, b \in \mathbb{Z},\ b \neq 0 \quad \Longrightarrow \quad \exists! q, r \in \mathbb{Z},\ a = bq + r,\ 0 \leqslant r < |b|$$

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$$r = a - bq$$

Zadatak 3

Odredite redukciju od 2015 *i* -2015 *modulo* 326*.*

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Rješenje

- $2015 \mod 326 =$

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$$q = \left\lfloor \frac{2015}{326} \right\rfloor$$

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Odredite redukciju od 2015 *i* −2015 *modulo* 326.

Rješenje

- $2015 \mod 326 =$

$$q = \left\lfloor \frac{2015}{326} \right\rfloor = \lfloor 6.1809 \cdots \rfloor$$

Zadatak 3

Odredite redukciju od 2015 *i* −2015 *modulo* 326.

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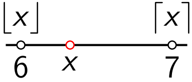
- $2015 \mod 326 =$



$$q = \left\lfloor \frac{2015}{326} \right\rfloor = \lfloor 6.1809\cdots \rfloor = 6$$

Zadatak 3

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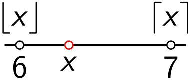
- $2015 \mod 326 =$



$$q = \left\lfloor \frac{2015}{326} \right\rfloor = \lfloor 6.1809 \cdots \rfloor = 6$$

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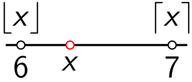

$$q = \left\lfloor \frac{2015}{326} \right\rfloor = \lfloor 6.1809 \cdots \rfloor = 6$$

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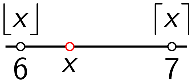

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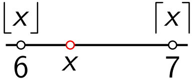
- $2015 \bmod 326 = 59 \quad \rightsquigarrow \quad 2015 \equiv 59 \pmod{326}$



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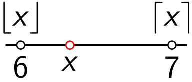

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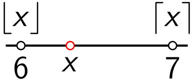
Odredite redukciju od 2015 *i* −2015 *modulo* 326.

Rješenje

- $2015 \bmod 326 = 59 \quad \leadsto \quad 2015 \equiv 59 \pmod{326}$



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- $-2015 \bmod 326 =$

$$q = \left\lfloor \frac{-2015}{326} \right\rfloor$$

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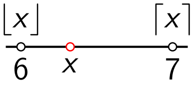

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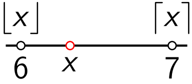

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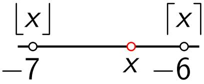
$$r = 2015 - 326 \cdot 6 = 59$$

- $-2015 \bmod 326 =$



$$q = \left\lfloor \frac{-2015}{326} \right\rfloor = \lfloor -6.1809\cdots \rfloor = -7$$

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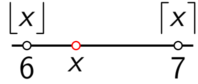

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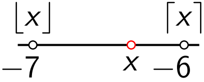

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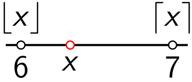

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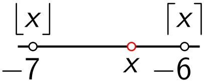

$$q = \left\lfloor \frac{-2015}{326} \right\rfloor = \lfloor -6.1809\cdots \rfloor = -7$$

$$r = -2015 - 326 \cdot (-7) = 267$$

Zadatak 3

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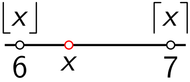
Rješenje

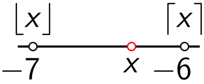
- $2015 \bmod 326 = 59 \quad \rightsquigarrow \quad 2015 \equiv 59 \pmod{326}$



$$q = \left\lfloor \frac{2015}{326} \right\rfloor = \lfloor 6.1809\cdots \rfloor = 6$$

$$r = 2015 - 326 \cdot 6 = 59$$

- $-2015 \bmod 326 = 267$



$$q = \left\lfloor \frac{-2015}{326} \right\rfloor = \lfloor -6.1809\cdots \rfloor = -7$$

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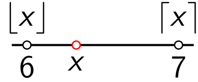
Rješenje

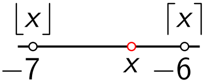
- $2015 \mod 326 = 59 \quad \rightsquigarrow \quad 2015 \equiv 59 \pmod{326}$



$$q = \left\lfloor \frac{2015}{326} \right\rfloor = \lfloor 6.1809\cdots \rfloor = 6$$

$$r = 2015 - 326 \cdot 6 = 59$$

- $-2015 \mod 326 = 267 \quad \rightsquigarrow \quad -2015 \equiv 267 \pmod{326}$



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četvrti zadatak

Zadatak 4

Odredite kvocijent i ostatak pri dijeljenju broja 3128 *s brojem* -219.

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Rješenje

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Rješenje

$$3128 = -219 \cdot q + r$$

$$q = \left\lceil \frac{3128}{-219} \right\rceil$$

Zadatak 4

Odredite kvocijent i ostatak pri dijeljenju broja 3128 *s brojem* −219.

Rješenje

$$3128 = -219 \cdot q + r$$

$$q = \left\lceil \frac{3128}{-219} \right\rceil = \lceil -14.2831 \cdots \rceil$$

Zadatak 4

Odredite kvocijent i ostatak pri dijeljenju broja 3128 *s brojem* −219.

Rješenje

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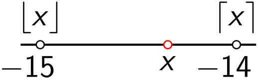
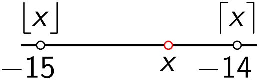


$$q = \left\lceil \frac{3128}{-219} \right\rceil = \lceil -14.2831\cdots \rceil = -14$$

Zadatak 4

Odredite kvocijent i ostatak pri dijeljenju broja 3128 *s brojem* −219.

Rješenje



$$3128 = -219 \cdot q + r$$

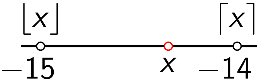
$$q = \left\lceil \frac{3128}{-219} \right\rceil = \lceil -14.2831 \cdots \rceil = -14$$

$$r = 3128 - (-219) \cdot (-14)$$

Zadatak 4

Odredite kvocijent i ostatak pri dijeljenju broja 3128 *s brojem* −219.

Rješenje



$$3128 = -219 \cdot q + r$$

$$q = \left\lceil \frac{3128}{-219} \right\rceil = \lceil -14.2831 \cdots \rceil = -14$$

$$r = 3128 - (-219) \cdot (-14) = 62$$

peti zadatak

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$a = bq_1 + r_1$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$a = bq_1 + r_1, \qquad 0 < r_1 < |b|$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, && 0 < r_1 < |b| \\ b &= r_1 q_2 + r_2 \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 < r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 < r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3 \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 < r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 &< r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\ r_2 &= r_3 q_4 + r_4 \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 &< r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\ r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 &< r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\
&\vdots
\end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 < r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\ r_2 &= r_3 q_4 + r_4, & 0 < r_4 < r_3 \\ &\vdots \\ r_{k-2} &= r_{k-1} q_k + r_k \end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 < r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 < r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 < r_k < r_{k-1}
\end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 &< r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 &< r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) =$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 < r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 < r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 < r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned} a &= bq_1 + r_1, & 0 &< r_1 < |b| \\ b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\ r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\ &\vdots \\ r_{k-2} &= r_{k-1} q_k + r_k, & 0 &< r_k < r_{k-1} \\ r_{k-1} &= r_k q_{k+1} \end{aligned}$$

$$M(a,b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a,b), \quad a, b \in \mathbb{Z}$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, && 0 < r_1 < |b| \\
b &= r_1 q_2 + r_2, && 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, && 0 < r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, && 0 < r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, && 0 < r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a,b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a,b), \qquad a, b \in \mathbb{Z}$$

$$r_i = r_{i-2} - q_i r_{i-1}$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 &< r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 &< r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$r_i = r_{i-2} - q_i r_{i-1}, \ r_{-1} = a, \ r_0 = b$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 &< r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 &< r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$r_i = r_{i-2} - q_i r_{i-1}, \;\; r_{-1} = a, \;\; r_0 = b$$

$$x_i = x_{i-2} - q_i x_{i-1}$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 < r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 < r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 < r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$r_i = r_{i-2} - q_i r_{i-1}, \; r_{-1} = a, \; r_0 = b$$

$$x_i = x_{i-2} - q_i x_{i-1}, \; x_{-1} = 1, \; x_0 = 0$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 &< r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 &< r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$r_i = r_{i-2} - q_i r_{i-1}, \; r_{-1} = a, \; r_0 = b$$

$$x_i = x_{i-2} - q_i x_{i-1}, \; x_{-1} = 1, \; x_0 = 0$$

$$y_i = y_{i-2} - q_i y_{i-1}$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 &< r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 &< r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 &< r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$r_i = r_{i-2} - q_i r_{i-1}, \; r_{-1} = a, \; r_0 = b$$

$$x_i = x_{i-2} - q_i x_{i-1}, \; x_{-1} = 1, \; x_0 = 0$$

$$y_i = y_{i-2} - q_i y_{i-1}, \; y_{-1} = 0, \; y_0 = 1$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 < r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 < r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 < r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$\begin{aligned}
r_i &= r_{i-2} - q_i r_{i-1}, \ r_{-1} = a, \ r_0 = b \\
x_i &= x_{i-2} - q_i x_{i-1}, \ x_{-1} = 1, \ x_0 = 0 \\
y_i &= y_{i-2} - q_i y_{i-1}, \ y_{-1} = 0, \ y_0 = 1
\end{aligned}$$

- Jedno cjelobrojno rješenje

$$x = x_k, \ y = y_k$$

Euklidov algoritam i prošireni Euklidov algoritam

Najveća zajednička mjera

$$\begin{aligned}
a &= bq_1 + r_1, & 0 < r_1 < |b| \\
b &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\
r_2 &= r_3 q_4 + r_4, & 0 < r_4 < r_3 \\
&\vdots \\
r_{k-2} &= r_{k-1} q_k + r_k, & 0 < r_k < r_{k-1} \\
r_{k-1} &= r_k q_{k+1}
\end{aligned}$$

$$M(a, b) = r_k$$

Cjelobrojno rješenje jednadžbe

$$ax + by = M(a, b), \quad a, b \in \mathbb{Z}$$

$$\begin{aligned}
r_i &= r_{i-2} - q_i r_{i-1},\ r_{-1} = a,\ r_0 = b \\
x_i &= x_{i-2} - q_i x_{i-1},\ x_{-1} = 1,\ x_0 = 0 \\
y_i &= y_{i-2} - q_i y_{i-1},\ y_{-1} = 0,\ y_0 = 1
\end{aligned}$$

- Jedno cjelobrojno rješenje

$$x = x_k, \ y = y_k$$

$$ax_i + by_i = r_i, \quad i = -1, 0, 1, 2, \ldots, k, k+1$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$28 = 2456 \cdot$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$28 = 2456 \cdot \ 0$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$28 = 2456 \cdot 0 +$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$28 = 2456 \cdot \; 0 \; + 28$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot \ 0 \ + 28 \\ 2456 &= \quad 28 \cdot \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot \ 0 \ + 28 \\ 2456 &= \quad 28 \cdot 87 + 20 \\ 28 &= \quad 20 \cdot \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned}
28 &= 2456 \cdot 0 + 28 \\
2456 &= 28 \cdot 87 + 20 \\
28 &= 20 \cdot 1 + 8 \\
20 &= 8 \cdot 2 +
\end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned}
28 &= 2456 \cdot 0 + 28\\
2456 &= 28 \cdot 87 + 20\\
28 &= 20 \cdot 1 + 8\\
20 &= 8 \cdot 2 + 4
\end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + 4 \\ 8 &= 4 \cdot \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način

$$\begin{aligned}
28 &= 2456 \cdot 0 + 28 \\
2456 &= 28 \cdot 87 + 20 \\
28 &= 20 \cdot 1 + 8 \\
20 &= 8 \cdot 2 + 4 \\
8 &= 4 \cdot 2
\end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + \boxed{4} \\ 8 &= 4 \cdot 2 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{aligned}
28 &= 2456 \cdot 0 + 28 \\
2456 &= 28 \cdot 87 + 20 \\
28 &= 20 \cdot 1 + 8 \\
20 &= 8 \cdot 2 + \boxed{4} \\
8 &= 4 \cdot 2
\end{aligned}$$

Zadatak 5

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje

1. način

$$\boxed{M(28, 2456) = 4}$$

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 &=& 2456 &\cdot& 0 &+& 28 \\ 2456 &=& 28 &\cdot& 87 &+& 20 \\ 28 &=& 20 &\cdot& 1 &+& 8 \\ 20 &=& 8 &\cdot& 2 &+& \boxed{4} \\ 8 &=& 4 &\cdot& 2 & & \end{array}$$

Zadatak 5

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcc|cr}
 & & & & q_i & & \\
28 & = & 2456 & \cdot & 0 & + & 28 \\
2456 & = & 28 & \cdot & 87 & + & 20 \\
28 & = & 20 & \cdot & 1 & + & 8 \\
20 & = & 8 & \cdot & 2 & + & \boxed{4} \\
8 & = & 4 & \cdot & 2 & &
\end{array}$$

i	-1	0	1	2	3	4
q_i						
x_i						
y_i						

Zadatak 5

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{array}{rcrc|c|cr} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i						
y_i						

Zadatak 5

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 & = & 2456 \cdot & 0 & + 28 \\ 2456 & = & 28 \cdot & 87 & + 20 \\ 28 & = & 20 \cdot & 1 & + 8 \\ 20 & = & 8 \cdot & 2 & + \boxed{4} \\ 8 & = & 4 \cdot & 2 & \end{array}$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0				
y_i						

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 &=& 2456 &\cdot& 0 &+& 28 \\ 2456 &=& 28 &\cdot& 87 &+& 20 \\ 28 &=& 20 &\cdot& 1 &+& 8 \\ 20 &=& 8 &\cdot& 2 &+& \boxed{4} \\ 8 &=& 4 &\cdot& 2 & & \end{array}$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0				
y_i	0	1				

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 &=& 2456 &\cdot& 0 &+& 28\\ 2456 &=& 28 &\cdot& 87 &+& 20\\ 28 &=& 20 &\cdot& 1 &+& 8\\ 20 &=& 8 &\cdot& 2 &+& \boxed{4}\\ 8 &=& 4 &\cdot& 2 & & \end{array}$$

$$1 - 0 \cdot 0 = 1$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1			
y_i	0	1				

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{array}{rcrcccc} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

$$0 - 87 \cdot 1 = -87$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87		
y_i	0	1				

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{aligned}
& && q_i && \\
28 &= 2456 \cdot && 0 &&+ 28 \\
2456 &= 28 \cdot && 87 &&+ 20 \\
28 &= 20 \cdot && 1 &&+ 8 \\
20 &= 8 \cdot && 2 &&+ \boxed{4} \\
8 &= 4 \cdot && 2 &&
\end{aligned}$$

$$1 - 1 \cdot (-87) = 88$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	
y_i	0	1				

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 &=& 2456 &\cdot& 0 &+& 28 \\ 2456 &=& 28 &\cdot& 87 &+& 20 \\ 28 &=& 20 &\cdot& 1 &+& 8 \\ 20 &=& 8 &\cdot& 2 &+& \boxed{4} \\ 8 &=& 4 &\cdot& 2 & & \end{array}$$

$$-87 - 2 \cdot 88 = -263$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1				

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 & = & 2456 \cdot & 0 & + 28 \\ 2456 & = & 28 \cdot & 87 & + 20 \\ 28 & = & 20 \cdot & 1 & + 8 \\ 20 & = & 8 \cdot & 2 & + \boxed{4} \\ 8 & = & 4 \cdot & 2 & \end{array}$$

$$0 - 0 \cdot 1 = 0$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0			

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način

$$M(28, 2456) = 4$$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$$1 - 87 \cdot 0 = 1$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1		

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način

$$M(28, 2456) = 4$$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + \ 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$$0 - 1 \cdot 1 = -1$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	

Zadatak 5

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje 1. način $M(28, 2456) = 4$

$$\begin{array}{rcrcc}
 & & & q_i & \\
28 & = & 2456 \cdot & 0 & + \ 28 \\
2456 & = & 28 \cdot & 87 & + \ 20 \\
28 & = & 20 \cdot & 1 & + \ 8 \\
20 & = & 8 \cdot & 2 & + \ \boxed{4} \\
8 & = & 4 \cdot & 2 &
\end{array}$$

$$1 - 2 \cdot (-1) = 3$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$$M(28, 2456) = 4$$

$$\begin{array}{rcrcccr}
 & & & & q_i & & \\
28 & = & 2456 & \cdot & 0 & + & 28 \\
2456 & = & 28 & \cdot & 87 & + & 20 \\
28 & = & 20 & \cdot & 1 & + & 8 \\
20 & = & 8 & \cdot & 2 & + & \boxed{4} \\
8 & = & 4 & \cdot & 2 & &
\end{array}$$

$$x = -263$$
$$y = 3$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{aligned}
& \qquad\qquad\quad q_i \\
28 &= 2456 \cdot 0 + 28 \\
2456 &= 28 \cdot 87 + 20 \\
28 &= 20 \cdot 1 + 8 \\
20 &= 8 \cdot 2 + \boxed{4} \\
8 &= 4 \cdot 2
\end{aligned}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$2456 = 28 \cdot$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

2. način

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

$$2456 = 28 \cdot 87$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

$$28x + 2456y = M(28, 2456).$$

Rješenje

1. način

$M(28, 2456) = 4$

2. način

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

$$2456 = 28 \cdot 87 +$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{aligned} & & & q_i & & \\ 28 &= 2456 &\cdot & \,0 &+& 28 \\ 2456 &= 28 &\cdot & \,87 &+& 20 \\ 28 &= 20 &\cdot & \,1 &+& 8 \\ 20 &= 8 &\cdot & \,2 &+& \boxed{4} \\ 8 &= 4 &\cdot & \,2 & & \end{aligned}$$

$x = -263$
$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$2456 = 28 \cdot 87 + 20$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$2456 = 28 \cdot 87 + 20$$
$$28 = 20 \cdot$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcc}
 & & & q_i & \\
28 &=& 2456 \cdot & 0 & + 28\\
2456 &=& 28 \cdot & 87 & + 20\\
28 &=& 20 \cdot & 1 & + 8\\
20 &=& 8 \cdot & 2 & + \boxed{4}\\
8 &=& 4 \cdot & 2 &
\end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{aligned}
2456 &= 28 \cdot 87 + 20\\
28 &= 20 \cdot 1
\end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$$M(28, 2456) = 4$$

$$\begin{aligned} & \quad q_i \\ 28 &= 2456 \cdot \boxed{0} + 28 \\ 2456 &= 28 \cdot \boxed{87} + 20 \\ 28 &= 20 \cdot \boxed{1} + 8 \\ 20 &= 8 \cdot \boxed{2} + \boxed{4} \\ 8 &= 4 \cdot 2 \end{aligned}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$2456 = 28 \cdot 87 + 20$$
$$28 = 20 \cdot 1 +$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcc|lr}
 & & & & q_i & & \\
28 & = & 2456 & \cdot & 0 & + & 28 \\
2456 & = & 28 & \cdot & 87 & + & 20 \\
28 & = & 20 & \cdot & 1 & + & 8 \\
20 & = & 8 & \cdot & 2 & + & \boxed{4} \\
8 & = & 4 & \cdot & 2 & &
\end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$2456 = 28 \cdot 87 + 20$$
$$28 = 20 \cdot 1 + 8$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$$M(28, 2456) = 4$$

$$\begin{array}{rcrcccl} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

$$x = -263$$
$$y = 3$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcrcl} 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & & & \end{array}$$

Zadatak 5

$$\begin{aligned} x_i &= x_{i-2} - q_i x_{i-1} \\ y_i &= y_{i-2} - q_i y_{i-1} \end{aligned}$$

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{aligned} 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcc|c}
 & & & & q_i & \\
28 & = & 2456 & \cdot & 0 & + 28 \\
2456 & = & 28 & \cdot & 87 & + 20 \\
28 & = & 20 & \cdot & 1 & + 8 \\
20 & = & 8 & \cdot & 2 & + \boxed{4} \\
8 & = & 4 & \cdot & 2 &
\end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcrl}
2456 & = & 28 & \cdot & 87 & + 20 \\
28 & = & 20 & \cdot & 1 & + 8 \\
20 & = & 8 & \cdot & 2 & +
\end{array}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

$$28x + 2456y = M(28, 2456).$$

Rješenje

1. način

$$M(28, 2456) = 4$$

$$\begin{array}{rcrcccl} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

$$x = -263$$
$$y = 3$$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{aligned} 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + 4 \end{aligned}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl}
 & & & q_i & \\
28 & = & 2456 \cdot & 0 & + 28 \\
2456 & = & 28 \cdot & 87 & + 20 \\
28 & = & 20 \cdot & 1 & + 8 \\
20 & = & 8 \cdot & 2 & + \boxed{4} \\
8 & = & 4 \cdot & 2 &
\end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcrcr}
2456 & = & 28 & \cdot & 87 & + & 20 \\
28 & = & 20 & \cdot & 1 & + & 8 \\
20 & = & 8 & \cdot & 2 & + & 4 \\
8 & = & 4 & \cdot & & &
\end{array}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$x = -263$

$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcl} 2456 &=& 28 \cdot 87 &+& 20 \\ 28 &=& 20 \cdot 1 &+& 8 \\ 20 &=& 8 \cdot 2 &+& 4 \\ 8 &=& 4 \cdot 2 & & \end{array}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$\begin{aligned} x_i &= x_{i-2} - q_i x_{i-1} \\ y_i &= y_{i-2} - q_i y_{i-1} \end{aligned}$$

Rješenje

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$M(28, 2456) = 4$

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 &=& 2456 &\cdot& 0 &+& 28 \\ 2456 &=& 28 &\cdot& 87 &+& 20 \\ 28 &=& 20 &\cdot& 1 &+& 8 \\ 20 &=& 8 &\cdot& 2 &+& \boxed{4} \\ 8 &=& 4 &\cdot& 2 & & \end{array}$$

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$y = 3$

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q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcrcr} 2456 &=& 28 &\cdot& 87 &+& 20 \\ 28 &=& 20 &\cdot& 1 &+& 8 \\ 20 &=& 8 &\cdot& 2 &+& \boxed{4} \\ 8 &=& 4 &\cdot& 2 & & \end{array}$$

Zadatak 5

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Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcl} & & & q_i & \\ 28 & = & 2456 \cdot & 0 & + 28 \\ 2456 & = & 28 \cdot & 87 & + 20 \\ 28 & = & 20 \cdot & 1 & + 8 \\ 20 & = & 8 \cdot & 2 & + \boxed{4} \\ 8 & = & 4 \cdot & 2 & \end{array}$$

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$y = 3$

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q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcl} & & & q_i & \\ 2456 & = & 28 \cdot & 87 & + 20 \\ 28 & = & 20 \cdot & 1 & + 8 \\ 20 & = & 8 \cdot & 2 & + \boxed{4} \\ 8 & = & 4 \cdot & 2 & \end{array}$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način

$M(28, 2456) = 4$

$$\begin{array}{rcrcc}
 & & & q_i & \\
28 & = & 2456 & \cdot\ 0 & +\ 28 \\
2456 & = & 28 & \cdot\ 87 & +\ 20 \\
28 & = & 20 & \cdot\ 1 & +\ 8 \\
20 & = & 8 & \cdot\ 2 & +\ \boxed{4} \\
8 & = & 4 & \cdot\ 2 &
\end{array}$$

$x = -263$
$y = 3$

i	-1	0	1	2	3	4
q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rcrcc}
 & & & q_i & \\
2456 & = & 28 & \cdot\ 87 & +\ 20 \\
28 & = & 20 & \cdot\ 1 & +\ 8 \\
20 & = & 8 & \cdot\ 2 & +\ \boxed{4} \\
8 & = & 4 & \cdot\ 2 &
\end{array}$$

i	-1	0	1	2	3
q_i					
y_i					
x_i					

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rclcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

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q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

$$\begin{array}{rclcl} & & & q_i & \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i					
x_i					

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

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Rješenje

1. način

$M(28, 2456) = 4$

$$\begin{array}{rclcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

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q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
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2. način

$$\begin{array}{rclcl} & & & q_i & \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0			
x_i					

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
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Rješenje

1. način

$M(28, 2456) = 4$

2. način

1. način

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 28 & = & 2456 & \cdot & 0 & + & 28 \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

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2. način

$$\begin{array}{rcrcccr} & & & & q_i & & \\ 2456 & = & 28 & \cdot & 87 & + & 20 \\ 28 & = & 20 & \cdot & 1 & + & 8 \\ 20 & = & 8 & \cdot & 2 & + & \boxed{4} \\ 8 & = & 4 & \cdot & 2 & & \end{array}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0			
x_i	0	1			

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$$M(28, 2456) = 4$$

q_i

$$\begin{array}{rcrcl}
28 & = & 2456 \cdot & 0 & + 28 \\
2456 & = & 28 \cdot & 87 & + 20 \\
28 & = & 20 \cdot & 1 & + 8 \\
20 & = & 8 \cdot & 2 & + \boxed{4} \\
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x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

q_i

$$\begin{array}{rcrcl}
2456 & = & 28 \cdot & 87 & + 20 \\
28 & = & 20 \cdot & 1 & + 8 \\
20 & = & 8 \cdot & 2 & + \boxed{4} \\
8 & = & 4 \cdot & 2 &
\end{array}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0	1		
x_i	0	1			

$$1 - 87 \cdot 0 = 1$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
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Rješenje

1. način

$M(28, 2456) = 4$

q_i

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + \boxed{4} \\ 8 &= 4 \cdot 2 \end{aligned}$$

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q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

2. način

q_i

$$\begin{aligned} 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + \boxed{4} \\ 8 &= 4 \cdot 2 \end{aligned}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0	1	-1	
x_i	0	1			

$$0 - 1 \cdot 1 = -1$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje

1. način

$M(28, 2456) = 4$

2. način

q_i

$$\begin{aligned} 28 &= 2456 \cdot 0 + 28 \\ 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + \boxed{4} \\ 8 &= 4 \cdot 2 \end{aligned}$$

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x_i	1	0	1	-87	88	-263
y_i	0	1	0	1	-1	3

q_i

$$\begin{aligned} 2456 &= 28 \cdot 87 + 20 \\ 28 &= 20 \cdot 1 + 8 \\ 20 &= 8 \cdot 2 + \boxed{4} \\ 8 &= 4 \cdot 2 \end{aligned}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0	1	-1	3
x_i	0	1			

$$1 - 2 \cdot (-1) = 3$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
$$y_i = y_{i-2} - q_i y_{i-1}$$

Rješenje 1. način $M(28, 2456) = 4$ 2. način

$$\begin{array}{rcrcl} & & & q_i & \\ 28 &=& 2456 \cdot & 0 & + 28 \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

$x = -263$
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$$\begin{array}{rcrcl} & & & q_i & \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0	1	-1	3
x_i	0	1	-87		

$$0 - 87 \cdot 1 = -87$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
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q_i			0	87	1	2
x_i	1	0	1	-87	88	-263
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$$\begin{array}{rcrcl} & & & q_i & \\ 2456 &=& 28 \cdot & 87 & + 20 \\ 28 &=& 20 \cdot & 1 & + 8 \\ 20 &=& 8 \cdot & 2 & + \boxed{4} \\ 8 &=& 4 \cdot & 2 & \end{array}$$

i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0	1	-1	3
x_i	0	1	-87	88	

$$1 - 1 \cdot (-87) = 88$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

$$x_i = x_{i-2} - q_i x_{i-1}$$
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Rješenje

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i	-1	0	1	2	3
q_i			87	1	2
y_i	1	0	1	-1	3
x_i	0	1	-87	88	-263

$$-87 - 2 \cdot 88 = -263$$

Zadatak 5

Odredite jedno cjelobrojno rješenje jednadžbe

$$28x + 2456y = M(28, 2456).$$

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Rješenje

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$M(28, 2456) = 4$

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q_i

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q_i

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y_i	1	0	1	-1	3
x_i	0	1	-87	88	-263

šesti zadatak

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$

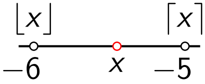
Rješenje

$2700 = -504 \cdot$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$



Rješenje

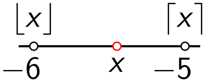
$2700 = -504 \cdot (-5)$

$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35\cdots \rceil = -5$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$



Rješenje

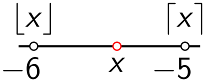
$$2700 = -504 \cdot (-5) +$$

$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35\cdots \rceil = -5$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$



Rješenje

$$2700 = -504 \cdot (-5) + 180$$

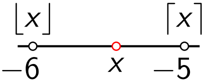
$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35\cdots \rceil = -5$$

$$r_1 = 2700 - (-504) \cdot (-5) = 180$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$



Rješenje

$$2700 = -504 \cdot (-5) + 180$$

$$-504 = 180 \cdot$$

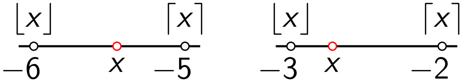
$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35\cdots \rceil = -5$$

$$r_1 = 2700 - (-504) \cdot (-5) = 180$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$



Rješenje

$$\begin{aligned} 2700 &= -504 \cdot (-5) + 180 \\ -504 &= \quad 180 \cdot (-3) \end{aligned}$$

$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35 \cdots \rceil = -5$$

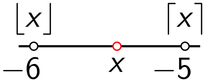
$$r_1 = 2700 - (-504) \cdot (-5) = 180$$

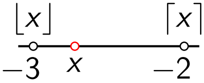
$$q_2 = \left\lfloor \frac{-504}{180} \right\rfloor = \lfloor -2.8 \rfloor = -3$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$





Rješenje

$$\begin{aligned} 2700 &= -504 \cdot (-5) + 180 \\ -504 &= \quad 180 \cdot (-3) + \end{aligned}$$

$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35 \cdots \rceil = -5$$

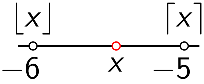
$$r_1 = 2700 - (-504) \cdot (-5) = 180$$

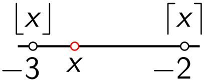
$$q_2 = \left\lfloor \frac{-504}{180} \right\rfloor = \lfloor -2.8 \rfloor = -3$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$





Rješenje

$$\begin{aligned} 2700 &= -504 \cdot (-5) + 180 \\ -504 &= 180 \cdot (-3) + 36 \end{aligned}$$

$$q_1 = \left\lceil \frac{2700}{-504} \right\rceil = \lceil -5.35\cdots \rceil = -5$$

$$r_1 = 2700 - (-504) \cdot (-5) = 180$$

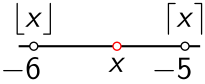
$$q_2 = \left\lfloor \frac{-504}{180} \right\rfloor = \lfloor -2.8 \rfloor = -3$$

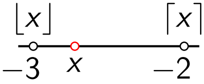
$$r_2 = -504 - 180 \cdot (-3) = 36$$

Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

$$2700x - 504y = M(2700, -504).$$





Rješenje

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Zadatak 6

Odredite jedno cjelobrojno rješenje jednadžbe

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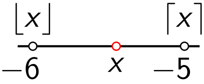

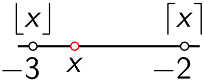

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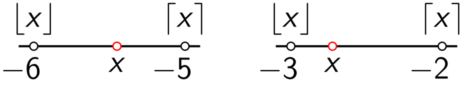

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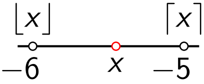

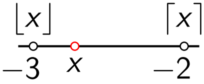

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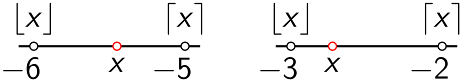

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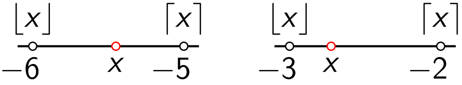
i	-1	0	1	2
q_i				
x_i				
y_i				



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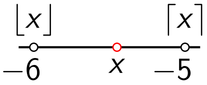

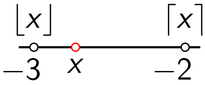

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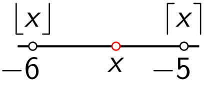
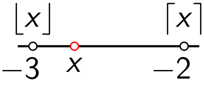
q_i			-5	-3
x_i	1	0		
y_i				





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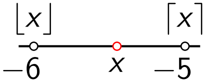

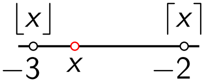

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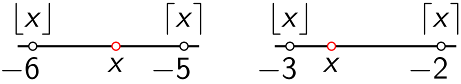
i	-1	0	1	2
q_i			-5	-3
x_i	1	0	1	
y_i	0	1		

$$1 - (-5) \cdot 0 = 1$$



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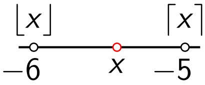
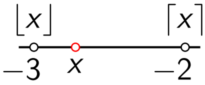
i	-1	0	1	2
q_i			-5	-3
x_i	1	0	1	3
y_i	0	1		

$$0 - (-3) \cdot 1 = 3$$





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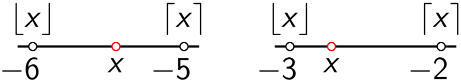
i	-1	0	1	2
q_i			-5	-3
x_i	1	0	1	3
y_i	0	1	5	

$$0 - (-5) \cdot 1 = 5$$



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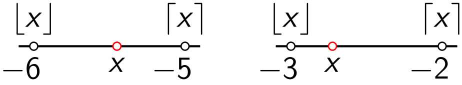

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i	-1	0	1	2
q_i			-5	-3
x_i	1	0	1	3
y_i	0	1	5	16

$$1 - (-3) \cdot 5 = 16$$

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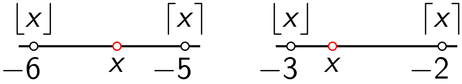
$\boxed{x = 3}$ $\boxed{y = 16}$



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sedmi zadatak

Svojstva kongruencija

- Ako je $a \equiv b \pmod{n}$ i $c \equiv d \pmod{n}$, tada je

$$a + c \equiv b + d \pmod{n}$$
$$a - c \equiv b - d \pmod{n}$$
$$ac \equiv bd \pmod{n}$$

- Ako je $a \equiv b \pmod{n}$ i $d \mid n$, tada je $a \equiv b \pmod{d}$.
- Ako je $a \equiv b \pmod{n}$, tada je $ac \equiv bc \pmod{nc}$ za svaki $c \in \mathbb{N}$.
- Ako je $a \equiv b \pmod{n}$ i $f \in \mathbb{Z}[x]$, tada je $f(a) \equiv f(b) \pmod{n}$.
- Dijeljenje kongruencija

$$ax \equiv ay \pmod{n} \quad \Longleftrightarrow \quad x \equiv y \left(\bmod\ \frac{n}{M(a,n)}\right)$$

Primjer

- Pretpostavimo da vrijedi $2a \equiv 11b \pmod{6}$.

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- Redukcija koeficijenata

$$2a \equiv \boxed{11}b \pmod{6} \quad \Longleftrightarrow \quad 2a \equiv \boxed{5}b \pmod{6}$$

$$11 \equiv 5 \pmod{6}$$

Zadatak 7

Na skupu $\mathbb{Z}$ *definirana je relacija* $\sim$ *s*

$$a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b.$$

a) *Dokažite da je* $\sim$ *relacija ekvivalencije na skupu* $\mathbb{Z}$.

b) *Odredite klasu broja* 1.

c) *Odredite kvocijentni skup* $\mathbb{Z}/\!\sim$.

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Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Zadatak 7

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a) *Dokažite da je $\sim$ relacija ekvivalencije na skupu $\mathbb{Z}$.*

b) *Odredite klasu broja* 1.

c) *Odredite kvocijentni skup $\mathbb{Z}/{\sim}$.*

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad (\forall a \in \mathbb{Z})\,(a \sim a)$

Zadatak 7

Na skupu $\mathbb{Z}$ *definirana je relacija* $\sim$ *s*

$$a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b.$$

a) *Dokažite da je* $\sim$ *relacija ekvivalencije na skupu* $\mathbb{Z}$*.*

b) *Odredite klasu broja* 1*.*

c) *Odredite kvocijentni skup* $\mathbb{Z}/{\sim}$*.*

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a$$

Zadatak 7

Na skupu $\mathbb{Z}$ definirana je relacija $\sim$ s

$$a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b.$$

a) *Dokažite da je $\sim$ relacija ekvivalencije na skupu $\mathbb{Z}$.*

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Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \ \Leftrightarrow\ 5 \mid 2a + 3a$$

Zadatak 7

Na skupu $\mathbb{Z}$ *definirana je relacija* $\sim$ *s*

$$a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b.$$

a) *Dokažite da je* $\sim$ *relacija ekvivalencije na skupu* $\mathbb{Z}$*.*

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c) *Odredite kvocijentni skup* $\mathbb{Z}/\!\sim$*.*

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 5 \mid 2a + 3a \;\Leftrightarrow\; 5 \mid 5a$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

Simetričnost $\left(\forall a, b \in \mathbb{Z}\right)\left(a \sim b \ \Rightarrow \ b \sim a\right)$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

Simetričnost $\big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$a \sim b$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

Simetričnost $\big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

$\boxed{5 \mid 2b + 3a}$

Simetričnost $\quad (\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b \;\Rightarrow$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

$5 \mid 2b + 3a$

Simetričnost $(\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b \;\Rightarrow\; 2a + 3b \equiv 0 \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a+3b}$$

$$\boxed{5 \mid 2b+3a}$$

Simetričnost

$$(\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; b \sim a\big)$$

$$a \sim b \;\Rightarrow\; 5 \mid 2a+3b \;\Rightarrow\; 2a+3b \equiv 0 \pmod{5} \;\Big/\cdot(-1) \;\Rightarrow$$

$$\Rightarrow\; -2a-3b \equiv 0 \pmod{5}$$

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Simetričnost $(\forall a, b \in \mathbb{Z})\big(a \sim b \ \Rightarrow\ b \sim a\big)$

$$a \sim b \ \Rightarrow\ 5 \mid 2a + 3b \ \Rightarrow\ 2a + 3b \equiv 0 \pmod 5 \ \big/ \cdot (-1) \ \Rightarrow$$

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$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

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Simetričnost $\quad \big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b \;\Rightarrow\; 2a + 3b \equiv 0 \pmod{5} \;\big/\cdot(-1) \;\Rightarrow$$

$$\Rightarrow\; -2a - 3b \equiv 0 \pmod{5} \;\Rightarrow\; 3a$$

$$-2 \equiv 3 \pmod{5}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

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Simetričnost $(\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b \;\Rightarrow\; 2a + 3b \equiv 0 \pmod{5} \;/\cdot(-1) \;\Rightarrow$$

$$-3 \equiv 2 \pmod{5}$$

$$\Rightarrow\; -2a - 3b \equiv 0 \pmod{5} \;\Rightarrow\; 3a + 2b$$

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$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a+3b}$$

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Simetričnost $(\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; b \sim a\big)$

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Simetričnost

$$(\forall a, b \in \mathbb{Z})\left(a \sim b \;\Rightarrow\; b \overset{\boxed{5 \mid 2b + 3a}}{\sim} a\right)$$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b \;\Rightarrow\; 2a + 3b \equiv 0 \pmod 5 \;/\cdot(-1) \;\Rightarrow$$

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$$\Rightarrow\; -2a - 3b \equiv 0 \pmod 5 \;\Rightarrow\; 3a + 2b \equiv 0 \pmod 5 \;\Rightarrow$$

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$$\Rightarrow\; 2b + 3a \equiv 0 \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

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Simetričnost $(\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; b \sim a\big)$

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$$\Rightarrow\; 2b + 3a \equiv 0 \pmod 5 \;\Rightarrow\; 5 \mid 2b + 3a$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

Simetričnost

$$(\forall a, b \in \mathbb{Z})\big(a \sim b \;\Rightarrow\; \overset{\boxed{5 \mid 2b + 3a}}{b \sim a}\big)$$

$$a \sim b \;\Rightarrow\; 5 \mid 2a + 3b \;\Rightarrow\; 2a + 3b \equiv 0 \pmod 5 \;\big/ \cdot (-1) \;\Rightarrow$$

$$-3 \equiv 2 \pmod 5$$

$$\Rightarrow\; -2a - 3b \equiv 0 \pmod 5 \;\Rightarrow\; 3a + 2b \equiv 0 \pmod 5 \;\Rightarrow$$

$$-2 \equiv 3 \pmod 5$$

$$\Rightarrow\; 2b + 3a \equiv 0 \pmod 5 \;\Rightarrow\; 5 \mid 2b + 3a \;\Rightarrow\; b \sim a$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

Tranzitivnost

$$\big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b$$

Tranzitivnost

$$(\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$(a \sim b) \wedge (b \sim c)$$

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Tranzitivnost $\big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a + 3b$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

Tranzitivnost

$$\big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a + 3b \quad \wedge \quad 5 \mid 2b + 3c$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

$$\boxed{5 \mid 2a + 3c}$$

Tranzitivnost

$$(\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a + 3b \quad \wedge \quad 5 \mid 2b + 3c \;\Rightarrow$$

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$$\boxed{5 \mid 2a + 3c}$$

Tranzitivnost

$$(\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a + 3b \;\;\wedge\;\; 5 \mid 2b + 3c \;\Rightarrow$$

$$\Rightarrow\; 5 \mid (2a + 3b) + (2b + 3c)$$

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \;\Longrightarrow\; c \mid k_1 a + k_2 b,\; \forall k_1, k_2 \in \mathbb{Z}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

$$\boxed{5 \mid 2a + 3c}$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a + 3b \;\;\wedge\;\; 5 \mid 2b + 3c \;\Rightarrow$$

$$\Rightarrow\; 5 \mid (2a + 3b) + (2b + 3c) \;\Rightarrow\; 5 \mid 2a + 3c + 5b$$

$$a, b, c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \;\Longrightarrow\; c \mid k_1 a + k_2 b,\;\; \forall k_1, k_2 \in \mathbb{Z}$$

$$\boxed{a \sim b \overset{\text{def}}{\Longleftrightarrow} 5 \mid 2a + 3b}$$

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Tranzitivnost

$$(\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a + 3b \;\;\wedge\;\; 5 \mid 2b + 3c \;\Rightarrow$$

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$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a+3b}$$

Tranzitivnost

$$(\forall a,b,c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \overset{\boxed{5 \mid 2a+3c}}{\sim} c\big)$$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 5 \mid 2a+3b \;\;\wedge\;\; 5 \mid 2b+3c \;\Rightarrow$$

$$\Rightarrow\; 5 \mid (2a+3b)+(2b+3c) \;\Rightarrow\; 5 \mid 2a+3c+5b \;\Rightarrow$$

$$\Rightarrow\; 5 \mid 2a+3c \;\Rightarrow\; a \sim c$$

$$\boxed{a,b,c \in \mathbb{Z},\; (c \mid a) \wedge (c \mid b) \;\Longrightarrow\; c \mid k_1 a + k_2 b,\;\; \forall k_1,k_2 \in \mathbb{Z}}$$

$$a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b$$

b)

$$[1]_\sim =$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_{\sim} = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\}$$

$$5 \mid 2x + 3$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \{x \in \mathbb{Z} : x \sim 1\} = \{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\} =$$

$$= \{x \in \mathbb{Z} : 5 \mid 2x + 3\}$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \{x \in \mathbb{Z} : x \sim 1\} = \{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\} =$$

$$= \{x \in \mathbb{Z} : 5 \mid 2x + 3\}$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv -3 \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\}$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv \overset{\overset{-3 \equiv 2 \pmod 5}{\uparrow}}{-3} \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\}$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv \overset{\overset{-3 \equiv 2 \pmod 5}{\uparrow}}{-3} \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2x \equiv 2 \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\}$$

$$-3 \equiv 2 \pmod 5$$

$$\uparrow$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv -3 \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2x \equiv 2 \pmod 5 \;\Big/ : 2$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\}$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv \overset{\overset{-3 \equiv 2 \pmod 5}{\uparrow}}{-3} \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2x \equiv 2 \pmod 5 \;\underset{M(2,5) = 1}{/ : 2} \;\Leftrightarrow\; x \equiv 1 \pmod 5$$

$$\boxed{ax \equiv ay \pmod n \iff x \equiv y \left(\mathrm{mod}\ \frac{n}{M(a,n)}\right)}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a+3b}$$

b)

$$[1]_{\sim} = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\} = \left\{x \in \mathbb{Z} : x \equiv 1 \pmod 5\right\}$$

$$5 \mid 2x+3 \;\Leftrightarrow\; 2x+3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv \overset{-3 \equiv 2 \pmod 5}{\overset{\uparrow}{-3}} \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2x \equiv 2 \pmod 5 \; \underset{M(2,5)=1}{\Big/ : 2} \;\Leftrightarrow\; x \equiv 1 \pmod 5$$

$$\boxed{ax \equiv ay \pmod n \iff x \equiv y \left(\bmod \frac{n}{M(a,n)}\right)}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

b)

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} = \left\{x \in \mathbb{Z} : 5 \mid 2x + 3 \cdot 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 5 \mid 2x + 3\right\} = \left\{x \in \mathbb{Z} : x \equiv 1 \pmod 5\right\} =$$

$$= 5\mathbb{Z} + 1$$

$$5 \mid 2x + 3 \;\Leftrightarrow\; 2x + 3 \equiv 0 \pmod 5 \;\Leftrightarrow\; 2x \equiv \underset{\substack{\uparrow \\ }}{} \overset{\substack{-3 \equiv 2 \pmod 5 \\ \uparrow}}{-3} \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2x \equiv 2 \pmod 5 \;\big/ : 2 \;\Leftrightarrow\; x \equiv 1 \pmod 5$$

$$M(2,5) = 1$$

$$\boxed{ax \equiv ay \pmod n \iff x \equiv y \left(\bmod \frac{n}{M(a,n)}\right)}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod{5}.$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod 5.$$

$$a \sim b$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod{5}.$$

$$a \sim b \Leftrightarrow 5 \mid 2a + 3b$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod 5.$$

$$a \sim b \;\Leftrightarrow\; 5 \mid 2a + 3b \;\Leftrightarrow\; 2a + 3b \equiv 0 \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod 5.$$

$$a \sim b \;\Leftrightarrow\; 5 \mid 2a + 3b \;\Leftrightarrow\; 2a + 3b \equiv 0 \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2a \equiv -3b \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a+3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod{5}.$$

$$a \sim b \;\Leftrightarrow\; 5 \mid 2a+3b \;\Leftrightarrow\; 2a+3b \equiv 0 \pmod{5} \;\Leftrightarrow$$

$$\Leftrightarrow\; 2a \equiv \underset{\substack{\big\downarrow \\ -3 \,\equiv\, 2 \pmod{5}}}{-3}b \pmod{5}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod 5.$$

$$a \sim b \Leftrightarrow 5 \mid 2a + 3b \Leftrightarrow 2a + 3b \equiv 0 \pmod 5 \Leftrightarrow$$

$$\Leftrightarrow 2a \equiv \underset{\underset{-3 \equiv 2 \pmod 5}{\downarrow}}{-3}b \pmod 5 \Leftrightarrow 2a \equiv 2b \pmod 5$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod{5}.$$

$$a \sim b \Leftrightarrow 5 \mid 2a + 3b \Leftrightarrow 2a + 3b \equiv 0 \pmod{5} \Leftrightarrow$$

$$\Leftrightarrow 2a \equiv \underset{\substack{\downarrow \\ -3 \equiv 2 \pmod{5}}}{-3}b \pmod{5} \Leftrightarrow 2a \equiv 2b \pmod{5} \underset{M(2,5) = 1}{\Leftrightarrow} a \equiv b \pmod{5}$$

$$\boxed{ax \equiv ay \pmod{n} \iff x \equiv y \left(\bmod \frac{n}{M(a,n)}\right)}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 5 \mid 2a + 3b}$$

c) Tvrdimo da vrijedi

$$a \sim b \iff a \equiv b \pmod 5.$$

$$a \sim b \;\Leftrightarrow\; 5 \mid 2a+3b \;\Leftrightarrow\; 2a+3b \equiv 0 \pmod 5 \;\Leftrightarrow$$

$$\Leftrightarrow\; 2a \equiv \underset{\substack{\downarrow \\ -3 \equiv 2 \pmod 5}}{-3}b \pmod 5 \;\Leftrightarrow\; 2a \equiv 2b \pmod 5 \;\underset{M(2,5)=1}{\Leftrightarrow}\; a \equiv b \pmod 5$$

$$\mathbb{Z}/\!\sim\; = \{5\mathbb{Z},\ 5\mathbb{Z}+1,\ 5\mathbb{Z}+2,\ 5\mathbb{Z}+3,\ 5\mathbb{Z}+4\}$$

$$\boxed{ax \equiv ay \pmod n \iff x \equiv y \left(\bmod \frac{n}{M(a,n)}\right)}$$

Rješavanje kongruencija

Teorem o rješenjima linearne kongruencije

Neka su $a, b \in \mathbb{Z}$, $a \neq 0$ i $n \in \mathbb{N} \setminus \{1\}$. Kongruencija

$$ax \equiv b \pmod{n}$$

ima rješenje akko $M(a, n) = d \mid b$. Ako je ovaj uvjet zadovoljen, tada gornja kongruencija ima d rješenja modulo n.

$ax \equiv b \pmod{n}$

$ax \equiv b \pmod{n}, \quad d = M(a, n)$

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

$$ax \equiv b \pmod{n} \,\Big/ : d$$

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

$$ax \equiv b \pmod{n} \Big/ : d$$

$$a'x \equiv b' \pmod{n'}$$

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

$$ax \equiv b \pmod{n} \Big/ : d$$

$$a'x \equiv b' \pmod{n'}$$

- $M(a', n') = 1$

$ax \equiv b \pmod{n}, \quad d = M(a, n)$

$a = a'd, \quad b = b'd, \quad n = n'd$

$$ax \equiv b \pmod{n} \ \Big/ : d$$

$$a'x \equiv b' \pmod{n'}$$

- $M(a', n') = 1$
- $a'x \equiv b' \pmod{n'}$ ima jedinstveno rješenje x_0 modulo n'.

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

$$ax \equiv b \pmod{n} \ \big/ : d$$

$$a'x \equiv b' \pmod{n'}$$

- $M(a', n') = 1$
- $a'x \equiv b' \pmod{n'}$ ima jedinstveno rješenje x_0 modulo n'.

Rješenja od $ax \equiv b \pmod{n}$

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

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$$x_k = x_0 + kn', \ k = 0, 1, \ldots, d-1$$

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<u>prošireni Euklidov algoritam</u>

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

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prošireni Euklidov algoritam

početak: n' dijelimo s a'

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

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Rješenja od $ax \equiv b \pmod{n}$

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<u>prošireni Euklidov algoritam</u>

početak: n' dijelimo s a'

$$u \rightsquigarrow y_i = y_{i-2} - q_i y_{i-1}$$

$$y_{-1} = 0, \;\; y_0 = 1$$

$$ax \equiv b \pmod{n}, \quad d = M(a, n)$$

$$a = a'd, \quad b = b'd, \quad n = n'd$$

Kako pronaći x_0?

$$M(a', n') = 1 \ \Rightarrow$$

Kvocijenti dobiveni primjenom Euklidovog algoritma na traženje $M(a, n)$ jednaki su kvocijentima koji se dobiju primjenom Euklidovog algoritma na traženje $M(a', n')$.

- $M(a', n') = 1$
- $a'x \equiv b' \pmod{n'}$ ima jedinstveno rješenje x_0 modulo n'.

Rješenja od $ax \equiv b \pmod{n}$

$$x_k = x_0 + kn', \ k = 0, 1, \ldots, d - 1$$

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$$n = aq_1 + r_1, \quad 0 < r_1 < a$$

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$$\vdots$$

$$\boxed{M(a', n') = 1}$$

$$n' = a'q_1 + r_1', \quad 0 < r_1' < a'$$

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$$n = aq_1 + r_1 \ \Rightarrow \ n'd = (a'd)q_1 + r_1 \Big/ : d \ \Rightarrow \ n' = a'q_1 + \frac{r_1}{d}$$

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$$r_1' < a'$$

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$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ / : d \ \Rightarrow\ n' = a'q_1 + \underbrace{\frac{r_1}{d}}_{= r_1'}$$

$$r_1' < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d}$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

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$$r_1' < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd, \; b = b'd, \; n = n'd$$

$$\boxed{M(a, n) = d}$$

$$\begin{aligned} n &= aq_1 + r_1, & 0 < r_1 < a \\ a &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\ &\vdots \end{aligned}$$

$$\boxed{M(a', n') = 1}$$

$$\begin{aligned} n' &= a'q_1 + r_1', & 0 < r_1' < a' \\ a' &= r_1' q_2 + r_2', & 0 < r_2' < r_1' \\ r_1' &= r_2' q_3 + r_3', & 0 < r_3' < r_2' \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \;\Rightarrow\; n'd = (a'd)q_1 + r_1 \;/ : d \;\Rightarrow\; n' = a'q_1 + \boxed{\frac{r_1}{d}}^{= r_1'}$$

$$r_1' < a' \;\Leftrightarrow\; \frac{r_1}{d} < \frac{a}{d} \;\Leftrightarrow\; r_1 < a$$

$$a = r_1 q_2 + r_2$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

$$\boxed{M(a,n) = d}$$

$$\begin{aligned} n &= aq_1 + r_1, & 0 < r_1 < a \\ a &= r_1q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2q_3 + r_3, & 0 < r_3 < r_2 \\ &\vdots \end{aligned}$$

$$\boxed{M(a',n') = 1}$$

$$\begin{aligned} n' &= a'q_1 + r_1', & 0 < r_1' < a' \\ a' &= r_1'q_2 + r_2', & 0 < r_2' < r_1' \\ r_1' &= r_2'q_3 + r_3', & 0 < r_3' < r_2' \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ /: d \ \Rightarrow\ n' = a'q_1 + \boxed{\frac{r_1}{d}}^{= r_1'}$$

$$r_1' < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$a = r_1q_2 + r_2 \ \Rightarrow\ a'd = (r_1'd)q_2 + r_2$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

$\boxed{M(a, n) = d}$

$$\begin{aligned} n &= aq_1 + r_1, & 0 < r_1 < a \\ a &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\ &\vdots \end{aligned}$$

$\boxed{M(a', n') = 1}$

$$\begin{aligned} n' &= a'q_1 + r'_1, & 0 < r'_1 < a' \\ a' &= r'_1 q_2 + r'_2, & 0 < r'_2 < r'_1 \\ r'_1 &= r'_2 q_3 + r'_3, & 0 < r'_3 < r'_2 \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ /: d \ \Rightarrow\ n' = a'q_1 + \boxed{\frac{r_1}{d}}^{= r'_1}$$

$$r'_1 < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$a = r_1 q_2 + r_2 \ \Rightarrow\ a'd = (r'_1 d)q_2 + r_2 \ /: d$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

$$\boxed{M(a, n) = d}$$

$$\begin{aligned} n &= aq_1 + r_1, & 0 &< r_1 < a \\ a &= r_1 q_2 + r_2, & 0 &< r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 &< r_3 < r_2 \\ &\vdots \end{aligned}$$

$$\boxed{M(a', n') = 1}$$

$$\begin{aligned} n' &= a'q_1 + r_1', & 0 &< r_1' < a' \\ a' &= r_1' q_2 + r_2', & 0 &< r_2' < r_1' \\ r_1' &= r_2' q_3 + r_3', & 0 &< r_3' < r_2' \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ /:d \ \Rightarrow\ n' = a'q_1 + \frac{r_1}{d}{}^{= r_1'}$$

$$r_1' < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$a = r_1 q_2 + r_2 \ \Rightarrow\ a'd = (r_1'd)q_2 + r_2 \ /:d \ \Rightarrow\ a' = r_1' q_2 + \frac{r_2}{d}$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

$$\boxed{M(a,n) = d}$$

$$\begin{aligned} n &= aq_1 + r_1, & 0 < r_1 < a \\ a &= r_1q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2q_3 + r_3, & 0 < r_3 < r_2 \\ &\vdots \end{aligned}$$

$$\boxed{M(a',n') = 1}$$

$$\begin{aligned} n' &= a'q_1 + r'_1, & 0 < r'_1 < a' \\ a' &= r'_1q_2 + r'_2, & 0 < r'_2 < r'_1 \\ r'_1 &= r'_2q_3 + r'_3, & 0 < r'_3 < r'_2 \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ /:d \ \Rightarrow\ n' = a'q_1 + \left(\frac{r_1}{d}\right)^{= r'_1}$$

$$r'_1 < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$a = r_1q_2 + r_2 \ \Rightarrow\ a'd = (r'_1d)q_2 + r_2 \ /:d \ \Rightarrow\ a' = r'_1q_2 + \left(\frac{r_2}{d}\right)^{= r'_2}$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

$$\boxed{M(a, n) = d}$$

$$\begin{aligned} n &= aq_1 + r_1, & 0 < r_1 < a \\ a &= r_1q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2q_3 + r_3, & 0 < r_3 < r_2 \\ &\vdots \end{aligned}$$

$$\boxed{M(a', n') = 1}$$

$$\begin{aligned} n' &= a'q_1 + r_1', & 0 < r_1' < a' \\ a' &= r_1'q_2 + r_2', & 0 < r_2' < r_1' \\ r_1' &= r_2'q_3 + r_3', & 0 < r_3' < r_2' \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ /:d \ \Rightarrow\ n' = a'q_1 + \boxed{\frac{r_1}{d}}^{= r_1'}$$

$$r_1' < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$a = r_1q_2 + r_2 \ \Rightarrow\ a'd = (r_1'd)q_2 + r_2 \ /:d \ \Rightarrow\ a' = r_1'q_2 + \boxed{\frac{r_2}{d}}^{= r_2'}$$

$$r_2' < r_1'$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\; b = b'd,\; n = n'd$$

$$\boxed{M(a, n) = d}$$

$$\begin{aligned} n &= aq_1 + r_1, & 0 < r_1 < a \\ a &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\ r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\ &\vdots \end{aligned}$$

$$\boxed{M(a', n') = 1}$$

$$\begin{aligned} n' &= a'q_1 + r_1', & 0 < r_1' < a' \\ a' &= r_1' q_2 + r_2', & 0 < r_2' < r_1' \\ r_1' &= r_2' q_3 + r_3', & 0 < r_3' < r_2' \\ &\vdots \end{aligned}$$

$$n = aq_1 + r_1 \;\Rightarrow\; n'd = (a'd)q_1 + r_1 \;/ : d \;\Rightarrow\; n' = a'q_1 + \boxed{\frac{r_1}{d}}^{= r_1'}$$

$$r_1' < a' \;\Leftrightarrow\; \frac{r_1}{d} < \frac{a}{d} \;\Leftrightarrow\; r_1 < a$$

$$a = r_1 q_2 + r_2 \;\Rightarrow\; a'd = (r_1'd)q_2 + r_2 \;/ : d \;\Rightarrow\; a' = r_1' q_2 + \boxed{\frac{r_2}{d}}^{= r_2'}$$

$$r_2' < r_1' \;\Leftrightarrow\; \frac{r_2}{d} < \frac{r_1}{d}$$

$$ax \equiv b \pmod{n}, \quad a'x \equiv b' \pmod{n'}, \quad a = a'd,\ b = b'd,\ n = n'd$$

$$\boxed{M(a, n) = d}$$

$$\begin{aligned}
n &= aq_1 + r_1, & 0 < r_1 < a \\
a &= r_1 q_2 + r_2, & 0 < r_2 < r_1 \\
r_1 &= r_2 q_3 + r_3, & 0 < r_3 < r_2 \\
&\vdots
\end{aligned}$$

$$\boxed{M(a', n') = 1}$$

$$\begin{aligned}
n' &= a'q_1 + r_1', & 0 < r_1' < a' \\
a' &= r_1' q_2 + r_2', & 0 < r_2' < r_1' \\
r_1' &= r_2' q_3 + r_3', & 0 < r_3' < r_2' \\
&\vdots
\end{aligned}$$

$$n = aq_1 + r_1 \ \Rightarrow\ n'd = (a'd)q_1 + r_1 \ /: d \ \Rightarrow\ n' = a'q_1 + \boxed{\frac{r_1}{d}}^{= r_1'}$$

$$r_1' < a' \ \Leftrightarrow\ \frac{r_1}{d} < \frac{a}{d} \ \Leftrightarrow\ r_1 < a$$

$$a = r_1 q_2 + r_2 \ \Rightarrow\ a'd = (r_1'd)q_2 + r_2 \ /: d \ \Rightarrow\ a' = r_1' q_2 + \boxed{\frac{r_2}{d}}^{= r_2'}$$

$$r_2' < r_1' \ \Leftrightarrow\ \frac{r_2}{d} < \frac{r_1}{d} \ \Leftrightarrow\ r_2 < r_1$$

osmi zadatak

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$527 = 4 \cdot$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$527 = 4 \cdot 131$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$527 = 4 \cdot 131 +$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$527 = 4 \cdot 131 + 3$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned}527 &= 4 \cdot 131 + 3\\ 4 &= 3 \cdot\end{aligned}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot 1 \end{aligned}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot \ \ 1 \ \ + \end{aligned}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot \ \ 1 \ \ + 1 \end{aligned}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned}
527 &= 4 \cdot 131 + 3 \\
4 &= 3 \cdot \ \ 1 \ \ + 1 \\
3 &= 1 \cdot
\end{aligned}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot 1 + 1 \\ 3 &= 1 \cdot 3 \end{aligned}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot 1 + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot 1 + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot \overset{q_i}{\boxed{131}} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4\cdot \overset{q_i}{\boxed{131}} + 3\\ 4 &= 3\cdot \boxed{1} + \boxed{1}\\ 3 &= 1\cdot 3\end{aligned}$$

$$\boxed{M(4,527) = 1}$$

i	-1	0	1	2
q_i				
y_i				

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{array}{rcl} & & \qquad q_i \\ 527 &=& 4 \cdot \boxed{131} + 3 \\ 4 &=& 3 \cdot \boxed{\;1\;} + \boxed{1} \\ 3 &=& 1 \cdot \;\; 3 \end{array}$$

$$\boxed{M(4,527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i				

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} 527 &= 4 \cdot \overset{q_i}{\boxed{131}} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1		

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{aligned} & \qquad\quad\;\; q_i \\ 527 &= 4 \cdot \boxed{131} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4,527) = 1}$$

$$0 - 131 \cdot 1 = -131$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

Rješenje

$$\begin{array}{rcl} & & \phantom{4\cdot{}}q_i \\ 527 &=& 4 \cdot \boxed{131} + 3 \\ 4 &=& 3 \cdot \boxed{1} + \boxed{1} \\ 3 &=& 1 \cdot 3 \end{array}$$

$$\boxed{M(4, 527) = 1}$$

$$1 - 1 \cdot (-131) = 132$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4\\ b' &= b = 89\\ n' &= n = 527\end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} 527 &= 4\cdot \overset{q_i}{\boxed{131}} + 3\\ 4 &= 3\cdot \boxed{1} + \boxed{1}\\ 3 &= 1\cdot 3\end{aligned}$$

$$\boxed{M(4,527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 =$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4 \\ b' &= b = 89 \\ n' &= n = 527 \end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} & \qquad\quad q_i \\ 527 &= 4 \cdot \boxed{131} + 3 \\ 4 &= 3 \cdot \boxed{\ 1\ } + \boxed{1} \\ 3 &= 1 \cdot \ 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\begin{aligned} a' &= a = 4 \\ b' &= b = 89 \\ n' &= n = 527 \end{aligned}$$

Rješenje

$$\boxed{x_0 = ub' \bmod n'}$$

q_i

$$\begin{aligned} 527 &= 4 \cdot 131 + 3 \\ 4 &= 3 \cdot 1 + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

$$\boxed{M(4, 527) = 1}$$

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132 \cdot 89$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4 \\ b' &= b = 89 \\ n' &= n = 527 \end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} 527 &= 4 \cdot \overset{q_i}{\boxed{131}} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132 \cdot 89 \mod 527$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4 \\ b' &= b = 89 \\ n' &= n = 527 \end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} 527 &= 4 \cdot \overset{q_i}{\boxed{131}} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132 \cdot 89 \mod 527 = 11\,748 \mod 527$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4 \\ b' &= b = 89 \\ n' &= n = 527 \end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} 527 &= 4 \cdot \overset{q_i}{\boxed{131}} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4,527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132 \cdot 89 \mod 527 = 11\,748 \mod 527 = 154$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4 \\ b' &= b = 89 \\ n' &= n = 527 \end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} & \qquad\quad q_i \\ 527 &= 4 \cdot \boxed{131} + 3 \\ 4 &= 3 \cdot \boxed{1} + \boxed{1} \\ 3 &= 1 \cdot 3 \end{aligned}$$

$$\boxed{M(4, 527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132 \cdot 89 \mod 527 = 11\,748 \mod 527 = 154$$

- Precizniji zapis rješenja

$$x \equiv 154 \pmod{527}$$

Zadatak 8

Riješite kongruenciju $4x \equiv 89 \pmod{527}$.

$$\boxed{\begin{aligned} a' &= a = 4\\ b' &= b = 89\\ n' &= n = 527\end{aligned}}$$

Rješenje

$$\boxed{x_0 = ub' \mod n'}$$

$$\begin{aligned} 527 &= 4\cdot \overset{q_i}{\boxed{131}} + 3\\ 4 &= 3\cdot \boxed{1} + \boxed{1}\\ 3 &= 1\cdot 3\end{aligned}$$

$$\boxed{M(4,527) = 1}$$

i	-1	0	1	2
q_i			131	1
y_i	0	1	-131	132

- Zadana kongruencija ima jedinstveno rješenje.

$$x_0 = 132\cdot 89 \mod 527 = 11\,748 \mod 527 = 154$$

- Precizniji zapis rješenja $\boxed{x = 527k + 154,\ k\in\mathbb{Z}}$

$$x \equiv 154 \pmod{527}$$

deveti zadatak

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$*.*

Rješenje

$5432 = 234 \cdot$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$5432 = 234 \cdot 23$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$5432 = 234 \cdot 23 +$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$*.*

Rješenje

$$5432 = 234 \cdot 23 + 50$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= \ \ 50 \cdot \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$*.*

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= \ \ 50 \cdot \ 4 \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= \ \ 50 \cdot \ 4 \ + 34 \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$*.*

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 + \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 + 16 \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50\\
234 &= 50 \cdot 4 + 34\\
50 &= 34 \cdot 1 + 16\\
34 &= 16 \cdot
\end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$*.*

Rješenje

$$
\begin{aligned}
5432 &= 234 \cdot 23 + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2
\end{aligned}
$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$*.*

Rješenje

$$
\begin{aligned}
5432 &= 234 \cdot 23 + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2 +
\end{aligned}
$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50\\
234 &= \ \ 50 \cdot \ 4 \ + 34\\
50 &= \ \ 34 \cdot \ 1 \ + 16\\
34 &= \ \ 16 \cdot \ 2 \ + \ 2
\end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$
\begin{array}{rcrcrcr}
5432 & = & 234 & \cdot & 23 & + & 50 \\
234 & = & 50 & \cdot & 4 & + & 34 \\
50 & = & 34 & \cdot & 1 & + & 16 \\
34 & = & 16 & \cdot & 2 & + & 2 \\
16 & = & 2 & \cdot & & &
\end{array}
$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50\\
234 &= 50 \cdot 4 + 34\\
50 &= 34 \cdot 1 + 16\\
34 &= 16 \cdot 2 + 2\\
16 &= 2 \cdot 8
\end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$\boxed{M(234, 5432) = 2}$

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 + 16 \\ 34 &= 16 \cdot 2 + \boxed{2} \\ 16 &= 2 \cdot 8 \end{aligned}$$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2 + \boxed{2} \\
16 &= 2 \cdot 8
\end{aligned}$$

- $2 \mid 54$

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2 + \boxed{2} \\
16 &= 2 \cdot 8
\end{aligned}$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje $\boxed{M(234, 5432) = 2}$ $234x \equiv 54 \pmod{5432}$

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 + 16 \\ 34 &= 16 \cdot 2 + \boxed{2} \\ 16 &= 2 \cdot 8 \end{aligned}$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$234x \equiv 54 \pmod{5432} \;\Big/ : 2$$

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2 + \boxed{2} \\
16 &= 2 \cdot 8
\end{aligned}$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$234x \equiv 54 \pmod{5432} \;\Big/ : 2$$

$$117x \equiv 27 \pmod{2716}$$

$$\begin{aligned} 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 + 16 \\ 34 &= 16 \cdot 2 + \boxed{2} \\ 16 &= 2 \cdot 8 \end{aligned}$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50\\
234 &= 50 \cdot 4 + 34\\
50 &= 34 \cdot 1 + 16\\
34 &= 16 \cdot 2 + \boxed{2}\\
16 &= 2 \cdot 8
\end{aligned}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$117x \equiv 27 \pmod{2716}$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{aligned}
5432 &= 234 \cdot 23 + 50\\
234 &= 50 \cdot 4 + 34\\
50 &= 34 \cdot 1 + 16\\
34 &= 16 \cdot 2 + \boxed{2}\\
16 &= 2 \cdot 8
\end{aligned}$$

- $2 \mid 54$ ⟶ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcll}
 & & & & q_i & \\
5432 &=& 234 &\cdot& 23 &+\ 50\\
234 &=& 50 &\cdot& 4 &+\ 34\\
50 &=& 34 &\cdot& 1 &+\ 16\\
34 &=& 16 &\cdot& 2 &+\ \boxed{2}\\
16 &=& 2 &\cdot& 8 &
\end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \ \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcc|c|cc}
 & & & & q_i & & \\
5432 & = & 234 & \cdot & 23 & + & 50 \\
234 & = & 50 & \cdot & 4 & + & 34 \\
50 & = & 34 & \cdot & 1 & + & 16 \\
34 & = & 16 & \cdot & 2 & + & \boxed{2} \\
16 & = & 2 & \cdot & 8 & &
\end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i						
y_i						

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \ (\text{mod } \overset{n}{5432}) \ \big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \ (\text{mod } \overset{n'}{2716})$$

$$\begin{aligned} & \quad\quad\quad\quad\ \ \ q_i \\ 5432 &= 234 \cdot 23 + 50 \\ 234 &= 50 \cdot 4 + 34 \\ 50 &= 34 \cdot 1 + 16 \\ 34 &= 16 \cdot 2 + \boxed{2} \\ 16 &= 2 \cdot 8 \end{aligned}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i						

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234,5432)=2}$$

$$\begin{array}{rcrcccl} & & & & q_i & & \\ 5432 &=& 234 &\cdot& 23 &+& 50 \\ 234 &=& 50 &\cdot& 4 &+& 34 \\ 50 &=& 34 &\cdot& 1 &+& 16 \\ 34 &=& 16 &\cdot& 2 &+& \boxed{2} \\ 16 &=& 2 &\cdot& 8 & & \end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ :2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1				

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcl} & & & q_i & \\ 5432 &=& 234 \cdot & 23 & + 50 \\ 234 &=& 50 \cdot & 4 & + 34 \\ 50 &=& 34 \cdot & 1 & + 16 \\ 34 &=& 16 \cdot & 2 & + \boxed{2} \\ 16 &=& 2 \cdot & 8 & \end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23			

$$0 - 23 \cdot 1 = -23$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$\boxed{M(234, 5432) = 2}$

$$\begin{aligned}
5432 &= 234 \cdot \overset{q_i}{23} + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2 + \boxed{2} \\
16 &= 2 \cdot 8
\end{aligned}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \ \big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93		

$$1 - 4 \cdot (-23) = 93$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{aligned}
5432 &= 234 \cdot \overset{q_i}{23} + 50 \\
234 &= 50 \cdot 4 + 34 \\
50 &= 34 \cdot 1 + 16 \\
34 &= 16 \cdot 2 + \boxed{2} \\
16 &= 2 \cdot 8
\end{aligned}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \ \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	

$$-23 - 1 \cdot 93 = -116$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcl} & & & q_i & \\ 5432 &=& 234 \cdot & \boxed{23} & +\ 50 \\ 234 &=& 50 \cdot & \boxed{4} & +\ 34 \\ 50 &=& 34 \cdot & \boxed{1} & +\ 16 \\ 34 &=& 16 \cdot & \boxed{2} & +\ \boxed{2} \\ 16 &=& 2 \cdot & 8 & \end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \ \big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

$$93 - 2 \cdot (-116) = 325$$

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{array}{rcrcl}
 & & & q_i & \\
5432 & = & 234 \cdot & 23 & + \; 50 \\
234 & = & 50 \cdot & 4 & + \; 34 \\
50 & = & 34 \cdot & 1 & + \; 16 \\
34 & = & 16 \cdot & 2 & + \; \boxed{2} \\
16 & = & 2 \cdot & 8 &
\end{array}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$x_0 =$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcc}
 & & & q_i & \\
5432 & = & 234 \cdot & 23 & + 50 \\
234 & = & 50 \cdot & 4 & + 34 \\
50 & = & 34 \cdot & 1 & + 16 \\
34 & = & 16 \cdot & 2 & + \boxed{2} \\
16 & = & 2 \cdot & 8 &
\end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \xrightarrow{\quad}$ kongruencija ima 2 rješenja

$$x_0 = 325$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{array}{rcrc|c|cl}
 & & & & q_i & & \\
5432 & = & 234 & \cdot & 23 & + & 50 \\
234 & = & 50 & \cdot & 4 & + & 34 \\
50 & = & 34 & \cdot & 1 & + & 16 \\
34 & = & 16 & \cdot & 2 & + & \boxed{2} \\
16 & = & 2 & \cdot & 8 & &
\end{array}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rclclcl}
 & & & & q_i & & \\
5432 & = & 234 & \cdot & 23 & + & 50 \\
234 & = & 50 & \cdot & 4 & + & 34 \\
50 & = & 34 & \cdot & 1 & + & 16 \\
34 & = & 16 & \cdot & 2 & + & \boxed{2} \\
16 & = & 2 & \cdot & 8 & &
\end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{array}{rcrcl} & & & q_i & \\ 5432 & = & 234 \cdot & 23 & + 50 \\ 234 & = & 50 \cdot & 4 & + 34 \\ 50 & = & 34 \cdot & 1 & + 16 \\ 34 & = & 16 \cdot & 2 & + \boxed{2} \\ 16 & = & 2 \cdot & 8 & \end{array}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcccl} & & & & q_i & & \\ 5432 & = & 234 & \cdot & 23 & + & 50 \\ 234 & = & 50 & \cdot & 4 & + & 34 \\ 50 & = & 34 & \cdot & 1 & + & 16 \\ 34 & = & 16 & \cdot & 2 & + & \boxed{2} \\ 16 & = & 2 & \cdot & 8 & & \end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716 = 627$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcccl} & & & & q_i & & \\ 5432 & = & 234 & \cdot & 23 & + & 50 \\ 234 & = & 50 & \cdot & 4 & + & 34 \\ 50 & = & 34 & \cdot & 1 & + & 16 \\ 34 & = & 16 & \cdot & 2 & + & \boxed{2} \\ 16 & = & 2 & \cdot & 8 & & \end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716 = 627$$

- Sva rješenja početne kongruencije:

Zadatak 9

$\boxed{x_0 = ub' \mod n'}$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$\boxed{M(234, 5432) = 2}$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{array}{rcrcl} & & & q_i & \\ 5432 &=& 234 \cdot & 23 & + 50 \\ 234 &=& 50 \cdot & 4 & + 34 \\ 50 &=& 34 \cdot & 1 & + 16 \\ 34 &=& 16 \cdot & 2 & + \boxed{2} \\ 16 &=& 2 \cdot & 8 & \end{array}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716 = 627$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \;\; k = 0, 1$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\begin{array}{rcrcl}
 & & & q_i & \\
5432 & = & 234 \cdot & 23 & + 50 \\
234 & = & 50 \cdot & 4 & + 34 \\
50 & = & 34 \cdot & 1 & + 16 \\
34 & = & 16 \cdot & 2 & + \boxed{2} \\
16 & = & 2 \cdot & 8 &
\end{array}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716 = 627$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \quad k = 0, 1$

$$x_k = 627 + 2716k, \quad k = 0, 1$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \;\Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{array}{rcr|c|l}
 & & & q_i & \\
5432 &=& 234 \cdot & 23 & + 50 \\
234 &=& 50 \cdot & 4 & + 34 \\
50 &=& 34 \cdot & 1 & + 16 \\
34 &=& 16 \cdot & 2 & + \boxed{2} \\
16 &=& 2 \cdot & 8 &
\end{array}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716 = 627$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \;\; k = 0, 1$

$$x_k = 627 + 2716k, \;\; k = 0, 1 \qquad \boxed{x_0 = 627}$$

Zadatak 9

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $234x \equiv 54 \pmod{5432}$.

Rješenje

$$\boxed{M(234, 5432) = 2}$$

$$\overset{a}{234}x \equiv \overset{b}{54} \pmod{\overset{n}{5432}} \ \Big/ : 2$$

$$\overset{a'}{117}x \equiv \overset{b'}{27} \pmod{\overset{n'}{2716}}$$

$$\begin{array}{rcrcccl} & & & & q_i & & \\ 5432 &=& 234 &\cdot& 23 &+& 50 \\ 234 &=& 50 &\cdot& 4 &+& 34 \\ 50 &=& 34 &\cdot& 1 &+& 16 \\ 34 &=& 16 &\cdot& 2 &+& \boxed{2} \\ 16 &=& 2 &\cdot& 8 & & \end{array}$$

i	-1	0	1	2	3	4
q_i			23	4	1	2
y_i	0	1	-23	93	-116	325

- $2 \mid 54 \longrightarrow$ kongruencija ima 2 rješenja

$$x_0 = 325 \cdot 27 \mod 2716 = 8775 \mod 2716 = 627$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \ \ k = 0, 1$

$$x_k = 627 + 2716k, \ \ k = 0, 1 \qquad \boxed{x_0 = 627} \qquad \boxed{x_1 = 3343}$$

deseti zadatak

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$2009 = 21 \cdot$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$2009 = 21 \cdot 95$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$2009 = 21 \cdot 95 +$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$2009 = 21 \cdot 95 + 14$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot \end{aligned}$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot \ 1 \end{aligned}$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot \ 1 \ + \end{aligned}$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot \ 1 \ + \ 7 \end{aligned}$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$
\begin{aligned}
2009 &= 21 \cdot 95 + 14 \\
21 &= 14 \cdot \ 1 \ + \ 7 \\
14 &= \ \ 7 \cdot
\end{aligned}
$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + 7 \\ 14 &= 7 \cdot 2 \end{aligned}$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49$

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot \ 1 \ + \boxed{7} \\ 14 &= \ \ 7 \cdot \ 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$21x \equiv 49 \pmod{2009}$$

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$21x \equiv 49 \pmod{2009} \Big/ : 7$$

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$21x \equiv 49 \pmod{2009} \;\Big/ : 7$$

$$3x \equiv 7 \pmod{287}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\begin{aligned} 2009 &= 21 \cdot 95 + 14 \\ 21 &= 14 \cdot 1 + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$3x \equiv 7 \pmod{287}$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$2009 = 21 \cdot 95 + 14$$

$$21 = 14 \cdot 1 + \boxed{7}$$

$$14 = 7 \cdot 2$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\boxed{M(21, 2009) = 7}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$
\begin{aligned}
2009 &= 21 \cdot \overset{q_i}{95} + 14\\
21 &= 14 \cdot 1 + \boxed{7}\\
14 &= 7 \cdot 2
\end{aligned}
$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i				
y_i				

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

i	-1	0	1	2
q_i			95	1
y_i				

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1		

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	

$$0 - 95 \cdot 1 = -95$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$$1 - 1 \cdot (-95) = 96$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

Zadatak 10

$\boxed{x_0 = ub' \mod n'}$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$\boxed{M(21, 2009) = 7}$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 =$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96$$

Zadatak 10

$\boxed{x_0 = ub' \mod n'}$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$\boxed{M(21, 2009) = 7}$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \boxed{\overset{q_i}{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287$$

Zadatak 10

$\boxed{x_0 = ub' \mod n'}$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$\boxed{M(21, 2009) = 7}$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \ \Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

Zadatak 10

$\boxed{x_0 = ub' \mod n'}$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$\boxed{M(21, 2009) = 7}$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije:

Zadatak 10

$$\boxed{x_0 = ub' \bmod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \ \big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \bmod 287 = 672 \bmod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \quad k = 0, 1, \ldots, 6$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ :7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \;\; k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \;\; k = 0, 1, \ldots, 6$$

Zadatak 10

$$\boxed{x_0 = ub' \bmod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \ \big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$x_0 = 98$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \bmod 287 = 672 \bmod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \ \ k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \ \ k = 0, 1, \ldots, 6$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21\cdot \overset{q_i}{\boxed{95}} + 14\\ 21 &= 14\cdot \boxed{1} + \boxed{7}\\ 14 &= 7\cdot 2\end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ :7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$x_0 = 98$

$x_1 = 385$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96\cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \;\; k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \;\; k = 0, 1, \ldots, 6$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$$\begin{aligned} x_0 &= 98 \\ x_1 &= 385 \\ x_2 &= 672 \end{aligned}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \ \ k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \ \ k = 0, 1, \ldots, 6$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$x_0 = 98$
$x_1 = 385$
$x_2 = 672$
$x_3 = 959$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn'$, $k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \quad k = 0, 1, \ldots, 6$$

Zadatak 10

$\boxed{x_0 = ub' \mod n'}$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$\boxed{M(21, 2009) = 7}$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$x_0 = 98$

$x_1 = 385$

$x_2 = 672$

$x_3 = 959$

$x_4 = 1246$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \;\; k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \;\; k = 0, 1, \ldots, 6$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$$\boxed{\begin{aligned} x_0 &= 98 \\ x_1 &= 385 \\ x_2 &= 672 \\ x_3 &= 959 \\ x_4 &= 1246 \\ x_5 &= 1533 \end{aligned}}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \;\; k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \;\; k = 0, 1, \ldots, 6$$

Zadatak 10

$$\boxed{x_0 = ub' \mod n'}$$

Riješite kongruenciju $21x \equiv 49 \pmod{2009}$.

Rješenje

$$\begin{aligned} 2009 &= 21 \cdot \overset{q_i}{\boxed{95}} + 14 \\ 21 &= 14 \cdot \boxed{1} + \boxed{7} \\ 14 &= 7 \cdot 2 \end{aligned}$$

$$\boxed{M(21, 2009) = 7}$$

$$\overset{a}{21}x \equiv \overset{b}{49} \pmod{\overset{n}{2009}} \;\Big/ : 7$$

$$\overset{a'}{3}x \equiv \overset{b'}{7} \pmod{\overset{n'}{287}}$$

i	-1	0	1	2
q_i			95	1
y_i	0	1	-95	96

$$\begin{aligned} x_0 &= 98 \\ x_1 &= 385 \\ x_2 &= 672 \\ x_3 &= 959 \\ x_4 &= 1246 \\ x_5 &= 1533 \\ x_6 &= 1820 \end{aligned}$$

- $7 \mid 49 \longrightarrow$ kongruencija ima 7 rješenja

$$x_0 = 96 \cdot 7 \mod 287 = 672 \mod 287 = 98$$

- Sva rješenja početne kongruencije: $x_k = x_0 + kn', \quad k = 0, 1, \ldots, 6$

$$x_k = 98 + 287k, \quad k = 0, 1, \ldots, 6$$

jedanaesti zadatak

Kineski teorem o ostacima

Neka su $n_1, n_2, \ldots, n_r$ u parovima relativno prosti prirodni brojevi, te neka su $a_1, a_2, \ldots, a_r$ cijeli brojevi. Tada sustav kongruencija

$$\begin{aligned} x &\equiv a_1 \pmod{n_1} \\ x &\equiv a_2 \pmod{n_2} \\ &\vdots \\ x &\equiv a_r \pmod{n_r} \end{aligned}$$

ima rješenje. Ako je x_0 jedno rješenje, tada su sva rješenja dana s

$$x \equiv x_0 \pmod{n_1 n_2 \cdots n_r}.$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod 7$$

$$x \equiv 5 \pmod 9$$

$$x \equiv 3 \pmod 4$$

$$x \equiv 9 \pmod{11}$$

Zadatak 11

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 1 \pmod{7} \\ x &\equiv 5 \pmod{9} \\ x &\equiv 3 \pmod{4} \\ x &\equiv 9 \pmod{11} \end{aligned}$$

Rješenje

Moduli jesu u parovima relativno prosti.

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4$$

Zadatak 11

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 1 \pmod{7} \\ x &\equiv 5 \pmod{9} \\ x &\equiv 3 \pmod{4} \\ x &\equiv 9 \pmod{11} \end{aligned}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$k_1 = \dfrac{n}{n_1} = 396$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod 7$$

$$x \equiv 5 \pmod 9$$

$$x \equiv 3 \pmod 4$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$
$$x \equiv 5 \pmod{9}$$
$$x \equiv 3 \pmod{4}$$
$$x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

Rješenje

$k_i x_i \equiv a_i \pmod{n_i}$

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396 x_1 \equiv 1 \pmod{7}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396 x_1 \equiv 1 \pmod{7}$$

Rješenje $396 \bmod 7 = 4$

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396 x_1 \equiv 1 \pmod{7}$$

$$4 x_1 \equiv 1 \pmod{7}$$

Rješenje $396 \mod 7 = 4$

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$
$$k_2 x \equiv 5 \pmod{9}$$
$$k_3 x \equiv 3 \pmod{4}$$
$$k_4 x \equiv 9 \pmod{11}$$

$$\begin{aligned} 396x_1 &\equiv 1 \pmod{7} \\ 4x_1 &\equiv 1 \pmod{7} \\ x_1 &= 2 \end{aligned}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396 x_1 \equiv 1 \pmod{7}$$

$$4 x_1 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 2}$$

Rješenje $308 \mod 9 = 2$

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$396x_1 \equiv 1 \pmod{7}$

$4x_1 \equiv 1 \pmod{7}$

$\boxed{x_1 = 2}$

$308x_2 \equiv 5 \pmod{9}$

$2x_2 \equiv 5 \pmod{9}$

Rješenje $308 \bmod 9 = 2$

Moduli jesu u parovima relativno prosti.

$k_i x_i \equiv a_i \pmod{n_i}$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad x_2 = 7$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396 x_1 \equiv 1 \pmod{7} \qquad 308 x_2 \equiv 5 \pmod{9}$$

$$4 x_1 \equiv 1 \pmod{7} \qquad 2 x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693 x_3 \equiv 3 \pmod{4}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$396x_1 \equiv 1 \pmod{7}$

$4x_1 \equiv 1 \pmod{7}$

$\boxed{x_1 = 2}$

$308x_2 \equiv 5 \pmod{9}$

$2x_2 \equiv 5 \pmod{9}$

$\boxed{x_2 = 7}$

$693x_3 \equiv 3 \pmod{4}$

Rješenje $693 \bmod 4 = 1$

Moduli jesu u parovima relativno prosti.

$k_i x_i \equiv a_i \pmod{n_i}$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7}$$

$$4x_1 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 2}$$

$$308x_2 \equiv 5 \pmod{9}$$

$$2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4}$$

$$1 \cdot x_3 \equiv 3 \pmod{4}$$

Rješenje $693 \bmod 4 = 1$

$$k_i x_i \equiv a_i \pmod{n_i}$$

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4}$$

$$1 \cdot x_3 \equiv 3 \pmod{4}$$

$$x_3 = 3$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4}$$

$$1 \cdot x_3 \equiv 3 \pmod{4}$$

$$\boxed{x_3 = 3}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396 x_1 \equiv 1 \pmod{7} \qquad 308 x_2 \equiv 5 \pmod{9}$$

$$4 x_1 \equiv 1 \pmod{7} \qquad 2 x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693 x_3 \equiv 3 \pmod{4} \qquad 252 x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4}$$

$$\boxed{x_3 = 3}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

Rješenje

$252 \bmod 11 = 10$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4}$$

$$\boxed{x_3 = 3}$$

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$

$$\boxed{x_3 = 3}$$

Rješenje

$252 \bmod 11 = 10$

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$

$$\boxed{x_3 = 3} \qquad x_4 = 2$$

Zadatak 11

Riješite sustav kongruencija

$$k_1 x \equiv 1 \pmod{7}$$

$$k_2 x \equiv 5 \pmod{9}$$

$$k_3 x \equiv 3 \pmod{4}$$

$$k_4 x \equiv 9 \pmod{11}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$\begin{aligned} 396x_1 &\equiv 1 \pmod{7} & 308x_2 &\equiv 5 \pmod{9} \\ 4x_1 &\equiv 1 \pmod{7} & 2x_2 &\equiv 5 \pmod{9} \\ &\boxed{x_1 = 2} & &\boxed{x_2 = 7} \end{aligned}$$

$$\begin{aligned} 693x_3 &\equiv 3 \pmod{4} & 252x_4 &\equiv 9 \pmod{11} \\ 1 \cdot x_3 &\equiv 3 \pmod{4} & 10x_4 &\equiv 9 \pmod{11} \\ &\boxed{x_3 = 3} & &\boxed{x_4 = 2} \end{aligned}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$
$$x \equiv 5 \pmod{9}$$
$$x \equiv 3 \pmod{4}$$
$$x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$
$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$
$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$
$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$
$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \bmod n$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

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$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

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$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$

$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$k_i x_i \equiv a_i \pmod{n_i}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$

$$x_0 = 396 \cdot 2 + 308 \cdot 7 + 693 \cdot 3 + 252 \cdot 2 \bmod 2772$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$
$$x \equiv 5 \pmod{9}$$
$$x \equiv 3 \pmod{4}$$
$$x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$
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$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$
$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$
$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$

$$x_0 = 396 \cdot 2 + 308 \cdot 7 + 693 \cdot 3 + 252 \cdot 2 \bmod 2772$$

$$x_0 = 5531 \bmod 2772$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$

$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$

$$x_0 = 396 \cdot 2 + 308 \cdot 7 + 693 \cdot 3 + 252 \cdot 2 \bmod 2772$$

$$x_0 = 5531 \bmod 2772 \qquad x_0 = 2759$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$

$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$

$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$

$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$

$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$

$$x_0 = 396 \cdot 2 + 308 \cdot 7 + 693 \cdot 3 + 252 \cdot 2 \bmod 2772$$

$$x_0 = 5531 \bmod 2772 \qquad \boxed{\boxed{x_0 = 2759}}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod{7}$$
$$x \equiv 5 \pmod{9}$$
$$x \equiv 3 \pmod{4}$$
$$x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod{7} \qquad 308x_2 \equiv 5 \pmod{9}$$
$$4x_1 \equiv 1 \pmod{7} \qquad 2x_2 \equiv 5 \pmod{9}$$
$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$
$$693x_3 \equiv 3 \pmod{4} \qquad 252x_4 \equiv 9 \pmod{11}$$
$$1 \cdot x_3 \equiv 3 \pmod{4} \qquad 10x_4 \equiv 9 \pmod{11}$$
$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \bmod n$$

$$x_0 = 396 \cdot 2 + 308 \cdot 7 + 693 \cdot 3 + 252 \cdot 2 \bmod 2772$$

$$x_0 = 5531 \bmod 2772 \qquad \boxed{\boxed{x_0 = 2759}} \qquad x \equiv 2759 \pmod{2772}$$

Zadatak 11

Riješite sustav kongruencija

$$x \equiv 1 \pmod 7$$
$$x \equiv 5 \pmod 9$$
$$x \equiv 3 \pmod 4$$
$$x \equiv 9 \pmod{11}$$

$$396x_1 \equiv 1 \pmod 7 \qquad 308x_2 \equiv 5 \pmod 9$$
$$4x_1 \equiv 1 \pmod 7 \qquad 2x_2 \equiv 5 \pmod 9$$
$$\boxed{x_1 = 2} \qquad \boxed{x_2 = 7}$$

$$693x_3 \equiv 3 \pmod 4 \qquad 252x_4 \equiv 9 \pmod{11}$$
$$1 \cdot x_3 \equiv 3 \pmod 4 \qquad 10x_4 \equiv 9 \pmod{11}$$
$$\boxed{x_3 = 3} \qquad \boxed{x_4 = 2}$$

Rješenje

Moduli jesu u parovima relativno prosti.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 7 \cdot 9 \cdot 4 \cdot 11 = 2772$$

$$k_1 = \frac{n}{n_1} = 396, \quad k_2 = \frac{n}{n_2} = 308, \quad k_3 = \frac{n}{n_3} = 693, \quad k_4 = \frac{n}{n_4} = 252$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \bmod n$$
$$x_0 = 396 \cdot 2 + 308 \cdot 7 + 693 \cdot 3 + 252 \cdot 2 \bmod 2772$$
$$x_0 = 5531 \bmod 2772 \qquad \boxed{\boxed{x_0 = 2759}}$$

sva rješenja

$$\boxed{x \equiv 2759 \pmod{2772}}$$

dvanaesti zadatak

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$x \equiv 3 \pmod{4}$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$x \equiv 3 \pmod{4}$

$x \equiv 1 \pmod{7}$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$x \equiv 3 \pmod{4}$

$x \equiv 1 \pmod{7}$

$x \equiv 9 \pmod{11}$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$

$$x \equiv 1 \pmod{7}$$

$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$

$$x \equiv 1 \pmod{7}$$

$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$

$$x \equiv 1 \pmod{7}$$

$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$
$$x \equiv 1 \pmod{7}$$
$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$

$$x \equiv 1 \pmod{7}$$

$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$
$$x \equiv 1 \pmod{7}$$
$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$
$$x \equiv 1 \pmod{7}$$
$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$\begin{aligned} x &\equiv 3 \pmod 4 \\ x &\equiv 1 \pmod 7 \\ x &\equiv 9 \pmod{11} \end{aligned}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

Rješenje

$$x \equiv 3 \pmod{4}$$
$$x \equiv 1 \pmod{7}$$
$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$x \equiv 3 \pmod{4}$$
$$x \equiv 1 \pmod{7}$$
$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$k_i x_i \equiv a_i \pmod{n_i}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$

$$k_2 x \equiv 1 \pmod{7}$$

$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$

$$k_2 x \equiv 1 \pmod{7}$$

$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod{4}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$k_i x_i \equiv a_i \pmod{n_i}$

Rješenje $77 \mod 4 = 1$

$$k_1 x \equiv 3 \pmod 4$$
$$k_2 x \equiv 1 \pmod 7$$
$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod 4$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje $77 \mod 4 = 1$

$$k_1 x \equiv 3 \pmod 4$$
$$k_2 x \equiv 1 \pmod 7$$
$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77 x_1 \equiv 3 \pmod 4$$
$$1 \cdot x_1 \equiv 3 \pmod 4$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$k_i x_i \equiv a_i \pmod{n_i}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$
$$k_2 x \equiv 1 \pmod{7}$$
$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77 x_1 \equiv 3 \pmod{4}$$
$$1 \cdot x_1 \equiv 3 \pmod{4}$$
$$x_1 = 3$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$k_i x_i \equiv a_i \pmod{n_i}$$

Rješenje

$$k_1 x \equiv 3 \pmod 4$$

$$k_2 x \equiv 1 \pmod 7$$

$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77 x_1 \equiv 3 \pmod 4$$

$$1 \cdot x_1 \equiv 3 \pmod 4$$

$$\boxed{x_1 = 3}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$

$$k_2 x \equiv 1 \pmod{7}$$

$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod{4} \qquad 44x_2 \equiv 1 \pmod{7}$$

$$1 \cdot x_1 \equiv 3 \pmod{4}$$

$$\boxed{x_1 = 3}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje $44 \mod 7 = 2$

$$k_1 x \equiv 3 \pmod{4}$$
$$k_2 x \equiv 1 \pmod{7}$$
$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod{4} \qquad 44x_2 \equiv 1 \pmod{7}$$

$$1 \cdot x_1 \equiv 3 \pmod{4}$$

$$\boxed{x_1 = 3}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje $44 \mod 7 = 2$

$$k_1 x \equiv 3 \pmod{4}$$

$$k_2 x \equiv 1 \pmod{7}$$

$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod{4} \qquad 44x_2 \equiv 1 \pmod{7}$$

$$1 \cdot x_1 \equiv 3 \pmod{4} \qquad 2x_2 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 3}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$k_i x_i \equiv a_i \pmod{n_i}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$
$$k_2 x \equiv 1 \pmod{7}$$
$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod{4} \qquad 44x_2 \equiv 1 \pmod{7}$$
$$1 \cdot x_1 \equiv 3 \pmod{4} \qquad 2x_2 \equiv 1 \pmod{7}$$
$$\boxed{x_1 = 3} \qquad x_2 = 4$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$k_1 x \equiv 3 \pmod 4$$

$$k_2 x \equiv 1 \pmod 7$$

$$k_3 x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod 4 \qquad 44x_2 \equiv 1 \pmod 7$$

$$1 \cdot x_1 \equiv 3 \pmod 4 \qquad 2x_2 \equiv 1 \pmod 7$$

$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$

$$k_2 x \equiv 1 \pmod{7}$$

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$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

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$$1 \cdot x_1 \equiv 3 \pmod{4} \qquad 2x_2 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje $28 \mod 11 = 6$

$k_1 x \equiv 3 \pmod{4}$

$k_2 x \equiv 1 \pmod{7}$

$k_3 x \equiv 9 \pmod{11}$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

$$k_1 = \frac{n}{n_1} = 77, \quad k_2 = \frac{n}{n_2} = 44, \quad k_3 = \frac{n}{n_3} = 28$$

$$77x_1 \equiv 3 \pmod{4} \qquad 44x_2 \equiv 1 \pmod{7} \qquad 28x_3 \equiv 9 \pmod{11}$$

$$1 \cdot x_1 \equiv 3 \pmod{4} \qquad 2x_2 \equiv 1 \pmod{7}$$

$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$

Rješenje $28 \bmod 11 = 6$

$$k_1 x \equiv 3 \pmod 4$$
$$k_2 x \equiv 1 \pmod 7$$
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$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

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$$1 \cdot x_1 \equiv 3 \pmod 4 \qquad 2x_2 \equiv 1 \pmod 7 \qquad 6x_3 \equiv 9 \pmod{11}$$

$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$k_i x_i \equiv a_i \pmod{n_i}$$

Rješenje

$$k_1 x \equiv 3 \pmod 4$$
$$k_2 x \equiv 1 \pmod 7$$
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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad x_3 = 7$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$k_1 x \equiv 3 \pmod{4}$$

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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

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Rješenje

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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 \bmod n$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

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Rješenje

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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 \bmod n$$

$$x_0 = 77 \cdot 3 + 44 \cdot 4 + 28 \cdot 7 \bmod 308$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 \bmod n$$

$$x_0 = 77 \cdot 3 + 44 \cdot 4 + 28 \cdot 7 \bmod 308$$

$$x_0 = 603 \bmod 308$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$x \equiv 3 \pmod{4}$$
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$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 \bmod n$$

$$x_0 = 77 \cdot 3 + 44 \cdot 4 + 28 \cdot 7 \bmod 308$$

$$x_0 = 603 \bmod 308 \qquad x_0 = 295$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

Rješenje

$$x \equiv 3 \pmod 4$$
$$x \equiv 1 \pmod 7$$
$$x \equiv 9 \pmod{11}$$

$$n = n_1 n_2 n_3 = 4 \cdot 7 \cdot 11 = 308$$

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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 \mod n$$

$$x_0 = 77 \cdot 3 + 44 \cdot 4 + 28 \cdot 7 \mod 308$$

$$x_0 = 603 \mod 308 \qquad \boxed{x_0 = 295}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

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Rješenje

$$x \equiv 3 \pmod{4}$$
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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 \mod n$$

$$x_0 = 77 \cdot 3 + 44 \cdot 4 + 28 \cdot 7 \mod 308$$

$$x_0 = 603 \mod 308 \qquad \boxed{x_0 = 295} \qquad x \equiv 295 \pmod{308}$$

Zadatak 12

Odredite sve prirodne brojeve između 900 *i* 999 *koji pri dijeljenju s* 4, 7 *i* 11 *daju redom ostatke* 3, 1 *i* 9.

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Rješenje

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$$\boxed{x_1 = 3} \qquad \boxed{x_2 = 4} \qquad \boxed{x_3 = 7}$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 \bmod n$$

$$x_0 = 77 \cdot 3 + 44 \cdot 4 + 28 \cdot 7 \bmod 308$$

$$x_0 = 603 \bmod 308 \qquad \boxed{\boxed{x_0 = 295}} \qquad \boxed{x \equiv 295 \pmod{308}}$$

sva rješenja

$$x \equiv 295 \pmod{308}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295,\ \ k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295,\ \ k \in \mathbb{Z}$$

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$$308k + 295 \geqslant 900$$

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$$x = 308k + 295, \ \ k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$308k + 295 \geqslant 900$$

$$308k \geqslant 900 - 295$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \; k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \end{aligned}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned}
308k + 295 &\geqslant 900 \\
308k &\geqslant 900 - 295 \\
308k &\geqslant 605 \\
k &\geqslant 1.96\cdots
\end{aligned}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295,\ \ k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$308k + 295 \leqslant 999$$

$$x \equiv 295 \pmod{308}$$

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$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$\begin{aligned} 308k + 295 &\leqslant 999 \\ 308k &\leqslant 999 - 295 \end{aligned}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$\begin{aligned} 308k + 295 &\leqslant 999 \\ 308k &\leqslant 999 - 295 \\ 308k &\leqslant 704 \end{aligned}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \ \ k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$\begin{aligned} 308k + 295 &\leqslant 999 \\ 308k &\leqslant 999 - 295 \\ 308k &\leqslant 704 \\ k &\leqslant 2.28\cdots \end{aligned}$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \ \ k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{array}{rlcrl}
308k + 295 & \geqslant 900 & \qquad & 308k + 295 & \leqslant 999 \\
308k & \geqslant 900 - 295 & & 308k & \leqslant 999 - 295 \\
308k & \geqslant 605 & & 308k & \leqslant 704 \\
k & \geqslant 1.96\cdots & & k & \leqslant 2.28\cdots
\end{array}$$

$$1.96\cdots \leqslant k \leqslant 2.28\cdots$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$\begin{aligned} 308k + 295 &\leqslant 999 \\ 308k &\leqslant 999 - 295 \\ 308k &\leqslant 704 \\ k &\leqslant 2.28\cdots \end{aligned}$$

$$1.96\cdots \leqslant k \leqslant 2.28\cdots$$

$$k = 2$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$\begin{aligned} 308k + 295 &\leqslant 999 \\ 308k &\leqslant 999 - 295 \\ 308k &\leqslant 704 \\ k &\leqslant 2.28\cdots \end{aligned}$$

$$1.96\cdots \leqslant k \leqslant 2.28\cdots$$

$$k = 2$$

$$x = 308 \cdot 2 + 295$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$\begin{tikzcd}
 & 900 \leqslant 308k + 295 \leqslant 999 \arrow[dl, squiggly] \arrow[dr, squiggly] & \\
308k + 295 \geqslant 900 & & 308k + 295 \leqslant 999
\end{tikzcd}$$

$$\begin{aligned} 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned} \qquad\qquad \begin{aligned} 308k &\leqslant 999 - 295 \\ 308k &\leqslant 704 \\ k &\leqslant 2.28\cdots \end{aligned}$$

$$1.96\cdots \leqslant k \leqslant 2.28\cdots$$

$$k = 2$$

$$x = 308 \cdot 2 + 295$$

$$x = 911$$

$$x \equiv 295 \pmod{308}$$

$$x = 308k + 295, \;\; k \in \mathbb{Z}$$

$$900 \leqslant 308k + 295 \leqslant 999$$

$$\begin{aligned} 308k + 295 &\geqslant 900 \\ 308k &\geqslant 900 - 295 \\ 308k &\geqslant 605 \\ k &\geqslant 1.96\cdots \end{aligned}$$

$$\begin{aligned} 308k + 295 &\leqslant 999 \\ 308k &\leqslant 999 - 295 \\ 308k &\leqslant 704 \\ k &\leqslant 2.28\cdots \end{aligned}$$

$$1.96\cdots \leqslant k \leqslant 2.28\cdots$$

$$k = 2$$

$$x = 308 \cdot 2 + 295$$

$$\boxed{x = 911}$$

trinaesti zadatak

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

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Riješite sustav kongruencija

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Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7, 15) = 1$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7, 15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

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- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

Zadatak 13

Riješite sustav kongruencija

$$
\begin{aligned}
7x &\equiv 6 \pmod{15}\\
x &\equiv 6 \pmod{12}\\
3x &\equiv 1 \pmod{7}
\end{aligned}
$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7, 15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3,7) = 1$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3,7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3,7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7, 15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3, 7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$, tj. $x \equiv 5 \pmod{7}$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \rightsquigarrow x \equiv 3 \pmod{15} \\ x &\equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod 7 \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod 7$.

  $M(3,7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$, tj. $x \equiv 5 \pmod 7$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \rightsquigarrow x \equiv 3 \pmod{15} \\ x &\equiv 6 \pmod{12} \rightsquigarrow x \equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7, 15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3, 7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$, tj. $x \equiv 5 \pmod{7}$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \rightsquigarrow x \equiv 3 \pmod{15} \\ x &\equiv 6 \pmod{12} \rightsquigarrow x \equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \rightsquigarrow x \equiv 5 \pmod{7} \end{aligned}$$

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7, 15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3, 7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$, tj. $x \equiv 5 \pmod{7}$

Zadatak 13

Riješite sustav kongruencija

$$7x \equiv 6 \pmod{15} \rightsquigarrow x \equiv 3 \pmod{15}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow x \equiv 6 \pmod{12}$$

$$3x \equiv 1 \pmod{7} \rightsquigarrow x \equiv 5 \pmod{7}$$

Rješenje

Što ako neka od kongruencija ima više rješenja?

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3,7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$, tj. $x \equiv 5 \pmod{7}$

Zadatak 13

Riješite sustav kongruencija

$$\begin{aligned} 7x &\equiv 6 \pmod{15} \rightsquigarrow x \equiv 3 \pmod{15} \\ x &\equiv 6 \pmod{12} \rightsquigarrow x \equiv 6 \pmod{12} \\ 3x &\equiv 1 \pmod{7} \rightsquigarrow x \equiv 5 \pmod{7} \end{aligned}$$

Moduli nisu
u parovima
relativno prosti

Rješenje

- Riješimo linearnu kongruenciju $7x \equiv 6 \pmod{15}$.

  $M(7,15) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 3$, tj. $x \equiv 3 \pmod{15}$

- Riješimo linearnu kongruenciju $3x \equiv 1 \pmod{7}$.

  $M(3,7) = 1 \longrightarrow$ kongruencija ima jedinstveno rješenje

  $x = 5$, tj. $x \equiv 5 \pmod{7}$

$x \equiv 3 \pmod{15}$

$x \equiv 6 \pmod{12}$

$x \equiv 5 \pmod{7}$

$$x \equiv 3 \pmod{15} \rightsquigarrow \left\{ \begin{array}{l} \\ \\ \end{array} \right.$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4}, \;\; x \equiv 1 \pmod{6}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\boxed{x \equiv 1 \pmod{24} \quad \begin{array}{c} \Longrightarrow \\ \not\Longleftarrow \end{array} \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6}}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4}, \ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } \ x = 13) \end{aligned}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12}$$

$$x \equiv 5 \pmod{7} \rightsquigarrow \begin{cases} \\ \\ \end{cases}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } x = 13) \end{aligned}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\boxed{\begin{array}{lcl} x \equiv 1 \pmod{24} & \Longrightarrow & x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ & \not\Longleftarrow & (\text{protuprimjer: } x = 13) \end{array}}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\boxed{\begin{array}{lcl} x \equiv 1 \pmod{24} & \Longrightarrow & x \equiv 1 \pmod{4},\ \ x \equiv 1 \pmod{6} \\ & \not\Longleftarrow & (\text{protuprimjer: } \ x = 13) \end{array}}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \left\{ \begin{array}{l} \\ \\ \end{array} \right.$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{array}{lcl} x \equiv 1 \pmod{24} & \Longrightarrow & x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ & \not\Longleftarrow & (\text{protuprimjer: } x = 13) \end{array}$$

$$\begin{tikzcd}
 & \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 0 \pmod{3} \\ \end{cases} \\
x \equiv 3 \pmod{15} \arrow[ur, rightsquigarrow] & & \\
x \equiv 6 \pmod{12} \arrow[dr, rightsquigarrow] & & \\
x \equiv 5 \pmod{7} & \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} &
\end{tikzcd}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } \ x = 13) \end{aligned}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\boxed{\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } x = 13) \end{aligned}}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \rightsquigarrow \begin{cases} \\ \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4}, \; x \equiv 1 \pmod{6}$$

$$\not\Longleftarrow \quad (\text{protuprimjer: } \; x = 13)$$

$$\begin{tikzcd}
 & \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \\
x \equiv 3 \pmod{15} \arrow[ur, rightsquigarrow] & & \\
x \equiv 6 \pmod{12} \arrow[dr, rightsquigarrow] & & \\
x \equiv 5 \pmod{7} & \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 0 \pmod{3} \\ \end{cases}
\end{tikzcd}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{array}{lll} x \equiv 1 \pmod{24} & \Longrightarrow & x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ & \not\Longleftarrow & (\text{protuprimjer: } x = 13) \end{array}$$

$$\begin{tikzcd}
 & \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \\
x \equiv 3 \pmod{15} \arrow[ur, rightsquigarrow] & & \\
x \equiv 6 \pmod{12} \arrow[dr, rightsquigarrow] & & \\
x \equiv 5 \pmod{7} & \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 2 \pmod{4} \end{cases}
\end{tikzcd}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } x = 13) \end{aligned}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 0 \pmod{3}} \\ x \equiv 3 \pmod{5} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 2 \pmod{4} \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 0 \pmod{3}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } x = 13) \end{aligned}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 0 \pmod{3}} \\ \boxed{x \equiv 3 \pmod{5}} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 2 \pmod{4} \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6}$$
$$\not\Longleftarrow \quad (\text{protuprimjer:}\ \ x = 13)$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 0 \pmod{3}} \\ \boxed{x \equiv 3 \pmod{5}} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ \boxed{x \equiv 2 \pmod{4}} \end{cases}$$

$$x \equiv 5 \pmod{7}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x &\equiv 0 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 2 \pmod{4} \end{aligned}$$

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6}$$

$$\not\Longleftarrow \quad \text{(protuprimjer: } x = 13\text{)}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 0 \pmod{3}} \\ \boxed{x \equiv 3 \pmod{5}} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ \boxed{x \equiv 2 \pmod{4}} \end{cases}$$

$$\boxed{x \equiv 5 \pmod{7}}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x &\equiv 0 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 2 \pmod{4} \\ x &\equiv 5 \pmod{7} \end{aligned}$$

$$\begin{aligned} x \equiv 1 \pmod{24} \quad &\Longrightarrow \quad x \equiv 1 \pmod{4},\ x \equiv 1 \pmod{6} \\ &\not\Longleftarrow \quad (\text{protuprimjer: } x = 13) \end{aligned}$$

$$x \equiv 3 \pmod{15} \rightsquigarrow \begin{cases} x \equiv 3 \pmod{3} \\ x \equiv 3 \pmod{5} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 0 \pmod{3}} \\ \boxed{x \equiv 3 \pmod{5}} \end{cases}$$

$$x \equiv 6 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 6 \pmod{3} \\ x \equiv 6 \pmod{4} \end{cases} \rightsquigarrow \begin{cases} x \equiv 0 \pmod{3} \\ \boxed{x \equiv 2 \pmod{4}} \end{cases}$$

$$\boxed{x \equiv 5 \pmod{7}}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x &\equiv 0 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 2 \pmod{4} \\ x &\equiv 5 \pmod{7} \end{aligned}$$

Moduli jesu
u parovima
relativno prosti

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4},\ \ x \equiv 1 \pmod{6}$$

$$\not\Longleftarrow \quad (\text{protuprimjer:}\ \ x = 13)$$

$$\left\{ x \equiv 3 \pmod{3} \right. \rightsquigarrow \left\{ \boxed{x \equiv 0 \pmod{3}} \right.$$

$$\begin{cases} 7x \equiv 6 \pmod{15} \\ x \equiv 6 \pmod{12} \\ 3x \equiv 1 \pmod{7} \end{cases} \iff \begin{cases} x \equiv 3 \pmod{15} \\ x \equiv 6 \pmod{12} \\ x \equiv 5 \pmod{7} \end{cases} \iff \begin{cases} x \equiv 0 \pmod{3} \\ x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{4} \\ x \equiv 5 \pmod{7} \end{cases}$$

$$\begin{aligned} x &= 24k + 1 \\ &= 4 \cdot (6k) + 1 \\ &= 6 \cdot (4k) + 1 \end{aligned}$$

$$\begin{aligned} x &\equiv 0 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 2 \pmod{4} \\ x &\equiv 5 \pmod{7} \end{aligned}$$

Moduli jesu
u parovima
relativno prosti

$$x \equiv 1 \pmod{24} \quad \Longrightarrow \quad x \equiv 1 \pmod{4},\ \ x \equiv 1 \pmod{6}$$

$$\not\Longleftarrow \quad (\text{protuprimjer:}\ \ x = 13)$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$k_1 = \frac{n}{n_1}$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$k_1 = \dfrac{n}{n_1} = 140$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2}$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3}$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4}$$

$x \equiv 0 \pmod 3$

$x \equiv 3 \pmod 5$

$x \equiv 2 \pmod 4$

$x \equiv 5 \pmod 7$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$x \equiv 0 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{4}$

$x \equiv 5 \pmod{7}$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$k_1 x \equiv 0 \pmod{3}$

$k_2 x \equiv 3 \pmod{5}$

$k_3 x \equiv 2 \pmod{4}$

$k_4 x \equiv 5 \pmod{7}$

$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$k_1 x \equiv 0 \pmod 3$

$k_2 x \equiv 3 \pmod 5$

$k_3 x \equiv 2 \pmod 4$

$k_4 x \equiv 5 \pmod 7$

$140 x_1 \equiv 0 \pmod 3$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 \mod 3 = 2$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$
$$k_2 x \equiv 3 \pmod{5}$$
$$k_3 x \equiv 2 \pmod{4}$$
$$k_4 x \equiv 5 \pmod{7}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$140 \mod 3 = 2$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$x_1 = 0$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$

$$2x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$k_1 x \equiv 0 \pmod{3}$

$k_2 x \equiv 3 \pmod{5}$

$k_3 x \equiv 2 \pmod{4}$

$k_4 x \equiv 5 \pmod{7}$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$140 x_1 \equiv 0 \pmod{3}$

$2 x_1 \equiv 0 \pmod{3}$

$\boxed{x_1 = 0}$

$84 x_2 \equiv 3 \pmod{5}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$84 \mod 5 = 4$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$k_1 x \equiv 0 \pmod{3}$

$k_2 x \equiv 3 \pmod{5}$

$k_3 x \equiv 2 \pmod{4}$

$k_4 x \equiv 5 \pmod{7}$

$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$

$84 \mod 5 = 4$

$140 x_1 \equiv 0 \pmod{3}$

$2 x_1 \equiv 0 \pmod{3}$

$\boxed{x_1 = 0}$

$84 x_2 \equiv 3 \pmod{5}$

$4 x_2 \equiv 3 \pmod{5}$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$
$$k_2 x \equiv 3 \pmod{5}$$
$$k_3 x \equiv 2 \pmod{4}$$
$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$
$$2 x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$
$$4 x_2 \equiv 3 \pmod{5}$$
$$x_2 = 2$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$
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$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$
$$2 x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$
$$4 x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

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$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$105 x_3 \equiv 2 \pmod{4}$$

$$84 x_2 \equiv 3 \pmod{5}$$

$$4 x_2 \equiv 3 \pmod{5}$$

$$\boxed{x_2 = 2}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$
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$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$
$$2 x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$
$$4 x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105 x_3 \equiv 2 \pmod{4}$$

$$105 \bmod 4 = 1$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$k_1 x \equiv 0 \pmod 3$

$k_2 x \equiv 3 \pmod 5$

$k_3 x \equiv 2 \pmod 4$

$k_4 x \equiv 5 \pmod 7$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$105 \mod 4 = 1$

$$140 x_1 \equiv 0 \pmod 3$$

$$2 x_1 \equiv 0 \pmod 3$$

$$\boxed{x_1 = 0}$$

$$105 x_3 \equiv 2 \pmod 4$$

$$1 \cdot x_3 \equiv 2 \pmod 4$$

$$84 x_2 \equiv 3 \pmod 5$$

$$4 x_2 \equiv 3 \pmod 5$$

$$\boxed{x_2 = 2}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

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$$k_2 x \equiv 3 \pmod{5}$$
$$k_3 x \equiv 2 \pmod{4}$$
$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$
$$2 x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$
$$4 x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105 x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$x_3 = 2$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

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$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$105x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$84x_2 \equiv 3 \pmod{5}$$
$$4x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

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$$k_3 x \equiv 2 \pmod{4}$$
$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$
$$2 x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$
$$4 x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105 x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60 x_4 \equiv 5 \pmod{7}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$60 \mod 7 = 4$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$

$$4 x_2 \equiv 3 \pmod{5}$$

$$\boxed{x_2 = 2}$$

$$105 x_3 \equiv 2 \pmod{4}$$

$$1 \cdot x_3 \equiv 2 \pmod{4}$$

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$$60 x_4 \equiv 5 \pmod{7}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$60 \mod 7 = 4$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$105 x_3 \equiv 2 \pmod{4}$$

$$1 \cdot x_3 \equiv 2 \pmod{4}$$

$$\boxed{x_3 = 2}$$

$$84 x_2 \equiv 3 \pmod{5}$$

$$4 x_2 \equiv 3 \pmod{5}$$

$$\boxed{x_2 = 2}$$

$$60 x_4 \equiv 5 \pmod{7}$$

$$4 x_4 \equiv 5 \pmod{7}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$105 x_3 \equiv 2 \pmod{4}$$

$$1 \cdot x_3 \equiv 2 \pmod{4}$$

$$\boxed{x_3 = 2}$$

$$84 x_2 \equiv 3 \pmod{5}$$

$$4 x_2 \equiv 3 \pmod{5}$$

$$\boxed{x_2 = 2}$$

$$60 x_4 \equiv 5 \pmod{7}$$

$$4 x_4 \equiv 5 \pmod{7}$$

$$x_4 = 3$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$k_1 x \equiv 0 \pmod{3}$$

$$k_2 x \equiv 3 \pmod{5}$$

$$k_3 x \equiv 2 \pmod{4}$$

$$k_4 x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$

$$2 x_1 \equiv 0 \pmod{3}$$

$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$

$$4 x_2 \equiv 3 \pmod{5}$$

$$\boxed{x_2 = 2}$$

$$105 x_3 \equiv 2 \pmod{4}$$

$$1 \cdot x_3 \equiv 2 \pmod{4}$$

$$\boxed{x_3 = 2}$$

$$60 x_4 \equiv 5 \pmod{7}$$

$$4 x_4 \equiv 5 \pmod{7}$$

$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{4}$$
$$x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84x_2 \equiv 3 \pmod{5}$$
$$4x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60x_4 \equiv 5 \pmod{7}$$
$$4x_4 \equiv 5 \pmod{7}$$
$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \bmod n$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{4}$$
$$x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140 x_1 \equiv 0 \pmod{3}$$
$$2 x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84 x_2 \equiv 3 \pmod{5}$$
$$4 x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105 x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60 x_4 \equiv 5 \pmod{7}$$
$$4 x_4 \equiv 5 \pmod{7}$$
$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$

$$x_0 = 140 \cdot 0 + 84 \cdot 2 + 105 \cdot 2 + 60 \cdot 3 \bmod 420$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{4}$$
$$x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84x_2 \equiv 3 \pmod{5}$$
$$4x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60x_4 \equiv 5 \pmod{7}$$
$$4x_4 \equiv 5 \pmod{7}$$
$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \bmod n$$

$$x_0 = 140 \cdot 0 + 84 \cdot 2 + 105 \cdot 2 + 60 \cdot 3 \bmod 420$$

$$x_0 = 558 \bmod 420$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{4}$$
$$x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84x_2 \equiv 3 \pmod{5}$$
$$4x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60x_4 \equiv 5 \pmod{7}$$
$$4x_4 \equiv 5 \pmod{7}$$
$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$

$$x_0 = 140 \cdot 0 + 84 \cdot 2 + 105 \cdot 2 + 60 \cdot 3 \bmod 420$$

$$x_0 = 558 \bmod 420 \qquad x_0 = 138$$

$$
\begin{aligned}
x &\equiv 0 \pmod{3}\\
x &\equiv 3 \pmod{5}\\
x &\equiv 2 \pmod{4}\\
x &\equiv 5 \pmod{7}
\end{aligned}
$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$
\begin{aligned}
140x_1 &\equiv 0 \pmod{3}\\
2x_1 &\equiv 0 \pmod{3}\\
&\boxed{x_1 = 0}
\end{aligned}
\qquad
\begin{aligned}
84x_2 &\equiv 3 \pmod{5}\\
4x_2 &\equiv 3 \pmod{5}\\
&\boxed{x_2 = 2}
\end{aligned}
$$

$$
\begin{aligned}
105x_3 &\equiv 2 \pmod{4}\\
1 \cdot x_3 &\equiv 2 \pmod{4}\\
&\boxed{x_3 = 2}
\end{aligned}
\qquad
\begin{aligned}
60x_4 &\equiv 5 \pmod{7}\\
4x_4 &\equiv 5 \pmod{7}\\
&\boxed{x_4 = 3}
\end{aligned}
$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x_0 = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \bmod n$$

$$x_0 = 140 \cdot 0 + 84 \cdot 2 + 105 \cdot 2 + 60 \cdot 3 \bmod 420$$

$$x_0 = 558 \bmod 420 \qquad \boxed{\boxed{x_0 = 138}}$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{4}$$
$$x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84x_2 \equiv 3 \pmod{5}$$
$$4x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60x_4 \equiv 5 \pmod{7}$$
$$4x_4 \equiv 5 \pmod{7}$$
$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$
$$x_0 = 140 \cdot 0 + 84 \cdot 2 + 105 \cdot 2 + 60 \cdot 3 \bmod 420$$
$$x_0 = 558 \bmod 420 \qquad \boxed{x_0 = 138} \qquad x \equiv 138 \pmod{420}$$

$$x \equiv 0 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{4}$$
$$x \equiv 5 \pmod{7}$$

$$\boxed{k_i x_i \equiv a_i \pmod{n_i}}$$

$$140x_1 \equiv 0 \pmod{3}$$
$$2x_1 \equiv 0 \pmod{3}$$
$$\boxed{x_1 = 0}$$

$$84x_2 \equiv 3 \pmod{5}$$
$$4x_2 \equiv 3 \pmod{5}$$
$$\boxed{x_2 = 2}$$

$$105x_3 \equiv 2 \pmod{4}$$
$$1 \cdot x_3 \equiv 2 \pmod{4}$$
$$\boxed{x_3 = 2}$$

$$60x_4 \equiv 5 \pmod{7}$$
$$4x_4 \equiv 5 \pmod{7}$$
$$\boxed{x_4 = 3}$$

$$n = n_1 n_2 n_3 n_4 = 3 \cdot 5 \cdot 4 \cdot 7 = 420$$

$$k_1 = \frac{n}{n_1} = 140, \quad k_2 = \frac{n}{n_2} = 84, \quad k_3 = \frac{n}{n_3} = 105, \quad k_4 = \frac{n}{n_4} = 60$$

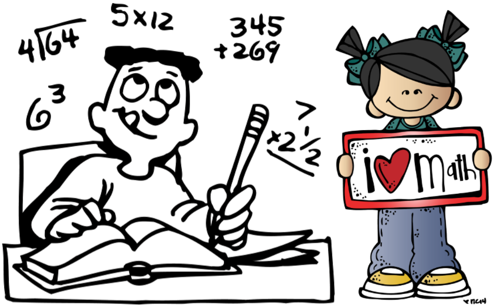
$$x_0 = k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 \bmod n$$
$$x_0 = 140 \cdot 0 + 84 \cdot 2 + 105 \cdot 2 + 60 \cdot 3 \bmod 420$$
$$x_0 = 558 \bmod 420 \qquad \boxed{x_0 = 138} \qquad \boxed{x \equiv 138 \pmod{420}}$$

sva rješenja

Domaća zadaća

Neka su $a, b \in \mathbb{N}$ takvi da je $M(a, b) = 1$ i $N \in \mathbb{Z}$. Dokažite da tada vrijedi:

$$x \equiv N \pmod{a},\ x \equiv N \pmod{b} \iff x \equiv N \pmod{ab}$$



četrnaesti zadatak

Zadatak 14

Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

Zadatak 14

Riješite sustav kongruencija

$$
\begin{aligned}
x &\equiv 2 \pmod{3}\\
x &\equiv 1 \pmod{12}\\
x &\equiv 7 \pmod{14}
\end{aligned}
$$

Rješenje

$$
\begin{aligned}
x &\equiv 2 \pmod{3}\\
x &\equiv 1 \pmod{12}\\
x &\equiv 7 \pmod{14}
\end{aligned}
$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \rightsquigarrow \left\{ \begin{array}{l} \\ \\ \end{array} \right. \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

$$\begin{array}{ll} x \equiv 2 \pmod{3} & \\ x \equiv 1 \pmod{12} & \rightsquigarrow \begin{cases} x \equiv 1 \pmod{3} \\ \end{cases} \\ x \equiv 7 \pmod{14} & \end{array}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 1 \pmod{4} \end{cases} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

$$x \equiv 2 \pmod{3}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 1 \pmod{4} \end{cases}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} \\ \end{cases}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

$$x \equiv 2 \pmod{3}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 1 \pmod{4} \end{cases}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ \end{cases}$$

Zadatak 14

Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

Rješenje

$$\begin{tikzcd}
 & \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 1 \pmod{4} \end{cases} \\
x \equiv 2 \pmod{3} & \\
x \equiv 1 \pmod{12} \arrow[ruu, rightsquigarrow] & \\
x \equiv 7 \pmod{14} \arrow[rd, rightsquigarrow] & \\
 & \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases}
\end{tikzcd}$$

Zadatak 14

Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

Rješenje

$$\begin{tikzcd}
 & \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 1 \pmod{4} \end{cases} & \\
x \equiv 2 \pmod{3} & & \\
x \equiv 1 \pmod{12} \arrow[uur, rightsquigarrow] & & \\
x \equiv 7 \pmod{14} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} \\ \end{cases}
\end{tikzcd}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod 3 \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

$$\begin{tikzcd}
 & \left\{\begin{array}{l} x \equiv 1 \pmod 3 \\ x \equiv 1 \pmod 4 \end{array}\right. & \\
x \equiv 2 \pmod 3 & & \\
x \equiv 1 \pmod{12} \arrow[ruu, rightsquigarrow] & & \\
x \equiv 7 \pmod{14} \arrow[r, rightsquigarrow] & \left\{\begin{array}{l} x \equiv 7 \pmod 2 \\ x \equiv 7 \pmod 7 \end{array}\right. \arrow[r, rightsquigarrow] & \left\{\begin{array}{l} x \equiv 1 \pmod 2 \\ \end{array}\right.
\end{tikzcd}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje

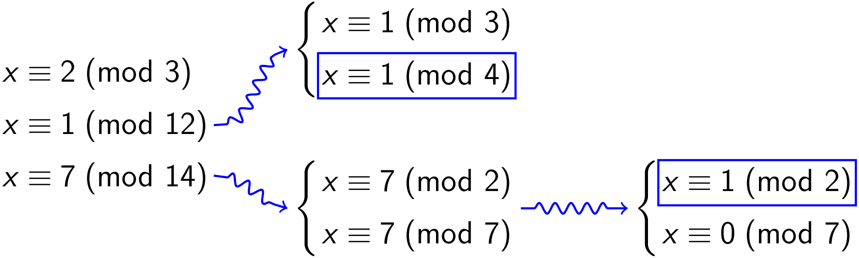
$$\begin{tikzcd}
 & \begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 1 \pmod{4} \end{cases} & \\
x \equiv 2 \pmod{3} & & \\
x \equiv 1 \pmod{12} \arrow[ruu, rightsquigarrow] & & \\
x \equiv 7 \pmod{14} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 1 \pmod{2} \\ x \equiv 0 \pmod{7} \end{cases}
\end{tikzcd}$$

Zadatak 14

Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

Rješenje



$$x \equiv 2 \pmod{3}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 1 \pmod{3} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ x \equiv 0 \pmod{7} \end{cases}$$

Zadatak 14

Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

Rješenje

$$\begin{tikzcd}
 & \begin{cases} x \equiv 1 \pmod{3} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \arrow[r, squiggly] & x \equiv 1 \pmod{4} \\
x \equiv 2 \pmod{3} & & \\
x \equiv 1 \pmod{12} \arrow[uur, squiggly] & & \\
x \equiv 7 \pmod{14} \arrow[r, squiggly] & \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \arrow[r, squiggly] & \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ x \equiv 0 \pmod{7} \end{cases} \arrow[uuu, squiggly]
\end{tikzcd}$$

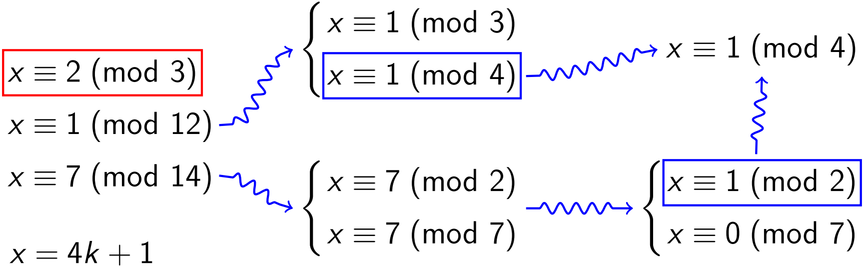
$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

Zadatak 14

Riješite sustav kongruencija $x \equiv 2 \pmod{3}$

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

Rješenje



$$\boxed{x \equiv 2 \pmod{3}}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} x \equiv 1 \pmod{3} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \rightsquigarrow x \equiv 1 \pmod{4}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ x \equiv 0 \pmod{7} \end{cases}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

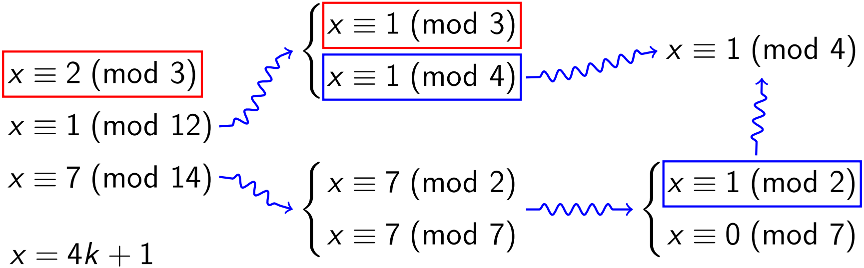
Zadatak 14

Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{3}$$

Rješenje



$$\boxed{x \equiv 2 \pmod{3}}$$
$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{3}} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \rightsquigarrow x \equiv 1 \pmod{4}$$
$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ x \equiv 0 \pmod{7} \end{cases}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{3} \\ x &\equiv 1 \pmod{4} \end{aligned}$$

Rješenje

$$\begin{tikzcd}
 & \begin{cases} \boxed{x \equiv 1 \pmod{3}} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \arrow[r, rightsquigarrow] & \boxed{x \equiv 1 \pmod{4}} \\
\boxed{x \equiv 2 \pmod{3}} & & \\
x \equiv 1 \pmod{12} \arrow[uur, rightsquigarrow] & & \\
x \equiv 7 \pmod{14} \arrow[r, rightsquigarrow] & \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \arrow[r, rightsquigarrow] & \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ x \equiv 0 \pmod{7} \end{cases} \arrow[uuu, rightsquigarrow]
\end{tikzcd}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

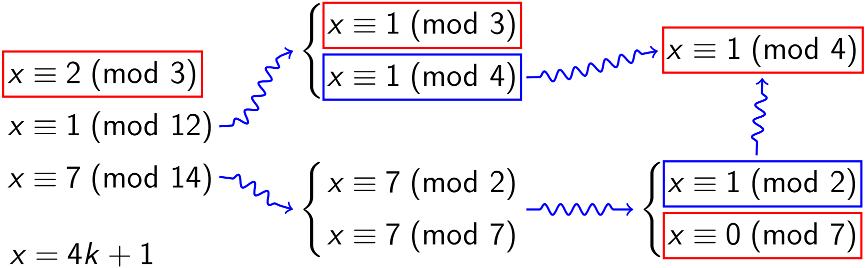
Zadatak 14

Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{3} \\ x &\equiv 1 \pmod{4} \\ x &\equiv 0 \pmod{7} \end{aligned}$$

Rješenje



$$\boxed{x \equiv 2 \pmod{3}}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{3}} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \rightsquigarrow \boxed{x \equiv 1 \pmod{4}}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ \boxed{x \equiv 0 \pmod{7}} \end{cases}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

Zadatak 14

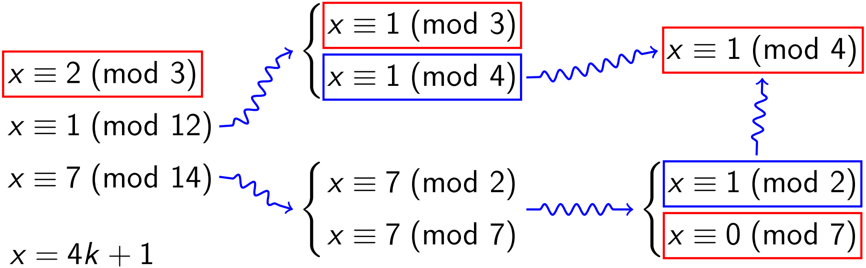
Riješite sustav kongruencija

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

$$\boxed{\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{3} \end{aligned}}$$

$$\begin{aligned} x &\equiv 1 \pmod{4} \\ x &\equiv 0 \pmod{7} \end{aligned}$$

Rješenje



$$\boxed{x \equiv 2 \pmod{3}}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{3}} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \rightsquigarrow \boxed{x \equiv 1 \pmod{4}}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ \boxed{x \equiv 0 \pmod{7}} \end{cases}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

Zadatak 14

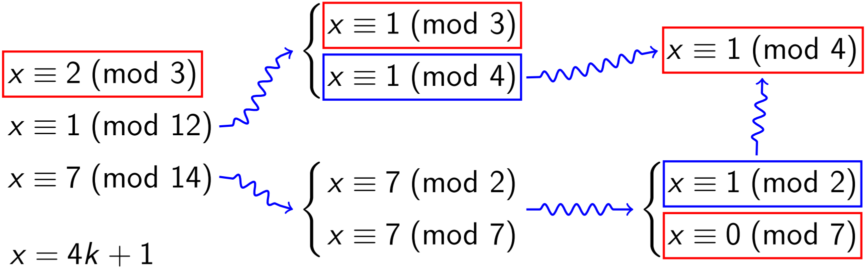
Riješite sustav kongruencija

$$x \equiv 2 \pmod{3}$$
$$x \equiv 1 \pmod{12}$$
$$x \equiv 7 \pmod{14}$$

$$\boxed{\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{3} \end{aligned}}$$
$$x \equiv 1 \pmod{4}$$
$$x \equiv 0 \pmod{7}$$

Kontradikcija

Rješenje



$$\boxed{x \equiv 2 \pmod{3}}$$
$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{3}} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \rightsquigarrow \boxed{x \equiv 1 \pmod{4}}$$
$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ \boxed{x \equiv 0 \pmod{7}} \end{cases}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

Zadatak 14

Riješite sustav kongruencija

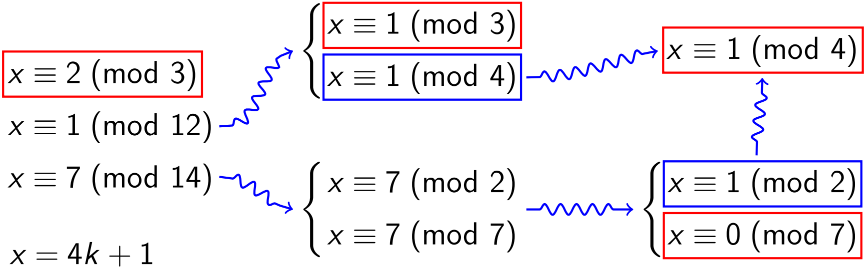
$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{12} \\ x &\equiv 7 \pmod{14} \end{aligned}$$

$$\boxed{\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{3} \end{aligned}}$$

Kontradikcija

$$\begin{aligned} x &\equiv 1 \pmod{4} \\ x &\equiv 0 \pmod{7} \end{aligned}$$

Rješenje



$$\boxed{x \equiv 2 \pmod{3}}$$

$$x \equiv 1 \pmod{12} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{3}} \\ \boxed{x \equiv 1 \pmod{4}} \end{cases} \rightsquigarrow \boxed{x \equiv 1 \pmod{4}}$$

$$x \equiv 7 \pmod{14} \rightsquigarrow \begin{cases} x \equiv 7 \pmod{2} \\ x \equiv 7 \pmod{7} \end{cases} \rightsquigarrow \begin{cases} \boxed{x \equiv 1 \pmod{2}} \\ \boxed{x \equiv 0 \pmod{7}} \end{cases}$$

$$\begin{aligned} x &= 4k + 1 \\ &= 2 \cdot (2k) + 1 \end{aligned}$$

Zadani sustav kongruencija nema rješenja.

Eulerova funkcija

Eulerova funkcija

$$\varphi : \mathbb{N} \to \mathbb{N}$$

- $\varphi(n)$ je jednak broju brojeva u nizu $1, 2, \ldots, n$ koji su relativno prosti s n
- φ je multiplikativna funkcija

$$\varphi(mn) = \varphi(m)\varphi(n), \quad m, n \in \mathbb{N},\ M(m,n) = 1$$

- Ako je p prosti broj, tada za svaki $i \in \mathbb{N}$ vrijedi

$$\varphi(p^i) = p^i - p^{i-1}$$

- Ako je $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ faktorizacija broja n na proste faktore, tada je

$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)$$

Primjer

1. način

$$\varphi(100) =$$

Primjer

1. način

$$\varphi(100) = \varphi(25 \cdot 4)$$

Primjer

1. način

$$\boxed{M(25,4)=1}$$

$$\varphi(100)=\varphi(25\cdot 4)\overset{\downarrow}{=}$$

Primjer

1. način

$$M(25,4)=1$$

$$\varphi(100)=\varphi(25\cdot 4)\overset{\downarrow}{=}\varphi(25)\cdot\varphi(4)$$

Primjer

1. način

$$\varphi(100) = \varphi(25 \cdot 4) \overset{\boxed{M(25,4)=1}}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right)$$

Primjer

1. način

$\boxed{M(25,4)=1}$

$\boxed{\varphi(p^i)=p^i-p^{i-1}}$

$$\varphi(100)=\varphi(25\cdot 4)=\varphi(25)\cdot\varphi(4)=\varphi\big(5^2\big)\cdot\varphi\big(2^2\big)=$$

Primjer

1. način

$M(25,4) = 1$

$\varphi(p^i) = p^i - p^{i-1}$

$$\varphi(100) = \varphi(25 \cdot 4) = \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right)$$

Primjer

1. način

$\varphi(p^i) = p^i - p^{i-1}$

$$\varphi(100) = \varphi(25 \cdot 4) \overset{M(25,4)=1}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot$$

Primjer

1. način

$$\boxed{\varphi(p^i)=p^i-p^{i-1}}$$

$$\varphi(100)=\varphi(25\cdot 4)\overset{\boxed{M(25,4)=1}}{=}\varphi(25)\cdot\varphi(4)=\varphi\left(5^2\right)\cdot\varphi\left(2^2\right)=$$

$$=\left(5^2-5^1\right)\cdot\left(2^2-2^1\right)$$

Primjer

1. način

$\boxed{M(25,4) = 1}$

$\boxed{\varphi(p^i) = p^i - p^{i-1}}$

$$\varphi(100) = \varphi(25 \cdot 4) \overset{M(25,4)=1}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2$$

Primjer

1. način

$\boxed{M(25,4) = 1}$

$\boxed{\varphi(p^i) = p^i - p^{i-1}}$

$$\varphi(100) = \varphi(25 \cdot 4) = \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

Primjer

1. način

$$\boxed{\varphi(p^i) = p^i - p^{i-1}}$$

$$\varphi(100) = \varphi(25 \cdot 4) \overset{M(25,4)=1}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

2. način

Primjer

$$\boxed{\varphi(p^i) = p^i - p^{i-1}}$$

1. način

$$\varphi(100) = \varphi(25 \cdot 4) \overset{M(25,4)=1}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

2. način

$$100 = 2^2 \cdot 5^2$$

Primjer

1. način

$\boxed{M(25,4) = 1}$

$\boxed{\varphi(p^i) = p^i - p^{i-1}}$

$$\varphi(100) = \varphi(25 \cdot 4) \overset{\downarrow}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

2. način

$$100 = 2^2 \cdot 5^2$$

$$\varphi(100) =$$

$$\boxed{\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)}$$

Primjer

1. način

$\boxed{M(25,4)=1}$

$\boxed{\varphi(p^i)=p^i-p^{i-1}}$

$$\varphi(100)=\varphi(25\cdot 4)\overset{\downarrow}{=}\varphi(25)\cdot\varphi(4)=\varphi\left(5^2\right)\cdot\varphi\left(2^2\right)=$$

$$=\left(5^2-5^1\right)\cdot\left(2^2-2^1\right)=20\cdot 2=40$$

2. način

$$100=2^2\cdot 5^2$$

$$\varphi(100)=100\cdot$$

$$\boxed{\varphi(n)=n\cdot\left(1-\frac{1}{p_1}\right)\cdot\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right)}$$

Primjer

1. način

$M(25,4)=1$

$\varphi(p^i)=p^i-p^{i-1}$

$$\varphi(100)=\varphi(25\cdot 4)\overset{\downarrow}{=}\varphi(25)\cdot\varphi(4)=\varphi\left(5^2\right)\cdot\varphi\left(2^2\right)=$$

$$=\left(5^2-5^1\right)\cdot\left(2^2-2^1\right)=20\cdot 2=40$$

2. način

$$100=2^2\cdot 5^2$$

$$\varphi(100)=100\cdot\left(1-\frac{1}{2}\right)$$

$$\varphi(n)=n\cdot\left(1-\frac{1}{p_1}\right)\cdot\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right)$$

Primjer

1. način

$M(25,4)=1$

$\varphi(p^i) = p^i - p^{i-1}$

$$\varphi(100) = \varphi(25 \cdot 4) = \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

2. način

$$100 = 2^2 \cdot 5^2$$

$$\varphi(100) = 100 \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{5}\right)$$

$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)$$

Primjer

1. način

$\boxed{M(25,4) = 1}$

$\boxed{\varphi(p^i) = p^i - p^{i-1}}$

$$\varphi(100) = \varphi(25 \cdot 4) \overset{\downarrow}{=} \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

2. način

$$100 = 2^2 \cdot 5^2$$

$$\varphi(100) = 100 \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{5}\right) = 100 \cdot \frac{1}{2} \cdot \frac{4}{5}$$

$$\boxed{\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)}$$

Primjer

$$\boxed{\varphi(p^i) = p^i - p^{i-1}}$$

1. način

$$\boxed{M(25,4) = 1}$$

$$\varphi(100) = \varphi(25 \cdot 4) = \varphi(25) \cdot \varphi(4) = \varphi\left(5^2\right) \cdot \varphi\left(2^2\right) =$$

$$= \left(5^2 - 5^1\right) \cdot \left(2^2 - 2^1\right) = 20 \cdot 2 = 40$$

2. način

$$100 = 2^2 \cdot 5^2$$

$$\varphi(100) = 100 \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{5}\right) = 100 \cdot \frac{1}{2} \cdot \frac{4}{5} = 40$$

$$\boxed{\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) =$$

$$= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) =$$

$$= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) =$$

$$= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right)$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\varphi(n) = n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) =$$

$$= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) =$$

$$= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) =$$

$$= \ \left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right)$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) = \\
&= \ \left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right) \cdot \left(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}\right)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) = \\
&= \ \left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right) \cdot \left(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}\right) \cdots \left(p_k^{\alpha_k} - p_k^{\alpha_k - 1}\right)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) = \\
&= \ \left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right) \cdot \left(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}\right) \cdots \left(p_k^{\alpha_k} - p_k^{\alpha_k - 1}\right) = \\
&= \varphi\big(p_1^{\alpha_1}\big)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) = \\
&= \ \left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right) \cdot \left(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}\right) \cdots \left(p_k^{\alpha_k} - p_k^{\alpha_k - 1}\right) = \\
&= \varphi\left(p_1^{\alpha_1}\right) \cdot \varphi\left(p_2^{\alpha_2}\right)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) = \\
&= \ \left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right) \cdot \left(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}\right) \cdots \left(p_k^{\alpha_k} - p_k^{\alpha_k - 1}\right) = \\
&= \varphi\left(p_1^{\alpha_1}\right) \cdot \varphi\left(p_2^{\alpha_2}\right) \cdots \varphi\left(p_k^{\alpha_k}\right)
\end{aligned}$$

Usporedba

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$

$$\begin{aligned}
\varphi(n) &= \boxed{n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)} = \\
&= p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right) = \\
&= p_1^{\alpha_1} \left(1 - \frac{1}{p_1}\right) \cdot p_2^{\alpha_2} \left(1 - \frac{1}{p_2}\right) \cdots p_k^{\alpha_k} \left(1 - \frac{1}{p_k}\right) = \\
&= \boxed{\left(p_1^{\alpha_1} - p_1^{\alpha_1 - 1}\right) \cdot \left(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}\right) \cdots \left(p_k^{\alpha_k} - p_k^{\alpha_k - 1}\right)} = \\
&= \varphi\left(p_1^{\alpha_1}\right) \cdot \varphi\left(p_2^{\alpha_2}\right) \cdots \varphi\left(p_k^{\alpha_k}\right)
\end{aligned}$$

Eulerov teorem

Ako je $M(a, n) = 1$, tada je $a^{\varphi(n)} \equiv 1 \pmod{n}$.

Mali Fermatov teorem

Neka je p prosti broj. Ako $p \nmid a$, tada je

$$a^{p-1} \equiv 1 \pmod{p}.$$

Nadalje, za svaki $a \in \mathbb{Z}$ vrijedi $a^p \equiv a \pmod{p}$.

- Obrat malog Fermatovog teorema ne vrijedi.
- Protuprimjer su **Carmichaelovi brojevi**.
- Najmanji Carmichaelov broj je 561.

petnaesti zadatak

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137*.*

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

Rješenje

$$\sqrt{137} \approx 11.7047$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

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Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

Zadatak 15

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Mali Fermatov teorem

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$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \;\Rightarrow\; 26^{137-1} \equiv 1 \pmod{137}$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

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$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \;\Rightarrow\; 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137*.*

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2,3,5,7,11$$

Niti jedan od prostih brojeva $2,3,5,7,11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $\;M(26,137)=1 \;\Rightarrow\; 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \,\big/^{\,2}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \ \Rightarrow\ 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \,\Big/^{2}$$

$$26^{272} \equiv 1 \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137*.*

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \ \Rightarrow\ 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \Big/^{2} \qquad\qquad 26^{3} \equiv 40 \pmod{137}$$

$$26^{272} \equiv 1 \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137*.*

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \;\Rightarrow\; 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \,\Big/^{2} \qquad\qquad 26^{3} \equiv 40 \pmod{137} \,\Big/^{3}$$

$$26^{272} \equiv 1 \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137*.*

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \;\Rightarrow\; 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \,\Big/^{\,2} \qquad\qquad 26^{3} \equiv 40 \pmod{137} \,\Big/^{\,3}$$

$$26^{272} \equiv 1 \pmod{137} \qquad\qquad 26^{9} \equiv 40^{3} \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \ \Rightarrow \ 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \Big/^{2} \qquad\qquad 26^{3} \equiv 40 \pmod{137} \Big/^{3}$$

$$26^{272} \equiv 1 \pmod{137} \qquad\qquad 26^{9} \equiv 40^{3} \pmod{137}$$

$$26^{9} \equiv 21 \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

Rješenje

$\sqrt{137} \approx 11.7047$ $\qquad$ $2, 3, 5, 7, 11$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

$\boxed{\text{Mali Fermatov teorem}}$ $M(26, 137) = 1 \ \Rightarrow \ 26^{137-1} \equiv 1 \pmod{137}$

$$26^{136} \equiv 1 \pmod{137} \Big/^{2} \qquad 26^{3} \equiv 40 \pmod{137} \Big/^{3}$$

$$26^{272} \equiv 1 \pmod{137} \qquad 26^{9} \equiv 40^{3} \pmod{137}$$

$$26^{9} \equiv 21 \pmod{137} \Big/ \cdot 26$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137.

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad\qquad 2, 3, 5, 7, 11$$

Niti jedan od prostih brojeva $2, 3, 5, 7, 11$ nije faktor od 137 pa zaključujemo da je 137 prosti broj.

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$$26^{272} \equiv 1 \pmod{137}$$

$$26^{3} \equiv 40 \pmod{137} \,\Big/^{\,3}$$

$$26^{9} \equiv 40^{3} \pmod{137}$$

$$26^{9} \equiv 21 \pmod{137} \,\Big/ \cdot 26$$

$$26^{10} \equiv 546 \pmod{137}$$

Zadatak 15

Dokažite da je 137 *prosti broj i odredite ostatak pri dijeljenju broja* 26^{282} *s brojem* 137*.*

Rješenje

$$\sqrt{137} \approx 11.7047 \qquad 2, 3, 5, 7, 11$$

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$$26^{272} \equiv 1 \pmod{137}$$
$$26^{10} \equiv 135 \pmod{137}$$

$$26^{3} \equiv 40 \pmod{137} \,\Big/^{3}$$
$$26^{9} \equiv 40^{3} \pmod{137}$$
$$26^{9} \equiv 21 \pmod{137} \,\Big/ \cdot 26$$
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$$26^{282} \equiv 135 \pmod{137}$$

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$$26^{9} \equiv 40^{3} \pmod{137}$$

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šesnaesti zadatak

Zadatak 16

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Zadatak 16

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

Zadatak 16

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

Eulerov teorem

Zadatak 16

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

Eulerov teorem $M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

Zadatak 16

$\boxed{\varphi(100) = 40}$

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

$\boxed{\text{Eulerov teorem}}$ $M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

$$3^{40} \equiv 1 \pmod{100}$$

Zadatak 16

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$$3^{40} \equiv 1 \pmod{100} \;\Big/^{12}$$

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$$3^{40} \equiv 1 \pmod{100} \;\Big/\,^{12}$$

$$3^{480} \equiv 1 \pmod{100}$$

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$\boxed{\varphi(100) = 40}$

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$\boxed{\text{Eulerov teorem}}$ $M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

$$3^{40} \equiv 1 \pmod{100} \;\big/^{\,12} \qquad\qquad 3^{10} \equiv 49 \pmod{100}$$

$$3^{480} \equiv 1 \pmod{100}$$

Zadatak 16

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Zadatak 16

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Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

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$$3^{40} \equiv 1 \pmod{100} \,/^{12}$$
$$3^{480} \equiv 1 \pmod{100}$$
$$3^{21} \equiv 3 \pmod{100}$$

$$3^{10} \equiv 49 \pmod{100} \,/^{2}$$
$$3^{20} \equiv 49^2 \pmod{100}$$
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Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

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$$\begin{aligned} 3^{40} &\equiv 1 \pmod{100} \,\big/^{\,12} \\ &\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100} \\ 3^{21} &\equiv 3 \pmod{100} \end{aligned}} \Bigg/ \cdot \end{aligned} \qquad\qquad \begin{aligned} 3^{10} &\equiv 49 \pmod{100} \,\big/^{\,2} \\ 3^{20} &\equiv 49^2 \pmod{100} \\ 3^{20} &\equiv 1 \pmod{100} \,\big/ \cdot 3 \end{aligned}$$

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Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

$\boxed{\text{Eulerov teorem}}$ $M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

$$3^{40} \equiv 1 \pmod{100} \;\big/^{12}$$

$$\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100} \\ 3^{21} &\equiv 3 \pmod{100} \end{aligned}} \;\Bigg/ \cdot$$

$$3^{501} \equiv 3 \pmod{100}$$

$$3^{10} \equiv 49 \pmod{100} \;\big/^{2}$$

$$3^{20} \equiv 49^2 \pmod{100}$$

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Zadatak 16

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Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

$\boxed{\text{Eulerov teorem}}$

$$M(3,100) = 1 \ \Rightarrow\ 3^{\varphi(100)} \equiv 1 \pmod{100}$$

$$M(7,100) = 1 \ \Rightarrow\ 7^{\varphi(100)} \equiv 1 \pmod{100}$$

$$3^{40} \equiv 1 \pmod{100} \Big/^{12}$$

$$\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100} \\ 3^{21} &\equiv 3 \pmod{100} \end{aligned}} \ \Big/ \cdot$$

$$3^{501} \equiv 3 \pmod{100}$$

$$3^{10} \equiv 49 \pmod{100} \Big/^{2}$$

$$3^{20} \equiv 49^2 \pmod{100}$$

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Zadatak 16

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Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

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$$3^{20} \equiv 49^2 \pmod{100}$$

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Zadatak 16

$\boxed{\varphi(100) = 40}$

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

Eulerov teorem $M(3, 100) = 1 \Rightarrow 3^{\varphi(100)} \equiv 1 \pmod{100}$

$M(7, 100) = 1 \Rightarrow 7^{\varphi(100)} \equiv 1 \pmod{100}$

$$3^{40} \equiv 1 \pmod{100} \;/^{12}$$

$$\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100} \\ 3^{21} &\equiv 3 \pmod{100} \end{aligned}} \;/\cdot$$

$$3^{501} \equiv 3 \pmod{100}$$

$$3^{10} \equiv 49 \pmod{100} \;/^{2}$$

$$3^{20} \equiv 49^2 \pmod{100}$$

$$3^{20} \equiv 1 \pmod{100} \;/\cdot 3$$

$$7^{40} \equiv 1 \pmod{100} \;/^{5}$$

Zadatak 16

$\boxed{\varphi(100) = 40}$

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

$\boxed{\text{Eulerov teorem}}$ $\quad M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

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$$3^{40} \equiv 1 \pmod{100} \;\Big/^{12}$$

$$\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100}\\ 3^{21} &\equiv 3 \pmod{100}\end{aligned}} \;\Bigg/\cdot$$

$$3^{501} \equiv 3 \pmod{100}$$

$$7^{200} \equiv 1 \pmod{100}$$

$$3^{10} \equiv 49 \pmod{100} \;\Big/^{2}$$

$$3^{20} \equiv 49^2 \pmod{100}$$

$$3^{20} \equiv 1 \pmod{100} \;\Big/ \cdot 3$$

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Zadatak 16

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Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

Eulerov teorem

$$M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$$

$$M(7, 100) = 1 \;\Rightarrow\; 7^{\varphi(100)} \equiv 1 \pmod{100}$$

$$3^{40} \equiv 1 \pmod{100} \;\Big/^{12}$$

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$$3^{20} \equiv 49^2 \pmod{100}$$

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Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

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Zadatak 16

$\boxed{\varphi(100) = 40}$

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

$\boxed{\text{Eulerov teorem}}$ $M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

$$M(7, 100) = 1 \;\Rightarrow\; 7^{\varphi(100)} \equiv 1 \pmod{100}$$

$$3^{40} \equiv 1 \pmod{100} \;\Big/^{12}$$

$$\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100} \\ 3^{21} &\equiv 3 \pmod{100} \end{aligned}} \;\Big/ \cdot$$

$$\boxed{\begin{aligned} 3^{501} &\equiv 3 \pmod{100} \\ 7^{200} &\equiv 1 \pmod{100} \end{aligned}} \;\Big/ \cdot$$

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$$3^{20} \equiv 49^2 \pmod{100}$$

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$$7^{40} \equiv 1 \pmod{100} \;\Big/^{5}$$

$$3^{501} \cdot 7^{200} \equiv 3 \pmod{100}$$

Zadatak 16

$\boxed{\varphi(100) = 40}$

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

$\boxed{\text{Eulerov teorem}}$

$$M(3, 100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$$

$$M(7, 100) = 1 \;\Rightarrow\; 7^{\varphi(100)} \equiv 1 \pmod{100}$$

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$$\boxed{3^{501} \cdot 7^{200} \equiv 3 \pmod{100}}$$

Zadatak 16

$\varphi(100) = 40$

Odredite zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$.

Zadnje dvije znamenke broja $3^{501} \cdot 7^{200}$ su 03.

Rješenje

Zanima nas ostatak pri dijeljenju broja $3^{501} \cdot 7^{200}$ s brojem 100.

Eulerov teorem $M(3,100) = 1 \;\Rightarrow\; 3^{\varphi(100)} \equiv 1 \pmod{100}$

$M(7,100) = 1 \;\Rightarrow\; 7^{\varphi(100)} \equiv 1 \pmod{100}$

$$3^{40} \equiv 1 \pmod{100} \;\Big/^{12}$$

$$\boxed{\begin{aligned} 3^{480} &\equiv 1 \pmod{100} \\ 3^{21} &\equiv 3 \pmod{100} \end{aligned}} \;\Big/ \cdot$$

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$$3^{10} \equiv 49 \pmod{100} \;\Big/^{2}$$

$$3^{20} \equiv 49^2 \pmod{100}$$

$$3^{20} \equiv 1 \pmod{100} \;\Big/ \cdot 3$$

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$$\boxed{3^{501} \cdot 7^{200} \equiv 3 \pmod{100}}$$

RSA kriptosustav

RSA kriptosustav

- $n = pq$, p i q su pažljivo odabrani veliki prosti brojevi
- $\varphi(n) = (p-1)(q-1)$
- biramo $e \in \mathbb{N}$, $1 < e < \varphi(n)$, $M(e, \varphi(n)) = 1$
- $d \in \mathbb{N}$ je rješenje kongruencije $de \equiv 1 \pmod{\varphi(n)}$
- javni dio ključa (n, e) tajni dio ključa (p, q, d)
- šifriranje $E(x) = x^e \mod n$
- dešifriranje $D(y) = y^d \mod n$
- digitalni potpis $S_B(x) = D_B(E_A(x))$

$$\text{Bob} \xrightarrow{(E_A(x),\ S_B(x))} \text{Alice}$$

sedamnaesti zadatak

Zadatak 17

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

a) *Šifrirajte za Alice poruku* $x = 10$.

b) *Odredite tajni RSA ključ koji pripada javnom ključu od Alice.*

c) *Bob je primio šifriranu poruku* $y = 172$ *i uz nju potpis* $S = 144$. *Je li poruku poslala Alice?*

Zadatak 17

$E(x) = x^e \mod n$

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

Zadatak 17

$$E(x) = x^e \mod n$$

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

Zadatak 17

$$E(x) = x^e \mod n$$

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

Zadatak 17

$E(x) = x^e \mod n$

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

$$q = \left\lfloor \frac{100\,000}{221} \right\rfloor$$

Zadatak 17

$$E(x) = x^e \mod n$$

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

$$q = \left\lfloor \frac{100\,000}{221} \right\rfloor = \lfloor 452.4886\cdots \rfloor$$

Zadatak 17

$$E(x) = x^{e} \mod n$$

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

$$q = \left\lfloor \frac{100\,000}{221} \right\rfloor = \lfloor 452.4886 \cdots \rfloor$$

$$q = 452$$

Zadatak 17

$$\boxed{E(x) = x^e \mod n}$$

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

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c) *Bob je primio šifriranu poruku* $y = 172$ *i uz nju potpis* $S = 144$. *Je li poruku poslala Alice?*

Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

$$q = \left\lfloor \frac{100\,000}{221} \right\rfloor = \lfloor 452.4886 \cdots \rfloor$$

$$q = 452$$

$$r = 100\,000 - 221 \cdot 452$$

Zadatak 17

$$E(x) = x^e \mod n$$

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

a) *Šifrirajte za Alice poruku* $x = 10$.

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Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

$$q = \left\lfloor \frac{100\,000}{221} \right\rfloor = \lfloor 452.4886\cdots \rfloor$$

$$q = 452$$

$$r = 100\,000 - 221 \cdot 452$$

$$r = 108$$

Zadatak 17

$$E(x) = x^e \mod n$$

Javni RSA ključ od Alice je $(n_A, e_A) = (221, 5)$.

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c) *Bob je primio šifriranu poruku* $y = 172$ *i uz nju potpis* $S = 144$. *Je li poruku poslala Alice?*

Rješenje

a)

$$y = x^{e_A} \mod n_A$$

$$y = 10^5 \mod 221$$

$$y = 100\,000 \mod 221$$

$$y = 108$$

$$q = \left\lfloor \frac{100\,000}{221} \right\rfloor = \lfloor 452.4886 \cdots \rfloor$$

$$q = 452$$

$$r = 100\,000 - 221 \cdot 452$$

$$r = 108$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 =$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = 13 \cdot 17$$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b) $$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) =$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17)$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$(n_A, e_A) = (221, 5)$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$192 = 5 \cdot$

$$(n_A, e_A) = (221, 5)$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$192 = 5 \cdot 38$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$192 = 5 \cdot 38 +$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$192 = 5 \cdot 38 + 2$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$192 = 5 \cdot 38 + 2$$

$$5 = 2 \cdot$$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$192 = 5 \cdot 38 + 2$$

$$5 = 2 \cdot \ 2$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$192 = 5 \cdot 38 + 2$$

$$5 = 2 \cdot \ 2 \ +$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$192 = 5 \cdot 38 + 2$$

$$5 = 2 \cdot 2 + 1$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot 38 + 2 \\ 5 &= 2 \cdot \ 2 \ + 1 \\ 2 &= 1 \cdot \end{aligned}$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot 38 + 2 \\ 5 &= 2 \cdot \ 2 \ + 1 \\ 2 &= 1 \cdot \ 2 \end{aligned}$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot 38 + 2 \\ 5 &= 2 \cdot 2 + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} & \qquad\quad\; q_i \\ 192 &= 5 \cdot \boxed{38} + 2 \\ 5 &= 2 \cdot \boxed{\;2\;} + \boxed{1} \\ 2 &= 1 \cdot \;\; 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

i	-1	0	1	2
q_i				
y_i				

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

i	-1	0	1	2
q_i			38	2
y_i				

$$\boxed{(n_A, e_A) = (221, 5)}$$

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$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

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$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

i	-1	0	1	2
q_i			38	2
y_i	0	1		

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$0 - 38 \cdot 1 = -38$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	

$$\boxed{(n_A, e_A) = (221, 5)}$$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$1 - 2 \cdot (-38) = 77$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$$\boxed{M(5, 192) = 1}$$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} & \qquad\quad q_i \\ 192 &= 5 \cdot \boxed{38} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$d =$

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} && q_i && \\ 192 &= 5 \cdot & \boxed{38} &+ 2 \\ 5 &= 2 \cdot & \boxed{2} &+ \boxed{1} \\ 2 &= 1 \cdot & 2 && \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$$d = 77$$

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$$d = 77 \cdot 1$$

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$$d = 77 \cdot 1 \mod 192$$

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$$d = 77 \cdot 1 \mod 192$$

$$d = 77$$

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} 192 &= 5 \cdot \overset{q_i}{\boxed{38}} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$$d = 77 \cdot 1 \mod 192$$

$$d = 77$$

Tajni RSA ključ od Alice

$\boxed{x_0 = ub' \mod n'}$

$\boxed{(n_A, e_A) = (221, 5)}$

b)

$$221 = \overset{p}{13} \cdot \overset{q}{17}$$

$$\varphi(221) = \varphi(13) \cdot \varphi(17) = 12 \cdot 16 = 192$$

$$ed \equiv 1 \pmod{\varphi(n)}$$

$$5d \equiv 1 \pmod{192}$$

$$\begin{aligned} & \qquad\quad q_i \\ 192 &= 5 \cdot \boxed{38} + 2 \\ 5 &= 2 \cdot \boxed{2} + \boxed{1} \\ 2 &= 1 \cdot 2 \end{aligned}$$

$\boxed{M(5, 192) = 1}$

i	-1	0	1	2
q_i			38	2
y_i	0	1	-38	77

$$d = 77 \cdot 1 \mod 192$$

$$d = 77$$

$\boxed{\text{Tajni RSA ključ od Alice}}$ $(p, q, d) = (13, 17, 77)$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{(E_B(x),\ S_A(x))} \text{Bob}$$

$$S_A(x) = D_A(E_B(x))$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{(E_B(x),\ S_A(x))} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

c)

$\boxed{(n_A, e_A) = (221, 5)}$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\, S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv \qquad \pmod{221}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\, S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221} \,\Big/^{2}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\, S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221} \, \Big/ ^2$$

$$144^4 \equiv 33\,489 \pmod{221}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{(E_B(x),\, S_A(x))} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221} \,\Big/^{\,2}$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221} \,\big/^{\,2}$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221} \,\big/ \cdot 144$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{(E_B(x),\ S_A(x))} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221} \,/\,^2$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221} \,/ \cdot 144$$

$$144^5 \equiv 16\,992 \pmod{221}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\, S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221$$

$$144^2 \equiv 183 \pmod{221} \,/\,^2$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221} \,/\cdot 144$$

$$144^5 \equiv 16\,992 \pmod{221}$$

$$144^5 \equiv 196 \pmod{221}$$

c)

$\boxed{(n_A, e_A) = (221, 5)}$

$$\text{Alice} \xrightarrow{\quad (E_B(x),\, S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221 = 196$$

$$144^2 \equiv 183 \pmod{221} \,\Big/^{\,2}$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221} \,\Big/ \cdot 144$$

$$144^5 \equiv 16\,992 \pmod{221}$$

$$144^5 \equiv 196 \pmod{221}$$

c)

$$\boxed{(n_A, e_A) = (221, 5)}$$

$$\text{Alice} \xrightarrow{(E_B(x),\, S_A(x))} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221 = 196 \neq 172$$

$$144^2 \equiv 183 \pmod{221} \,/\,^2$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221} \,/ \cdot 144$$

$$144^5 \equiv 16\,992 \pmod{221}$$

$$144^5 \equiv 196 \pmod{221}$$

c) Poruku nije poslala Alice.

$(n_A, e_A) = (221, 5)$

$$\text{Alice} \xrightarrow{(E_B(x),\ S_A(x))} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172, \quad S = 144$$

Alice je poslala poruku jedino ako je $E_A(S) = y$.

$$E_A(144) = 144^5 \mod 221 = 196 \neq 172$$

$$144^2 \equiv 183 \pmod{221} \,/^{2}$$

$$144^4 \equiv 33\,489 \pmod{221}$$

$$144^4 \equiv 118 \pmod{221} \,/\cdot 144$$

$$144^5 \equiv 16\,992 \pmod{221}$$

$$144^5 \equiv 196 \pmod{221}$$

$\boxed{(n_A, e_A) = (221, 5)}$

Kako bi izgledao potpis da je Alice poslala poruku?

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

$(n_A, e_A) = (221, 5)$

Kako bi izgledao potpis da je Alice poslala poruku?

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

$S_A(x) = D_A(y)$

$$\boxed{(n_A, e_A) = (221, 5)}$$

Kako bi izgledao potpis da je Alice poslala poruku?

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

$$S_A(x) = D_A(y) = D_A(172)$$

$\boxed{d = 77}$ $\boxed{(n_A, e_A) = (221, 5)}$

Kako bi izgledao potpis da je Alice poslala poruku?

$$\text{Alice} \xrightarrow{\qquad (E_B(x),\ S_A(x)) \qquad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

$$S_A(x) = D_A(y) = D_A(172) = 172^{77} \mod 221$$

$\boxed{d = 77}$ $\boxed{(n_A, e_A) = (221, 5)}$

Kako bi izgledao potpis da je Alice poslala poruku?

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

$$S_A(x) = D_A(y) = D_A(172) = 172^{77} \mod 221 = 100$$

$\boxed{d = 77}$ $\boxed{(n_A, e_A) = (221, 5)}$

Kako bi izgledao potpis da je Alice poslala poruku?

$$\text{Alice} \xrightarrow{\quad (E_B(x),\ S_A(x)) \quad} \text{Bob}$$

$$S_A(x) = D_A(E_B(x)), \quad y = E_B(x) = 172$$

$$S_A(x) = D_A(y) = D_A(172) = 172^{77} \mod 221 = 100$$

Modularno potenciranje
(binarna metoda)

Potenciranje – binarna metoda

$$x^y$$

- $x, y \in \mathbb{N}$

Potenciranje – binarna metoda

$$x^y = x^{\sum_{i=0}^{D-1} y_i 2^i}$$

- $x, y \in \mathbb{N}$
- $y_i \in \{0, 1\},\ i = 0, 1, \ldots, D-1$

Potenciranje – binarna metoda

$$x^y = x^{\sum\limits_{i=0}^{D-1} y_i 2^i} = \prod_{i=0}^{D-1} x^{y_i 2^i}$$

- $x, y \in \mathbb{N}$
- $y_i \in \{0, 1\},\ i = 0, 1, \ldots, D-1$

Potenciranje – binarna metoda

$$x^y = x^{\sum\limits_{i=0}^{D-1} y_i 2^i} = \prod_{i=0}^{D-1} x^{y_i 2^i} = \prod_{i=0}^{D-1} \left(x^{2^i}\right)^{y_i}$$

- $x, y \in \mathbb{N}$
- $y_i \in \{0, 1\},\ i = 0, 1, \ldots, D-1$

Potenciranje – binarna metoda

$$x^y = x^{\sum\limits_{i=0}^{D-1} y_i 2^i} = \prod_{i=0}^{D-1} x^{y_i 2^i} = \prod_{i=0}^{D-1} \left(x^{2^i}\right)^{y_i}$$

- $x, y \in \mathbb{N}$
- $y_i \in \{0, 1\},\ i = 0, 1, \ldots, D-1$
- Složenost potenciranja klasičnim načinom je $O(y)$.

Potenciranje – binarna metoda

$$x^y = x^{\sum\limits_{i=0}^{D-1} y_i 2^i} = \prod_{i=0}^{D-1} x^{y_i 2^i} = \prod_{i=0}^{D-1} \left(x^{2^i}\right)^{y_i}$$

- $x, y \in \mathbb{N}$
- $y_i \in \{0, 1\},\ i = 0, 1, \ldots, D-1$
- Složenost potenciranja klasičnim načinom je $O(y)$.
- Složenost potenciranja binarnom metodom je $O(\log y)$.

Potenciranje – binarna metoda

$$x^y = x^{\sum\limits_{i=0}^{D-1} y_i 2^i} = \prod_{i=0}^{D-1} x^{y_i 2^i} = \prod_{i=0}^{D-1} \left(x^{2^i}\right)^{y_i}$$

- $x, y \in \mathbb{N}$
- $y_i \in \{0, 1\},\ i = 0, 1, \ldots, D-1$
- Složenost potenciranja klasičnim načinom je $O(y)$.
- Složenost potenciranja binarnom metodom je $O(\log y)$.
- Na primjer, ako je $y = 2^{30}$, tada je broj množenja
  - kod klasičnog potenciranja reda veličine 1 073 741 824
  - kod binarne metode reda veličine 30

Primjer: $y = 13$

$$x^{13} =$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0} =$$

$$= x^{2^0}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

x

1

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

x

$$1 \xrightarrow{\cdot x}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

x

$$1 \xrightarrow{\cdot x} x$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \quad \xrightarrow{\text{kvadriraj}}$$

$$1 \xrightarrow{\cdot x} x$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2$$

$$1 \xrightarrow{\cdot x} x$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}}$$

$$1 \xrightarrow{\cdot x} x$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4$$

$$1 \xrightarrow{\cdot x} x$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4$$

$$1 \xrightarrow{\cdot\, x} x \xrightarrow{\cdot\, x^4}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4$$

$$1 \xrightarrow{\cdot x} x \xrightarrow{\cdot x^4} x^5$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4 \xrightarrow{\text{kvadriraj}}$$

$$1 \xrightarrow{\cdot x} x \xrightarrow{\cdot x^4} x^5$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4 \xrightarrow{\text{kvadriraj}} x^8$$

$$1 \xrightarrow{\cdot x} x \xrightarrow{\cdot x^4} x^5$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4 \xrightarrow{\text{kvadriraj}} x^8$$

$$1 \xrightarrow{\cdot x} x \xrightarrow{\cdot x^4} x^5 \xrightarrow{\cdot x^8}$$

Primjer: $y = 13$

$$x^{13} = x^{(1101)_2} = x^{1\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0} =$$

$$= x^{2^0} \cdot x^{2^2} \cdot x^{2^3} = x \cdot x^4 \cdot x^8$$

$$x \xrightarrow{\text{kvadriraj}} x^2 \xrightarrow{\text{kvadriraj}} x^4 \xrightarrow{\text{kvadriraj}} x^8$$

$$1 \xrightarrow{\cdot x} x \xrightarrow{\cdot x^4} x^5 \xrightarrow{\cdot x^8} x^{13}$$

Algoritam: Modularno potenciranje – binarna metoda s desna na lijevo

Ulaz: $x, y, n \in \mathbb{N},\ y = (y_{D-1} \cdots y_1 y_0)_2$

Izlaz: $x^y \mod n$

```
$z := x \mod n$;
$a := 1$;
for $0 \leqslant j < D - 1$ do
    if $y_j = 1$ then
        $a := az \mod n$;
    end
    $z := z^2 \mod n$;
end
$a := az \pmod{n}$;
return $a$
```

$172^{77} \mod 221$

korak	a	z
0		
1		
2		
3		
4		
5		
6		
7		

$$77 = \left(\begin{array}{ccccccc} y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)_2$$

$172^{77} \mod 221$

korak	a	z
0		
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$172^{77} \mod 221$

korak	a	z
0		
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$a := 1$

$172^{77} \mod 221$

korak	a	z
0	1	
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$a := 1$

$172^{77} \mod 221$

korak	a	z
0	1	
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$a := 1$

$z := x \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$a := 1$

$z := x \mod n$

$z := 172 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$a := 1$

$z := x \mod n$

$z := 172 \mod 221$

$z := 172$

$172^{77} \mod 221$

korak	a	z
0	1	172
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

0. korak

$a := 1$

$z := x \mod n$

$z := 172 \mod 221$

$z := 172$

$172^{77} \mod 221$

korak	a	z
0	1	172
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak $\quad y_0 = 1$

$172^{77} \mod 221$

korak	a	z
0	1	172
1		
2		
3		
4		
5		
6		
7		

$$\begin{array}{cccccccc} & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \\ 77 = (& 1 & 0 & 0 & 1 & 1 & 0 & 1 \ )_2 \end{array}$$

1. korak $y_0 = 1$

$a := az \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1		
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak $y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1		
2		
3		
4		
5		
6		
7		

$$\begin{array}{rl} & \begin{array}{ccccccc} y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \end{array} \\ 77 = & (\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{array})_2 \end{array}$$

1. korak $y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$a := 172$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	
2		
3		
4		
5		
6		
7		

$$\begin{array}{ccccccccc} & & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \\ 77 & = & (1 & 0 & 0 & 1 & 1 & 0 & 1)_2 \end{array}$$

1. korak $y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$a := 172$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak $y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$a := 172$

$z := z^2 \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak $\quad y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$a := 172$

$z := z^2 \mod n$

$z := 172^2 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak $\quad y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$a := 172$

$z := z^2 \mod n$

$z := 172^2 \mod 221$

$z := 191$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

1. korak $\quad y_0 = 1$

$a := az \mod n$

$a := 1 \cdot 172 \mod 221$

$a := 172$

$z := z^2 \mod n$

$z := 172^2 \mod 221$

$z := 191$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

2. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

$\boxed{\text{2. korak}}$ $y_1 = 0$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2		
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

2. korak $y_1 = 0$

$a := 172$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

2. korak $y_1 = 0$

$a := 172$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

2. korak $y_1 = 0$

$a := 172$

$z := z^2 \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

2. korak $\quad y_1 = 0$

$a := 172$

$z := z^2 \mod n$

$z := 191^2 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

2. korak $y_1 = 0$

$a := 172$

$z := z^2 \mod n$

$z := 191^2 \mod 221$

$z := 16$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

$\boxed{\text{2. korak}}$ $y_1 = 0$

$a := 172$

$z := z^2 \mod n$

$z := 191^2 \mod 221$

$z := 16$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $y_2 = 1$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $\quad y_2 = 1$

$a := az \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $\quad y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3		
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$a := 100$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$a := 100$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$a := 100$

$z := z^2 \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$a := 100$

$z := z^2 \mod n$

$z := 16^2 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

3. korak $y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$a := 100$

$z := z^2 \mod n$

$z := 16^2 \mod 221$

$z := 35$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4		
5		
6		
7		

$$\begin{array}{l} \quad\;\; y_6\, y_5\, y_4\, y_3\, y_2\, y_1\, y_0 \\ 77 = (1\ 0\ 0\ 1\ 1\ 0\ 1)_2 \end{array}$$

3. korak $y_2 = 1$

$a := az \mod n$

$a := 172 \cdot 16 \mod 221$

$a := 100$

$z := z^2 \mod n$

$z := 16^2 \mod 221$

$z := 35$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $\quad y_3 = 1$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak

$y_3 = 1$

$a := az \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4		
5		
6		
7		

$$\begin{array}{ccccccccc} & & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \\ 77 & = & (1 & 0 & 0 & 1 & 1 & 0 & 1)_2 \end{array}$$

4. korak $y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4		
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $\quad y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$a := 185$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$a := 185$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$a := 185$

$z := z^2 \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$a := 185$

$z := z^2 \mod n$

$z := 35^2 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$a := 185$

$z := z^2 \mod n$

$z := 35^2 \mod 221$

$z := 120$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

4. korak $y_3 = 1$

$a := az \mod n$

$a := 100 \cdot 35 \mod 221$

$a := 185$

$z := z^2 \mod n$

$z := 35^2 \mod 221$

$z := 120$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak $y_4 = 0$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5		
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak $y_4 = 0$

$a := 185$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak $y_4 = 0$

$a := 185$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak $y_4 = 0$

$a := 185$

$z := z^2 \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	
6		
7		

$$\begin{array}{c} y_6\ y_5\ y_4\ y_3\ y_2\ y_1\ y_0 \\ 77 = (1\ 0\ 0\ 1\ 1\ 0\ 1)_2 \end{array}$$

$\boxed{\text{5. korak}}$ $y_4 = 0$

$a := 185$

$z := z^2 \mod n$

$z := 120^2 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak $y_4 = 0$

$a := 185$

$z := z^2 \mod n$

$z := 120^2 \mod 221$

$z := 35$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

5. korak $y_4 = 0$

$a := 185$

$z := z^2 \mod n$

$z := 120^2 \mod 221$

$z := 35$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6		
7		

$$\begin{array}{ccccccccc} & & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \\ 77 & = & (1 & 0 & 0 & 1 & 1 & 0 & 1)_2 \end{array}$$

6. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6		
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

6. korak $y_5 = 0$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6		
7		

$$\begin{array}{ccccccccc} & & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \\ 77 = & (& 1 & 0 & 0 & 1 & 1 & 0 & 1 &)_2 \end{array}$$

6. korak $\quad y_5 = 0$

$a := 185$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

6. korak $y_5 = 0$

$a := 185$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

6. korak $y_5 = 0$

$a := 185$

$z := z^2 \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

6. korak $y_5 = 0$

$a := 185$

$z := z^2 \mod n$

$z := 35^2 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

6. korak $\quad y_5 = 0$

$a := 185$

$z := z^2 \mod n$

$z := 35^2 \mod 221$

$z := 120$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

6. korak $y_5 = 0$

$a := 185$

$z := z^2 \mod n$

$z := 35^2 \mod 221$

$z := 120$

$$172^{77} \mod 221$$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

7. korak

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

7. korak $\quad y_6 = 1$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

7. korak $y_6 = 1$

$a := az \mod n$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7		

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

7. korak $\quad y_6 = 1$

$a := az \mod n$

$a := 185 \cdot 120 \mod 221$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7		

$$\begin{array}{ccccccc} y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \end{array}$$

$$77 = (1\ 0\ 0\ 1\ 1\ 0\ 1)_2$$

7. korak $y_6 = 1$

$a := az \mod n$

$a := 185 \cdot 120 \mod 221$

$a := 100$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7	100	

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

7. korak $y_6 = 1$

$a := az \mod n$

$a := 185 \cdot 120 \mod 221$

$a := 100$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7	100	–

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

7. korak $y_6 = 1$

$a := az \mod n$

$a := 185 \cdot 120 \mod 221$

$a := 100$

$172^{77} \mod 221$

korak	a	z
0	1	172
1	172	191
2	172	16
3	100	35
4	185	120
5	185	35
6	185	120
7	$\boxed{100}$	–

$$77 = (\overset{y_6}{1}\ \overset{y_5}{0}\ \overset{y_4}{0}\ \overset{y_3}{1}\ \overset{y_2}{1}\ \overset{y_1}{0}\ \overset{y_0}{1})_2$$

$$172^{77} \mod 221 = 100$$

RSA u stvarnoj primjeni

RSA u stvarnoj primjeni

Generiranje velikih prostih brojeva

- Generiramo slučajni broj s n bitova i na njega primijenimo neki vjerojatnosni test za ispitivanje prostosti (Eulerov kriterij, Miller-Rabinov test, . . .).
- Ako broj ne prođe test, tada generiramo novi slučajni broj s n bitova i ponovimo postupak.
- Postupak ponavljamo tako dugo dok ne dobijemo prosti broj.
- Zahvaljujući teoremu o prostim brojevima nakon razumnog broja koraka dobit ćemo prosti broj.

$$\pi(x) \sim \frac{x}{\ln x}$$

RSA u stvarnoj primjeni

Množenje velikih prirodnih brojeva

- Složenost *školskog množenja* dva n-znamenkasta prirodna broja jednaka je $O(n^2)$.
- *Školsko množenje* nije efikasno na brojevima sa stotinjak i više znamenaka.
- Postoje razni moderni algoritmi za brzo množenje jako velikih prirodnih brojeva.

RSA u stvarnoj primjeni

- Jedna ideja je da se veliki prirodni brojevi podijele na manje dijelove, manji dijelovi se pomnože *školskim množenjem*, a nakon toga se spoje u cjelinu da se dobije traženi produkt.

Karatsubina metoda

- Svaki od brojeva se podijeli na dva jednaka dijela, manji dijelovi se na odgovarajući način *školski* pomnože i zatim spoje u cjelinu.
- Složenost je $O(n^{\log_2 3}) = O(n^{1.585})$ pri čemu je n broj znamenaka.
- Metoda je pogodna za brojeve sa stotinjak znamenaka.

RSA u stvarnoj primjeni

Toom-Cook metoda

- Poopćenje Karatsubine metode, brojevi se dijele na više manjih dijelova, a množenje brojeva se povezuje s množenjem polinoma.
- Polinomi se evaluiraju u određenom broju točaka tako da se dobije dovoljno podataka za njihov produkt. Tu se koristi *školsko množenje* manjih brojeva.
- Na temelju tih podataka rješavanjem sustava linearnih jednadžbi dobivaju se koeficijenti produkta dva polinoma iz kojih se dobije traženi produkt prirodnih brojeva. Rješavanje sustava se svodi na množenje matrice i vektora pri čemu opet množimo i zbrajamo manje brojeve.

RSA u stvarnoj primjeni

Toom-Cook metoda

- Složenost je $O(n^{1+\varepsilon})$ pri čemu je n broj znamenaka. Za dovoljno veliki stupanj polinoma, $\varepsilon > 0$ može biti proizvoljno blizu nule.
- Međutim, to je samo teorijska složenost jer u ovoj složenosti nisu brojana zbrajanja i množenja konstantama koja znatno rastu s povećanjem stupnja polinoma.

RSA u stvarnoj primjeni

Diskretna Fourierova transformacija

- Množenje prirodnih brojeva se temelji na diskretnoj Fourierovoj transformaciji signala i povezanosti množenja prirodnih brojeva s acikličkom konvolucijom signala.
- FFT algoritam (*Fast Fourier Transform*) je efikasan algoritam koji daje diskretnu Fourierovu transformaciju signala. Složenost mu je $O(D \ln D)$ pri čemu je D duljina signala.
- Množenje prirodnih brojeva se temelji na teoremu o konvoluciji, a složenost je jednaka $O(n \cdot \ln n \cdot \ln(\ln n))$ pri čemu je n broj znamenaka (bitova).

RSA u stvarnoj primjeni

Diskretna Fourierova transformacija

- Prirodni brojevi se poistovjete sa signalima i pronađe se diskretna Fourirerova transformacija oba signala preko FFT algoritma.
- Transformirani signali se pomnože po komponentama i pronađe se inverzna diskretna Fourierova transformacija tog produkta ponovo pomoću FFT algoritma.
- Napravimo zaokruživanje dobivenog signala na cijele brojeve, a komponente tog signala daju traženi produkt prirodnih brojeva (uz dodatno napravljeni prijenos znamenaka).
- Ovdje ulazimo u aritmetiku realnih brojeva pa treba paziti na preciznost da kod zaokruživanja ne dobijemo pogrešni rezultat.

RSA u stvarnoj primjeni

Modularno potenciranje $x^y \mod n$

- Šifriranje i dešifriranje u RSA algoritmu je također efikasno.
- Postoje efikasni algoritmi za modularno potenciranje.
- Jedna od tih metoda je binarna metoda čija složenost je $O(\log y)$.
- Druga dobra metoda se temelji na **Montgomerijevom produktu**.

RSA u stvarnoj primjeni

Montgomerijevo potenciranje $x^y \mod n$

- Ideja Montgomerijevog potenciranja je izbjegavanje dijeljenja s modulom n.
- Modularno potenciranje se zapravo ne obavlja u klasičnom potpunom sustavu ostataka modulo n, već u transformiranom potpunom sustavu ostataka modulo n.
- U transformiranom sustavu ostataka se primijenjuje Montgomerijev produkt koji se u svakom koraku obavlja sa svega dva množenja.