Relacije

Diskretne strukture s teorijom grafova

Damir Horvat

FOI, Varaždin

Sadržaj

prvi zadatak

Zadatak 1

Neka je $\mathcal{A} = \{U, W, X, Y, Z\}$ *skup od pet knjiga koje se prodaju na fakultetu. Pretpostavimo da knjige imaju sljedeća svojstva:*

knjiga	*cijena*	*debljina*
U	70 *kn*	100 *stranica*
W	175 *kn*	125 *stranica*
X	140 *kn*	150 *stranica*
Y	70 *kn*	200 *stranica*
Z	35 *kn*	100 *stranica*

Na skupu $\mathcal{A}$ *definiramo relaciju* $\mathcal{R}$ *na sljedeći način:*

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

pri čemu je $c(K)$ *cijena knjige* K*, a* $d(K)$ *debljina knjige* K.

a) *Ispišite elemente relacije* $\mathcal{R}$.

b) *Odredite matricu incidencije relacije* $\mathcal{R}$ *i nacrtajte graf relacije* $\mathcal{R}$.

c) *Ispitajte je li relacija* $\mathcal{R}$ *refleksivna, simetrična, antisimetrična i tranzitivna.*

a) *Ispišite elemente relacije* $\mathcal{R}$.

b) *Odredite matricu incidencije relacije* $\mathcal{R}$ *i nacrtajte graf relacije* $\mathcal{R}$.

c) *Ispitajte je li relacija* $\mathcal{R}$ *refleksivna, simetrična, antisimetrična i tranzitivna.*

Rješenje

$$(a, b) \in \mathcal{R} \quad \leftrightsquigarrow \quad a \, \mathcal{R} \, b$$

Matrica incidencije

$$\begin{array}{c|ccccc}
{}_a\diagdown^b & U & W & X & Y & Z \\
\hline
U & & & & & \\
W & & & & & \\
X & & & & & \\
Y & & & & & \\
Z & & & & &
\end{array}$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U					
W					
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\Longleftrightarrow} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1				
W					
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0			
W					
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0		
W					
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	
W					
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W					
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1				
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1			
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0		
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a,b) \in \mathcal{R} \ \overset{\text{def}}{\Longleftrightarrow}\ \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X					
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1				
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0			
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a,b) \in \mathcal{R} \;\overset{\text{def}}{\iff}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1		
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y					
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1				
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0			
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \stackrel{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0		
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z					

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0				

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0			

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0		

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	

knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

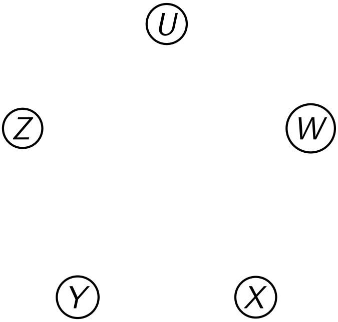
knjiga	cijena	debljina
U	70 kn	100 stranica
W	175 kn	125 stranica
X	140 kn	150 stranica
Y	70 kn	200 stranica
Z	35 kn	100 stranica

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

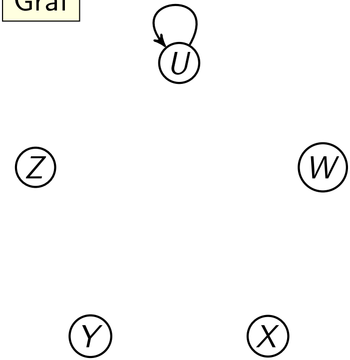


$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

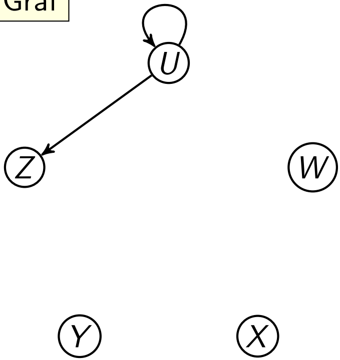


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

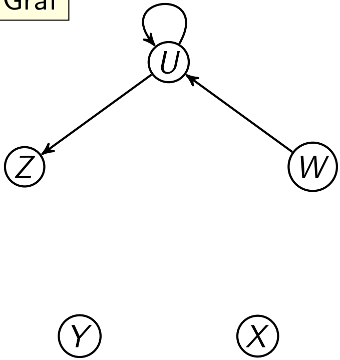


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

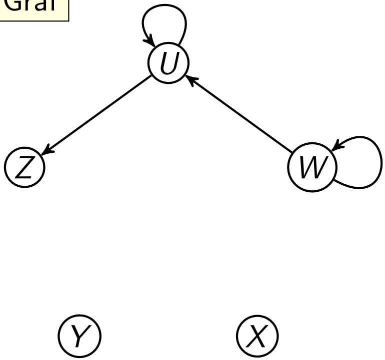


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

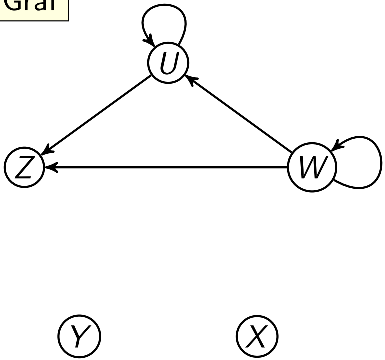


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

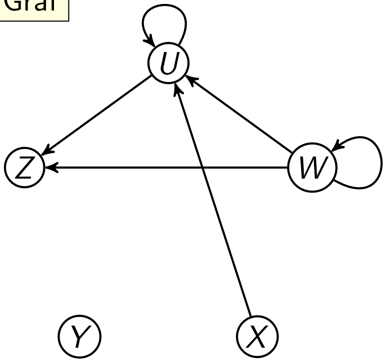


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

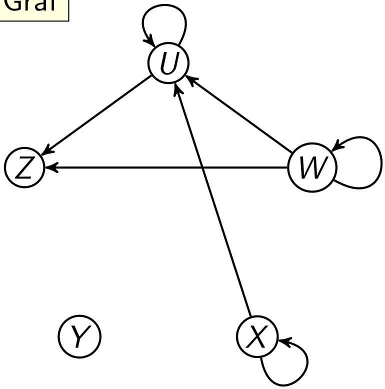


$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

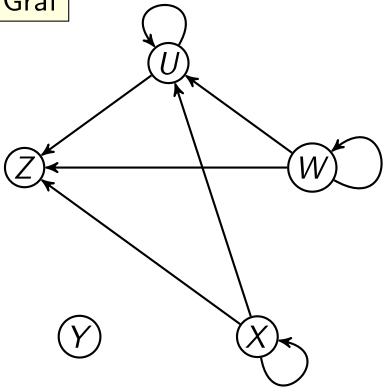


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

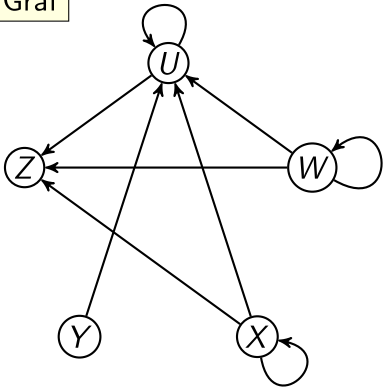


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

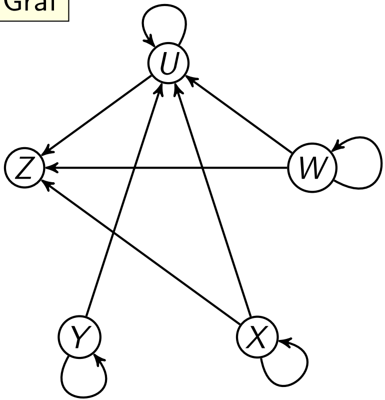


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

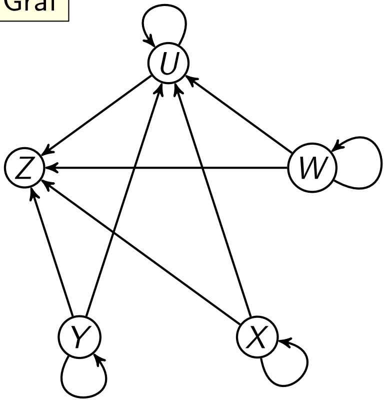


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

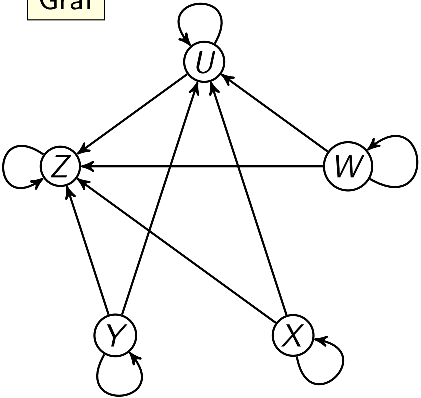


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

a \ b	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

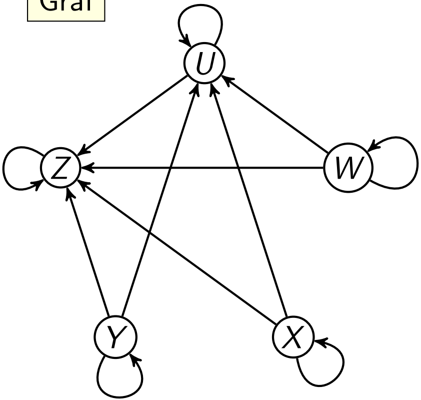


$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

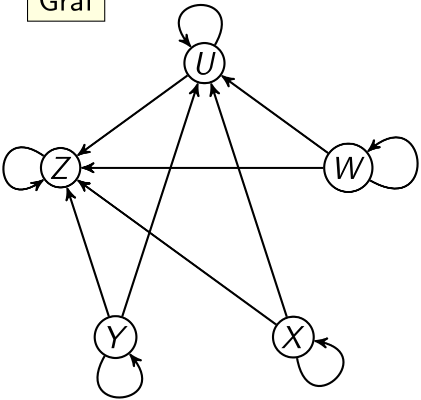
$$\mathcal{R} = \{$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

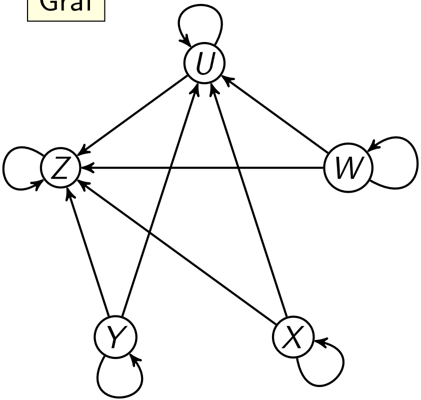
$$\mathcal{R} = \{(U, U)$$

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

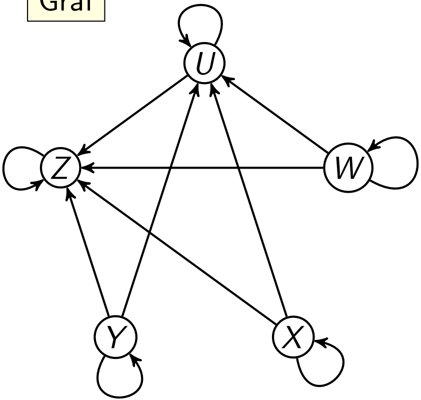
$$\mathcal{R} = \{(U, U), (U, Z)$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

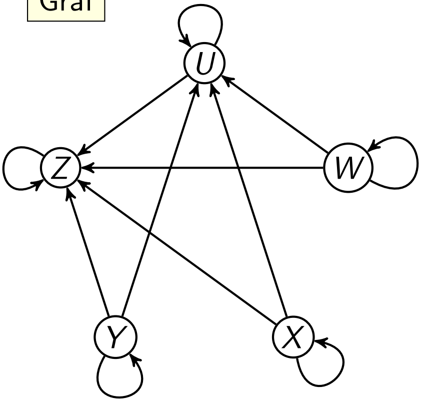
$$\mathcal{R} = \{(U, U), (U, Z), (W, U)$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

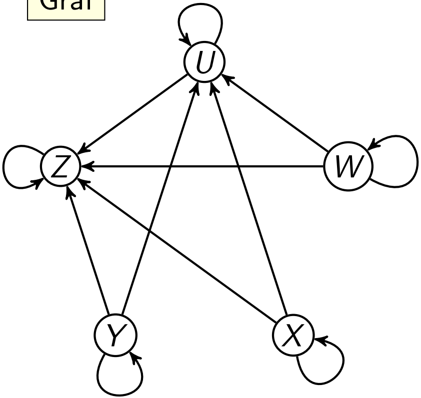
$$\mathcal{R} = \{(U, U), (U, Z), (W, U), (W, W)$$

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

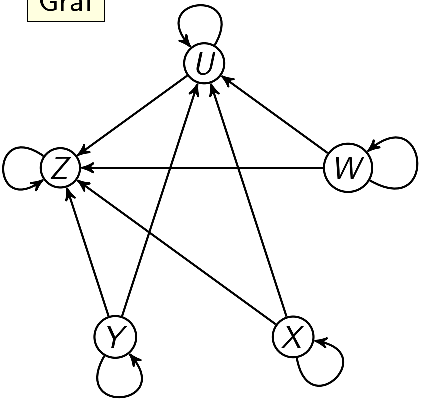
$$\mathcal{R} = \big\{(U,U),(U,Z),(W,U),(W,W),(W,Z)$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

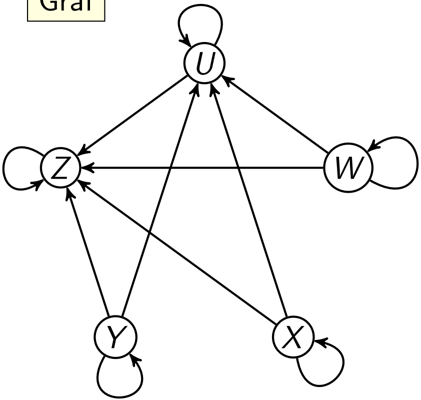
$$\mathcal{R} = \{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U)$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

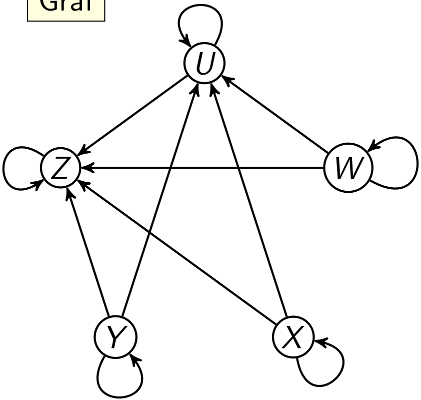
$$\mathcal{R} = \big\{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U),$$
$$(X,X)$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

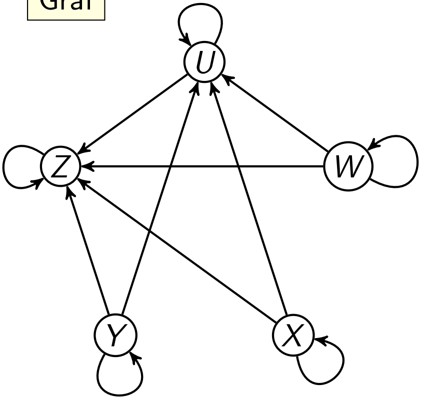
$$\mathcal{R} = \{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U),$$
$$(X,X),(X,Z)$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

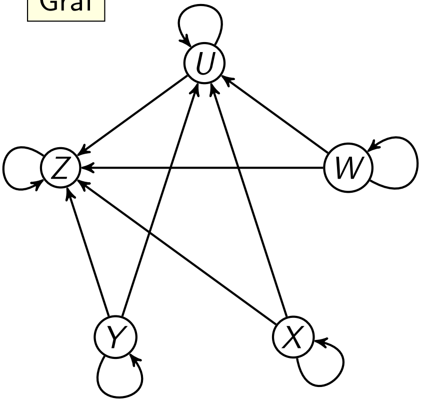
$$\mathcal{R} = \{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U),$$
$$(X,X),(X,Z),(Y,U)$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

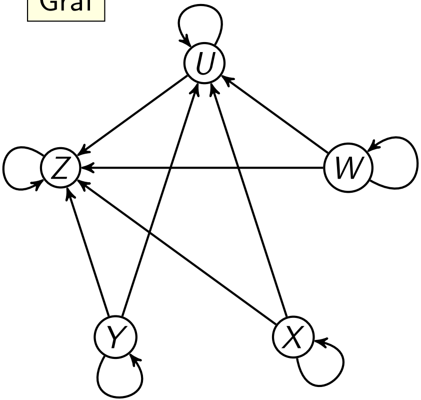
$$\mathcal{R} = \big\{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U),$$
$$(X,X),(X,Z),(Y,U),(Y,Y)$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

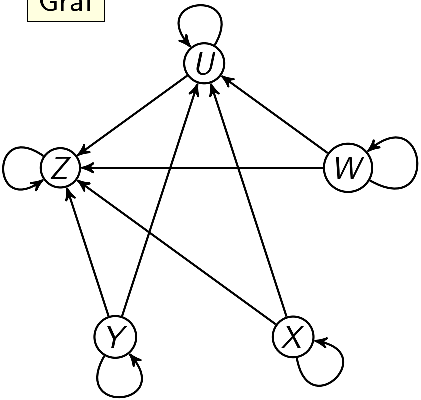
$$\mathcal{R} = \{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U),$$
$$(X,X),(X,Z),(Y,U),(Y,Y),(Y,Z)\}$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\Longleftrightarrow} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Elementi relacije

$$\mathcal{R} = \big\{(U,U),(U,Z),(W,U),(W,W),(W,Z),(X,U),$$
$$(X,X),(X,Z),(Y,U),(Y,Y),(Y,Z),(Z,Z)\big\}$$

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\left(\forall x \in \mathcal{A}\right)\left(x \,\mathcal{R}\, x\right)$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\big(\forall x \in \mathcal{A}\big)\big(x\,\mathcal{R}\,x\big)$

- **Pomoću matrice incidencije**

$$\boxed{(a,b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\big(\forall x \in \mathcal{A}\big)\big(x \,\mathcal{R}\, x\big)$

- **Pomoću matrice incidencije**

  Na glavnoj dijagonali se nalaze jedinice.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\big(\forall x \in \mathcal{A}\big)\big(x \mathrel{\mathcal{R}} x\big)$

- **Pomoću matrice incidencije**

  Na glavnoj dijagonali se nalaze jedinice.

- **Pomoću grafa relacije**

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\quad (\forall x \in \mathcal{A})\,(x\ \mathcal{R}\ x)$

- **Pomoću matrice incidencije**
  Na glavnoj dijagonali se nalaze jedinice.
- **Pomoću grafa relacije**
  Kod svakog vrha u grafu se nalazi petlja.

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\big(\forall x \in \mathcal{A}\big)\big(x \, \mathcal{R} \, x\big)$

- **Pomoću matrice incidencije**

  Na glavnoj dijagonali se nalaze jedinice.

- **Pomoću grafa relacije**

  Kod svakog vrha u grafu se nalazi petlja.

- **Pomoću definicije**

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\quad (\forall x \in \mathcal{A})\,(x\ \mathcal{R}\ x)$

- **Pomoću matrice incidencije**

  Na glavnoj dijagonali se nalaze jedinice.

- **Pomoću grafa relacije**

  Kod svakog vrha u grafu se nalazi petlja.

- **Pomoću definicije**

  Za svaki $x \in \mathcal{A}$ je $c(x) \geqslant c(x)$ i $d(x) \geqslant d(x)$ pa vrijedi $x\ \mathcal{R}\ x$.

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\Longleftrightarrow} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\quad (\forall x \in \mathcal{A})(x \,\mathcal{R}\, x)$

- **Pomoću matrice incidencije**

  Na glavnoj dijagonali se nalaze jedinice.

- **Pomoću grafa relacije**

  Kod svakog vrha u grafu se nalazi petlja.

- **Pomoću definicije**

  Za svaki $x \in \mathcal{A}$ je $c(x) \geqslant c(x)$ i $d(x) \geqslant d(x)$ pa vrijedi $x \,\mathcal{R}\, x$.

Relacija $\mathcal{R}$ je refleksivna relacija na skupu $\mathcal{A}$.

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)}$$

Refleksivnost $\quad (\forall x \in \mathcal{A})\big(x\,\mathcal{R}\,x\big)$

- **Pomoću matrice incidencije**

  Na glavnoj dijagonali se nalaze jedinice.

- **Pomoću grafa relacije**

  Kod svakog vrha u grafu se nalazi petlja.

- **Pomoću definicije**

  Za svaki $x \in \mathcal{A}$ je $c(x) \geqslant c(x)$ i $d(x) \geqslant d(x)$ pa vrijedi $x\,\mathcal{R}\,x$.

Relacija $\mathcal{R}$ je refleksivna relacija na skupu $\mathcal{A}$.

Općenito je $\mathcal{R}$ refleksivna relacija na bilo kojem skupu knjiga.

$$\boxed{(a,b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\big(\forall x, y \in \mathcal{A}\big)\big(x \,\mathcal{R}\, y \;\Rightarrow\; y \,\mathcal{R}\, x\big)$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\quad (\forall x, y \in \mathcal{A})\big(x \,\mathcal{R}\, y \;\Rightarrow\; y \,\mathcal{R}\, x\big)$

- **Pomoću matrice incidencije**

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\big(\forall x, y \in \mathcal{A}\big)\big(x \,\mathcal{R}\, y \;\Rightarrow\; y \,\mathcal{R}\, x\big)$

- **Pomoću matrice incidencije**

  Matrica incidencije nije simetrična matrica.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big(x \,\mathcal{R}\, y \;\Rightarrow\; y \,\mathcal{R}\, x\big)$

- **Pomoću matrice incidencije**

  Matrica incidencije nije simetrična matrica. Na primjer,

$$W \,\mathcal{R}\, U, \;\; U \not\mathcal{R}\, W$$

$$\boxed{(a, b) \in \mathcal{R} \;\stackrel{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\quad (\forall x, y \in \mathcal{A})\left(x \, \mathcal{R} \, y \;\Rightarrow\; y \, \mathcal{R} \, x\right)$

- **Pomoću matrice incidencije**

  Matrica incidencije nije simetrična matrica. Na primjer,

$$W \, \mathcal{R} \, U, \;\; U \not\mathcal{R} \, W$$

- **Pomoću grafa relacije**

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big(x \,\mathcal{R}\, y \;\Rightarrow\; y \,\mathcal{R}\, x\big)$

- **Pomoću matrice incidencije**

  Matrica incidencije nije simetrična matrica. Na primjer,

$$W \,\mathcal{R}\, U, \;\; U \,\not{\mathcal{R}}\, W$$

- **Pomoću grafa relacije**

  Na primjer, postoji luk (W, U), ali ne postoji luk (U, W).

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Simetričnost $\quad (\forall x, y \in \mathcal{A})\big(x \mathrel{\mathcal{R}} y \;\Rightarrow\; y \mathrel{\mathcal{R}} x\big)$

- **Pomoću matrice incidencije**
  Matrica incidencije nije simetrična matrica. Na primjer,
  $$W \mathrel{\mathcal{R}} U, \;\; U \not\mathrel{\mathcal{R}} W$$
- **Pomoću grafa relacije**
  Na primjer, postoji luk (W, U), ali ne postoji luk (U, W).

Relacija $\mathcal{R}$ nije simetrična relacija na skupu $\mathcal{A}$.

$$\boxed{(a, b) \in \mathcal{R} \;\stackrel{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\left(\forall x, y \in \mathcal{A}\right)\left((x\,\mathcal{R}\,y) \wedge (y\,\mathcal{R}\,x) \;\Rightarrow\; x = y\right)$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\big(\forall x, y \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$x \,\mathcal{R}\, y \quad \Longrightarrow$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\big(\forall x, y \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$x \,\mathcal{R}\, y \quad\Longrightarrow\quad c(x) \geqslant c(y),$$

$$\boxed{(a, b) \in \mathcal{R} \;\stackrel{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad (\forall x, y \in \mathcal{A})\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} x$.

$$x \mathrel{\mathcal{R}} y \quad\Longrightarrow\quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y)$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$
\begin{aligned}
x \,\mathcal{R}\, y \quad &\Longrightarrow \quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y) \\
y \,\mathcal{R}\, x \quad &\Longrightarrow
\end{aligned}
$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big((x\ \mathcal{R}\ y) \wedge (y\ \mathcal{R}\ x) \ \Rightarrow\ x = y\big)$

Pretpostavimo da je $x\ \mathcal{R}\ y$ i $y\ \mathcal{R}\ x$.

$$
\begin{aligned}
x\ \mathcal{R}\ y \quad &\Longrightarrow \quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y) \\
y\ \mathcal{R}\ x \quad &\Longrightarrow \quad c(y) \geqslant c(x),
\end{aligned}
$$

$$\boxed{(a, b) \in \mathcal{R} \ \stackrel{\text{def}}{\Longleftrightarrow}\ \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$x \,\mathcal{R}\, y \quad\Longrightarrow\quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y)$$
$$y \,\mathcal{R}\, x \quad\Longrightarrow\quad c(y) \geqslant c(x), \quad d(y) \geqslant d(x)$$

$$\boxed{(a, b) \in \mathcal{R} \;\stackrel{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad \big(\forall x, y \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$\begin{array}{rcl} x \,\mathcal{R}\, y & \Longrightarrow & \boxed{\begin{array}{l} c(x) \geqslant c(y), \\ c(y) \geqslant c(x), \end{array}} \begin{array}{l} d(x) \geqslant d(y) \\ d(y) \geqslant d(x) \end{array} \\ y \,\mathcal{R}\, x & \Longrightarrow & \end{array}$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad (\forall x, y \in \mathcal{A})\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} x) \ \Rightarrow\ x = y\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} x$.

$$
\begin{array}{lcll}
x \mathrel{\mathcal{R}} y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y \mathrel{\mathcal{R}} x & \Longrightarrow & c(y) \geqslant c(x), & d(y) \geqslant d(x)
\end{array}
$$

$$c(x) = c(y)$$

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)$$

Antisimetričnost

$$(\forall x, y \in \mathcal{A})\big((x \mathcal{R} y) \wedge (y \mathcal{R} x) \;\Rightarrow\; x = y\big)$$

Pretpostavimo da je $x \mathcal{R} y$ i $y \mathcal{R} x$.

$$\begin{aligned} x \mathcal{R} y &\implies \boxed{\begin{aligned} c(x) &\geqslant c(y), \\ c(y) &\geqslant c(x), \end{aligned}} \; \boxed{\begin{aligned} d(x) &\geqslant d(y) \\ d(y) &\geqslant d(x) \end{aligned}} \\ y \mathcal{R} x &\implies \end{aligned}$$

$$c(x) = c(y)$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad (\forall x, y \in \mathcal{A})\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, x$.

$$
\begin{array}{lcll}
x \,\mathcal{R}\, y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y \,\mathcal{R}\, x & \Longrightarrow & c(y) \geqslant c(x), & d(y) \geqslant d(x) \\
 & & \downarrow & \downarrow \\
 & & c(x) = c(y) & d(x) = d(y)
\end{array}
$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\Longleftrightarrow} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad (\forall x, y \in \mathcal{A})\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} x$.

$$\begin{array}{rcll}
x \mathrel{\mathcal{R}} y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y \mathrel{\mathcal{R}} x & \Longrightarrow & c(y) \geqslant c(x), & d(y) \geqslant d(x) \\
 & & \downarrow & \downarrow \\
 & & c(x) = c(y) & d(x) = d(y)
\end{array}$$

- Dakle, knjige x i y imaju jednaku cijenu i jednaki broj stranica.

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\Longleftrightarrow} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Antisimetričnost $\quad (\forall x, y \in \mathcal{A})\big((x \mathrel{\mathcal{R}} y) \land (y \mathrel{\mathcal{R}} x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} x$.

$$
\begin{aligned}
x \mathrel{\mathcal{R}} y &\implies c(x) \geqslant c(y), \quad d(x) \geqslant d(y) \\
y \mathrel{\mathcal{R}} x &\implies c(y) \geqslant c(x), \quad d(y) \geqslant d(x)
\end{aligned}
$$

$$c(x) = c(y) \qquad d(x) = d(y)$$

- Dakle, knjige x i y imaju jednaku cijenu i jednaki broj stranica.
- U našem slučaju slijedi da je $x = y$ pa je $\mathcal{R}$ antisimetrična relacija.

$$(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \land \big(d(a) \geqslant d(b)\big)$$

Antisimetričnost $\quad (\forall x, y \in \mathcal{A})\big((x\, \mathcal{R}\, y) \wedge (y\, \mathcal{R}\, x) \;\Rightarrow\; x = y\big)$

Pretpostavimo da je $x\, \mathcal{R}\, y$ i $y\, \mathcal{R}\, x$.

$$\begin{array}{lcll}
x\, \mathcal{R}\, y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y\, \mathcal{R}\, x & \Longrightarrow & c(y) \geqslant c(x), & d(y) \geqslant d(x) \\
 & & \downarrow & \downarrow \\
 & & c(x) = c(y) & d(x) = d(y)
\end{array}$$

- Dakle, knjige x i y imaju jednaku cijenu i jednaki broj stranica.
- U našem slučaju slijedi da je $x = y$ pa je $\mathcal{R}$ antisimetrična relacija.
- Općenito to ne mora biti antisimetrična relacija jer dvije različite knjige mogu imati jednaku cijenu i jednaki broj stranica.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\big(\forall x, y, z \in \mathcal{A}\big)\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\big)$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\big(\forall x, y, z \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, z) \;\Rightarrow\; x \,\mathcal{R}\, z\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, z$.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, z) \;\Rightarrow\; x \,\mathcal{R}\, z\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, z$.

$$x \,\mathcal{R}\, y \quad \Longrightarrow$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} z$.

$$x \mathrel{\mathcal{R}} y \quad\Longrightarrow\quad c(x) \geqslant c(y),$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\big(\forall x, y, z \in \mathcal{A}\big)\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} z$.

$$x \mathrel{\mathcal{R}} y \quad\Longrightarrow\quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y)$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, z) \;\Rightarrow\; x \,\mathcal{R}\, z\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, z$.

$$x \,\mathcal{R}\, y \quad \Longrightarrow \quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y)$$

$$y \,\mathcal{R}\, z \quad \Longrightarrow$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\big(\forall x, y, z \in \mathcal{A}\big)\big((x\ \mathcal{R}\ y) \wedge (y\ \mathcal{R}\ z) \ \Rightarrow\ x\ \mathcal{R}\ z\big)$

Pretpostavimo da je $x\ \mathcal{R}\ y$ i $y\ \mathcal{R}\ z$.

$$
\begin{aligned}
x\ \mathcal{R}\ y &\implies c(x) \geqslant c(y), \quad d(x) \geqslant d(y) \\
y\ \mathcal{R}\ z &\implies c(y) \geqslant c(z),
\end{aligned}
$$

$$\boxed{(a, b) \in \mathcal{R} \ \overset{\text{def}}{\Longleftrightarrow}\ \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\big(\forall x, y, z \in \mathcal{A}\big)\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} z$.

$$\begin{aligned} x \mathrel{\mathcal{R}} y \quad &\Longrightarrow \quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y) \\ y \mathrel{\mathcal{R}} z \quad &\Longrightarrow \quad c(y) \geqslant c(z), \quad d(y) \geqslant d(z) \end{aligned}$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\left(\forall x, y, z \in \mathcal{A}\right)\left((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\right)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} z$.

$$\begin{array}{rcll} x \mathrel{\mathcal{R}} y & \Longrightarrow & \boxed{\begin{array}{l} c(x) \geqslant c(y), \\ c(y) \geqslant c(z), \end{array}} & \begin{array}{l} d(x) \geqslant d(y) \\ d(y) \geqslant d(z) \end{array} \\ y \mathrel{\mathcal{R}} z & \Longrightarrow & & \end{array}$$

$$\boxed{(a, b) \in \mathcal{R} \;\stackrel{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x\, \mathcal{R}\, y) \wedge (y\, \mathcal{R}\, z) \;\Rightarrow\; x\, \mathcal{R}\, z\big)$

Pretpostavimo da je $x\, \mathcal{R}\, y$ i $y\, \mathcal{R}\, z$.

$$\begin{aligned} x\, \mathcal{R}\, y \quad &\Longrightarrow \quad c(x) \geqslant c(y), \quad d(x) \geqslant d(y) \\ y\, \mathcal{R}\, z \quad &\Longrightarrow \quad c(y) \geqslant c(z), \quad d(y) \geqslant d(z) \end{aligned}$$

$$c(x) \geqslant c(z)$$

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} z$.

$$\begin{array}{lcll}
x \mathrel{\mathcal{R}} y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y \mathrel{\mathcal{R}} z & \Longrightarrow & c(y) \geqslant c(z), & d(y) \geqslant d(z)
\end{array}$$

$$c(x) \geqslant c(z)$$

$$\boxed{(a, b) \in \mathcal{R} \overset{\text{def}}{\iff} \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x \,\mathcal{R}\, y) \wedge (y \,\mathcal{R}\, z) \;\Rightarrow\; x \,\mathcal{R}\, z\big)$

Pretpostavimo da je $x \,\mathcal{R}\, y$ i $y \,\mathcal{R}\, z$.

$$\begin{array}{lcll}
x \,\mathcal{R}\, y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y \,\mathcal{R}\, z & \Longrightarrow & c(y) \geqslant c(z), & d(y) \geqslant d(z) \\
 & & \downarrow & \downarrow \\
 & & c(x) \geqslant c(z) & d(x) \geqslant d(z)
\end{array}$$

$$\boxed{(a, b) \in \mathcal{R} \;\stackrel{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x \mathrel{\mathcal{R}} y) \wedge (y \mathrel{\mathcal{R}} z) \;\Rightarrow\; x \mathrel{\mathcal{R}} z\big)$

Pretpostavimo da je $x \mathrel{\mathcal{R}} y$ i $y \mathrel{\mathcal{R}} z$.

$$
\begin{array}{lcll}
x \mathrel{\mathcal{R}} y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y \mathrel{\mathcal{R}} z & \Longrightarrow & c(y) \geqslant c(z), & d(y) \geqslant d(z) \\
 & & \downarrow & \downarrow \\
 & & c(x) \geqslant c(z) & d(x) \geqslant d(z)
\end{array}
$$

- Iz $c(x) \geqslant c(z)$ i $d(x) \geqslant d(z)$ slijedi $x \mathrel{\mathcal{R}} z$.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad \big(\forall x, y, z \in \mathcal{A}\big)\big((x\,\mathcal{R}\,y) \wedge (y\,\mathcal{R}\,z) \;\Rightarrow\; x\,\mathcal{R}\,z\big)$

Pretpostavimo da je $x\,\mathcal{R}\,y$ i $y\,\mathcal{R}\,z$.

$$\begin{array}{lcll}
x\,\mathcal{R}\,y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y\,\mathcal{R}\,z & \Longrightarrow & c(y) \geqslant c(z), & d(y) \geqslant d(z) \\
 & & \downarrow & \downarrow \\
 & & c(x) \geqslant c(z) & d(x) \geqslant d(z)
\end{array}$$

- Iz $c(x) \geqslant c(z)$ i $d(x) \geqslant d(z)$ slijedi $x\,\mathcal{R}\,z$. Stoga je $\mathcal{R}$ tranzitivna relacija.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Tranzitivnost $\quad (\forall x, y, z \in \mathcal{A})\big((x\,\mathcal{R}\,y) \wedge (y\,\mathcal{R}\,z) \;\Rightarrow\; x\,\mathcal{R}\,z\big)$

Pretpostavimo da je $x\,\mathcal{R}\,y$ i $y\,\mathcal{R}\,z$.

$$\begin{array}{rcll}
x\,\mathcal{R}\,y & \Longrightarrow & c(x) \geqslant c(y), & d(x) \geqslant d(y) \\
y\,\mathcal{R}\,z & \Longrightarrow & c(y) \geqslant c(z), & d(y) \geqslant d(z) \\
 & & \downarrow & \downarrow \\
 & & c(x) \geqslant c(z) & d(x) \geqslant d(z)
\end{array}$$

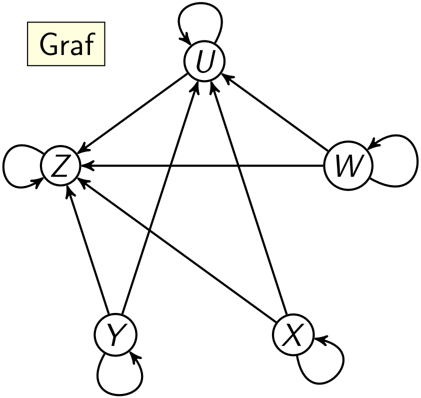
- Iz $c(x) \geqslant c(z)$ i $d(x) \geqslant d(z)$ slijedi $x\,\mathcal{R}\,z$. Stoga je $\mathcal{R}$ tranzitivna relacija.
- Općenito, relacija $\mathcal{R}$ je tranzitivna relacija na proizvoljnom skupu knjiga.

$$\boxed{(a, b) \in \mathcal{R} \;\overset{\text{def}}{\Longleftrightarrow}\; \big(c(a) \geqslant c(b)\big) \wedge \big(d(a) \geqslant d(b)\big)}$$

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

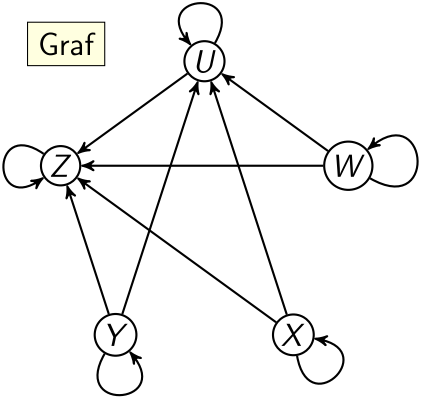
Graf



Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

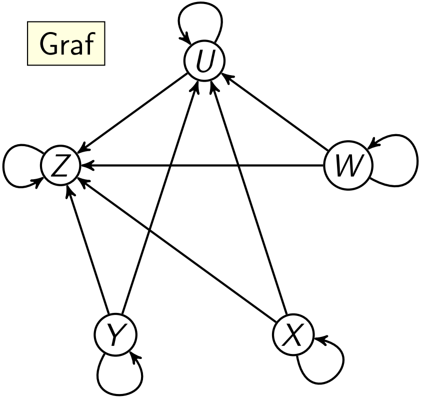


Najmanji element

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf

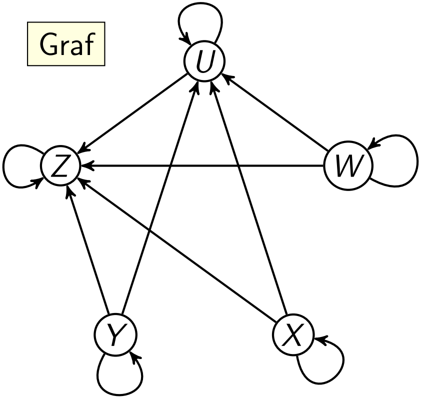


Najmanji element ne postoji

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



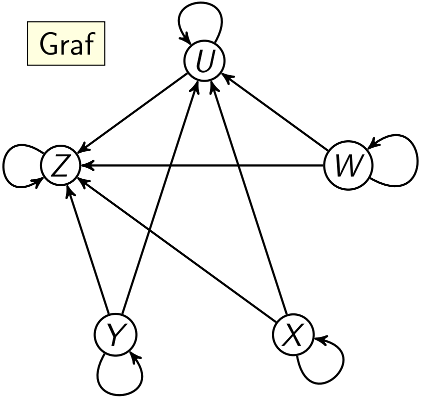
Najmanji element ne postoji

Minimalni elementi

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



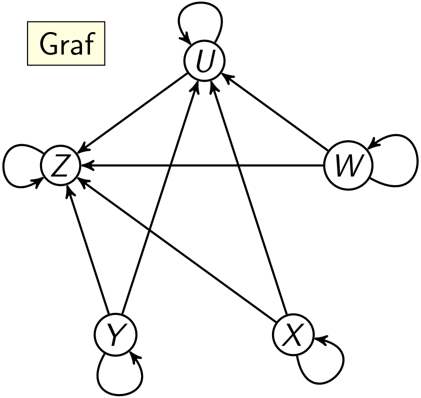
Najmanji element ne postoji

Minimalni elementi X, Y, W

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Najmanji element ne postoji

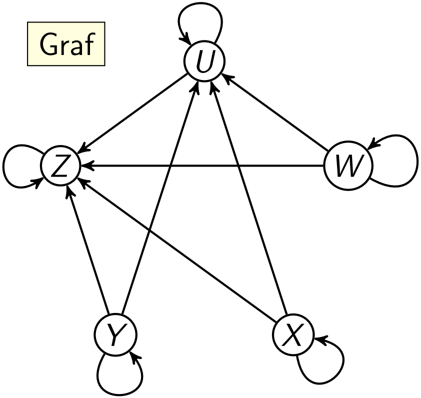
Minimalni elementi X, Y, W

Najveći element

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Najmanji element ne postoji

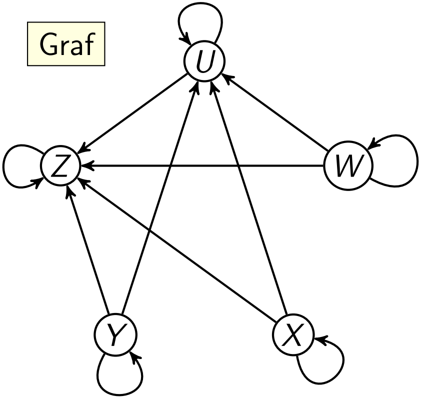
Minimalni elementi X, Y, W

Najveći element Z

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Najmanji element ne postoji

Minimalni elementi X, Y, W

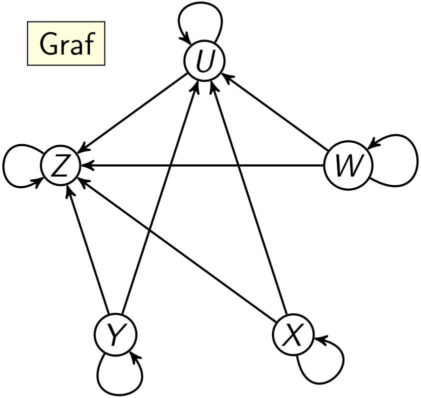
Najveći element Z

Maksimalni elementi

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Najmanji element ne postoji

Minimalni elementi X, Y, W

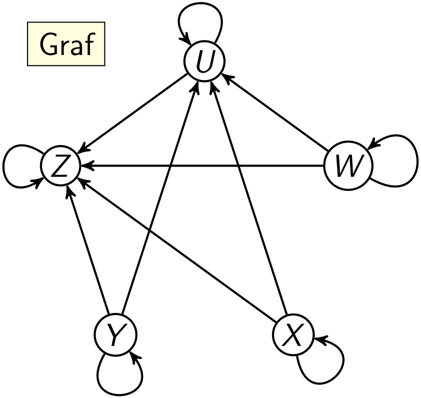
Najveći element Z

Maksimalni elementi Z

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

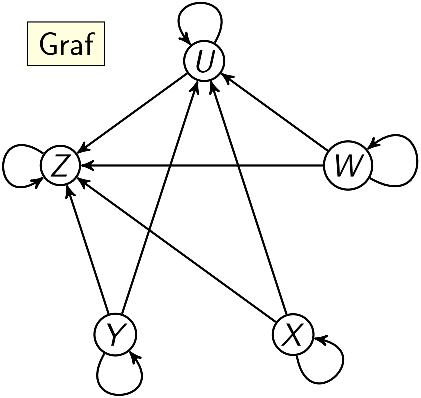
Maksimalni elementi Z

Hasseov dijagram

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

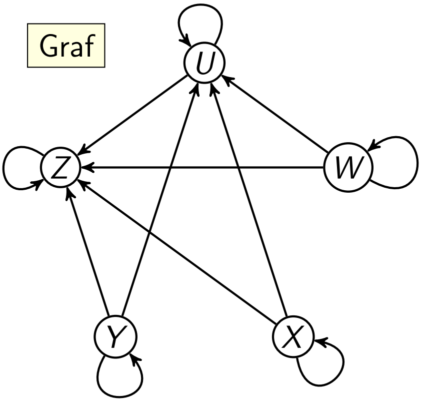
Maksimalni elementi Z

Hasseov dijagram

Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



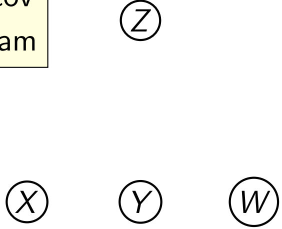
Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

Maksimalni elementi Z

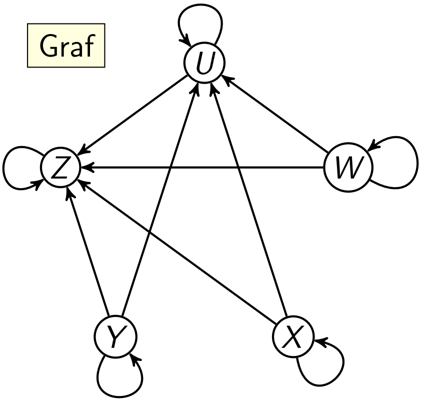
Hasseov dijagram



Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



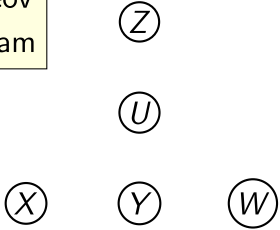
Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

Maksimalni elementi Z

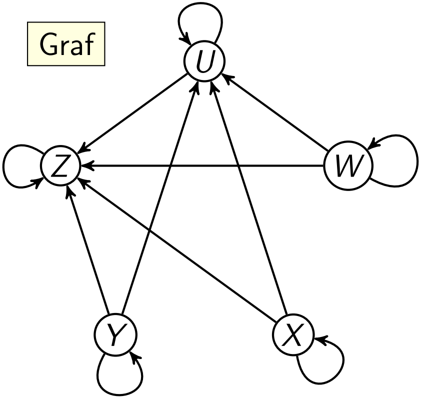
Hasseov dijagram



Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



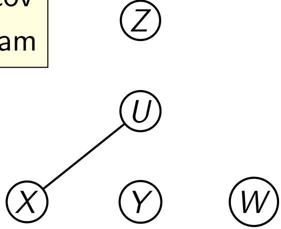
Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

Maksimalni elementi Z

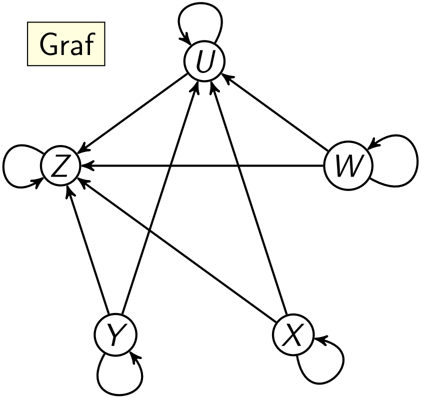
Hasseov dijagram



Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



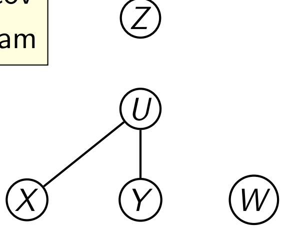
Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

Maksimalni elementi Z

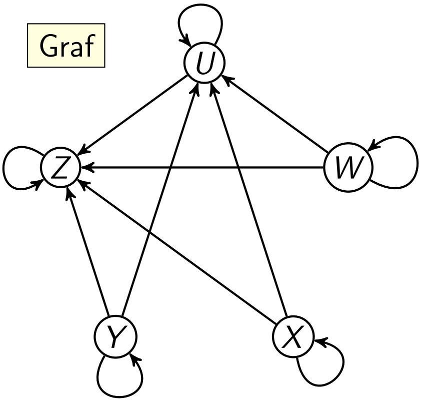
Hasseov dijagram



Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



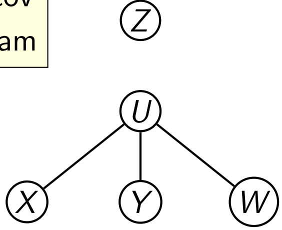
Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

Maksimalni elementi Z

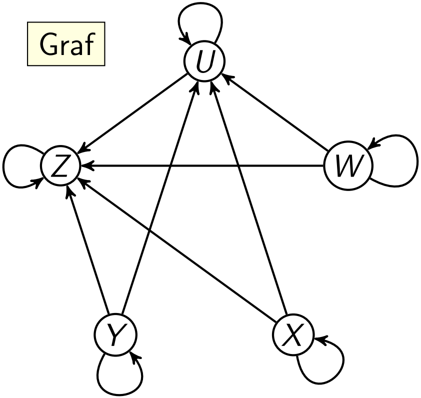
Hasseov dijagram



Matrica incidencije

$a \backslash b$	U	W	X	Y	Z
U	1	0	0	0	1
W	1	1	0	0	1
X	1	0	1	0	1
Y	1	0	0	1	1
Z	0	0	0	0	1

Graf



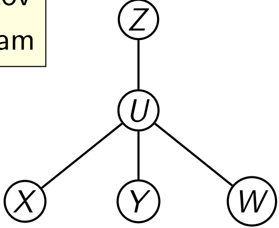
Najmanji element ne postoji

Minimalni elementi X, Y, W

Najveći element Z

Maksimalni elementi Z

Hasseov dijagram



drugi zadatak

Zadatak 2

Zadana je particija

$$\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}$$

skupa $A = \{1,2,3,4,5,6,7,8\}$. *Napišite matricu incidencije i nacrtajte graf relacije ekvivalencije* ρ *na skupu* A *koju prirodno definira zadana particija* $\mathcal{P}$.

Zadatak 2

Zadana je particija

$$\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}$$

skupa $A = \{1,2,3,4,5,6,7,8\}$. *Napišite matricu incidencije i nacrtajte graf relacije ekvivalencije* ρ *na skupu* A *koju prirodno definira zadana particija* $\mathcal{P}$.

Rješenje

$$x \;\rho\; y \overset{\text{def}}{\iff} x \text{ i } y \text{ pripadaju istom elementu particije } \mathcal{P}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

$$\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2								
3								
4								
5								
6								
7								
8								

$$\boxed{\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3								
4								
5								
6								
7								
8								

$$\boxed{\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4								
5								
6								
7								
8								

$$\boxed{\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5								
6								
7								
8								

$$\boxed{\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6								
7								
8								

$$\boxed{\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7								
8								

$$\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8								

$$\boxed{\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}}$$

Matrica incidencije

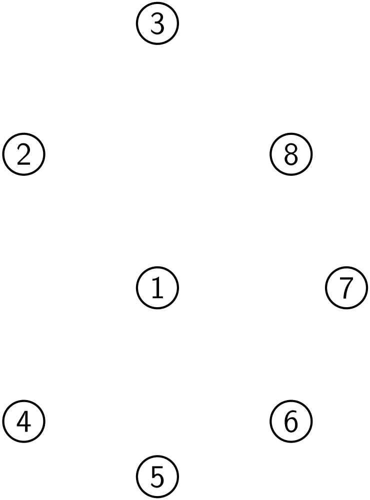
$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

$$\mathcal{P} = \big\{\{1,4,5\},\{2,3,8\},\{6,7\}\big\}$$

Graf

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

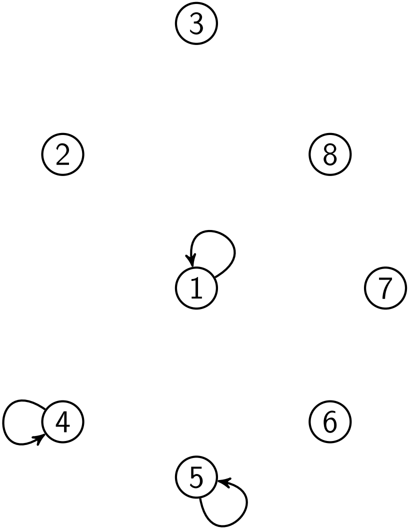


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Graf

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

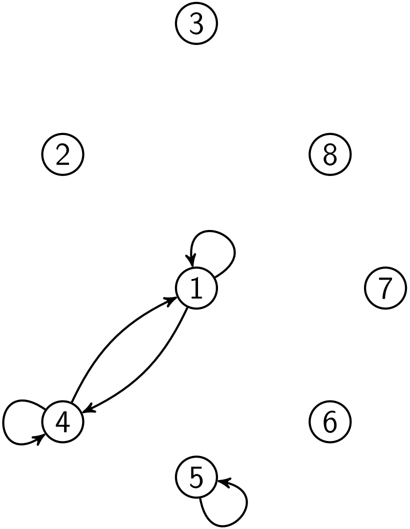


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

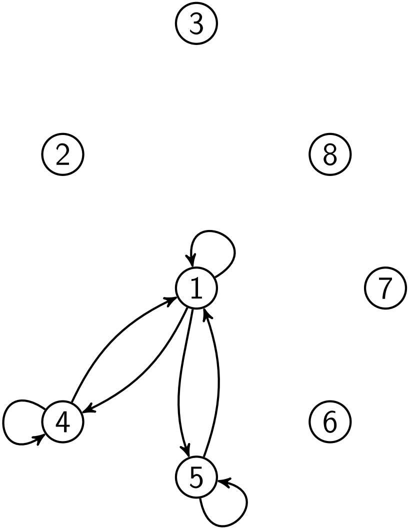


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Graf

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

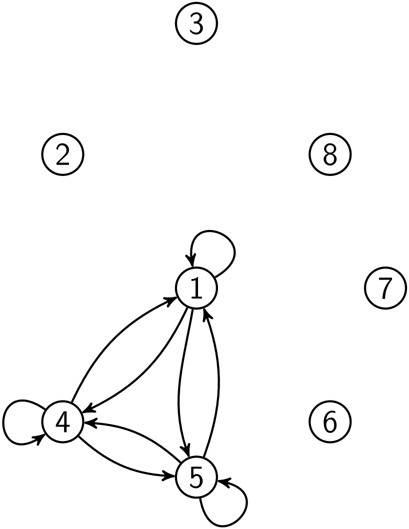


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

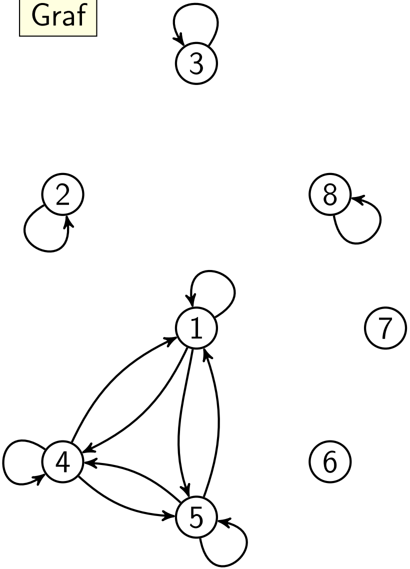


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

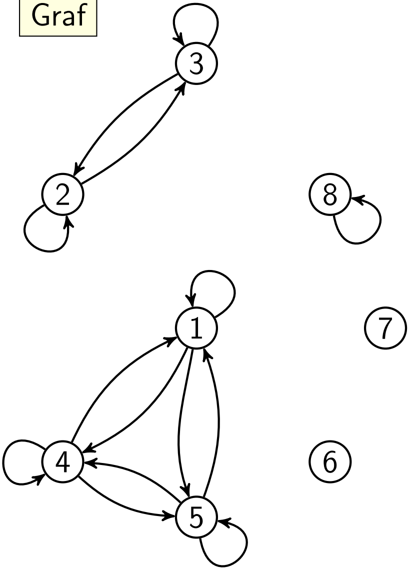


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

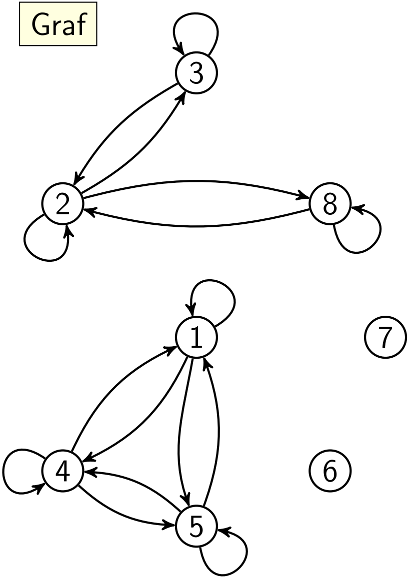


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

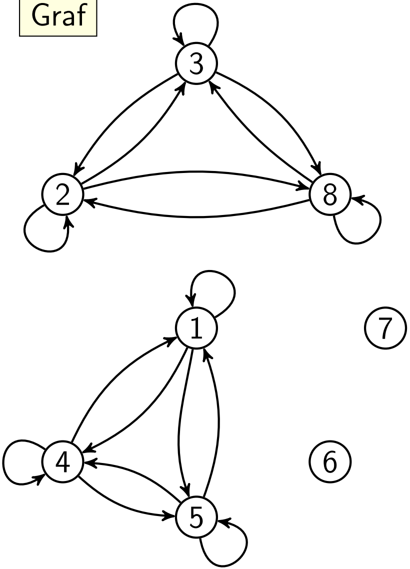


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

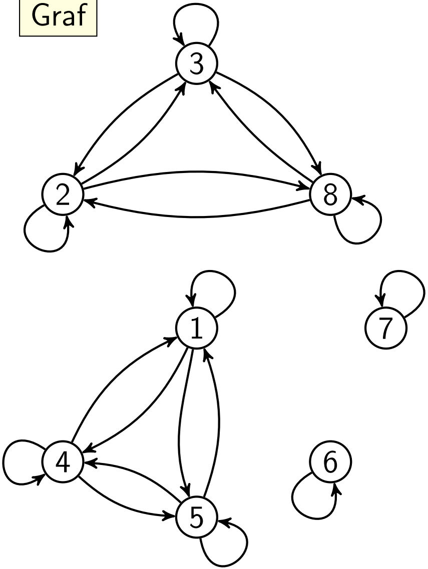


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf

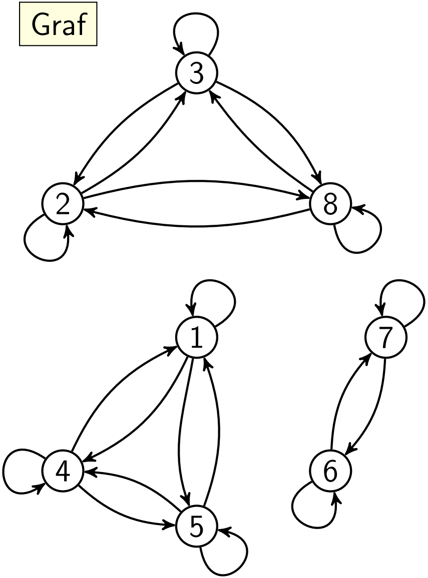


$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

Matrica incidencije

$x \backslash y$	1	2	3	4	5	6	7	8
1	1	0	0	1	1	0	0	0
2	0	1	1	0	0	0	0	1
3	0	1	1	0	0	0	0	1
4	1	0	0	1	1	0	0	0
5	1	0	0	1	1	0	0	0
6	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	0
8	0	1	1	0	0	0	0	1

Graf



$$\mathcal{P} = \{\{1,4,5\},\{2,3,8\},\{6,7\}\}$$

treći zadatak

Definicija kongruencije modulo n

Neka su $a, b \in \mathbb{Z}$ i $n \in \mathbb{N}$, $n > 1$.

$$a \equiv b \pmod{n} \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad \exists k \in \mathbb{Z},\ a - b = nk$$

Definicija kongruencije modulo n

Neka su $a, b \in \mathbb{Z}$ i $n \in \mathbb{N}$, $n > 1$.

$$a \equiv b \pmod{n} \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad \exists k \in \mathbb{Z},\ a - b = nk$$

$a \equiv b \pmod{n} \quad \Longleftrightarrow \quad$ a i b daju isti ostatak pri dijeljenju s n

Definicija kongruencije modulo n

Neka su $a, b \in \mathbb{Z}$ i $n \in \mathbb{N}$, $n > 1$.

$$a \equiv b \pmod{n} \quad \overset{\text{def}}{\Longleftrightarrow} \quad \exists k \in \mathbb{Z},\ a - b = nk$$

$a \equiv b \pmod{n} \quad \Longleftrightarrow \quad a$ i b daju isti ostatak pri dijeljenju s n

- $10 \equiv 1 \pmod{3}$ jer $3 \mid 10 - 1$

Definicija kongruencije modulo n

Neka su $a, b \in \mathbb{Z}$ i $n \in \mathbb{N}$, $n > 1$.

$$a \equiv b \pmod{n} \quad \overset{\text{def}}{\iff} \quad \exists k \in \mathbb{Z},\ a - b = nk$$

$$a \equiv b \pmod{n} \quad \iff \quad a \text{ i } b \text{ daju isti ostatak pri dijeljenju s } n$$

- $10 \equiv 1 \pmod{3}$ jer $3 \mid 10 - 1$
- $-15 \equiv 13 \pmod{7}$ jer $7 \mid -15 - 13$

Definicija kongruencije modulo n

Neka su $a, b \in \mathbb{Z}$ i $n \in \mathbb{N}$, $n > 1$.

$$a \equiv b \pmod{n} \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad \exists k \in \mathbb{Z},\ a - b = nk$$

$$a \equiv b \pmod{n} \quad \Longleftrightarrow \quad a \text{ i } b \text{ daju isti ostatak pri dijeljenju s } n$$

- $10 \equiv 1 \pmod{3}$ jer $3 \mid 10 - 1$
- $-15 \equiv 13 \pmod{7}$ jer $7 \mid -15 - 13$
- $2 \not\equiv 7 \pmod{11}$ jer $11 \nmid 2 - 7$

Definicija kongruencije modulo n

Neka su $a, b \in \mathbb{Z}$ i $n \in \mathbb{N}$, $n > 1$.

$$a \equiv b \pmod{n} \quad \overset{\text{def}}{\Longleftrightarrow} \quad \exists k \in \mathbb{Z},\ a - b = nk$$

$a \equiv b \pmod{n} \quad \Longleftrightarrow \quad a$ i b daju isti ostatak pri dijeljenju s n

- $10 \equiv 1 \pmod{3}$ jer $3 \mid 10 - 1$
- $-15 \equiv 13 \pmod{7}$ jer $7 \mid -15 - 13$
- $2 \not\equiv 7 \pmod{11}$ jer $11 \nmid 2 - 7$
- $-15 \not\equiv -13 \pmod{7}$ jer $7 \nmid -15 - (-13)$

Zadatak 3

Na skupu $B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ *zadana je relacija* $\sim$ *s*

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5.$$

a) *Dokažite da je* $\sim$ *relacija ekvivalencije na skupu* B.

b) *Odredite sve elemente kvocijentnog skupa* $B/\sim$.

$$m \sim n \overset{\text{def}}{\Longleftrightarrow} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

Refleksivnost $\quad (\forall a \in B)(a \sim a)$

$$m \sim n \stackrel{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

Refleksivnost $\quad (\forall a \in B)(a \sim a)$

$$a \sim a$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

Refleksivnost $\quad \big(\forall a \in B\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; a^2 \equiv a^2 \pmod 5$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

Refleksivnost $\quad (\forall a \in B)(a \sim a)$

$$a \sim a \;\Leftrightarrow\; a^2 \equiv a^2 \pmod 5$$

Simetričnost $\quad (\forall a, b \in B)(a \sim b \;\Rightarrow\; b \sim a)$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod{5}$$

Rješenje

a) 1. način

Refleksivnost $\quad (\forall a \in B)(a \sim a)$

$$a \sim a \;\Leftrightarrow\; a^2 \equiv a^2 \pmod{5}$$

Simetričnost $\quad (\forall a, b \in B)(a \sim b \;\Rightarrow\; b \sim a)$

$$a \sim b$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

Refleksivnost $\quad \big(\forall a \in B\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; a^2 \equiv a^2 \pmod 5$$

Simetričnost $\quad \big(\forall a, b \in B\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; a^2 \equiv b^2 \pmod 5$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Rješenje

a) 1. način

Refleksivnost $\quad (\forall a \in B)(a \sim a)$

$$a \sim a \Leftrightarrow a^2 \equiv a^2 \pmod 5$$

Simetričnost $\quad (\forall a, b \in B)(a \sim b \Rightarrow b \sim a)$

$$a \sim b \Rightarrow a^2 \equiv b^2 \pmod 5 \Rightarrow b^2 \equiv a^2 \pmod 5$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

Rješenje

a) $\boxed{\text{1. način}}$

$\boxed{\text{Refleksivnost}}$ $\quad \big(\forall a \in B\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; a^2 \equiv a^2 \pmod 5$$

$\boxed{\text{Simetričnost}}$ $\quad \big(\forall a, b \in B\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\Rightarrow\; b^2 \equiv a^2 \pmod 5 \;\Rightarrow\; b \sim a$$

$$m \sim n \stackrel{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost

$$\big(\forall a, b, c \in B\big)\big((a \sim b) \land (b \sim c) \;\Rightarrow\; a \sim c\big)$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad (\forall a, b, c \in B)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$(a \sim b) \wedge (b \sim c)$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad (\forall a, b, c \in B)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad (\forall a, b, c \in B)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\; \wedge$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad (\forall a, b, c \in B)\big((a \sim b) \wedge (b \sim c) \Rightarrow a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \Rightarrow a^2 \equiv b^2 \pmod 5 \ \wedge \ b^2 \equiv c^2 \pmod 5$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad \big(\forall a, b, c \in B\big)\big((a \sim b) \wedge (b \sim c) \ \Rightarrow \ a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \ \Rightarrow \ a^2 \equiv b^2 \pmod 5 \ \wedge \ b^2 \equiv c^2 \pmod 5 \ \Rightarrow$$

$$\Rightarrow \ \exists u, v \in \mathbb{Z},$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

Tranzitivnost $\quad \big(\forall a, b, c \in B\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\wedge\; b^2 \equiv c^2 \pmod 5 \;\Rightarrow$$

$$\Rightarrow\; \exists u, v \in \mathbb{Z},\; a^2 - b^2 = 5u,$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

Tranzitivnost $\quad \big(\forall a, b, c \in B\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\wedge\; b^2 \equiv c^2 \pmod 5 \;\Rightarrow$$

$$\Rightarrow\; \exists u, v \in \mathbb{Z},\; a^2 - b^2 = 5u,\; b^2 - c^2 = 5v$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

Tranzitivnost $\quad \big(\forall a, b, c \in B\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\wedge\; b^2 \equiv c^2 \pmod 5 \;\Rightarrow$$

$$\Rightarrow\; \exists u, v \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 5u,\;\; b^2 - c^2 = 5v}_{+} \;\Rightarrow$$

$$\Rightarrow\; \big(a^2 - b^2\big) + \big(b^2 - c^2\big) = 5u + 5v$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad (\forall a, b, c \in B)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\;\wedge\;\; b^2 \equiv c^2 \pmod 5 \;\Rightarrow$$

$$\Rightarrow\; \exists u, v \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 5u,\;\; b^2 - c^2 = 5v}_{+} \;\Rightarrow$$

$$\Rightarrow\; \left(a^2 - b^2\right) + \left(b^2 - c^2\right) = 5u + 5v \;\Rightarrow\; a^2 - c^2 = 5(u + v)$$

$$m \sim n \overset{\text{def}}{\Longleftrightarrow} m^2 \equiv n^2 \pmod{5}$$

Tranzitivnost $\quad \big(\forall a, b, c \in B\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod{5} \;\wedge\; b^2 \equiv c^2 \pmod{5} \;\Rightarrow$$

$$\Rightarrow\; \exists u, v \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 5u,\; b^2 - c^2 = 5v}_{+} \;\Rightarrow$$

$$\Rightarrow\; \big(a^2 - b^2\big) + \big(b^2 - c^2\big) = 5u + 5v \;\Rightarrow\; a^2 - c^2 = 5(\underbrace{u + v}_{\in \mathbb{Z}}) \;\Rightarrow$$

$$\Rightarrow\; a^2 \equiv c^2 \pmod{5}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

Tranzitivnost $\quad (\forall a, b, c \in B)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; a^2 \equiv b^2 \pmod 5 \;\wedge\; b^2 \equiv c^2 \pmod 5 \;\Rightarrow$$

$$\Rightarrow\; \exists u, v \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 5u,\; b^2 - c^2 = 5v}_{+} \;\Rightarrow$$

$$\Rightarrow\; \left(a^2 - b^2\right) + \left(b^2 - c^2\right) = 5u + 5v \;\Rightarrow\; a^2 - c^2 = 5(\underbrace{u + v}_{\in \mathbb{Z}}) \;\Rightarrow$$

$$\Rightarrow\; a^2 \equiv c^2 \pmod 5 \;\Rightarrow\; a \sim c$$

$$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

2. način

$$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$$

Odredimo klase svih elemenata.

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim =$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

2. način

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim =$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

2. način

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$

$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod{5}}$

2. način

$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim =$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim =$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

2. način

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1,$$

$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$

$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$

2. način

$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \stackrel{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

2. način

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim =$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim =$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1,$$

$$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

2. način

$$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim =$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2,$$

$$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

2. način

$$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim =$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3,$$

$$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

2. način

$$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\Longleftrightarrow} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

2. način

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$$[4]_\sim =$$

$$\boxed{B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

$$\boxed{m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5}$$

2. način

$$\boxed{[a]_\sim = \{x \in B : x \sim a\}}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$$[4]_\sim = \{-4,$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$$[4]_\sim = \{-4, -1,$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$[0]_\sim = \{0\}$

$[-4]_\sim = \{-4, -1, 1, 4\}$

$[-3]_\sim = \{-3, -2, 2, 3\}$

$[-2]_\sim = \{-3, -2, 2, 3\}$

$[-1]_\sim = \{-4, -1, 1, 4\}$

$[1]_\sim = \{-4, -1, 1, 4\}$

$[2]_\sim = \{-3, -2, 2, 3\}$

$[3]_\sim = \{-3, -2, 2, 3\}$

$[4]_\sim = \{-4, -1, 1,$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$$[4]_\sim = \{-4, -1, 1, 4\}$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \stackrel{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

2. način

$$[a]_\sim = \{x \in B : x \sim a\}$$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

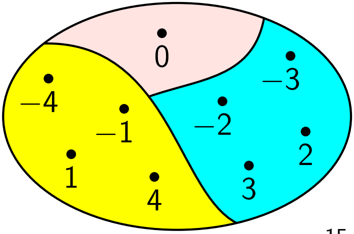
$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$$[4]_\sim = \{-4, -1, 1, 4\}$$



$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\Longleftrightarrow} m^2 \equiv n^2 \pmod 5$

2. način

$[a]_\sim = \{x \in B : x \sim a\}$

Odredimo klase svih elemenata.

$$[-4]_\sim = \{-4, -1, 1, 4\}$$

$$[-3]_\sim = \{-3, -2, 2, 3\}$$

$$[-2]_\sim = \{-3, -2, 2, 3\}$$

$$[-1]_\sim = \{-4, -1, 1, 4\}$$

$$[0]_\sim = \{0\}$$

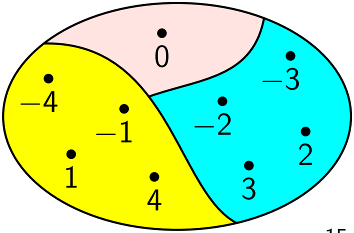
$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$[3]_\sim = \{-3, -2, 2, 3\}$$

$$[4]_\sim = \{-4, -1, 1, 4\}$$

Dobili smo particiju skupa B koja prirodno definira relaciju $\sim$. Stoga je $\sim$ relacija ekvivalencije na skupu B.

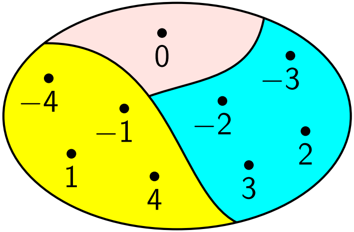


$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

$[a]_\sim = \{x \in B : x \sim a\}$

b)

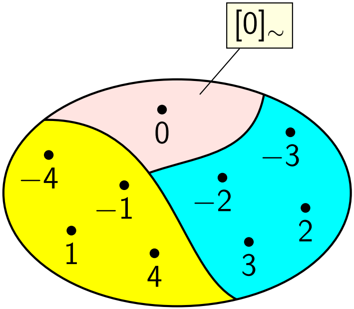


$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

b)



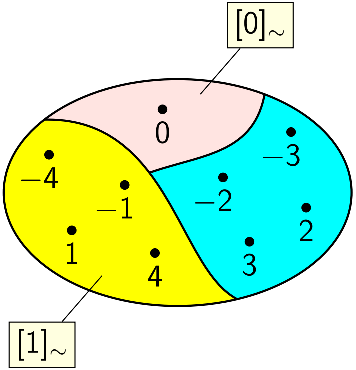
$$[0]_\sim = \{0\}$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

b)



$$[0]_\sim = \{0\}$$

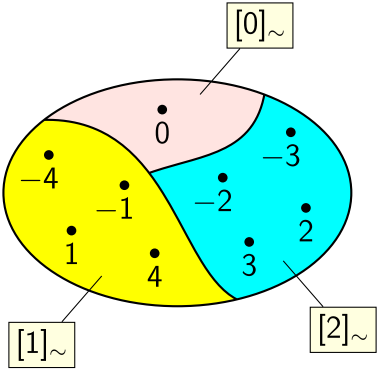
$$[1]_\sim = \{-4, -1, 1, 4\}$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

$[a]_\sim = \{x \in B : x \sim a\}$

b)



$[0]_\sim = \{0\}$

$[1]_\sim = \{-4, -1, 1, 4\}$

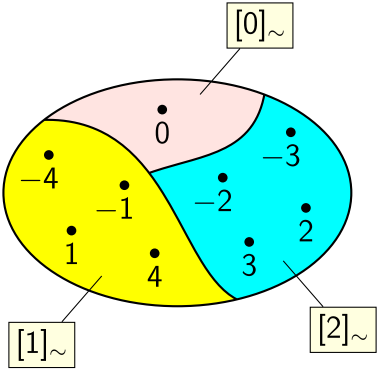
$[2]_\sim = \{-3, -2, 2, 3\}$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

b)



$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

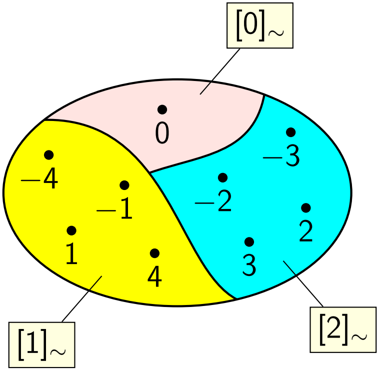
$$B/\!\sim\; =$$

$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$

$[a]_\sim = \{x \in B : x \sim a\}$

b)



$[0]_\sim = \{0\}$

$[1]_\sim = \{-4, -1, 1, 4\}$

$[2]_\sim = \{-3, -2, 2, 3\}$

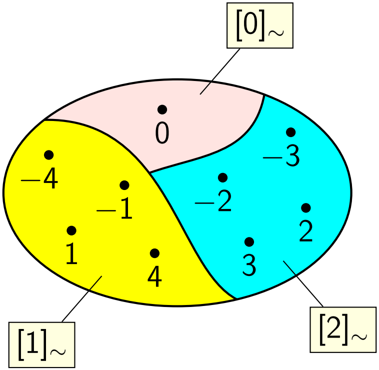
$B/\!\sim\; = \big\{[0]_\sim,\ [1]_\sim,\ [2]_\sim\big\}$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

$$m \sim n \overset{\text{def}}{\iff} m^2 \equiv n^2 \pmod 5$$

$$[a]_\sim = \{x \in B : x \sim a\}$$

b)



$$[0]_\sim = \{0\}$$

$$[1]_\sim = \{-4, -1, 1, 4\}$$

$$[2]_\sim = \{-3, -2, 2, 3\}$$

$$B/\!\sim\; = \{[0]_\sim, [1]_\sim, [2]_\sim\}$$

$$k(B/\!\sim) = 3$$

Napomena

- U slučaju da relacija ρ nije relacija ekvivalencije na skupu A, također možemo govoriti o "klasi" pojedinog elementa.

Napomena

- U slučaju da relacija ρ nije relacija ekvivalencije na skupu A, također možemo govoriti o "klasi" pojedinog elementa.
- "Klase" u tom slučaju ne moraju dati particiju skupa A. Štoviše, moguće je da "klasa" nekog elementa bude prazan skup.

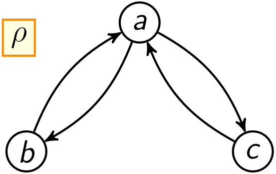
Napomena

- U slučaju da relacija ρ nije relacija ekvivalencije na skupu A, također možemo govoriti o "klasi" pojedinog elementa.
- "Klase" u tom slučaju ne moraju dati particiju skupa A. Štoviše, moguće je da "klasa" nekog elementa bude prazan skup.
- Ako ρ nije simetrična relacija, tada su skupovi

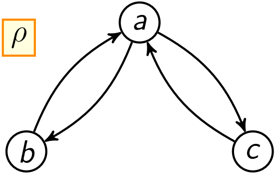
$$[a]_\rho^{(left)} = \{x \in A : x \ \rho \ a\}, \qquad [a]_\rho^{(right)} = \{x \in A : a \ \rho \ x\}$$

općenito različiti. U tom slučaju govorimo o lijevoj i desnoj klasi pojedinog elementa $a \in A$.

Relacija ρ na skupu $A = \{a, b, c\}$



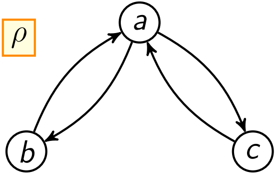
Relacija ρ na skupu $A = \{a, b, c\}$



(Lijeve) klase elemenata

$$[a]_\rho =$$

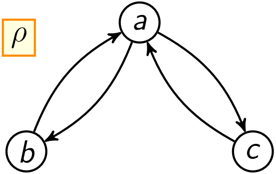
Relacija ρ na skupu $A = \{a, b, c\}$



(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$

Relacija ρ na skupu $A = \{a, b, c\}$

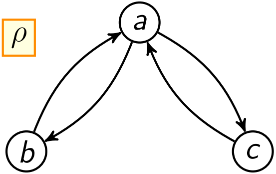


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$

$$[b]_\rho =$$

Relacija ρ na skupu $A = \{a, b, c\}$

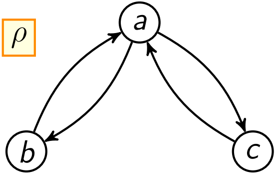


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$

$$[b]_\rho = \{a\}$$

Relacija ρ na skupu $A = \{a, b, c\}$



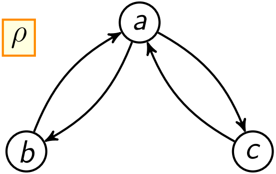
(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$

$$[b]_\rho = \{a\}$$

$$[c]_\rho =$$

Relacija ρ na skupu $A = \{a, b, c\}$

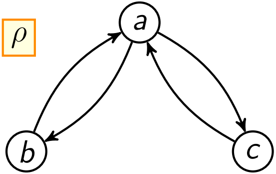


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

Relacija ρ na skupu $A = \{a, b, c\}$

$\boxed{\rho}$

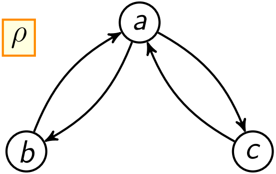


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.

Relacija ρ na skupu $A = \{a, b, c\}$

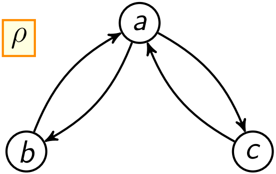


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija ρ na skupu $A = \{a, b, c\}$

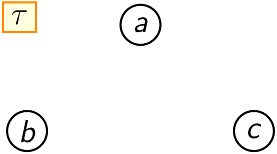


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

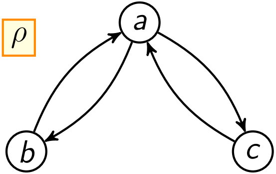
- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$



Relacija ρ na skupu $A = \{a, b, c\}$

$\boxed{\rho}$



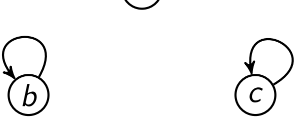
(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

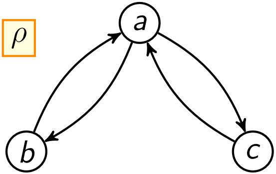
Relacija τ na skupu $A = \{a, b, c\}$

$\boxed{\tau}$



Relacija ρ na skupu $A = \{a, b, c\}$

$\boxed{\rho}$



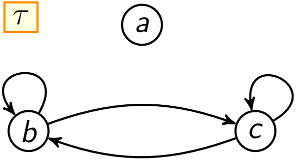
(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

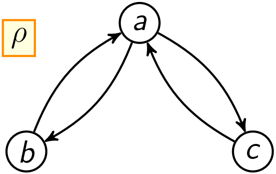
- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$

$\boxed{\tau}$



Relacija ρ na skupu $A = \{a, b, c\}$

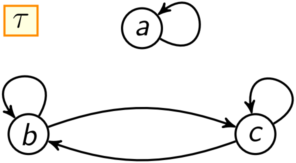


(Lijeve) klase elemenata

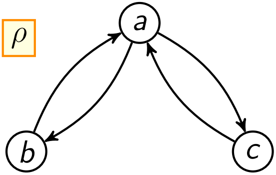
$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$



Relacija ρ na skupu $A = \{a, b, c\}$

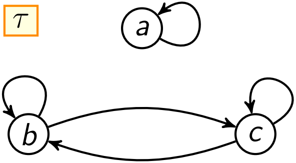


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$

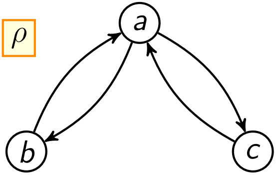


Klase elemenata

$$[a]_\tau =$$

Relacija ρ na skupu $A = \{a, b, c\}$

$\boxed{\rho}$



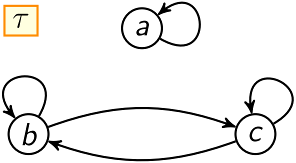
(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$

$\boxed{\tau}$

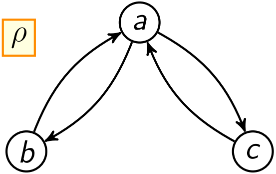


Klase elemenata

$$[a]_\tau = \{a\}$$

Relacija ρ na skupu $A = \{a, b, c\}$

$\boxed{\rho}$



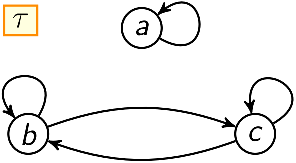
(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \{\{b, c\}, \{a\}\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$

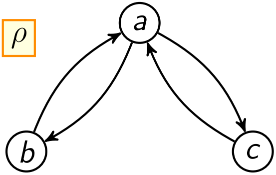
$\boxed{\tau}$



Klase elemenata

$$[a]_\tau = \{a\}$$
$$[b]_\tau =$$

Relacija ρ na skupu $A = \{a, b, c\}$

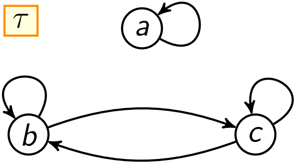


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

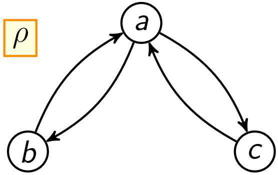
Relacija τ na skupu $A = \{a, b, c\}$



Klase elemenata

$$[a]_\tau = \{a\}$$
$$[b]_\tau = \{b, c\}$$

Relacija ρ na skupu $A = \{a, b, c\}$

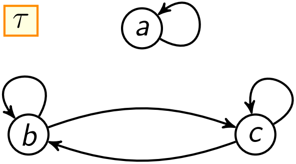


(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$

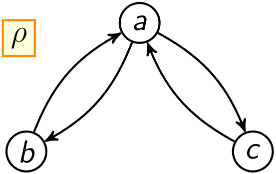


Klase elemenata

$$[a]_\tau = \{a\}$$
$$[b]_\tau = \{b, c\}$$
$$[c]_\tau =$$

Relacija ρ na skupu $A = \{a, b, c\}$

$\boxed{\rho}$



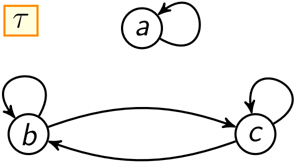
(Lijeve) klase elemenata

$$[a]_\rho = \{b, c\}$$
$$[b]_\rho = \{a\}$$
$$[c]_\rho = \{a\}$$

- Klase čine particiju $\mathcal{P} = \big\{\{b, c\}, \{a\}\big\}$ skupa $A = \{a, b, c\}$.
- Particija $\mathcal{P}$ ne definira relaciju ρ, nego relaciju ekvivalencije τ.

Relacija τ na skupu $A = \{a, b, c\}$

$\boxed{\tau}$



Klase elemenata

$$[a]_\tau = \{a\}$$
$$[b]_\tau = \{b, c\}$$
$$[c]_\tau = \{b, c\}$$

Neka je ρ refleksivna relacija na skupu A koja zadovoljava sljedeći uvjet:

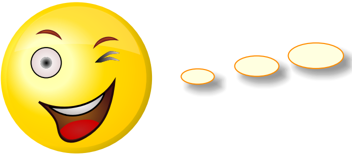
($\clubsuit$) Lijeve klase svaka dva elementa iz skupa A su međusobno jednake ili disjunktne.

Tada je ρ relacija ekvivalencije na skupu A.

Neka je ρ refleksivna relacija na skupu A koja zadovoljava sljedeći uvjet:

(♣) Lijeve klase svaka dva elementa iz skupa A su međusobno jednake ili disjunktne.

Tada je ρ relacija ekvivalencije na skupu A.



Domaća zadaća. Dokažite navedenu simpatičnu tvrdnju.

četvrti zadatak

Zadatak 4

Na skupu $\mathbb{Z}$ definirana je relacija $\sim$ s

$$a \sim b \overset{\text{def}}{\iff} a^2 - b^2 \text{ je djeljiv s } 3.$$

a) *Dokažite da je $\sim$ relacija ekvivalencije na skupu $\mathbb{Z}$.*

b) *Odredite klasu elementa 0 i klasu elementa 1.*

c) *Odredite kvocijentni skup $\mathbb{Z}/\sim$.*

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad (\forall a \in \mathbb{Z})\,(a \sim a)$

$$a \sim a$$

$$a \sim b \overset{\text{def}}{\Longleftrightarrow} 3 \mid a^2 - b^2$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad (\forall a \in \mathbb{Z})\,(a \sim a)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

$$\boxed{a \sim b \overset{\text{def}}{\Longleftrightarrow} 3 \mid a^2 - b^2}$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad (\forall a \in \mathbb{Z})\,(a \sim a)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad (\forall a, b \in \mathbb{Z})\,(a \sim b \;\Rightarrow\; b \sim a)$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad \big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad \big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 3 \mid a^2 - b^2$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad \big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 3 \mid a^2 - b^2 \;\Rightarrow\; \exists k \in \mathbb{Z},\; a^2 - b^2 = 3k$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad (\forall a \in \mathbb{Z})(a \sim a)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad (\forall a, b \in \mathbb{Z})(a \sim b \;\Rightarrow\; b \sim a)$

$$a \sim b \;\Rightarrow\; 3 \mid a^2 - b^2 \;\Rightarrow\; \exists k \in \mathbb{Z},\ a^2 - b^2 = 3k \;\Rightarrow$$

$$\Rightarrow\; b^2 - a^2 = 3 \cdot (-k)$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad \big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 3 \mid a^2 - b^2 \;\Rightarrow\; \exists k \in \mathbb{Z},\; a^2 - b^2 = 3k \;\Rightarrow$$

$$\Rightarrow\; b^2 - a^2 = 3 \cdot \underbrace{(-k)}_{\in \mathbb{Z}} \;\Rightarrow\; 3 \mid b^2 - a^2$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Rješenje

a) Treba provjeriti da je $\sim$ refleksivna, simetrična i tranzitivna relacija.

Refleksivnost $\quad \big(\forall a \in \mathbb{Z}\big)\big(a \sim a\big)$

$$a \sim a \;\Leftrightarrow\; 3 \mid a^2 - a^2 \;\Leftrightarrow\; 3 \mid 0$$

Simetričnost $\quad \big(\forall a, b \in \mathbb{Z}\big)\big(a \sim b \;\Rightarrow\; b \sim a\big)$

$$a \sim b \;\Rightarrow\; 3 \mid a^2 - b^2 \;\Rightarrow\; \exists k \in \mathbb{Z},\; a^2 - b^2 = 3k \;\Rightarrow$$

$$\Rightarrow\; b^2 - a^2 = 3 \cdot \underbrace{(-k)}_{\in \mathbb{Z}} \;\Rightarrow\; 3 \mid b^2 - a^2 \;\Rightarrow\; b \sim a$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c)$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\; \wedge$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \ \Rightarrow \ a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \ \Rightarrow \ 3 \mid a^2 - b^2 \ \wedge \ 3 \mid b^2 - c^2$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\wedge\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\;\wedge\;\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},\;\; a^2 - b^2 = 3k_1,$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\wedge\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},\; a^2 - b^2 = 3k_1,\; b^2 - c^2 = 3k_2$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\wedge\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 3k_1,\; b^2 - c^2 = 3k_2}_{+} \;\Rightarrow$$

$$\Rightarrow\; \big(a^2 - b^2\big) + \big(b^2 - c^2\big) = 3k_1 + 3k_2$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{Z}\big)\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\wedge\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 3k_1,\; b^2 - c^2 = 3k_2}_{+} \;\Rightarrow$$

$$\Rightarrow\; \big(a^2 - b^2\big) + \big(b^2 - c^2\big) = 3k_1 + 3k_2 \;\Rightarrow$$

$$\Rightarrow\; a^2 - c^2 = 3\big(k_1 + k_2\big)$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\wedge\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 3k_1,\;\; b^2 - c^2 = 3k_2}_{+} \;\Rightarrow$$

$$\Rightarrow\; \left(a^2 - b^2\right) + \left(b^2 - c^2\right) = 3k_1 + 3k_2 \;\Rightarrow$$

$$\Rightarrow\; a^2 - c^2 = 3\big(\underbrace{k_1 + k_2}_{\in \mathbb{Z}}\big) \;\Rightarrow\; 3 \mid a^2 - c^2$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{Z})\big((a \sim b) \wedge (b \sim c) \;\Rightarrow\; a \sim c\big)$

$$(a \sim b) \wedge (b \sim c) \;\Rightarrow\; 3 \mid a^2 - b^2 \;\wedge\; 3 \mid b^2 - c^2 \;\Rightarrow$$

$$\Rightarrow\; \exists k_1, k_2 \in \mathbb{Z},\; \underbrace{a^2 - b^2 = 3k_1,\;\; b^2 - c^2 = 3k_2}_{+} \;\Rightarrow$$

$$\Rightarrow\; \left(a^2 - b^2\right) + \left(b^2 - c^2\right) = 3k_1 + 3k_2 \;\Rightarrow$$

$$\Rightarrow\; a^2 - c^2 = 3\big(\underbrace{k_1 + k_2}_{\in \mathbb{Z}}\big) \;\Rightarrow\; 3 \mid a^2 - c^2 \;\Rightarrow\; a \sim c$$

$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$ $\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$

b)

$$[0]_\sim =$$

$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$ $\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$

b) $$[0]_\sim = \left\{x \in \mathbb{Z} : x \sim 0\right\}$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

b)

$$[0]_\sim = \left\{x \in \mathbb{Z} : x \sim 0\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\right\}$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

b)

$$[0]_\sim = \left\{x \in \mathbb{Z} : x \sim 0\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2\right\}$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

b)

$$[0]_\sim = \{x \in \mathbb{Z} : x \sim 0\} = \{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\} =$$

$$= \{x \in \mathbb{Z} : 3 \mid x^2\}$$

Cijeli broj x je djeljiv s 3 ako i samo ako je x^2 djeljiv s 3.

$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$ $\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$

b)

$$[0]_\sim = \left\{x \in \mathbb{Z} : x \sim 0\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\right\} =$$
$$= \left\{x \in \mathbb{Z} : 3 \mid x^2\right\}$$

Cijeli broj x je djeljiv s 3 ako i samo ako je x^2 djeljiv s 3.

- Tvrdnju smo dokazali ranije za prirodne brojeve.

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}} \qquad \boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

b)

$$[0]_\sim = \left\{x \in \mathbb{Z} : x \sim 0\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2\right\}$$

Cijeli broj x je djeljiv s 3 ako i samo ako je x^2 djeljiv s 3.

- Tvrdnju smo dokazali ranije za prirodne brojeve.
- Svi ponuđeni dokazi od ranije potpuno analogno prolaze i u slučaju cijelih brojeva.

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}} \qquad \boxed{a \sim b \overset{\text{def}}{\Longleftrightarrow} 3 \mid a^2 - b^2}$$

b)

$$[0]_\sim = \{x \in \mathbb{Z} : x \sim 0\} = \{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\} =$$

$$= \{x \in \mathbb{Z} : 3 \mid x^2\} = \{x \in \mathbb{Z} : 3 \mid x\}$$

Cijeli broj x je djeljiv s 3 ako i samo ako je x^2 djeljiv s 3.

- Tvrdnju smo dokazali ranije za prirodne brojeve.
- Svi ponuđeni dokazi od ranije potpuno analogno prolaze i u slučaju cijelih brojeva.

$$[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

b)

$$[0]_\sim = \left\{x \in \mathbb{Z} : x \sim 0\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 0^2\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2\right\} = \left\{x \in \mathbb{Z} : 3 \mid x\right\} = 3\mathbb{Z}$$

Cijeli broj x je djeljiv s 3 ako i samo ako je x^2 djeljiv s 3.

- Tvrdnju smo dokazali ranije za prirodne brojeve.
- Svi ponuđeni dokazi od ranije potpuno analogno prolaze i u slučaju cijelih brojeva.

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

$$[1]_\sim =$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$$

$$\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\}$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}} \quad \boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\right\}$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}} \quad \boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1\right\}$$

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}} \qquad \boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

$$\begin{aligned}
[1]_\sim &= \left\{x \in \mathbb{Z} : x \sim 1\right\} = \\
&= \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1\right\} = \\
&= \left\{x \in \mathbb{Z} : \exists k \in \mathbb{Z},\ x^2 = 3k + 1\right\}
\end{aligned}$$

$$[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}$$

$$a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2$$

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : \exists k \in \mathbb{Z},\ x^2 = 3k + 1\right\}$$

Cijeli broj x nije djeljiv s 3 ako i samo ako x^2 pri dijeljenju s 3 daje ostatak 1.

$$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}} \qquad \boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$$

$$\begin{aligned} [1]_\sim &= \{x \in \mathbb{Z} : x \sim 1\} = \\ &= \{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\} = \{x \in \mathbb{Z} : 3 \mid x^2 - 1\} = \\ &= \{x \in \mathbb{Z} : \exists k \in \mathbb{Z},\ x^2 = 3k + 1\} = \{x \in \mathbb{Z} : 3 \nmid x\} \end{aligned}$$

Cijeli broj x nije djeljiv s 3 ako i samo ako x^2 pri dijeljenju s 3 daje ostatak 1.

$$[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}$$

$$a \sim b \overset{\text{def}}{\Longleftrightarrow} 3 \mid a^2 - b^2$$

$$\begin{aligned}
[1]_\sim &= \left\{x \in \mathbb{Z} : x \sim 1\right\} = \\
&= \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1\right\} = \\
&= \left\{x \in \mathbb{Z} : \exists k \in \mathbb{Z},\ x^2 = 3k + 1\right\} = \left\{x \in \mathbb{Z} : 3 \nmid x\right\} = \\
&= (3\mathbb{Z} + 1) \cup (3\mathbb{Z} + 2)
\end{aligned}$$

Cijeli broj x nije djeljiv s 3 ako i samo ako x^2 pri dijeljenju s 3 daje ostatak 1.

$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$ $\boxed{a \sim b \overset{\text{def}}{\iff} 3 \mid a^2 - b^2}$

$$[1]_\sim = \left\{x \in \mathbb{Z} : x \sim 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\right\} = \left\{x \in \mathbb{Z} : 3 \mid x^2 - 1\right\} =$$

$$= \left\{x \in \mathbb{Z} : \exists k \in \mathbb{Z},\ x^2 = 3k + 1\right\} = \left\{x \in \mathbb{Z} : 3 \nmid x\right\} =$$

$$= (3\mathbb{Z} + 1) \cup (3\mathbb{Z} + 2) = \mathbb{Z} \setminus 3\mathbb{Z}$$

Cijeli broj x nije djeljiv s 3 ako i samo ako x^2 pri dijeljenju s 3 daje ostatak 1.

$\boxed{[a]_\sim = \{x \in \mathbb{Z} : x \sim a\}}$ $\boxed{a \sim b \overset{\text{def}}{\Longleftrightarrow} 3 \mid a^2 - b^2}$

$$[1]_\sim = \{x \in \mathbb{Z} : x \sim 1\} =$$

$$= \{x \in \mathbb{Z} : 3 \mid x^2 - 1^2\} = \{x \in \mathbb{Z} : 3 \mid x^2 - 1\} =$$

$$= \{x \in \mathbb{Z} : \exists k \in \mathbb{Z},\ x^2 = 3k + 1\} = \{x \in \mathbb{Z} : 3 \nmid x\} =$$

$$= (3\mathbb{Z} + 1) \cup (3\mathbb{Z} + 2) = \mathbb{Z} \setminus 3\mathbb{Z}$$

Cijeli broj x nije djeljiv s 3 ako i samo ako x^2 pri dijeljenju s 3 daje ostatak 1.

- Dokažimo još navedenu tvrdnju koju smo ovdje koristili.

$$\boxed{x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$$

$\boxed{\Rightarrow}$ $\left(3 \nmid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}\right)$

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3.

$x \in \mathbb{Z},\, 3 \nmid x \;\Leftrightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Rightarrow}$ $(3 \nmid x \;\Rightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$\boxed{x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1$)$

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

  $x = 3k + 1$

$x \in \mathbb{Z},\ 3 \nmid x \;\Leftrightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Rightarrow}$ $(3 \nmid x \;\Rightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \;\Rightarrow\; x^2 = 9k^2 + 6k + 1$$

$x \in \mathbb{Z},\ 3 \nmid x \;\Leftrightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Rightarrow}$ $(3 \nmid x \;\Rightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \;\Rightarrow\; x^2 = 9k^2 + 6k + 1 = 3 \cdot (3k^2 + 2k) + 1$$

$$\boxed{x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$$

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1$)$

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \ \Rightarrow\ x^2 = 9k^2 + 6k + 1 = 3 \cdot (\underbrace{3k^2 + 2k}_{\in \mathbb{Z}}) + 1 \ \Rightarrow$$

$\Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow \ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow \ x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \ \Rightarrow \ x^2 = 9k^2 + 6k + 1 = 3 \cdot (\underbrace{3k^2 + 2k}_{\in \mathbb{Z}}) + 1 \ \Rightarrow$$

$\Rightarrow \ x^2$ pri dijeljenju s 3 daje ostatak 1

- $x = 3k + 2$ za neki $k \in \mathbb{Z}$

$\boxed{x \in \mathbb{Z},\ 3 \nmid x \iff x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$

$\boxed{\Rightarrow}$ ($3 \nmid x \Rightarrow x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \;\Rightarrow\; x^2 = 9k^2 + 6k + 1 = 3 \cdot (\underbrace{3k^2 + 2k}_{\in \mathbb{Z}}) + 1 \;\Rightarrow$$

$\Rightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1

- $x = 3k + 2$ za neki $k \in \mathbb{Z}$

$$x = 3k + 2$$

$\boxed{x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1$)$

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

  $$x = 3k + 1 \ \Rightarrow\ x^2 = 9k^2 + 6k + 1 = 3 \cdot (\underbrace{3k^2 + 2k}_{\in \mathbb{Z}}) + 1 \ \Rightarrow$$

  $\Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

- $x = 3k + 2$ za neki $k \in \mathbb{Z}$

  $$x = 3k + 2 \ \Rightarrow\ x^2 = 9k^2 + 12k + 4$$

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Rightarrow}$ ($3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1)

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \ \Rightarrow\ x^2 = 9k^2 + 6k + 1 = 3 \cdot (\underbrace{3k^2 + 2k}_{\in \mathbb{Z}}) + 1 \ \Rightarrow$$

$\Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

- $x = 3k + 2$ za neki $k \in \mathbb{Z}$

$$x = 3k + 2 \ \Rightarrow\ x^2 = 9k^2 + 12k + \overset{3+1}{4} = 3 \cdot (3k^2 + 4k + 1) + 1$$

$$\boxed{x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2 \text{ pri dijeljenju s 3 daje ostatak 1}}$$

$\boxed{\Rightarrow}$ $(3 \nmid x \ \Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1$)$

Pretpostavimo da $x \in \mathbb{Z}$ nije djeljiv s 3. Razlikujemo dva slučaja.

- $x = 3k + 1$ za neki $k \in \mathbb{Z}$

$$x = 3k + 1 \ \Rightarrow\ x^2 = 9k^2 + 6k + 1 = 3 \cdot (\underbrace{3k^2 + 2k}_{\in \mathbb{Z}}) + 1 \ \Rightarrow$$

$\Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

- $x = 3k + 2$ za neki $k \in \mathbb{Z}$

$$x = 3k + 2 \ \Rightarrow\ x^2 = 9k^2 + 12k + \overset{3+1}{4} = 3 \cdot (\underbrace{3k^2 + 4k + 1}_{\in \mathbb{Z}}) + 1 \ \Rightarrow$$

$\Rightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$x \in \mathbb{Z},\ 3 \nmid x \iff x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\ \Rightarrow\ 3 \nmid x$)

Dokazujemo kontrapoziciju:

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\ \Rightarrow\ 3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3.

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3. Tada je $x = 3k$ za neki $k \in \mathbb{Z}$.

$x \in \mathbb{Z},\ 3 \nmid x \;\Leftrightarrow\; x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow\; 3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \;\Rightarrow\; x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3. Tada je $x = 3k$ za neki $k \in \mathbb{Z}$.

$$x = 3k$$

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \;\Rightarrow\; x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3. Tada je $x = 3k$ za neki $k \in \mathbb{Z}$.

$$x = 3k \;\Rightarrow\; x^2 = 9k^2$$

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3. Tada je $x = 3k$ za neki $k \in \mathbb{Z}$.

$$x = 3k \ \Rightarrow\ x^2 = 9k^2 \ \Rightarrow\ x^2 = 3 \cdot \left(3k^2\right)$$

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3. Tada je $x = 3k$ za neki $k \in \mathbb{Z}$.

$$x = 3k \ \Rightarrow\ x^2 = 9k^2 \ \Rightarrow\ x^2 = 3 \cdot \underbrace{\left(3k^2\right)}_{\in \mathbb{Z}}$$

Dakle, x^2 je djeljiv s 3.

$x \in \mathbb{Z},\ 3 \nmid x \ \Leftrightarrow\ x^2$ pri dijeljenju s 3 daje ostatak 1

$\boxed{\Leftarrow}$ (x^2 pri dijeljenju s 3 daje ostatak 1 $\Rightarrow$ $3 \nmid x$)

Dokazujemo kontrapoziciju:

$$3 \mid x \ \Rightarrow\ x^2 \text{ pri dijeljenju s 3 ne daje ostatak 1}$$

Pretpostavimo da je $x \in \mathbb{Z}$ djeljiv s 3. Tada je $x = 3k$ za neki $k \in \mathbb{Z}$.

$$x = 3k \ \Rightarrow\ x^2 = 9k^2 \ \Rightarrow\ x^2 = 3 \cdot \underbrace{\left(3k^2\right)}_{\in \mathbb{Z}}$$

Dakle, x^2 je djeljiv s 3. Stoga x^2 pri dijeljenju s 3 ne daje ostatak 1.

c) Odredili smo ranije klase elemenata 0 i 1.

$$[0]_\sim = 3\mathbb{Z}$$

$$[1]_\sim = \mathbb{Z} \setminus 3\mathbb{Z}$$

c) Odredili smo ranije klase elemenata 0 i 1.

$$[0]_\sim = 3\mathbb{Z}$$

$$[1]_\sim = \mathbb{Z} \setminus 3\mathbb{Z}$$

Kako je $\sim$ relacija ekvivalencije na skupu $\mathbb{Z}$ i

$$[0]_\sim \cup [1]_\sim = \mathbb{Z},$$

c) Odredili smo ranije klase elemenata 0 i 1.

$$[0]_\sim = 3\mathbb{Z}$$

$$[1]_\sim = \mathbb{Z} \setminus 3\mathbb{Z}$$

Kako je $\sim$ relacija ekvivalencije na skupu $\mathbb{Z}$ i

$$[0]_\sim \cup [1]_\sim = \mathbb{Z},$$

zaključujemo da se kvocijentni skup sastoji od samo dvije klase, tj.

c) Odredili smo ranije klase elemenata 0 i 1.

$$[0]_\sim = 3\mathbb{Z}$$

$$[1]_\sim = \mathbb{Z} \setminus 3\mathbb{Z}$$

Kako je $\sim$ relacija ekvivalencije na skupu $\mathbb{Z}$ i

$$[0]_\sim \cup [1]_\sim = \mathbb{Z},$$

zaključujemo da se kvocijentni skup sastoji od samo dvije klase, tj.

$$\mathbb{Z}/\sim = \big\{3\mathbb{Z}, \, \mathbb{Z} \setminus 3\mathbb{Z}\big\}.$$

Parcijalni uređaj i Hasseovi dijagrami

Algoritam za ručno crtanje Hasseovih dijagrama

1) Pronađi matricu incidencije zadanog parcijalnog uređaja.
2) Dvostruko zaokruži sve jedinice na glavnoj dijagonali.
3) Ponavljaj redom sljedeće korake tako dugo dok sve jedinice u matrici incidencije ne budu dvostruko zaokružene.

    3.1 Traži stupce u matrici incidencije čije su sve jedinice jednostruko ili dvostruko zaokružene i ubaci pripadne elemente na sljedeći nivo u Hasseovom dijagramu.

    3.2 Jednostruko zaokruži sve nezaokružene jedinice u svim retcima od elemenata koji su upravo ubačeni u Hasseov dijagram.

    3.3 Provjeri sve jednostruko zaokružene jedinice koje se nalaze u retcima i stupcima do sada ubačenih elemenata u Hasseov dijagram i po potrebi dodaj odgovarajuće bridove u Hasseov dijagram, a nakon provjere sve takve jedinice dvostruko zaokruži.

peti zadatak

Zadatak 5

Na skupu $T = \{2, 3, 4, 8, 9, 16, 32, 64, 81\}$ *definirana je relacija parcijalnog uređaja* $\preccurlyeq$ *na sljedeći način:*

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}.$$

a) *Odredite matricu incidencije zadanog parcijalnog uređaja.*

b) *Odredite najveći, najmanji, minimalne i maksimalne elemente u parcijalno uređenom skupu* T.

c) *Nacrtajte Hasseov dijagram parcijalno uređenog skupa* T.

d) *Odredite supremum i infimum podskupova* $\{4, 8\}$ *i* $\{4, 8, 32\}$.

e) *Je li parcijalno uređen skup* T *mreža? Objasnite.*

Rješenje

$a \setminus b$	2	3	4	8	9	16	32	64	81
2									
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1								
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0							
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1						
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1					
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0				
3									
4									
8									
9									
16									
32									
64									
81									

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1			
3									
4									
8									
9									
16									
32									
64									
81									

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1		
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \ \stackrel{\text{def}}{\Longleftrightarrow}\ b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \ \stackrel{\text{def}}{\Longleftrightarrow}\ b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3									
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0								
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1							
4									
8									
9									
16									
32									
64									
81									

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0						
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0					
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1				
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0			
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0		
4									
8									
9									
16									
32									
64									
81									

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4									
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0								
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0							
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \ \overset{\text{def}}{\Longleftrightarrow}\ b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1						
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0					
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \stackrel{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0				
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1			
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \stackrel{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0		
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \ \stackrel{\text{def}}{\Longleftrightarrow}\ b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8									
9									
16									
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9									
16									
32									
64									
81									

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16									
32									
64									
81									

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32									
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64									
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81									

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	$\boxed{2}$	$\boxed{3}$	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

$$a \preccurlyeq b \stackrel{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	$\boxed{2}$	$\boxed{3}$	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

2 3

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	$\boxed{2}$	$\boxed{3}$	4	8	9	16	32	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

2 3

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

2 3

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	$\boxed{2}$	$\boxed{3}$	4	8	9	16	32	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

2 3

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

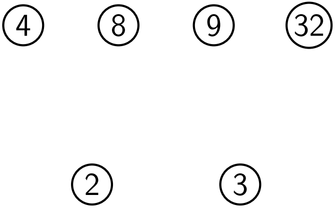
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

2 3

$$\boxed{a \preccurlyeq b \quad \overset{\text{def}}{\Longleftrightarrow} \quad b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

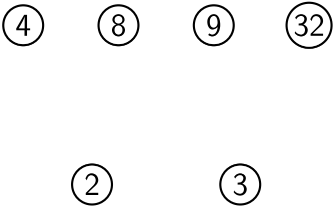
4 8 9 32

2 3

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

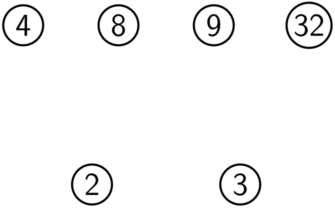
a \ b	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

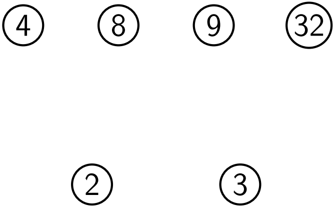
a \ b	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

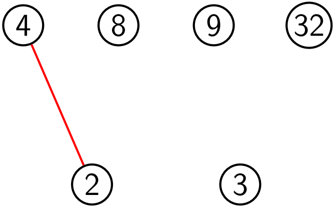
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

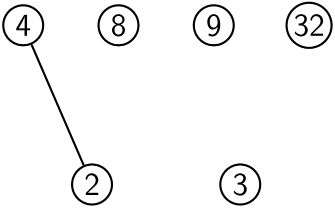
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

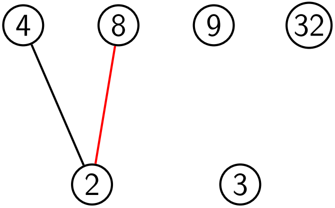
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

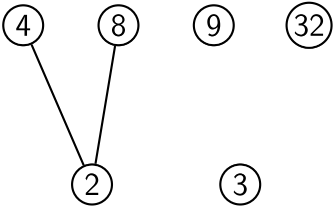
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

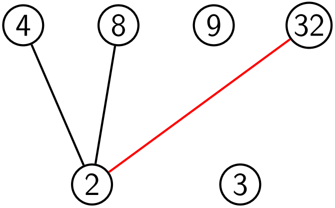
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$\boxed{a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

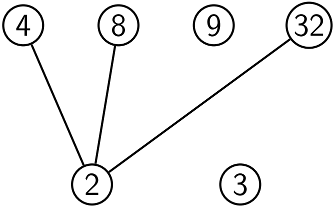
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

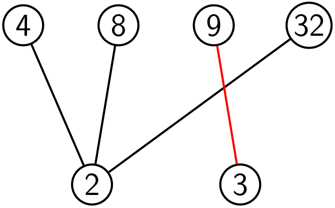
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

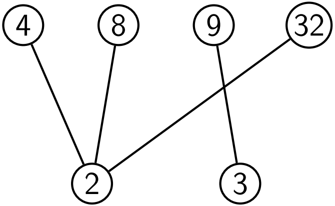
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

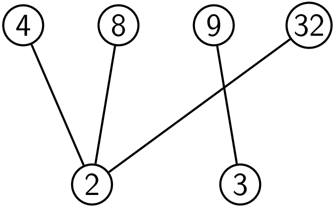
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	16	$\boxed{32}$	64	81
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

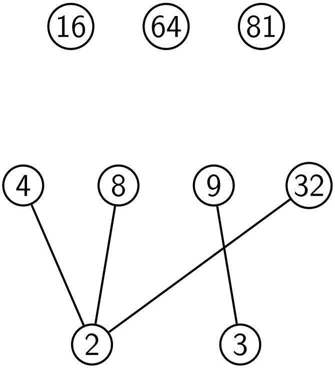
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	$\boxed{16}$	$\boxed{32}$	$\boxed{64}$	$\boxed{81}$
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

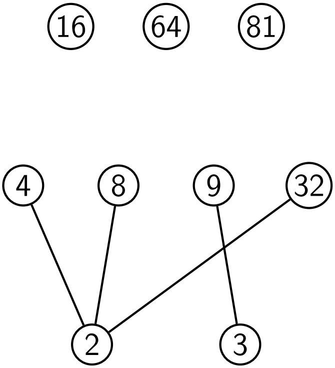
$a \backslash b$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{8}$	$\boxed{9}$	$\boxed{16}$	$\boxed{32}$	$\boxed{64}$	$\boxed{81}$
$\boxed{2}$	1	0	1	1	0	1	1	1	0
$\boxed{3}$	0	1	0	0	1	0	0	0	1
$\boxed{4}$	0	0	1	0	0	1	0	1	0
$\boxed{8}$	0	0	0	1	0	0	0	1	0
$\boxed{9}$	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
$\boxed{32}$	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

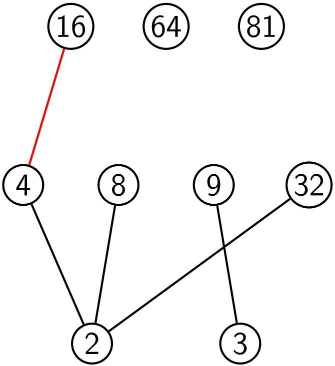
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

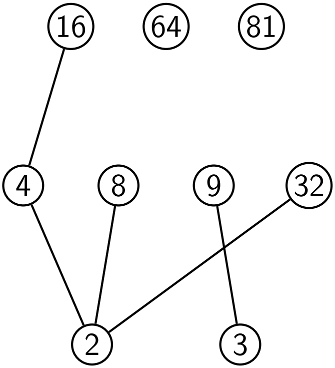
a \ b	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

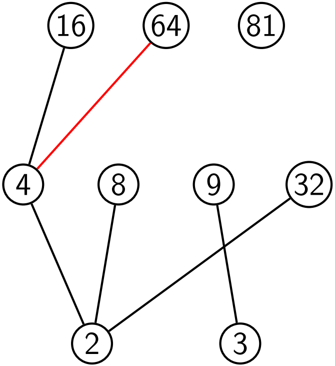
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \stackrel{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

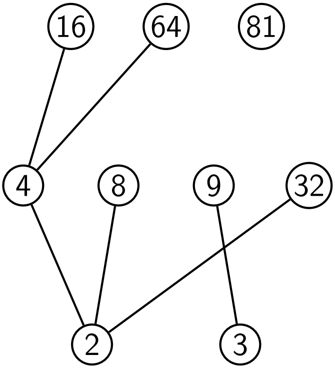
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

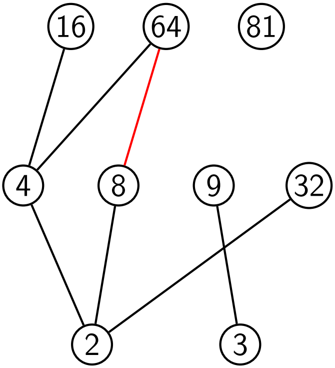
a \ b	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \stackrel{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

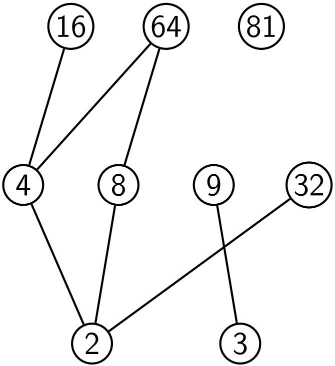
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \;\overset{\text{def}}{\Longleftrightarrow}\; b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

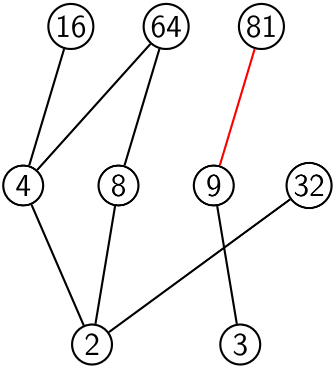
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

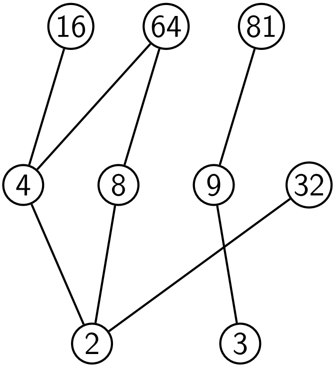
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

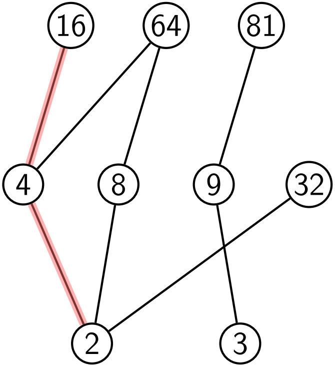
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

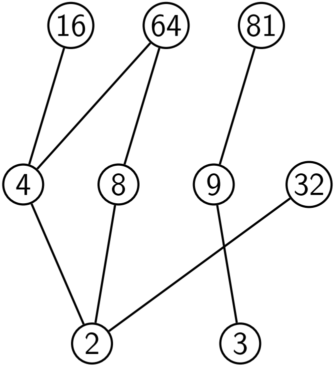
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \stackrel{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

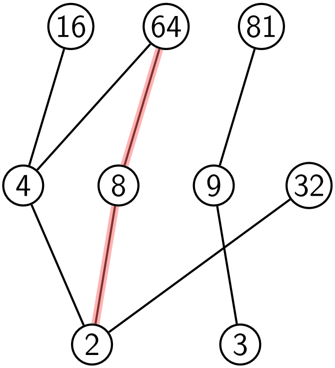
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

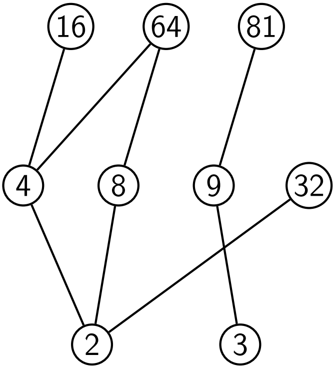
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

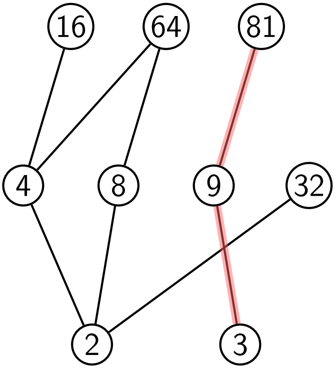
a \ b	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \stackrel{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

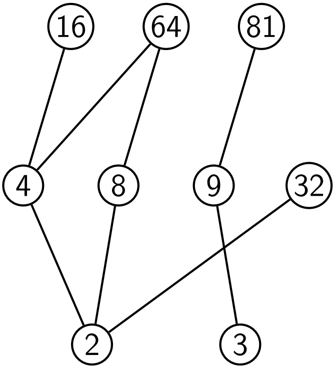
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1



$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

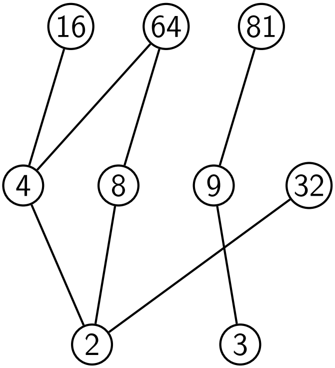


$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

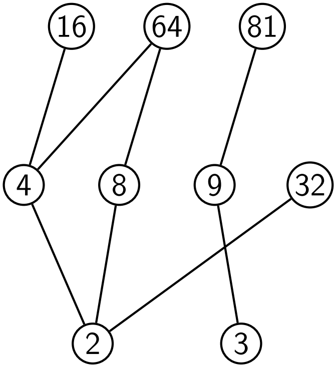
Rješenje

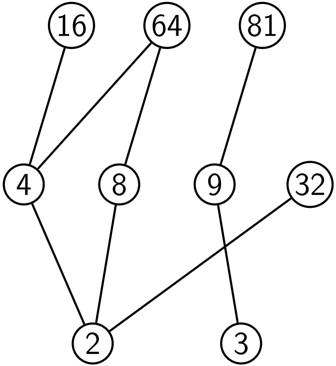
$a \backslash b$	2	3	4	8	9	16	32	64	81
2	1	0	1	1	0	1	1	1	0
3	0	1	0	0	1	0	0	0	1
4	0	0	1	0	0	1	0	1	0
8	0	0	0	1	0	0	0	1	0
9	0	0	0	0	1	0	0	0	1
16	0	0	0	0	0	1	0	0	0
32	0	0	0	0	0	0	1	0	0
64	0	0	0	0	0	0	0	1	0
81	0	0	0	0	0	0	0	0	1

Hasseov dijagram

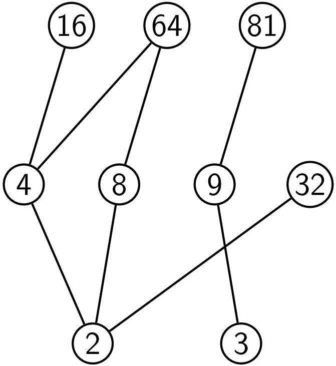


$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

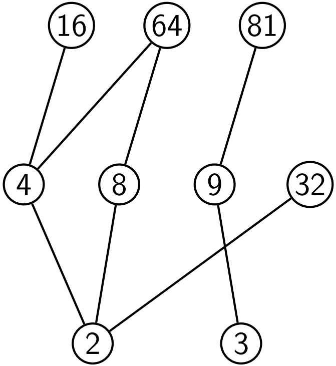

16
64
81
4
8
9
32
2
3



Najmanji element

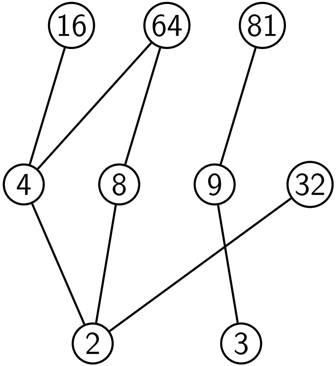


$\boxed{\text{Najmanji element}}$ ne postoji



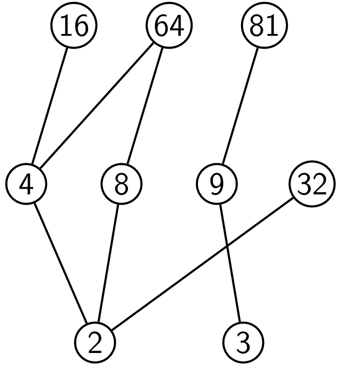
$\boxed{\text{Najmanji element}}$ ne postoji

$\boxed{\text{Minimalni elementi}}$



$\boxed{\text{Najmanji element}}$ ne postoji

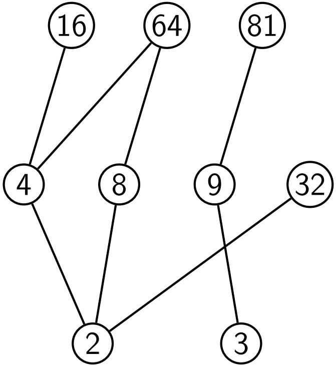
$\boxed{\text{Minimalni elementi}}$ $2, 3$



Najmanji element ne postoji

Minimalni elementi $2, 3$

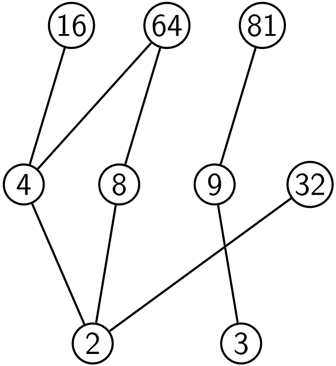
Najveći element



$\boxed{\text{Najmanji element}}$ ne postoji

$\boxed{\text{Minimalni elementi}}$ $2, 3$

$\boxed{\text{Najveći element}}$ ne postoji

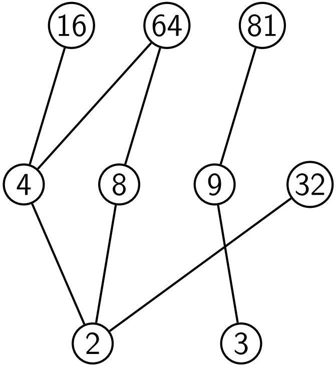


$\boxed{\text{Najmanji element}}$ ne postoji

$\boxed{\text{Minimalni elementi}}$ $2, 3$

$\boxed{\text{Najveći element}}$ ne postoji

$\boxed{\text{Maksimalni elementi}}$

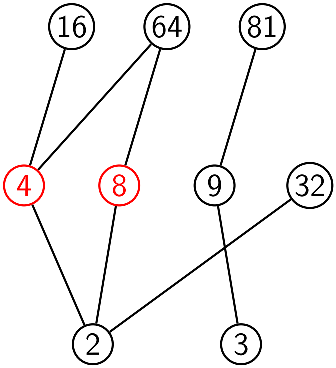


Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$



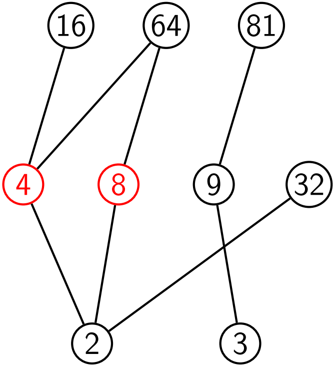
Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$



Najmanji element ne postoji

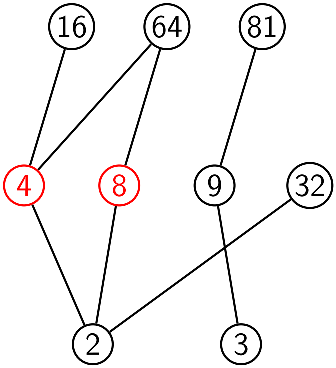
Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe



Najmanji element ne postoji

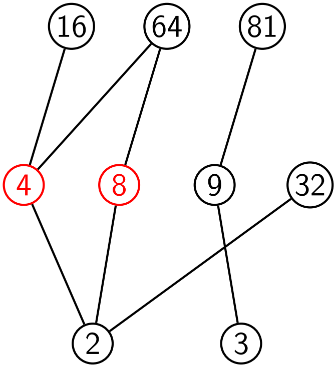
Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2



Najmanji element ne postoji

Minimalni elementi $2, 3$

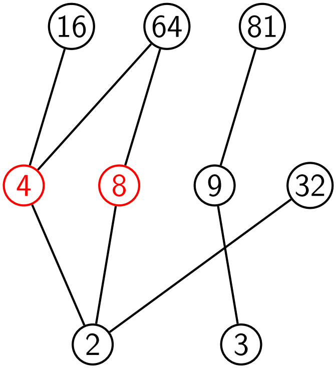
Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

  Gornje međe



Najmanji element ne postoji

Minimalni elementi $2, 3$

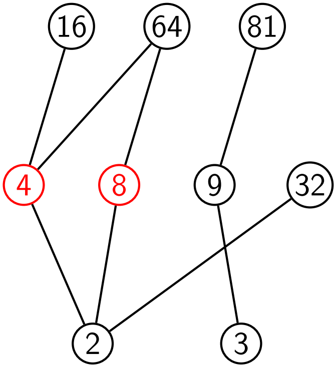
Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

  Gornje međe 64



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

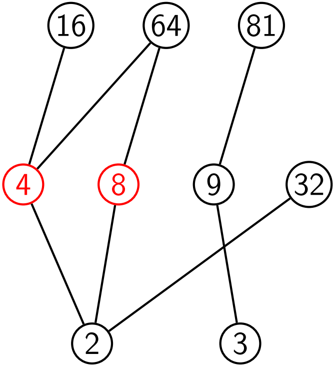
Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

  Gornje međe 64

  $\inf\{4, 8\} = 2$



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

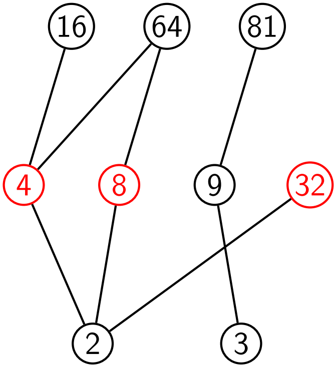
- Podskup $\{4, 8\}$

Donje međe 2

Gornje međe 64

$$\inf\{4, 8\} = 2$$

$$\sup\{4, 8\} = 64$$



$\boxed{\text{Najmanji element}}$ ne postoji

$\boxed{\text{Minimalni elementi}}$ $2, 3$

$\boxed{\text{Najveći element}}$ ne postoji

$\boxed{\text{Maksimalni elementi}}$ $16, 64, 81, 32$

- Podskup $\{4, 8\}$

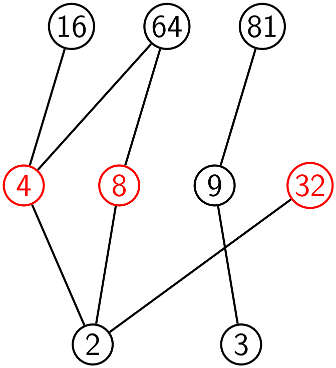
  $\boxed{\text{Donje međe}}$ 2

  $\boxed{\text{Gornje međe}}$ 64

  $\inf \{4, 8\} = 2$

  $\sup \{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

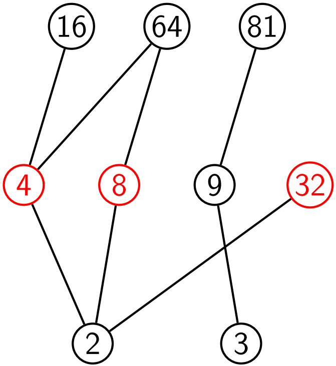
  Gornje međe 64

  $\inf\{4, 8\} = 2$

  $\sup\{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$

  Donje međe



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

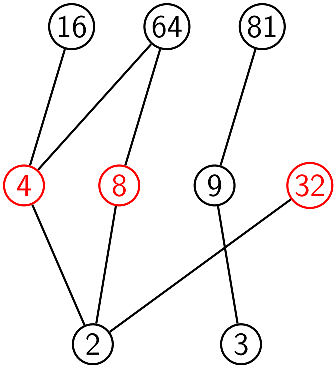
  Gornje međe 64

  $\inf\{4, 8\} = 2$

  $\sup\{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$

  Donje međe 2



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

  Gornje međe 64

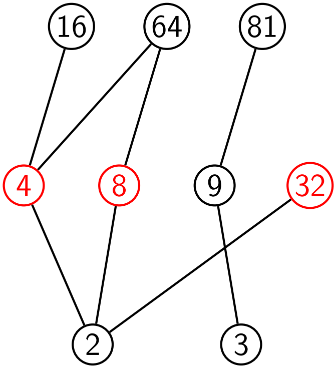
  $\inf\{4, 8\} = 2$

  $\sup\{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$

  Donje međe 2

  Gornje međe



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

  Gornje međe 64

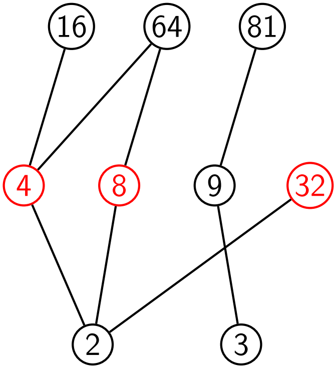
  $\inf\{4, 8\} = 2$

  $\sup\{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$

  Donje međe 2

  Gornje međe ne postoje



$\boxed{\text{Najmanji element}}$ ne postoji

$\boxed{\text{Minimalni elementi}}$ $2, 3$

$\boxed{\text{Najveći element}}$ ne postoji

$\boxed{\text{Maksimalni elementi}}$ $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  $\boxed{\text{Donje međe}}$ 2

  $\boxed{\text{Gornje međe}}$ 64

  $\inf\{4, 8\} = 2$

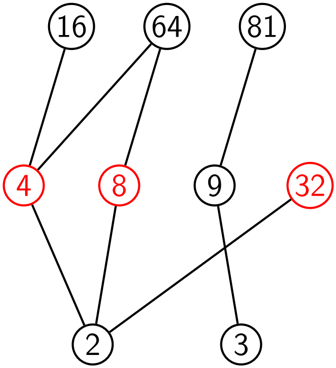
  $\sup\{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$

  $\boxed{\text{Donje međe}}$ 2

  $\boxed{\text{Gornje međe}}$ ne postoje

  $\inf\{4, 8, 32\} = 2$



Najmanji element ne postoji

Minimalni elementi $2, 3$

Najveći element ne postoji

Maksimalni elementi $16, 64, 81, 32$

- Podskup $\{4, 8\}$

  Donje međe 2

  Gornje međe 64

  $\inf \{4, 8\} = 2$

  $\sup \{4, 8\} = 64$

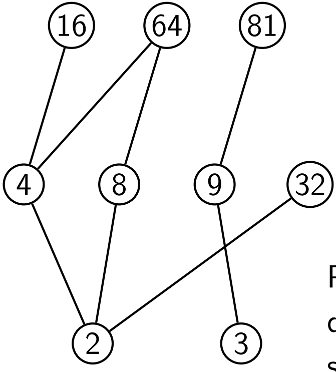
- Podskup $\{4, 8, 32\}$

  Donje međe 2

  Gornje međe ne postoje

  $\inf \{4, 8, 32\} = 2$

  $\sup \{4, 8, 32\}$ ne postoji



$\boxed{\text{Najmanji element}}$ ne postoji

$\boxed{\text{Minimalni elementi}}$ $2, 3$

$\boxed{\text{Najveći element}}$ ne postoji

$\boxed{\text{Maksimalni elementi}}$ $16, 64, 81, 32$

Parcijalno uređen skup T nije mreža jer npr. dvočlani podskup $\{2, 3\}$ nema infimum (niti supremum).

- Podskup $\{4, 8\}$

  $\boxed{\text{Donje međe}}$ 2

  $\boxed{\text{Gornje međe}}$ 64

  $\inf\{4, 8\} = 2$

  $\sup\{4, 8\} = 64$

- Podskup $\{4, 8, 32\}$

  $\boxed{\text{Donje međe}}$ 2

  $\boxed{\text{Gornje međe}}$ ne postoje

  $\inf\{4, 8, 32\} = 2$

  $\sup\{4, 8, 32\}$ ne postoji

šesti zadatak

Zadatak 6

Na skupu $B = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ *definirana je relacija parcijalnog uređaja* $\preccurlyeq$ *na sljedeći način:*

$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

a) *Odredite matricu incidencije zadanog parcijalnog uređaja.*

b) *Odredite najveći, najmanji, minimalne i maksimalne elemente u parcijalno uređenom skupu* B.

c) *Nacrtajte Hasseov dijagram parcijalno uređenog skupa* B.

d) *Odredite supremum, infimum, maksimum i minimum podskupa* $\{4, 5, 6\}$.

e) *Napišite nekoliko lanaca u parcijalno uređenom skupu* B.

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2									
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \;\stackrel{\text{def}}{\Longleftrightarrow}\; (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$$(2 \mid 2) \vee (2 \text{ je prost } \wedge (2 < 2)) \longrightarrow 1 \vee (1 \wedge 0) = 1 \vee 0 = 1$$

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1								
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$$(2 \mid 3) \vee (2 \text{ je prost } \wedge (2 < 3)) \longrightarrow 0 \vee (1 \wedge 1) = 0 \vee 1 = 1$$

x \ y	2	3	4	5	6	7	8	9	10
2	1	1							
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$$(2 \mid 4) \vee (2 \text{ je prost } \wedge (2 < 4)) \longrightarrow 1 \vee (1 \wedge 1) = 1 \vee 1 = 1$$

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1						
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1					
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1				
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1			
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1		
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \;\overset{\text{def}}{\iff}\; (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3									
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$$(3 \mid 2) \vee (3 \text{ je prost } \wedge (3 < 2)) \longrightarrow 0 \vee (1 \wedge 0) = 0 \vee 0 = 0$$

$x \setminus y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0								
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1							
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1						
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1					
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1				
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1			
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1		
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \ \stackrel{\text{def}}{\Longleftrightarrow}\ (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4									
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0								
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$$(4 \mid 3) \vee (4 \text{ je prost } \wedge (4 < 3)) \longrightarrow 0 \vee (0 \wedge 0) = 0 \vee 0 = 0$$

x \ y	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0							
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \ \stackrel{\text{def}}{\Longleftrightarrow}\ (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$(4 \mid 4) \vee (4 \text{ je prost } \wedge (4 < 4)) \longrightarrow 1 \vee (0 \wedge 0) = 1 \vee 0 = 1$

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1						
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0					
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0				
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0			
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1		
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \ \stackrel{\text{def}}{\Longleftrightarrow}\ (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5									
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6									
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\Longleftrightarrow} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7									
8									
9									
10									

$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8									
9									
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9									
10									

$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10									

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	$\boxed{2}$	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

$x \backslash y$	$\boxed{2}$	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

2

$$\boxed{x \preccurlyeq y \ \stackrel{\text{def}}{\Longleftrightarrow}\ (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	$\boxed{2}$	3	4	5	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

2

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

2

$$\boxed{x \preccurlyeq y \ \stackrel{\text{def}}{\Longleftrightarrow}\ (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	$\boxed{2}$	$\boxed{3}$	4	5	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

2

$$\boxed{x \preccurlyeq y \quad \overset{\text{def}}{\iff} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	$\boxed{2}$	$\boxed{3}$	4	5	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

3

2

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

$x \backslash y$	$\boxed{2}$	$\boxed{3}$	4	5	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

3

2

$$x \preccurlyeq y \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

$x \backslash y$	$\boxed{2}$	$\boxed{3}$	4	5	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

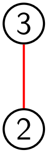
3

2

$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

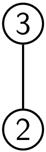
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

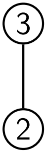
x \ y	$\boxed{2}$	$\boxed{3}$	4	5	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Riješenje

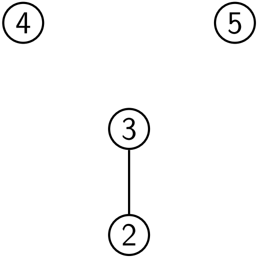
x \ y	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

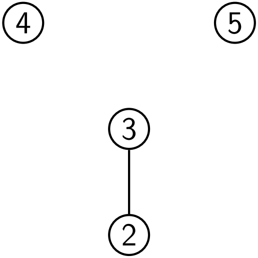
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

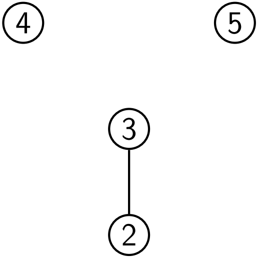
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

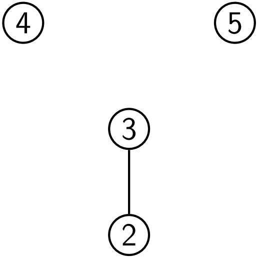
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

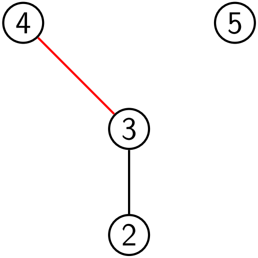
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))$$

Rješenje

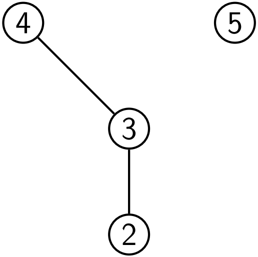
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

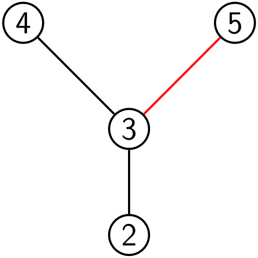
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

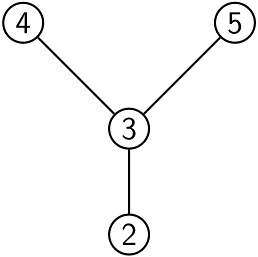
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

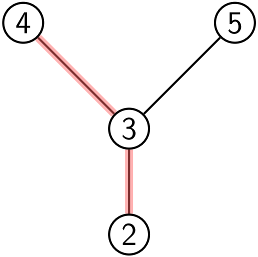
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \;\overset{\text{def}}{\iff}\; (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

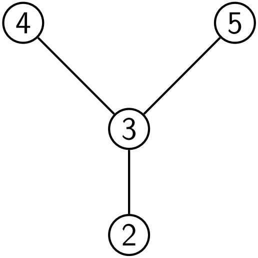
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

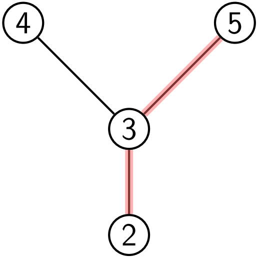
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

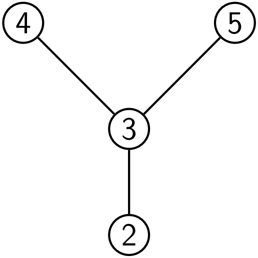
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

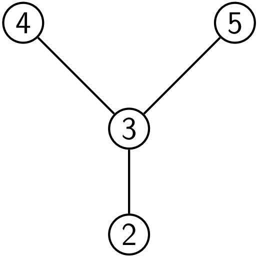
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	6	7	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \quad \overset{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

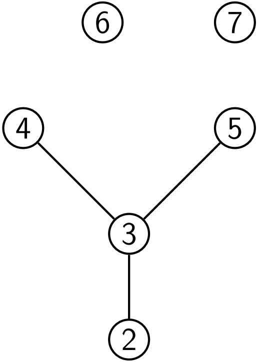
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

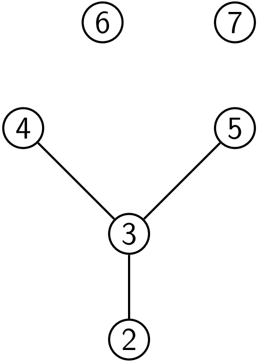
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

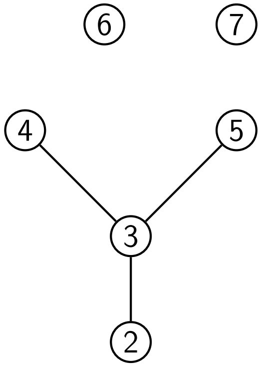
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

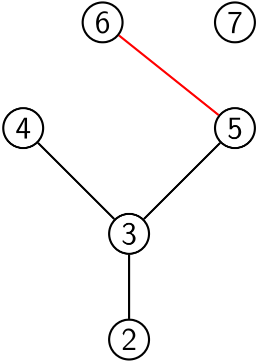
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

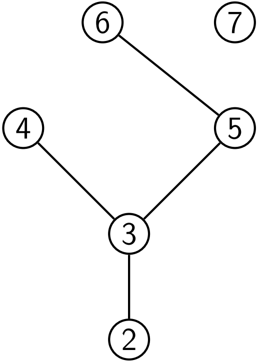
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

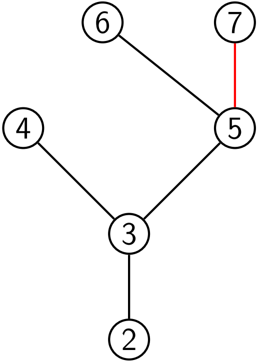
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

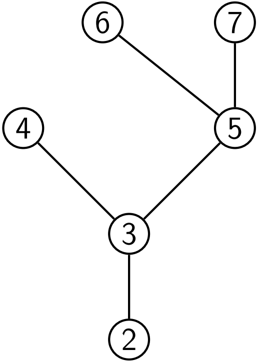
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

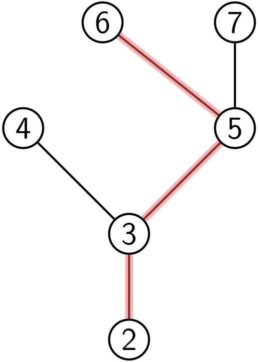
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

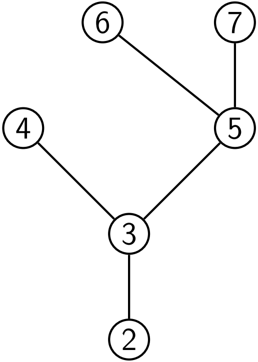
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

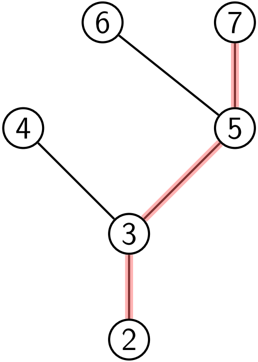
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

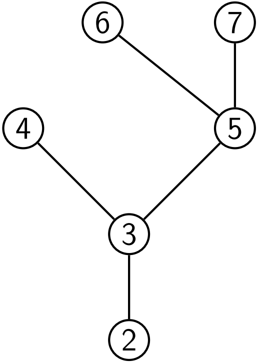
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \stackrel{\text{def}}{\Longleftrightarrow} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

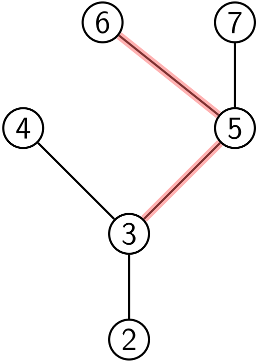
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \quad \stackrel{\text{def}}{\iff} \quad (x \mid y) \lor (x \text{ je prost } \land (x < y))$$

Rješenje

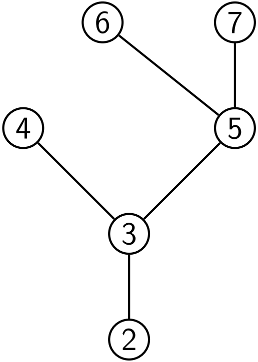
x \ y	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

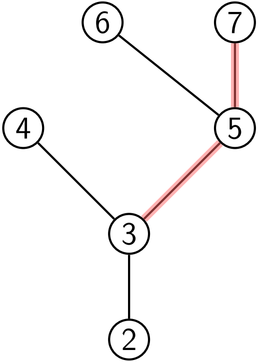
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \;\stackrel{\text{def}}{\Longleftrightarrow}\; (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

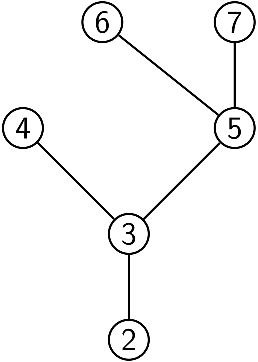
$x \backslash y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

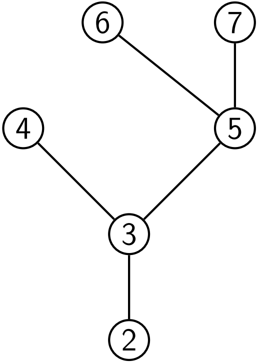
$x \setminus y$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	8	9	10
$\boxed{2}$	1	1	1	1	1	1	1	1	1
$\boxed{3}$	0	1	1	1	1	1	1	1	1
$\boxed{4}$	0	0	1	0	0	0	1	0	0
$\boxed{5}$	0	0	0	1	1	1	1	1	1
$\boxed{6}$	0	0	0	0	1	0	0	0	0
$\boxed{7}$	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

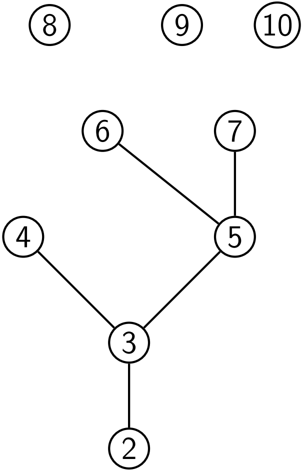
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

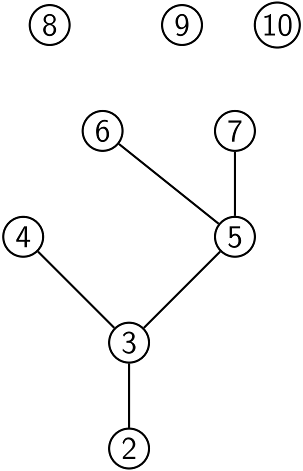
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

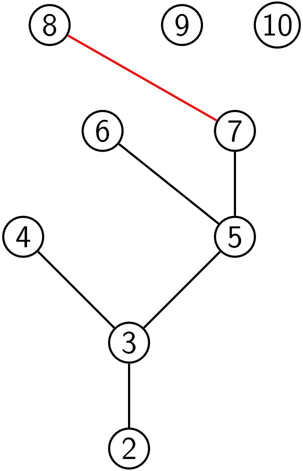
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \quad \overset{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

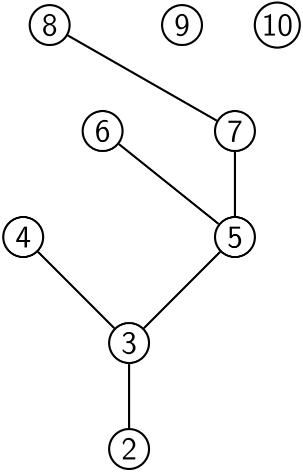
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

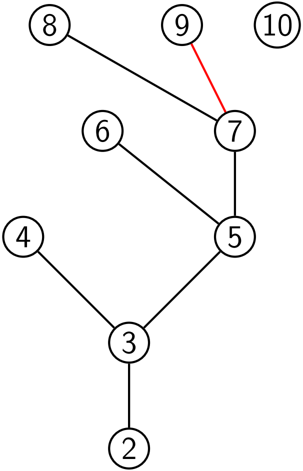
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

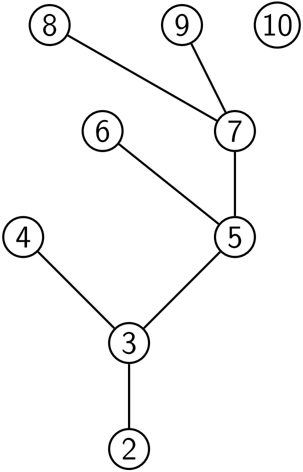
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \quad \overset{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

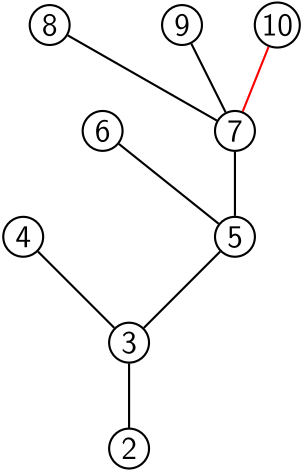
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \stackrel{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

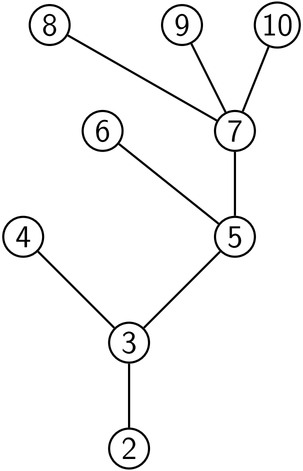
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

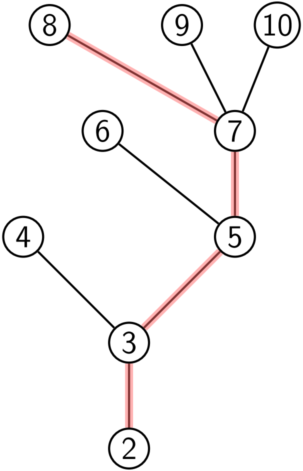
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \quad \stackrel{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

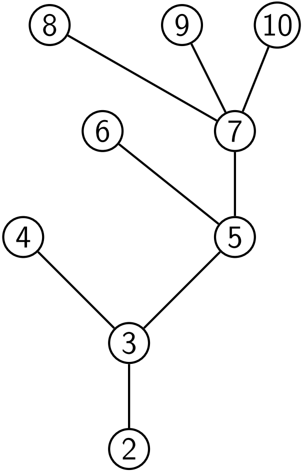
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

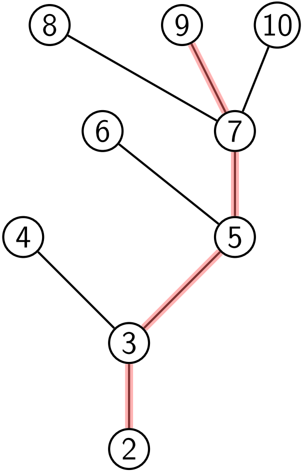
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

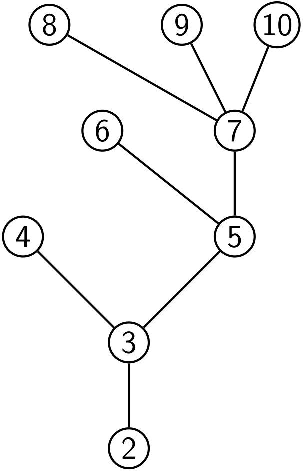
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

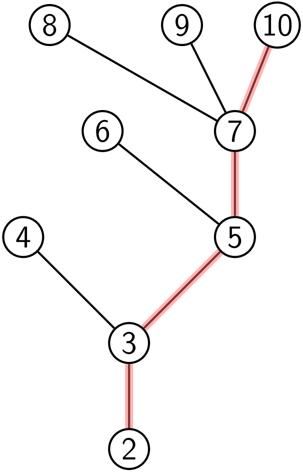
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

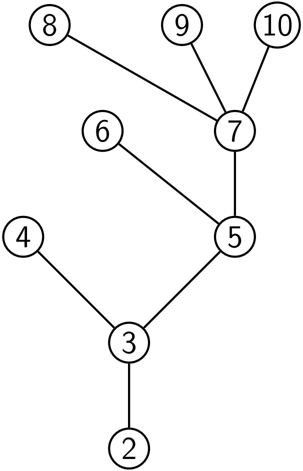
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))$$

Rješenje

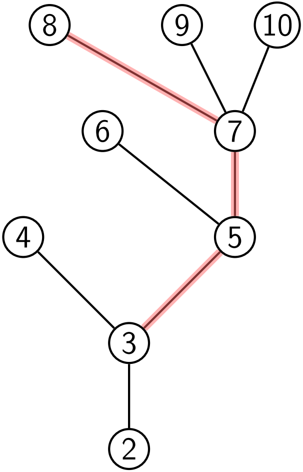
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\Longleftrightarrow} (x \mid y) \lor (x \text{ je prost } \land (x < y))}$$

Rješenje

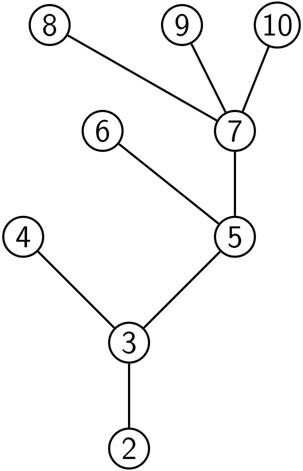
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

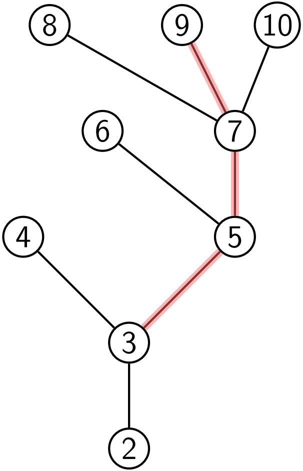
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

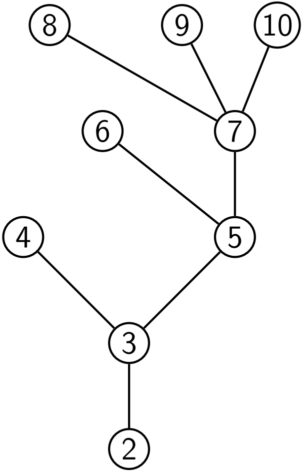
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

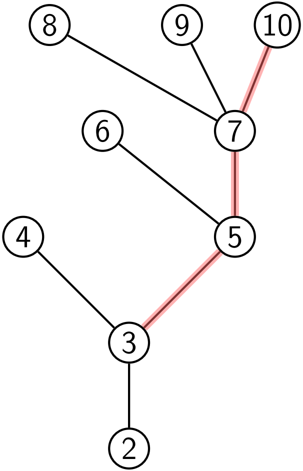
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))}$$

Rješenje

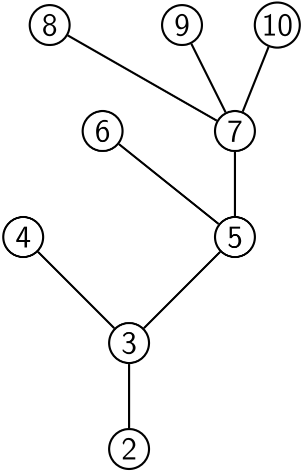
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

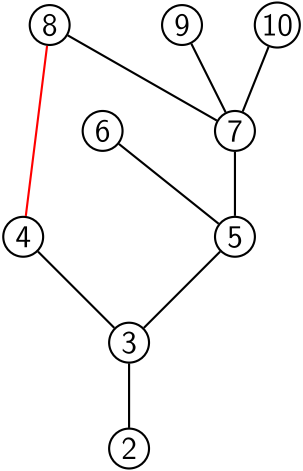
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \ \overset{\text{def}}{\iff}\ (x \mid y) \lor (x \text{ je prost} \land (x < y))}$$

Rješenje

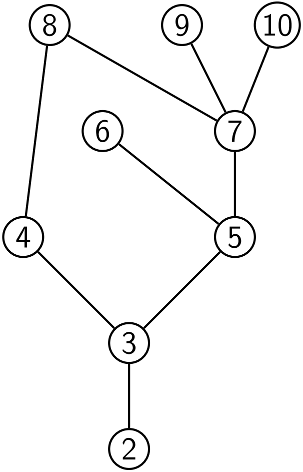
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$\boxed{x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))}$$

Rješenje

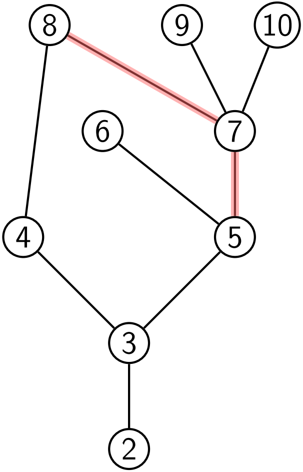
x \ y	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

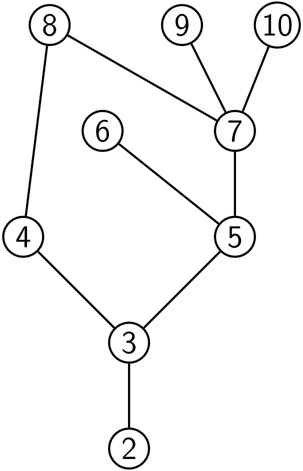
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \quad \overset{\text{def}}{\Longleftrightarrow} \quad (x \mid y) \vee (x \text{ je prost } \wedge (x < y))$$

Rješenje

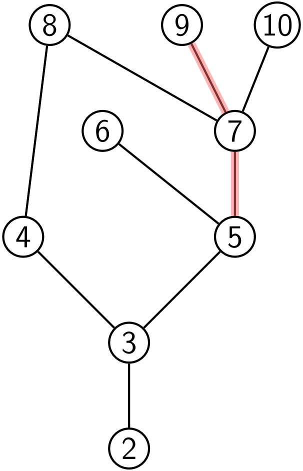
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \stackrel{\text{def}}{\Longleftrightarrow} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

Rješenje

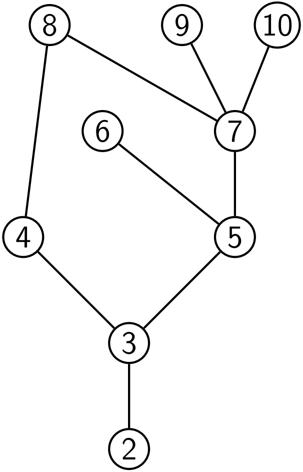
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))$$

Rješenje

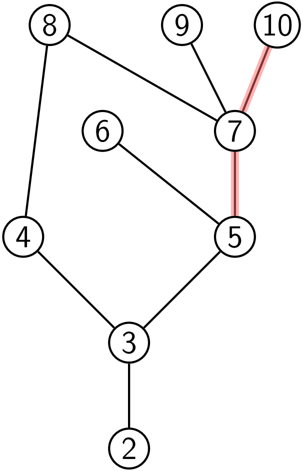
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost } \land (x < y))$$

Rješenje

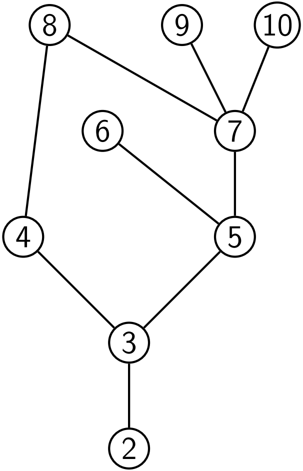
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1



$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

Rješenje

$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

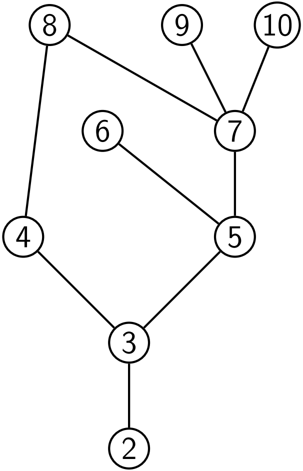


$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \lor (x \text{ je prost} \land (x < y))$$

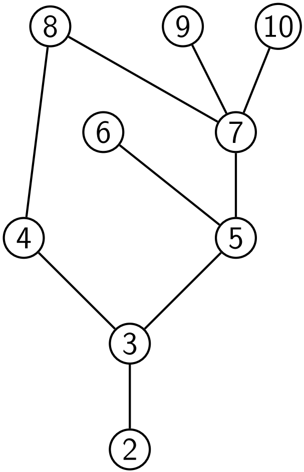
Rješenje

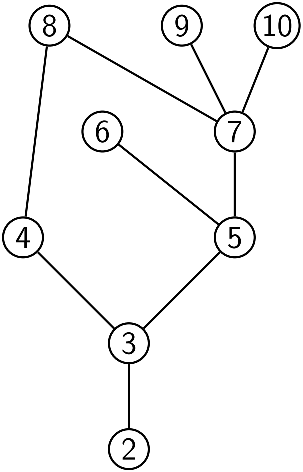
$x \backslash y$	2	3	4	5	6	7	8	9	10
2	1	1	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	1	0	0
5	0	0	0	1	1	1	1	1	1
6	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	1	1	1	1
8	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	1

Hasseov dijagram

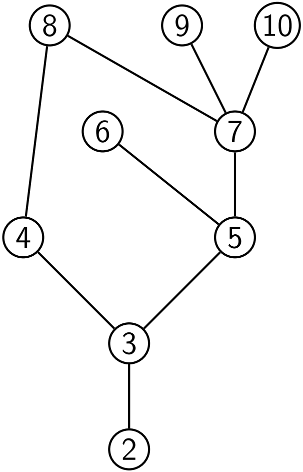


$$x \preccurlyeq y \overset{\text{def}}{\iff} (x \mid y) \vee (x \text{ je prost} \wedge (x < y))$$

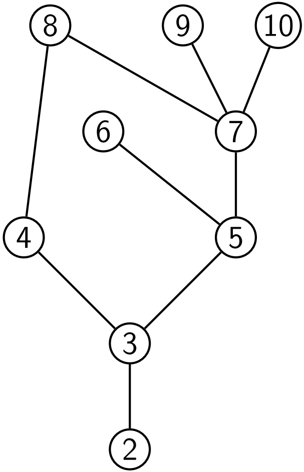

8
9
10
6
7
4
5
3
2



Najmanji element

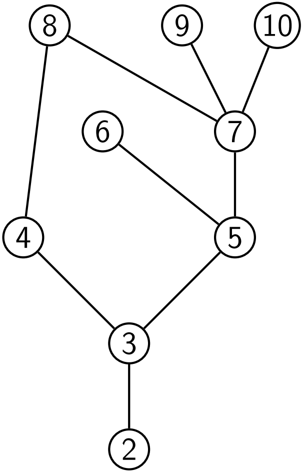


$\boxed{\text{Najmanji element}}$ 2



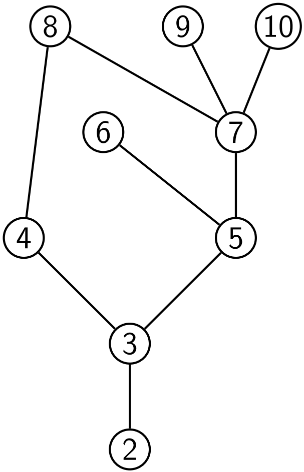
Najmanji element 2

Minimalni elementi



$\boxed{\text{Najmanji element}}$ 2

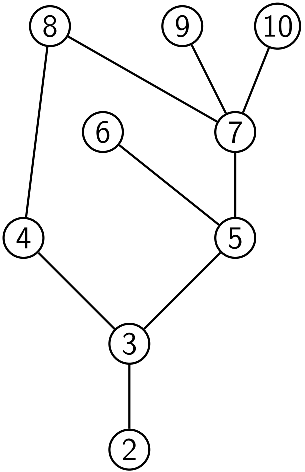
$\boxed{\text{Minimalni elementi}}$ 2



Najmanji element 2

Minimalni elementi 2

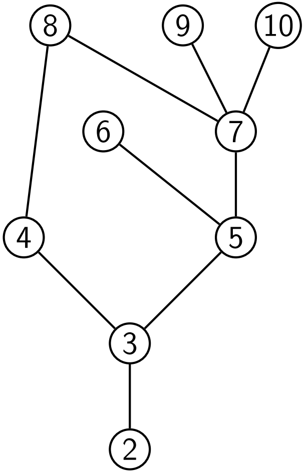
Najveći element



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

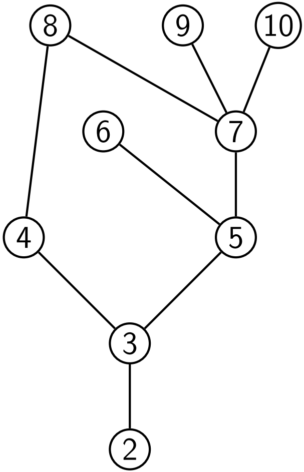


$\boxed{\text{Najmanji element}}$ 2

$\boxed{\text{Minimalni elementi}}$ 2

$\boxed{\text{Najveći element}}$ ne postoji

$\boxed{\text{Maksimalni elementi}}$

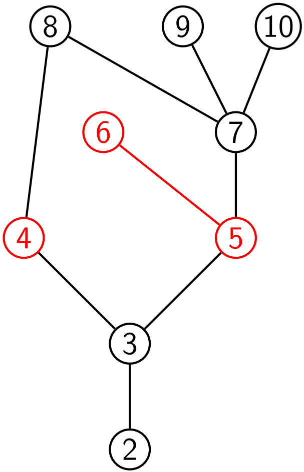


Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$



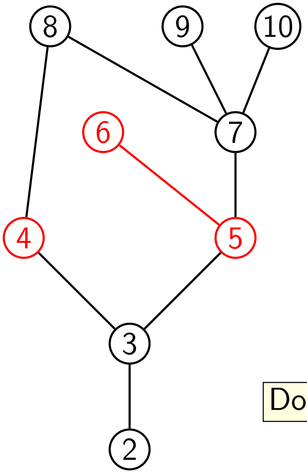
Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$



Najmanji element 2

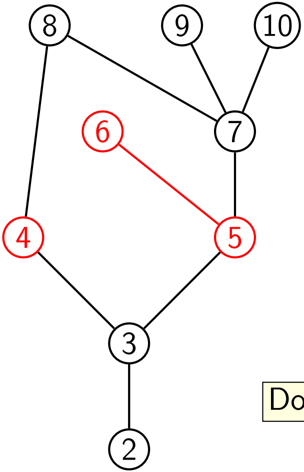
Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe



Najmanji element 2

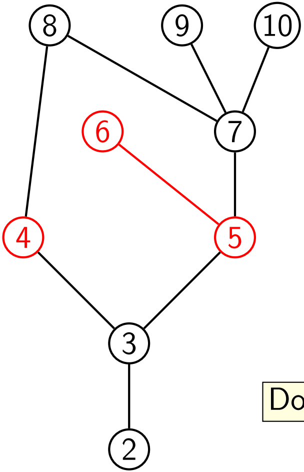
Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$



Najmanji element 2

Minimalni elementi 2

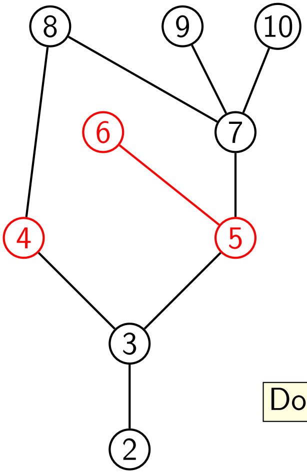
Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe



Najmanji element 2

Minimalni elementi 2

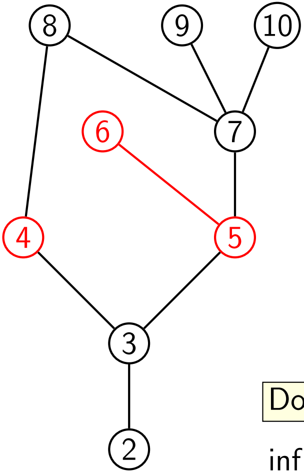
Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

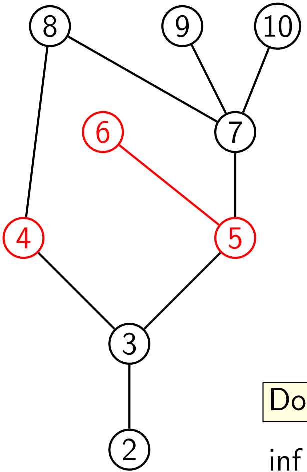
Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje

$\inf\{4, 5, 6\} = 3$



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

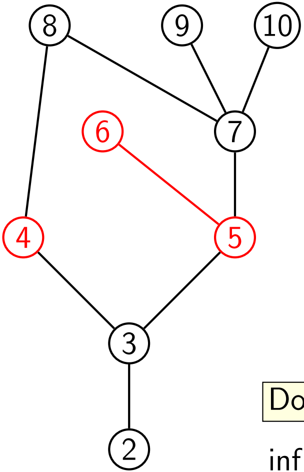
Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje

$\inf\{4, 5, 6\} = 3$

$\sup\{4, 5, 6\}$ ne postoji



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

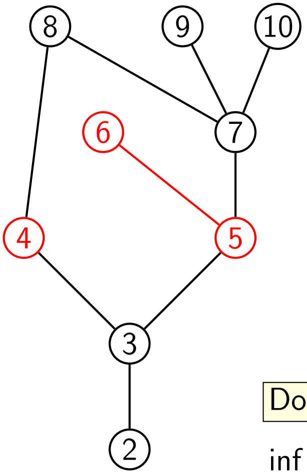
Donje međe $2, 3$

$\inf \{4, 5, 6\} = 3$

$\min \{4, 5, 6\}$ ne postoji

Gornje međe ne postoje

$\sup \{4, 5, 6\}$ ne postoji



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

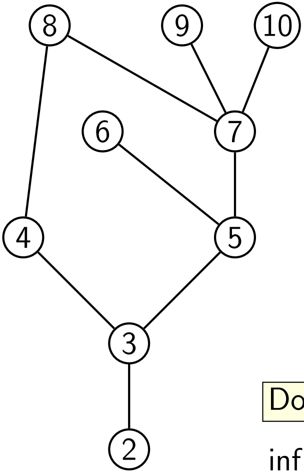
Gornje međe ne postoje

$\inf\{4, 5, 6\} = 3$

$\sup\{4, 5, 6\}$ ne postoji

$\min\{4, 5, 6\}$ ne postoji

$\max\{4, 5, 6\}$ ne postoji



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje

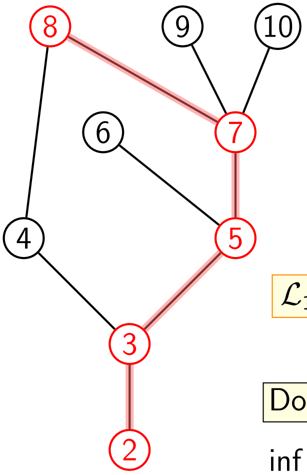
$\inf \{4, 5, 6\} = 3$

$\sup \{4, 5, 6\}$ ne postoji

$\min \{4, 5, 6\}$ ne postoji

$\max \{4, 5, 6\}$ ne postoji

Neki lanci u B



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

$\mathcal{L}_1$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje

$\inf\{4, 5, 6\} = 3$

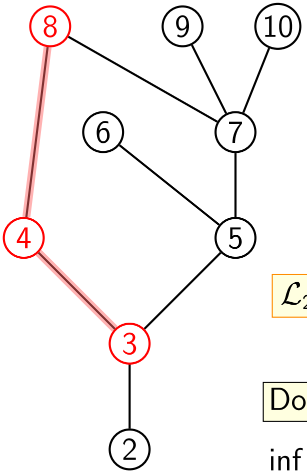
$\sup\{4, 5, 6\}$ ne postoji

$\min\{4, 5, 6\}$ ne postoji

$\max\{4, 5, 6\}$ ne postoji

Neki lanci u B

$$\mathcal{L}_1 = \{2, 3, 5, 7, 8\}$$



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

$\mathcal{L}_2$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje

$\inf \{4, 5, 6\} = 3$

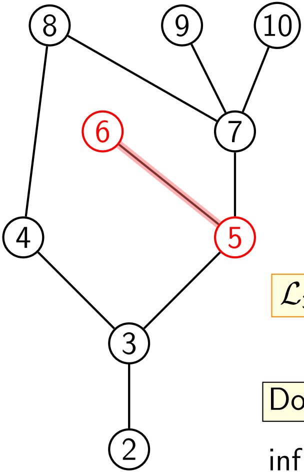
$\sup \{4, 5, 6\}$ ne postoji

$\min \{4, 5, 6\}$ ne postoji

$\max \{4, 5, 6\}$ ne postoji

Neki lanci u B

$$\mathcal{L}_1 = \{2, 3, 5, 7, 8\}, \quad \mathcal{L}_2 = \{3, 4, 8\}$$



$\mathcal{L}_3$

Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

Gornje međe ne postoje

$\inf\{4, 5, 6\} = 3$

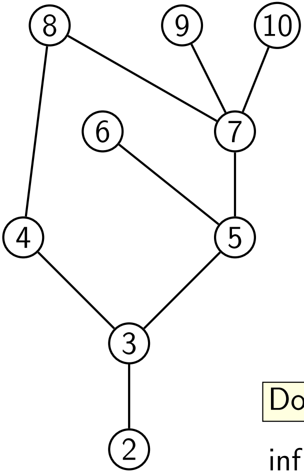
$\sup\{4, 5, 6\}$ ne postoji

$\min\{4, 5, 6\}$ ne postoji

$\max\{4, 5, 6\}$ ne postoji

Neki lanci u B

$$\mathcal{L}_1 = \{2, 3, 5, 7, 8\}, \quad \mathcal{L}_2 = \{3, 4, 8\}, \quad \mathcal{L}_3 = \{5, 6\}$$



Najmanji element 2

Minimalni elementi 2

Najveći element ne postoji

Maksimalni elementi $8, 9, 10, 6$

Podskup $\{4, 5, 6\}$

Donje međe $2, 3$

$\inf\{4, 5, 6\} = 3$

$\min\{4, 5, 6\}$ ne postoji

Gornje međe ne postoje

$\sup\{4, 5, 6\}$ ne postoji

$\max\{4, 5, 6\}$ ne postoji

Neki lanci u B

$$\mathcal{L}_1 = \{2, 3, 5, 7, 8\}, \quad \mathcal{L}_2 = \{3, 4, 8\}, \quad \mathcal{L}_3 = \{5, 6\}$$

sedmi zadatak

Zadatak 7

Neka je $A = \{1, 2, 3\}$. *Na skupu* $\mathcal{P}(A)$ *definirana je relacija parcijalnog uređaja* $\preccurlyeq$ *na sljedeći način:*

$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

pri čemu su $k(X)$ *i* $k(Y)$ *kardinalni brojevi skupova* X *i* Y.

a) *Odredite matricu incidencije zadanog parcijalnog uređaja.*

b) *Odredite najveći, najmanji, minimalne i maksimalne elemente u parcijalno uređenom skupu* $\mathcal{P}(A)$.

c) *Nacrtajte Hasseov dijagram parcijalno uređenog skupa* $\mathcal{P}(A)$.

d) *Odredite supremum, infimum, maksimum i minimum podskupa* $C = \big\{\{1, 2\}, \{1, 3\}\big\}$.

e) *Je li* $\mathcal{P}(A)$ *linearno uređen skup? Je li* $\mathcal{P}(A)$ *mreža? Obrazložite svoje odgovore.*

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$								
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1							
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1						
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1					
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1				
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1			
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1		
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0							
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1						
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0					
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0				
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1			
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1		
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0							
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \stackrel{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0						
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1					
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0				
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1			
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1		
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$								
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$								
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$								

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\{1\}$ $\{2\}$ $\{3\}$

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\boxed{\{1\}}$	$\boxed{\{2\}}$	$\boxed{\{3\}}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\boxed{\{1\}}$	0	1	0	0	1	1	1	1
$\boxed{\{2\}}$	0	0	1	0	1	1	1	1
$\boxed{\{3\}}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\{1\}$ $\qquad$ $\{2\}$ $\qquad$ $\{3\}$

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\{1\}$ $\{2\}$ $\{3\}$

$\emptyset$

$$X \preccurlyeq Y \overset{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\boxed{\{1\}}$	$\boxed{\{2\}}$	$\boxed{\{3\}}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\boxed{\{1\}}$	0	1	0	0	1	1	1	1
$\boxed{\{2\}}$	0	0	1	0	1	1	1	1
$\boxed{\{3\}}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

$\{1\}$ $\{2\}$ $\{3\}$

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

$X \backslash Y$	$\boxed{\emptyset}$	$\boxed{\{1\}}$	$\boxed{\{2\}}$	$\boxed{\{3\}}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\boxed{\{1\}}$	0	1	0	0	1	1	1	1
$\boxed{\{2\}}$	0	0	1	0	1	1	1	1
$\boxed{\{3\}}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

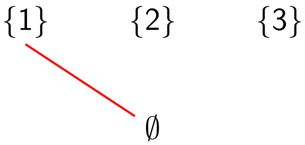
$\{1\}$ $\{2\}$ $\{3\}$

$\emptyset$

$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))}$$

Rješenje

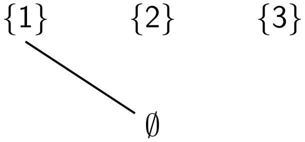
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \stackrel{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

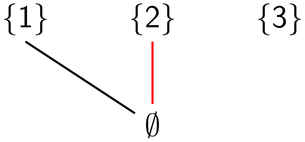
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

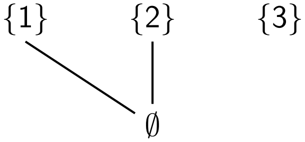
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

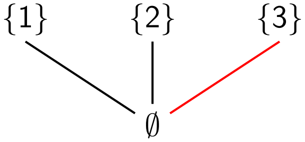
$X \backslash Y$	$\boxed{\emptyset}$	$\boxed{\{1\}}$	$\boxed{\{2\}}$	$\boxed{\{3\}}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\boxed{\{1\}}$	0	1	0	0	1	1	1	1
$\boxed{\{2\}}$	0	0	1	0	1	1	1	1
$\boxed{\{3\}}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))}$$

Rješenje

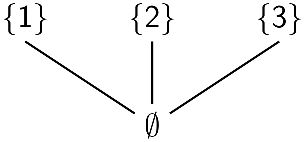
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

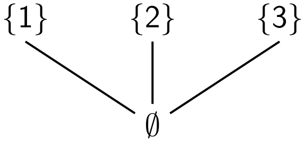
$X \backslash Y$	$\boxed{\emptyset}$	$\boxed{\{1\}}$	$\boxed{\{2\}}$	$\boxed{\{3\}}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\boxed{\{1\}}$	0	1	0	0	1	1	1	1
$\boxed{\{2\}}$	0	0	1	0	1	1	1	1
$\boxed{\{3\}}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \ \stackrel{\text{def}}{\iff}\ (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

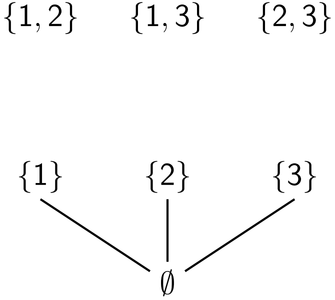
$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

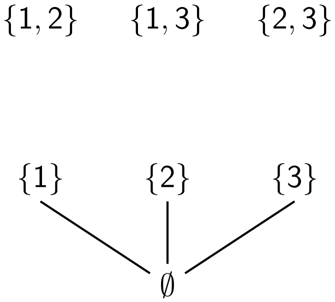
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

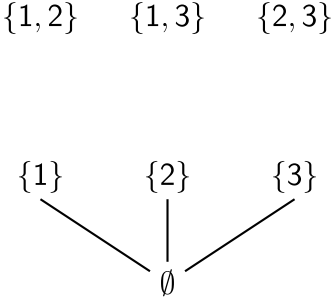
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

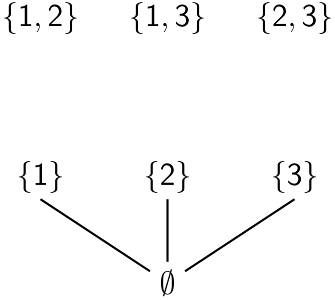
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

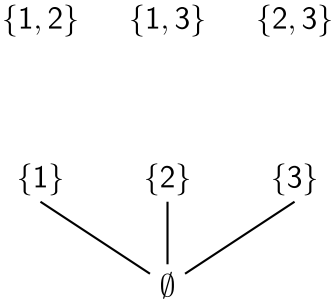
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))}$$

Rješenje

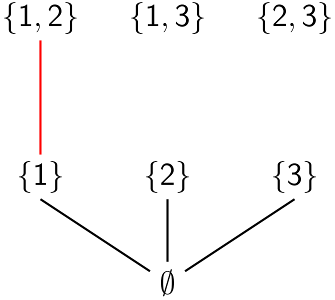
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

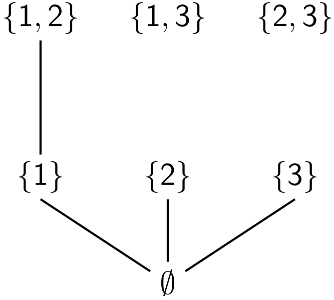
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

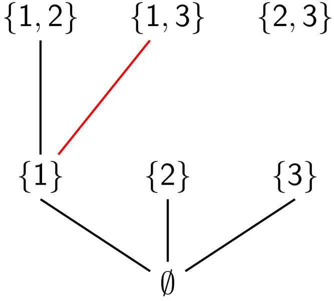
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

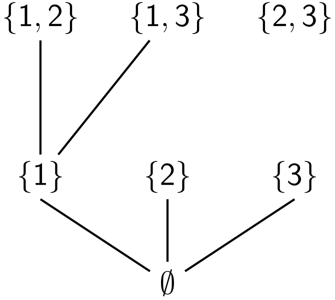
$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

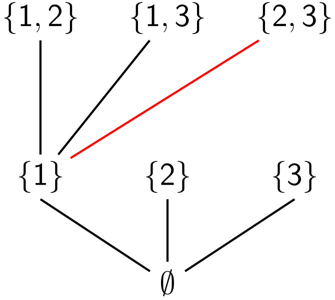
$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

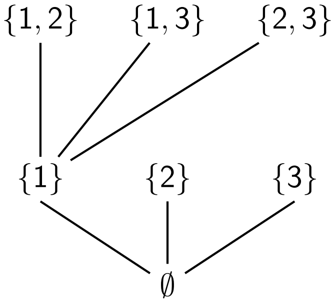
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

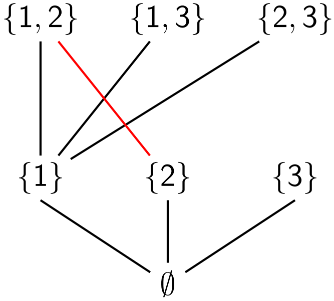
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

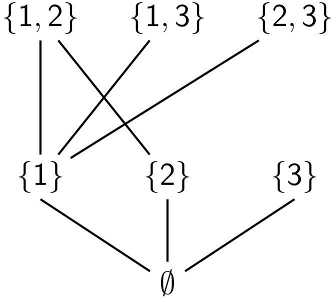
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

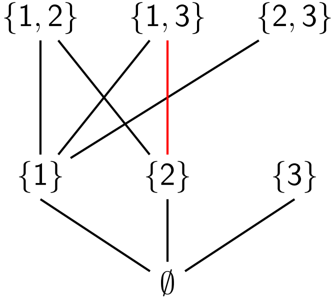
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

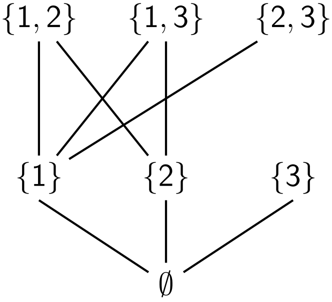
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

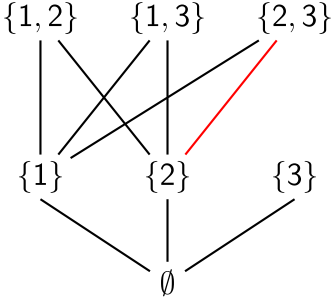
X \ Y	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

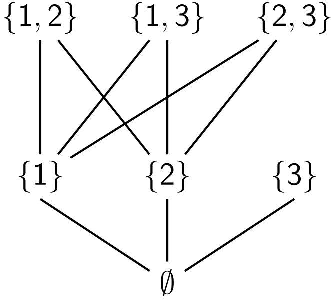
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

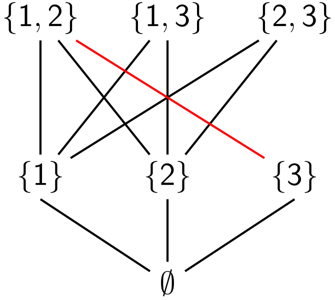
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))$$

Rješenje

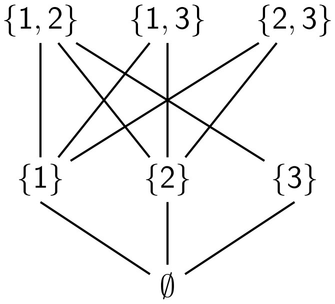
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

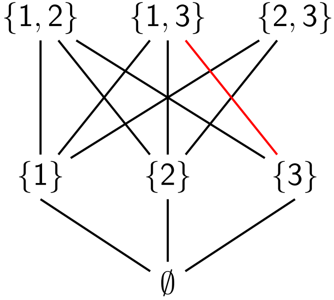
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

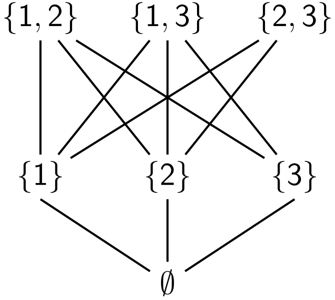
$X \backslash Y$	$\boxed{\emptyset}$	$\boxed{\{1\}}$	$\boxed{\{2\}}$	$\boxed{\{3\}}$	$\boxed{\{1,2\}}$	$\boxed{\{1,3\}}$	$\boxed{\{2,3\}}$	$\{1,2,3\}$
$\boxed{\emptyset}$	1	1	1	1	1	1	1	1
$\boxed{\{1\}}$	0	1	0	0	1	1	1	1
$\boxed{\{2\}}$	0	0	1	0	1	1	1	1
$\boxed{\{3\}}$	0	0	0	1	1	1	1	1
$\boxed{\{1,2\}}$	0	0	0	0	1	0	0	1
$\boxed{\{1,3\}}$	0	0	0	0	0	1	0	1
$\boxed{\{2,3\}}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

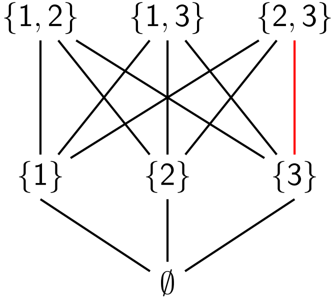
$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

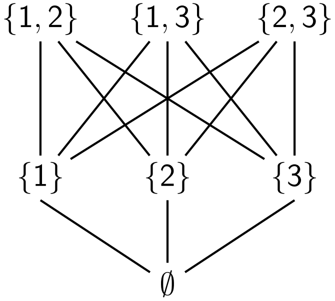
$X \setminus Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

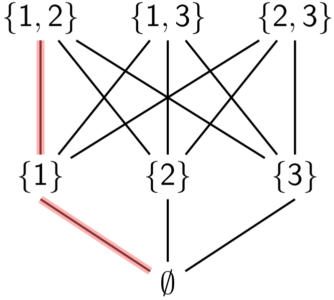
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

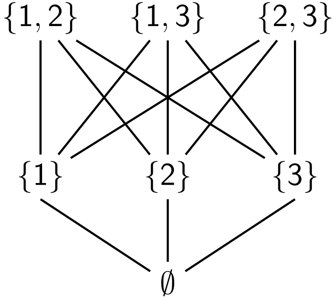
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

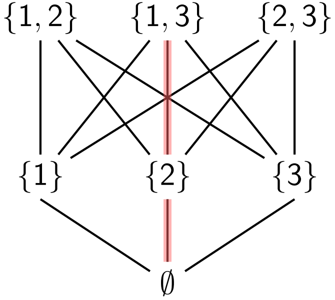
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

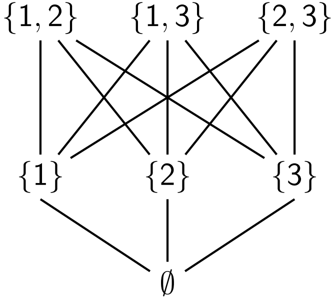
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))$$

Rješenje

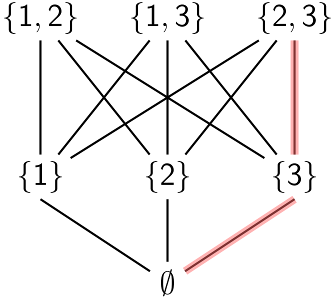
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \stackrel{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

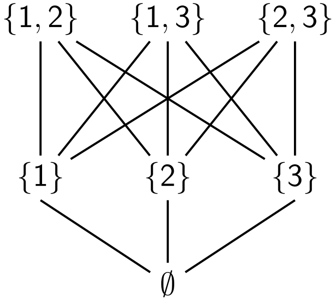
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

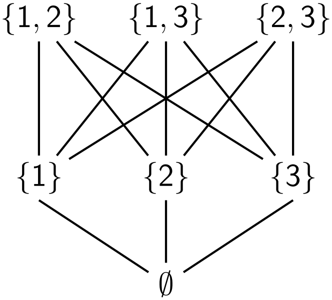
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

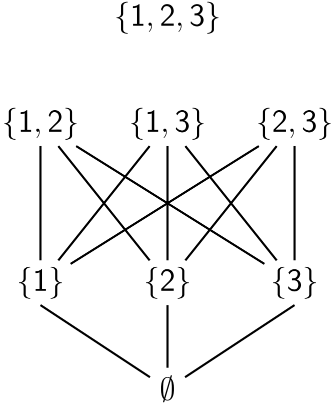
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))$$

Rješenje

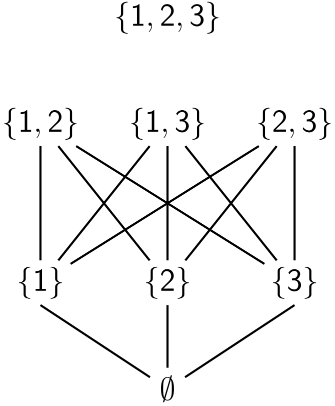
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

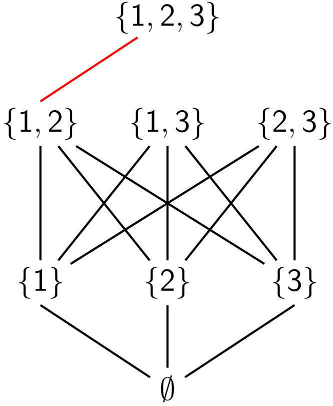
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

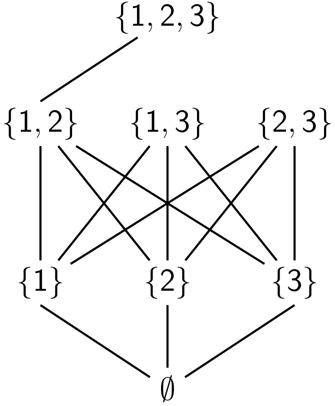
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

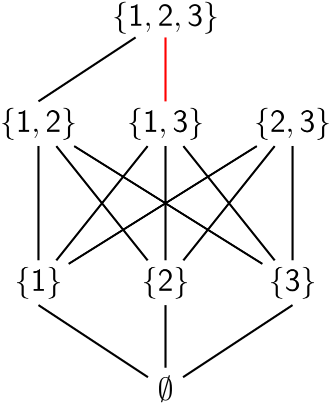
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

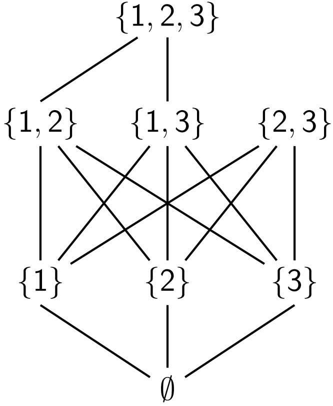
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

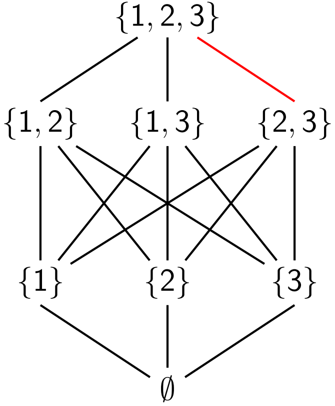
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

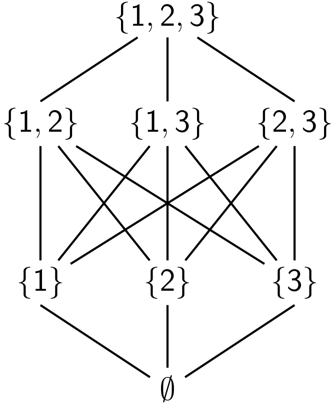
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

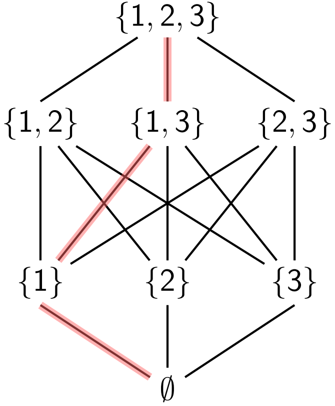
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

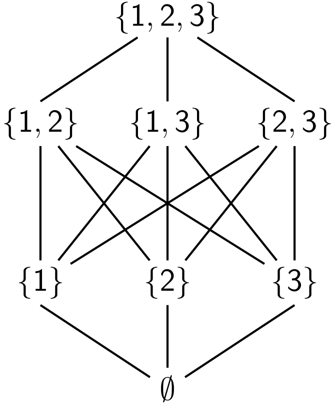
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

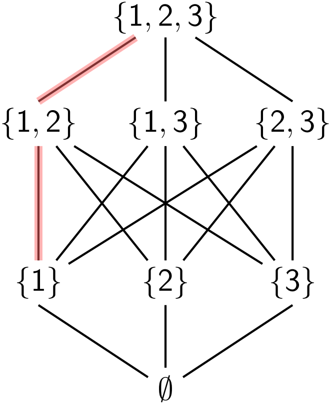
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \stackrel{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

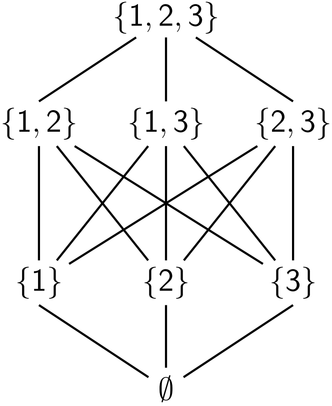
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \lor (k(X) < k(Y))$$

Rješenje

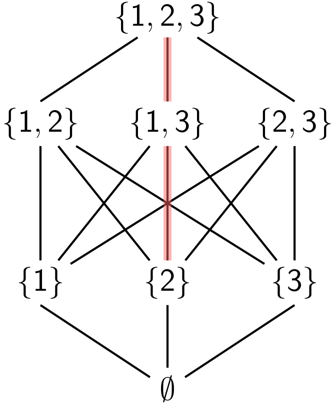
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \stackrel{\text{def}}{\Longleftrightarrow} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

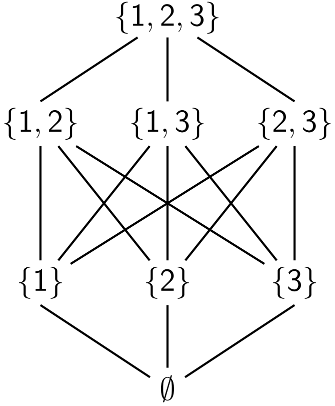
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

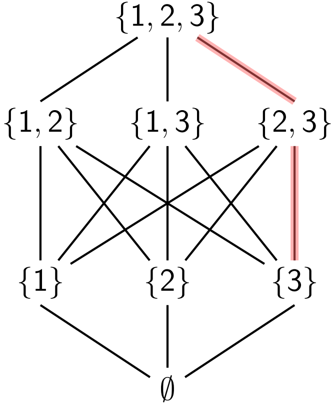
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$\boxed{X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))}$$

Rješenje

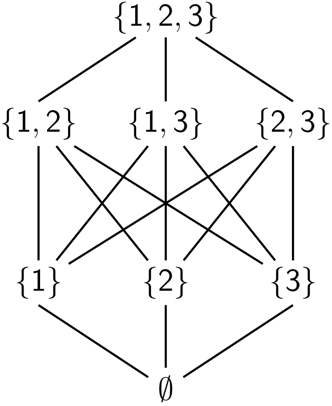
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1



$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

Rješenje

$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

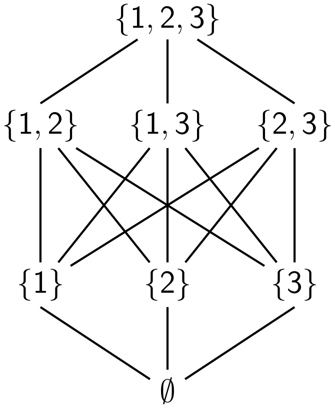


$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

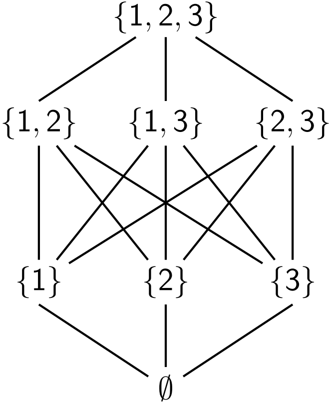
Rješenje

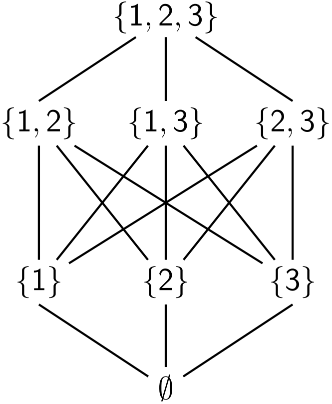
$X \backslash Y$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	1	1	1	1	1	1	1	1
$\{1\}$	0	1	0	0	1	1	1	1
$\{2\}$	0	0	1	0	1	1	1	1
$\{3\}$	0	0	0	1	1	1	1	1
$\{1,2\}$	0	0	0	0	1	0	0	1
$\{1,3\}$	0	0	0	0	0	1	0	1
$\{2,3\}$	0	0	0	0	0	0	1	1
$\{1,2,3\}$	0	0	0	0	0	0	0	1

Hasseov dijagram

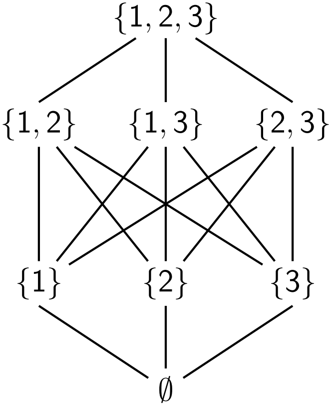


$$X \preccurlyeq Y \overset{\text{def}}{\iff} (X = Y) \vee (k(X) < k(Y))$$

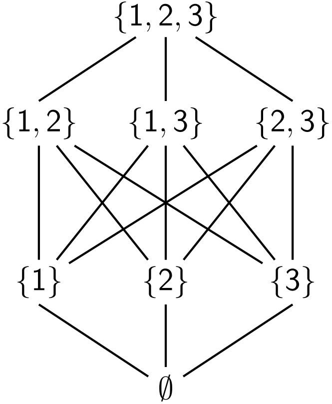

{1, 2, 3}
{1, 2}
{1, 3}
{2, 3}
{1}
{2}
{3}
∅



$\boxed{\text{Najmanji element}}$

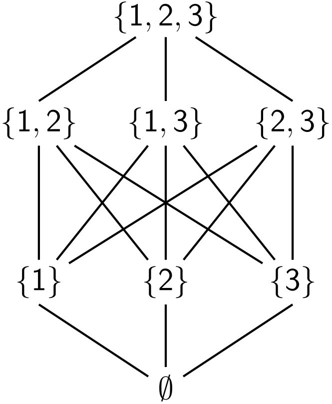


$\boxed{\text{Najmanji element}}$ $\emptyset$



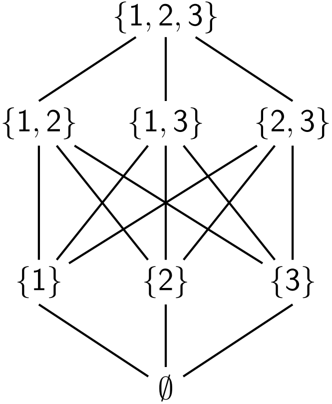
$\boxed{\text{Najmanji element}}$ $\emptyset$

$\boxed{\text{Minimalni elementi}}$



$\boxed{\text{Najmanji element}}$ $\emptyset$

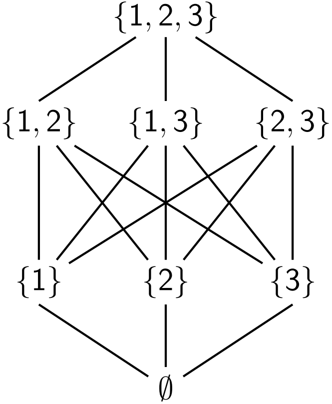
$\boxed{\text{Minimalni elementi}}$ $\emptyset$



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

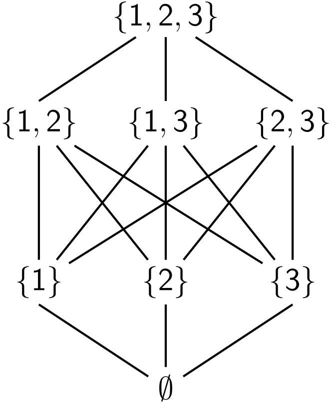
Najveći element



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

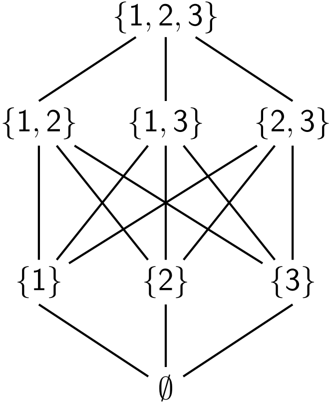


Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1, 2, 3\}$

Maksimalni elementi

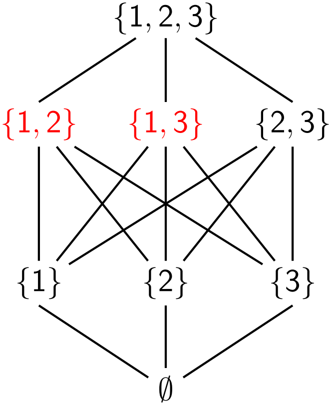


Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$



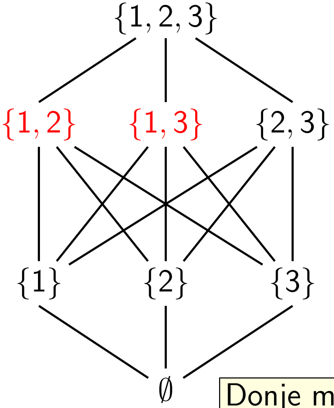
Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \{\{1,2\},\{1,3\}\}$$



Donje međe

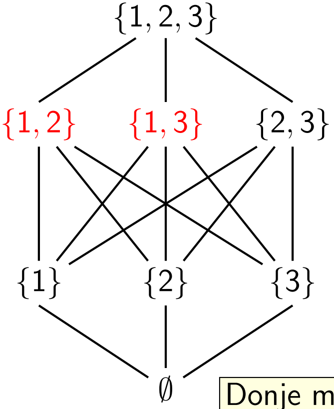
Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1, 2, 3\}$

Maksimalni elementi $\{1, 2, 3\}$

$$C = \big\{\{1, 2\}, \{1, 3\}\big\}$$



Najmanji element $\emptyset$

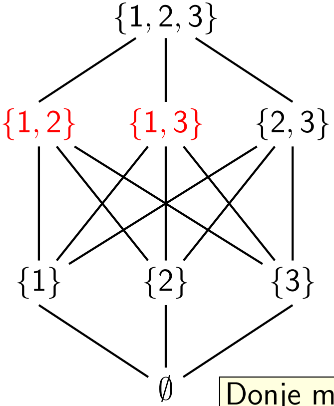
Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \{\{1,2\},\{1,3\}\}$$

Donje međe $\{1\},\{2\},\{3\},\emptyset$



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

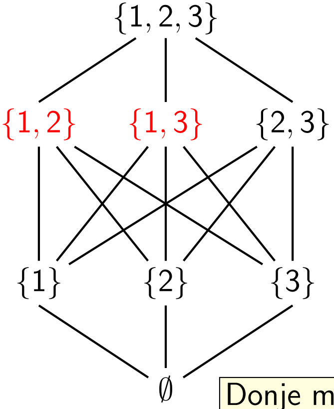
Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \big\{\{1,2\},\{1,3\}\big\}$$

Donje međe $\{1\},\{2\},\{3\},\emptyset$

Gornje međe



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

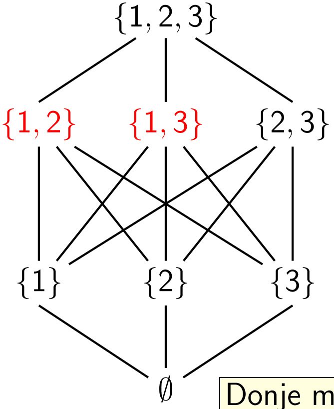
Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \{\{1,2\},\{1,3\}\}$$

Donje međe $\{1\},\{2\},\{3\},\emptyset$

Gornje međe $\{1,2,3\}$



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

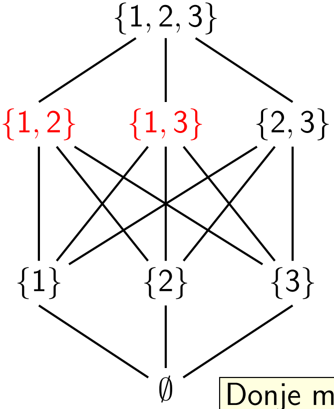
Najveći element $\{1, 2, 3\}$

Maksimalni elementi $\{1, 2, 3\}$

$$C = \{\{1, 2\}, \{1, 3\}\}$$

Donje međe $\{1\}, \{2\}, \{3\}, \emptyset$ Gornje međe $\{1, 2, 3\}$

inf C ne postoji



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1, 2, 3\}$

Maksimalni elementi $\{1, 2, 3\}$

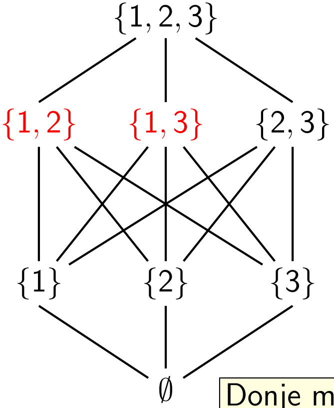
$$C = \{\{1, 2\}, \{1, 3\}\}$$

Donje međe $\{1\}, \{2\}, \{3\}, \emptyset$

Gornje međe $\{1, 2, 3\}$

inf C ne postoji

$\sup C = \{1, 2, 3\}$



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \{\{1,2\},\{1,3\}\}$$

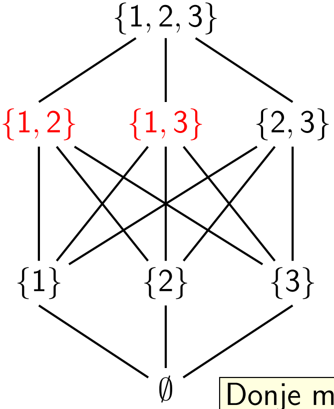
Donje međe $\{1\},\{2\},\{3\},\emptyset$

Gornje međe $\{1,2,3\}$

inf C ne postoji

sup $C = \{1,2,3\}$

min C ne postoji



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \{\{1,2\},\{1,3\}\}$$

Donje međe $\{1\},\{2\},\{3\},\emptyset$

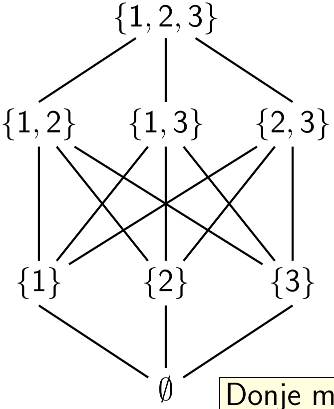
inf C ne postoji

min C ne postoji

Gornje međe $\{1,2,3\}$

$\sup C = \{1,2,3\}$

max C ne postoji



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \big\{\{1,2\},\{1,3\}\big\}$$

Donje međe $\{1\},\{2\},\{3\},\emptyset$

inf C ne postoji

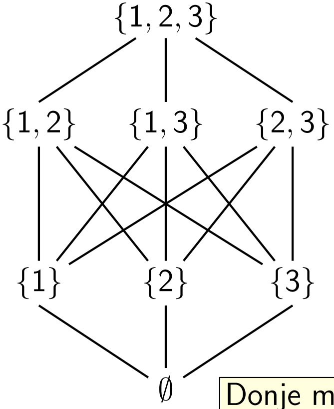
min C ne postoji

Gornje međe $\{1,2,3\}$

$\sup C = \{1,2,3\}$

max C ne postoji

- $\big(\mathcal{P}(A), \preccurlyeq\big)$ nije linearno uređen skup jer npr. elementi $\{1\}$ i $\{2\}$ nisu usporedivi.



Najmanji element $\emptyset$

Minimalni elementi $\emptyset$

Najveći element $\{1,2,3\}$

Maksimalni elementi $\{1,2,3\}$

$$C = \big\{\{1,2\},\{1,3\}\big\}$$

Donje međe $\{1\},\{2\},\{3\},\emptyset$

inf C ne postoji

min C ne postoji

Gornje međe $\{1,2,3\}$

$\sup C = \{1,2,3\}$

max C ne postoji

- $\big(\mathcal{P}(A),\preccurlyeq\big)$ nije linearno uređen skup jer npr. elementi $\{1\}$ i $\{2\}$ nisu usporedivi.
- $\big(\mathcal{P}(A),\preccurlyeq\big)$ nije mreža jer podskup C nema infimum.

osmi zadatak

Zadatak 8

Na skupu $\mathbb{N}$ *definirana je relacija* $\preccurlyeq$ *s*

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}.$$

a) *Dokažite da je* $\big(\mathbb{N}, \preccurlyeq\big)$ *parcijalno uređen skup.*

b) *Odredite najveći, najmanji, minimalne i maksimalne elemente u parcijalno uređenom skupu* $\mathbb{N}$.

c) *Odredite maksimalne elemente u* $\big(\mathbb{N} \setminus \{1\}, \preccurlyeq\big)$.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^1 \;\Rightarrow\; a \preccurlyeq a$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\,\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\qquad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\qquad (\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a)$$

Rješenje

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

a) **Refleksivnost** $\quad (\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^{1} \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = a^{r_1},$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\,\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = a^{r_1},\ a = b^{r_2}$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) **Refleksivnost** $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\,\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a}^{r_1},\ a = \boxed{b^{r_2}}$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^{1} \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a}^{r_1},\ a = \boxed{b^{r_2}} \Rightarrow$$

$$\Rightarrow \left(b^{r_2}\right)^{r_1} = b$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) **Refleksivnost** $(\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $(\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \land (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \land (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a}^{r_1},\ a = \boxed{b^{r_2}} \Rightarrow$$

$$\Rightarrow \left(b^{r_2}\right)^{r_1} = b \Rightarrow b^{r_1 r_2} = b$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) **Refleksivnost** $(\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^{1} \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $(\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a}^{r_1},\ a = \boxed{b^{r_2}} \Rightarrow$$

$$\Rightarrow \left(b^{r_2}\right)^{r_1} = b \Rightarrow b^{r_1 r_2} = b \Rightarrow r_1 r_2 = 1$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) **Refleksivnost** $\quad (\forall a \in \mathbb{N})(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$$\boxed{r = 1}$$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a}^{r_1},\ a = \boxed{b^{r_2}} \Rightarrow$$

$$\Rightarrow \left(b^{r_2}\right)^{r_1} = b \Rightarrow b^{r_1 r_2} = b \Rightarrow r_1 r_2 = 1 \Rightarrow r_1 = 1, r_2 = 1$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^{1} \Rightarrow a \preccurlyeq a$$

$r = 1$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\,\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\; b = \boxed{a}^{r_1},\; a = \boxed{b^{r_2}} \Rightarrow$$

$$\Rightarrow \left(b^{r_2}\right)^{r_1} = b \Rightarrow b^{r_1 r_2} = b \Rightarrow r_1 r_2 = 1 \Rightarrow r_1 = 1, r_2 = 1$$

jer su $r_1, r_2 \in \mathbb{N}$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Rješenje

a) Refleksivnost $\quad (\forall a \in \mathbb{N})\,(a \preccurlyeq a)$

$$a = a^1 \Rightarrow a \preccurlyeq a$$

$r = 1$

Antisimetričnost $\quad (\forall a, b \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow a = b\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq a) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a}^{r_1},\ a = \boxed{b^{r_2}} \Rightarrow$$

$$\Rightarrow \left(b^{r_2}\right)^{r_1} = b \Rightarrow b^{r_1 r_2} = b \Rightarrow r_1 r_2 = 1 \Rightarrow r_1 = 1, r_2 = 1 \Rightarrow$$

$$\Rightarrow a = b$$

jer su $r_1, r_2 \in \mathbb{N}$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost

$$(\forall a, b, c \in \mathbb{N})\big((a \preccurlyeq b) \land (b \preccurlyeq c) \;\Rightarrow\; a \preccurlyeq c\big)$$

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{N}\big)\big((a \preccurlyeq b) \wedge (b \preccurlyeq c) \;\Rightarrow\; a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq c)$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{N}\big)\big((a \preccurlyeq b) \wedge (b \preccurlyeq c) \;\Rightarrow\; a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq c) \;\Rightarrow\; \exists r_1, r_2 \in \mathbb{N},$$

$$a \preccurlyeq b \stackrel{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{N}\big)\big((a \preccurlyeq b) \land (b \preccurlyeq c) \;\Rightarrow\; a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \land (b \preccurlyeq c) \;\Rightarrow\; \exists r_1, r_2 \in \mathbb{N},\; b = a^{r_1},$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{N}\big)\big((a \preccurlyeq b) \land (b \preccurlyeq c) \Rightarrow a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \land (b \preccurlyeq c) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = a^{r_1},\ c = b^{r_2}$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{N})\big((a \preccurlyeq b) \land (b \preccurlyeq c) \;\Rightarrow\; a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \land (b \preccurlyeq c) \;\Rightarrow\; \exists r_1, r_2 \in \mathbb{N},\; b = \boxed{a^{r_1}},\; c = \boxed{b}^{r_2}$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq c) \Rightarrow a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq c) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\; b = \boxed{a^{r_1}},\; c = \boxed{b}^{r_2} \Rightarrow$$

$$\Rightarrow c = \left(a^{r_1}\right)^{r_2}$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq c) \Rightarrow a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq c) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a^{r_1}},\ c = \boxed{b}^{r_2} \Rightarrow$$

$$\Rightarrow c = \left(a^{r_1}\right)^{r_2} \Rightarrow c = a^{r_1 r_2}$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Tranzitivnost $\quad (\forall a, b, c \in \mathbb{N})\big((a \preccurlyeq b) \wedge (b \preccurlyeq c) \Rightarrow a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq c) \Rightarrow \exists r_1, r_2 \in \mathbb{N},\ b = \boxed{a^{r_1}},\ c = \boxed{b}^{r_2} \Rightarrow$$

$$\Rightarrow c = \left(a^{r_1}\right)^{r_2} \Rightarrow c = a^{r_1 r_2} \Rightarrow a \preccurlyeq c$$

$a \preccurlyeq b \stackrel{\text{def}}{\iff} b = a^r$ za neki $r \in \mathbb{N}$

Tranzitivnost $\quad \big(\forall a, b, c \in \mathbb{N}\big)\big((a \preccurlyeq b) \wedge (b \preccurlyeq c) \;\Rightarrow\; a \preccurlyeq c\big)$

$$(a \preccurlyeq b) \wedge (b \preccurlyeq c) \;\Rightarrow\; \exists r_1, r_2 \in \mathbb{N},\;\; b = \boxed{a^{r_1}},\;\; c = \boxed{b}^{r_2} \;\Rightarrow$$

$$\Rightarrow\;\; c = \left(a^{r_1}\right)^{r_2} \;\Rightarrow\; c = a^{r_1 r_2} \;\Rightarrow\; a \preccurlyeq c$$

jer je $r_1 r_2 \in \mathbb{N}$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

b) Najmanji element

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

b) Najmanji element

Pretpostavimo da je m najmanji element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r$ za neki $r \in \mathbb{N}$

b) Najmanji element

Pretpostavimo da je m najmanji element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(m \preccurlyeq a)$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

b) Najmanji element

Pretpostavimo da je m najmanji element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(m \preccurlyeq a)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(\exists r \in \mathbb{N})(a = m^r)$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

b) Najmanji element

Pretpostavimo da je m najmanji element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(m \preccurlyeq a)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(\exists r \in \mathbb{N})(a = m^r)$$

Dakle, svaki prirodni broj se može napisati kao prirodna potencija broja m.

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}}$$

b) Najmanji element

Pretpostavimo da je m najmanji element u parcijalno uređenom skupu $\left(\mathbb{N}, \preccurlyeq\right)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(m \preccurlyeq a)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})(\exists r \in \mathbb{N})(a = m^r)$$

Dakle, svaki prirodni broj se može napisati kao prirodna potencija broja m. To je kontradikcija pa najmanji element u $\left(\mathbb{N}, \preccurlyeq\right)$ ne postoji.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Najveći element

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Najveći element

Pretpostavimo da je M najveći element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Najveći element

Pretpostavimo da je M najveći element u parcijalno uređenom skupu $\big(\mathbb{N}, \preccurlyeq\big)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(a \preccurlyeq M)$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Najveći element

Pretpostavimo da je M najveći element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(a \preccurlyeq M)$$

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(\exists r \in \mathbb{N})(M = a^r)$$

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Najveći element

Pretpostavimo da je M najveći element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(a \preccurlyeq M)$$

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(\exists r \in \mathbb{N})(M = a^r)$$

Dakle, M se može napisati kao prirodna potencija svakog prirodnog broja.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Najveći element

Pretpostavimo da je M najveći element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(a \preccurlyeq M)$$

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})(\exists r \in \mathbb{N})(M = a^r)$$

Dakle, M se može napisati kao prirodna potencija svakog prirodnog broja. To je kontradikcija pa najveći element u $(\mathbb{N}, \preccurlyeq)$ ne postoji.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Maksimalni elementi

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Maksimalni elementi

Pretpostavimo da je M maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$

Maksimalni elementi

Pretpostavimo da je M maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big(M \preccurlyeq a \;\Rightarrow\; a = M\big)$$

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Maksimalni elementi

Pretpostavimo da je M maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big(M \preccurlyeq a \;\Rightarrow\; a = M\big)$$

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(a = M^r) \;\Rightarrow\; a = M\big)$$

Maksimalni elementi

Pretpostavimo da je M maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big(M \preccurlyeq a \;\Rightarrow\; a = M\big)$$

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(a = M^r) \;\Rightarrow\; a = M\big)$$

Ako je $M \neq 1$, tada je $M \neq M^2$ i $M \preccurlyeq M^2$.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Maksimalni elementi

Pretpostavimo da je M maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big(M \preccurlyeq a \;\Rightarrow\; a = M\big)$$

$$(\exists M \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(a = M^r) \;\Rightarrow\; a = M\big)$$

Ako je $M \neq 1$, tada je $M \neq M^2$ i $M \preccurlyeq M^2$. Stoga je broj 1 jedini maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$a \preccurlyeq b \ \overset{\text{def}}{\Longleftrightarrow}\ b = a^r \text{ za neki } r \in \mathbb{N}$$

Minimalni elementi

$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r$ za neki $r \in \mathbb{N}$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r$ za neki $r \in \mathbb{N}$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(m = a^r) \;\Rightarrow\; a = m\big)$$

$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r$ za neki $r \in \mathbb{N}$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(m = a^r) \;\Rightarrow\; a = m\big)$$

Dakle, m se ne može napisati kao prirodna potencija prirodnog broja $(r \neq 1)$.

$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r$ za neki $r \in \mathbb{N}$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(m = a^r) \;\Rightarrow\; a = m\big)$$

Dakle, m se ne može napisati kao prirodna potencija prirodnog broja ($r \neq 1$). Minimalni elementi su prirodni brojevi koji nisu potencije prirodnog broja.

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(m = a^r) \;\Rightarrow\; a = m\big)$$

Dakle, m se ne može napisati kao prirodna potencija prirodnog broja $(r \neq 1)$. Minimalni elementi su prirodni brojevi koji nisu potencije prirodnog broja. Na primjer: $6, 15, 17, 34, \ldots$

$$a \preccurlyeq b \stackrel{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(m = a^r) \;\Rightarrow\; a = m\big)$$

Dakle, m se ne može napisati kao prirodna potencija prirodnog broja ($r \neq 1$). Minimalni elementi su prirodni brojevi koji nisu potencije prirodnog broja. Na primjer: $6, 15, 17, 34, \ldots$

Specijalno, prosti brojevi su minimalni elementi.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

Minimalni elementi

Pretpostavimo da je m minimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big(a \preccurlyeq m \;\Rightarrow\; a = m\big)$$

$$(\exists m \in \mathbb{N})(\forall a \in \mathbb{N})\big((\exists r \in \mathbb{N})(m = a^r) \;\Rightarrow\; a = m\big)$$

Dakle, m se ne može napisati kao prirodna potencija prirodnog broja $(r \neq 1)$. Minimalni elementi su prirodni brojevi koji nisu potencije prirodnog broja. Na primjer: $6, 15, 17, 34, \ldots$

Specijalno, prosti brojevi su minimalni elementi.

Broj 1 je jedini element koji je minimalni i maksimalni.

$$\boxed{a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}}$$

c) Broj 1 je jedini maksimalni element u parcijalno uređenom skupu $\big(\mathbb{N}, \preccurlyeq\big)$.

$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r$ za neki $r \in \mathbb{N}$

c) Broj 1 je jedini maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

Stoga parcijalno uređeni skup $(\mathbb{N} \setminus \{1\}, \preccurlyeq)$ nema niti jedan maksimalni element.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

c) Broj 1 je jedini maksimalni element u parcijalno uređenom skupu $\big(\mathbb{N}, \preccurlyeq\big)$.

Stoga parcijalno uređeni skup $\big(\mathbb{N} \setminus \{1\}, \preccurlyeq\big)$ nema niti jedan maksimalni element.

Zornova lema

Svaki neprazni parcijalno uređeni skup u kojemu svaki lanac ima gornju među, sadrži barem jedan maksimalni element.

$$a \preccurlyeq b \overset{\text{def}}{\iff} b = a^r \text{ za neki } r \in \mathbb{N}$$

c) Broj 1 je jedini maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

Stoga parcijalno uređeni skup $(\mathbb{N} \setminus \{1\}, \preccurlyeq)$ nema niti jedan maksimalni element.

Zornova lema

Svaki neprazni parcijalno uređeni skup u kojemu svaki lanac ima gornju među, sadrži barem jedan maksimalni element.

- Obrat Zornove leme ne vrijedi. Jedan protuprimjer je upravo $(\mathbb{N}, \preccurlyeq)$.

$$a \preccurlyeq b \overset{\text{def}}{\Longleftrightarrow} b = a^r \text{ za neki } r \in \mathbb{N}$$

c) Broj 1 je jedini maksimalni element u parcijalno uređenom skupu $(\mathbb{N}, \preccurlyeq)$.

Stoga parcijalno uređeni skup $(\mathbb{N} \setminus \{1\}, \preccurlyeq)$ nema niti jedan maksimalni element.

Zornova lema

Svaki neprazni parcijalno uređeni skup u kojemu svaki lanac ima gornju među, sadrži barem jedan maksimalni element.

- Obrat Zornove leme ne vrijedi. Jedan protuprimjer je upravo $(\mathbb{N}, \preccurlyeq)$.
- Na konačnim skupovima Zornova lema je trivijalna činjenica.