Stabla

Diskretne strukture s teorijom grafova

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Sadržaj

prvi zadatak

Zadatak 1

U stablu T je $\Delta(T) = 5$. Pritom, stablo T ima četiri vrha stupnja 2, jedan vrh stupnja 3, dva vrha stupnja 4 i jedan vrh stupnja 5. Koliko listova ima stablo T i koliki je ukupni broj vrhova i bridova u promatranom stablu?

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

Rješenje

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$$\boldsymbol{\ell} \cdot 1$$

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$$\boldsymbol{\ell} \cdot 1 +$$

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$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3$$

Rješenje

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$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4$$

Rješenje

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$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 +$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 =$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

Rješenje

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$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

$$\boldsymbol{\ell} + 24$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

$$\boldsymbol{\ell} + 24 =$$

Rješenje

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$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 8 - 1)$$

Rješenje

$\boldsymbol{\ell} \longleftarrow$ broj listova u stablu T

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$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 8 - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 8 - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

Rješenje

$\boldsymbol{\ell} \longleftarrow$ broj listova u stablu T

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 8 - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

$$\boldsymbol{\ell} = 10$$

Rješenje

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

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$$\boxed{\boldsymbol{\ell} = 10}$$

$\nu = \boldsymbol{\ell} + 8$

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Rješenje

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$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 8 - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

$$\boxed{\boldsymbol{\ell} = 10}$$

$\nu = \boldsymbol{\ell} + 8$

$\nu = 10 + 8$

$\nu = 18$

Rješenje

$\boldsymbol{\ell} \longleftarrow$ broj listova u stablu T

$\Delta(T) = 5, \quad \nu = \boldsymbol{\ell} + 8, \quad \varepsilon = \nu - 1$

$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 8 - 1)$$

$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

$$\boxed{\boldsymbol{\ell} = 10}$$

$\nu = \boldsymbol{\ell} + 8$

$\nu = 10 + 8$

$\boxed{\nu = 18}$

Rješenje

$\boldsymbol{\ell} \longleftarrow$ broj listova u stablu T

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$$\sum_{v \in V(T)} d(v) = 2\varepsilon$$

$$\boldsymbol{\ell} \cdot 1 + 4 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5 = 2(\nu - 1)$$

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$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

$$\boxed{\boldsymbol{\ell} = 10}$$

$$\nu = \boldsymbol{\ell} + 8$$

$$\nu = 10 + 8$$

$$\boxed{\nu = 18}$$

$$\varepsilon = \nu - 1$$

Rješenje

$\boldsymbol{\ell} \longleftarrow$ broj listova u stablu T

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$\nu = \boldsymbol{\ell} + 8$

$\nu = 10 + 8$

$\boxed{\nu = 18}$

$\varepsilon = \nu - 1$

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Rješenje

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$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

$$\boxed{\boldsymbol{\ell} = 10}$$

$$\nu = \boldsymbol{\ell} + 8$$
$$\nu = 10 + 8$$
$$\boxed{\nu = 18}$$

$$\varepsilon = \nu - 1$$
$$\varepsilon = 18 - 1$$
$$\varepsilon = 17$$

Rješenje

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$$\boldsymbol{\ell} + 24 = 2(\boldsymbol{\ell} + 7)$$

$$\boldsymbol{\ell} + 24 = 2\boldsymbol{\ell} + 14$$

$$\boxed{\boldsymbol{\ell} = 10}$$

$$\nu = \boldsymbol{\ell} + 8$$

$$\nu = 10 + 8$$

$$\boxed{\nu = 18}$$

$$\varepsilon = \nu - 1$$

$$\varepsilon = 18 - 1$$

$$\boxed{\varepsilon = 17}$$

drugi zadatak

Teorem (rekurzija za broj razapinjućih stabala)

Neka je $\tau(G)$ broj razapinjućih stabala grafa G. Ako je $e \in E(G)$ karika, tada vrijedi

$$\tau(G) = \tau(G - e) + \tau(G \cdot e).$$

Matrični teorem o stablima

Neka je G povezani graf bez petlji sa skupom vrhova $V = \{v_1, v_2, \ldots, v_n\}$. Neka je $A = [a_{ij}]$ matrica susjedstva grafa G. Neka je $Q = [q_{ij}]$ $n \times n$ matrica za koju vrijedi

$$q_{ij} = \begin{cases} -a_{ij}, & i \neq j \\ d_G(v_i), & i = j \end{cases}.$$

Tada je $\tau(G)$ jednak bilo kojem kofaktoru matrice Q.

Zadatak 2

Graf G zadan je matricom susjedstva

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}.$$

a) *Bez crtanja grafa G odredite ukupni broj petlji u grafu i stupnjeve njegovih vrhova.*

b) *Nacrtajte graf G i pomoću rekurzije odredite ukupni broj razapinjućih stabala grafa G.*

c) *Pomoću matričnog teorema o stablima odredite ukupni broj razapinjućih stabala grafa G.*

d) *Je li G Hamiltonov graf? Postoji li Hamiltonov put u grafu G?*

Rješenje

a)

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} =$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) =$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} \boxed{1 \quad 3 \quad 0 \quad 0} \\ 3 \quad 0 \quad 1 \quad 1 \\ 0 \quad 1 \quad 2 \quad 2 \\ 0 \quad 1 \quad 2 \quad 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$

$$d(v_2) =$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ \boxed{3 \quad 0 \quad 1 \quad 1} \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$

$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$
$$d(v_3) =$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$
$$d(v_3) = 0 + 1 + 2 \cdot 2 + 2$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$
$$d(v_3) = 0 + 1 + 2 \cdot 2 + 2 = 7$$

Rješenje

a)

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$
$$d(v_3) = 0 + 1 + 2 \cdot 2 + 2 = 7$$
$$d(v_4) =$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ \boxed{0 \quad 1 \quad 2 \quad 0} \end{bmatrix} \end{array}$$

$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$
$$d(v_3) = 0 + 1 + 2 \cdot 2 + 2 = 7$$
$$d(v_4) = 0 + 1 + 2 + 2 \cdot 0$$

Rješenje

a)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

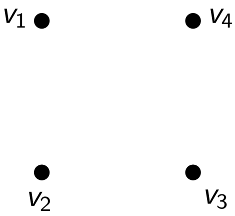
$$\text{broj petlji} = 1 + 0 + 2 + 0 = 3$$

$$d(v_1) = 2 \cdot 1 + 3 + 0 + 0 = 5$$
$$d(v_2) = 3 + 2 \cdot 0 + 1 + 1 = 5$$
$$d(v_3) = 0 + 1 + 2 \cdot 2 + 2 = 7$$
$$d(v_4) = 0 + 1 + 2 + 2 \cdot 0 = 3$$

b)

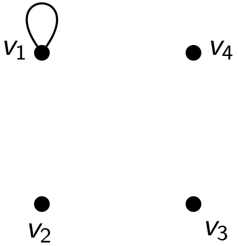
$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)



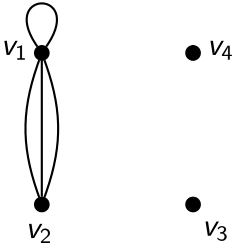
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)



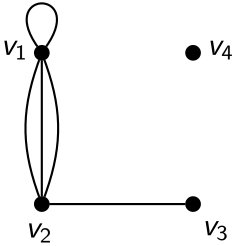
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)



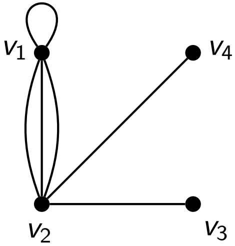
$$A = \begin{array}{c c} & \begin{array}{cccc} \color{blue}{v_1} & \color{blue}{v_2} & \color{blue}{v_3} & \color{blue}{v_4} \end{array} \\ \begin{array}{c} \color{blue}{v_1} \\ \color{blue}{v_2} \\ \color{blue}{v_3} \\ \color{blue}{v_4} \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)



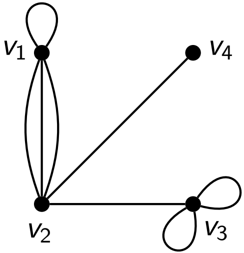
$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)



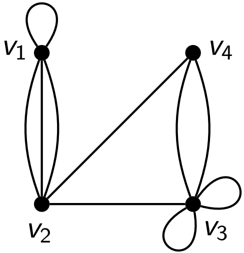
$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)



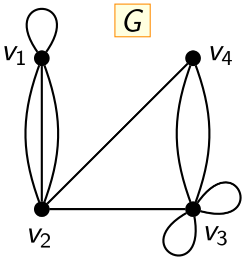
$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)

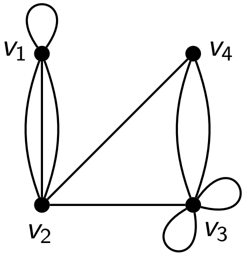


$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

b)

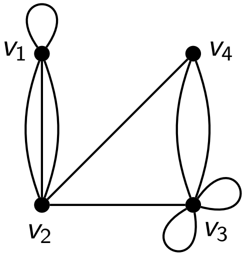


$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$



$=$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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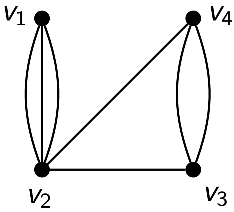
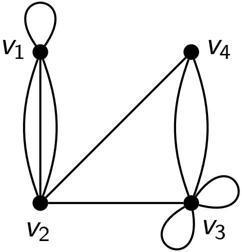


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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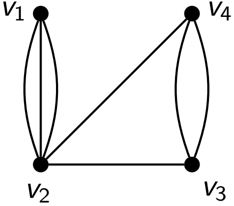

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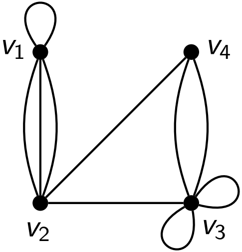
$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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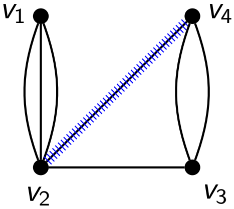

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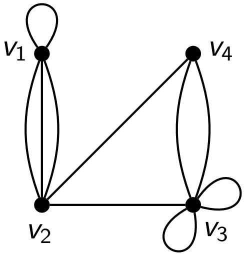
$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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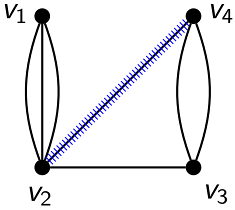

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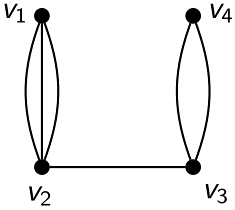
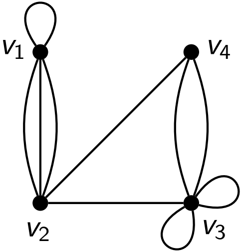


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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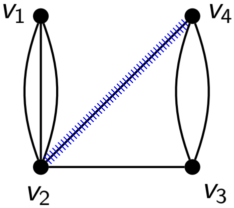

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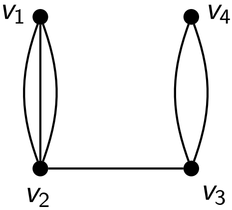

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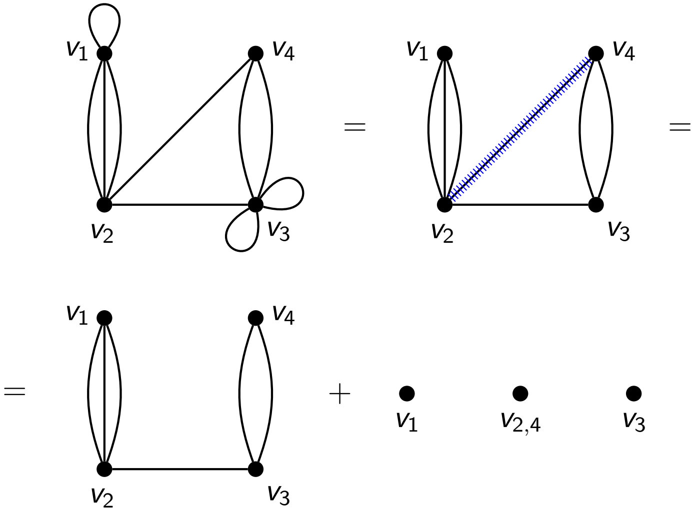
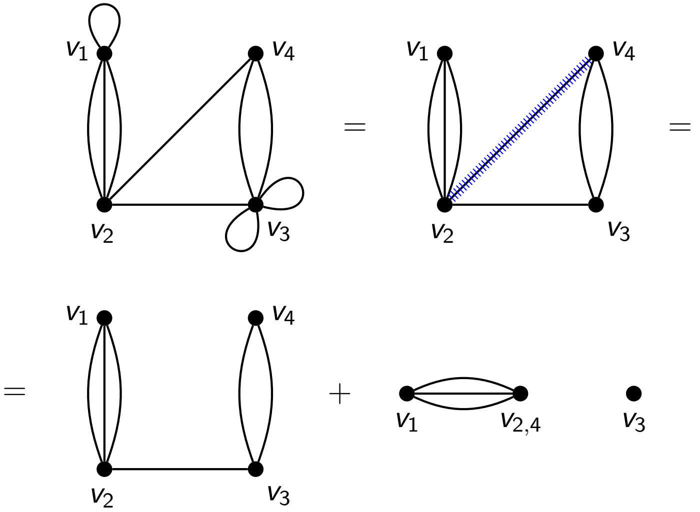
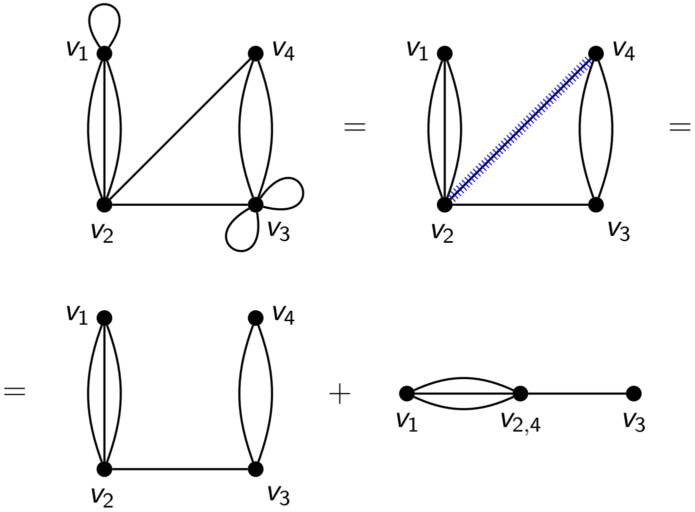
$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



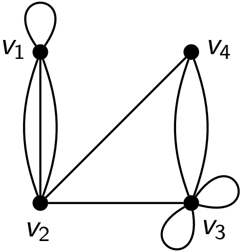
$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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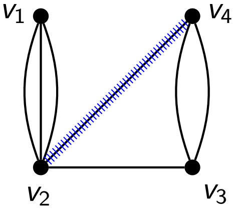

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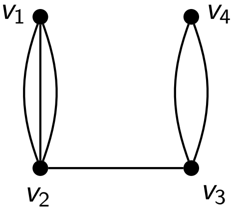

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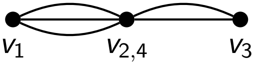
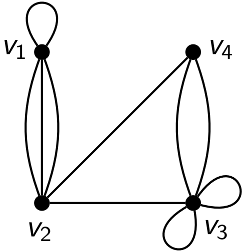


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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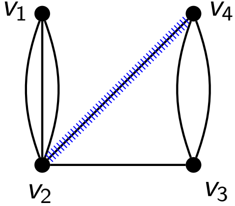

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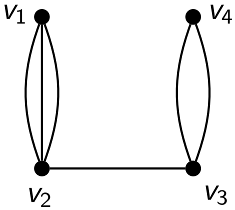

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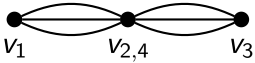
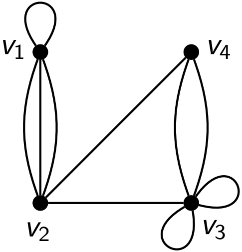


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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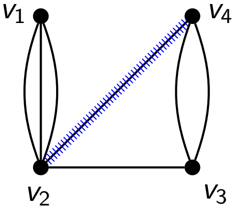

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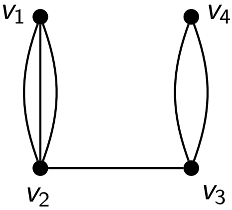

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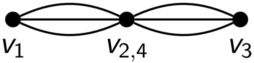

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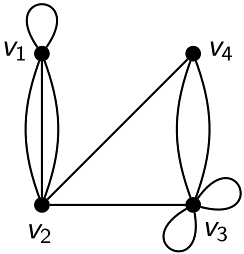
$=$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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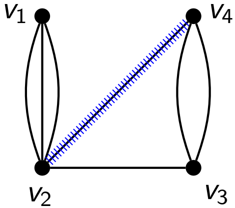

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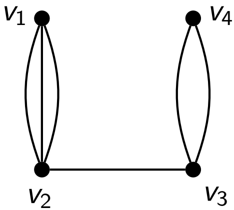

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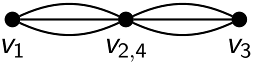
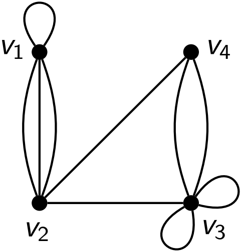


$=$

$$= 3 \cdot 1 \cdot 2$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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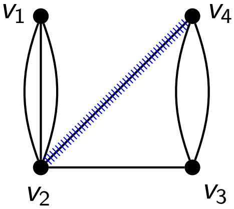

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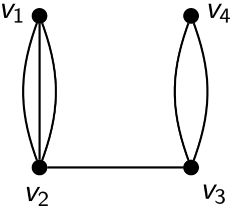

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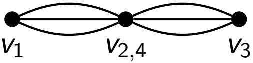
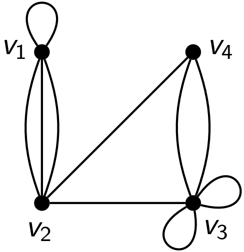


$=$

$= 3 \cdot 1 \cdot 2 +$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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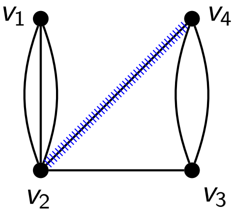

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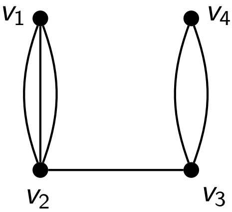

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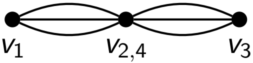
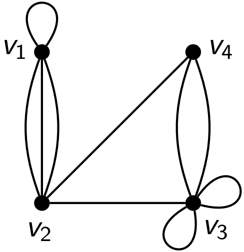


$=$

$$= 3 \cdot 1 \cdot 2 + 3 \cdot 3$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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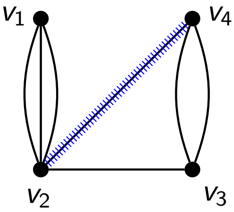

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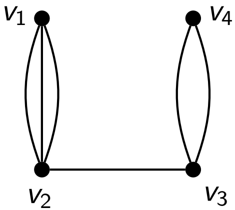

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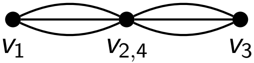
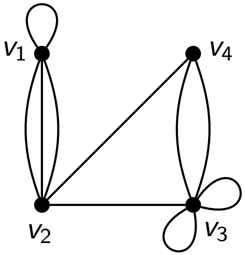


$=$

$$= 3 \cdot 1 \cdot 2 + 3 \cdot 3 = 15$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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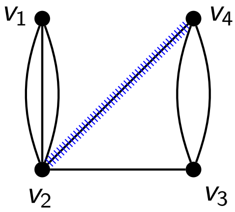

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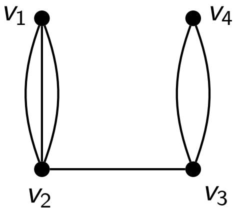

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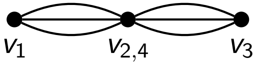


$=$

$$= 3 \cdot 1 \cdot 2 + 3 \cdot 3 = 15$$

$$\boxed{\tau(G) = 15}$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

c)

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ & 0 & 1 & 1 \\ & & 0 & 2 \\ & & & 0 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & & & \\ & 5 & & \\ & & & \\ & & & \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & & & \\ & 5 & & \\ & & 3 & \\ & & & \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & & & \\ & 5 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & & \\ & 5 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & \\ & 5 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ & 5 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & & 3 \end{bmatrix}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$Q_{21} =$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$Q_{21} = (-1)^{2+1}$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$Q_{21} = (-1)^{2+1} \cdot$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$Q_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 0 & 0 \\ -1 & 3 & -2 \\ -1 & -2 & 3 \end{vmatrix}$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$Q_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 0 & 0 \\ -1 & 3 & -2 \\ -1 & -2 & 3 \end{vmatrix} = -1 \cdot (-15)$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

$$Q_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 0 & 0 \\ -1 & 3 & -2 \\ -1 & -2 & 3 \end{vmatrix} = -1 \cdot (-15) = 15$$

$$\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$$

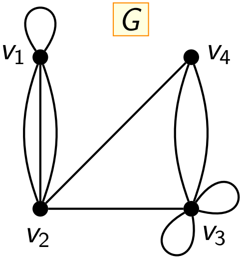
c) Najprije treba ukloniti sve petlje iz grafa G.

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 5 & -1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

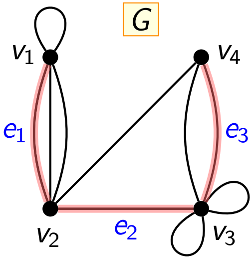
$$Q_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 0 & 0 \\ -1 & 3 & -2 \\ -1 & -2 & 3 \end{vmatrix} = -1 \cdot (-15) = 15$$

$$\boxed{\tau(G) = 15}$$

d)

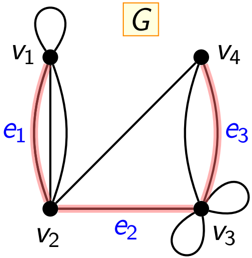


d) G ima Hamiltonov put, npr. $v_1 e_1 v_2 e_2 v_3 e_3 v_4$.



d) G ima Hamiltonov put, npr. $v_1 e_1 v_2 e_2 v_3 e_3 v_4$.

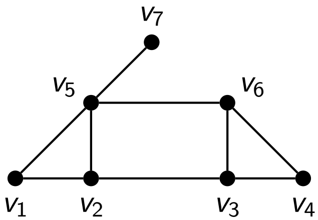
G nije Hamiltonov graf jer je v_1 problematični vrh.



treći zadatak

Zadatak 3

Zadan je graf G.



a) *Pomoću rekurzije odredite ukupni broj razapinjućih stabala grafa G.*

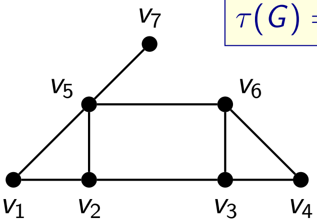
b) *Pomoću BFS algoritma pronađite jedno razapinjuće stablo grafa G.*

c) *Pomoću DFS algoritma pronađite jedno razapinjuće stablo grafa G.*

Rješenje

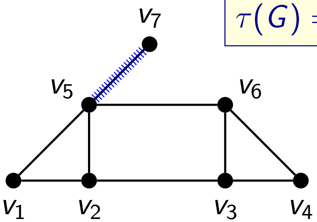
$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

a)



Rješenje

a)



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$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

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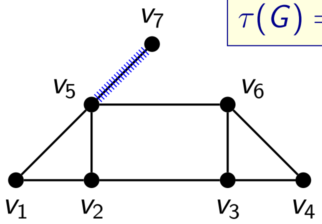
Rješenje

a)

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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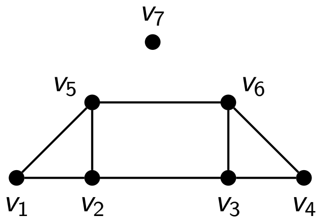
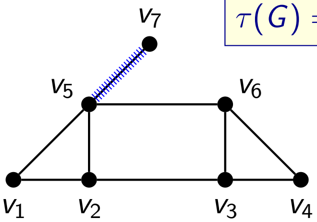


Rješenje

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

a)



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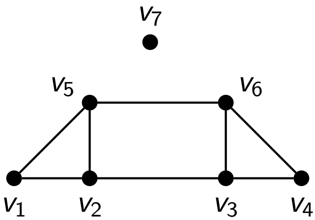

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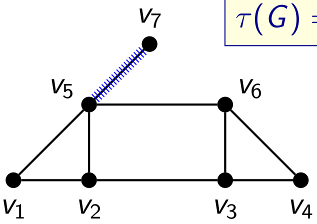
Rješenje

a)

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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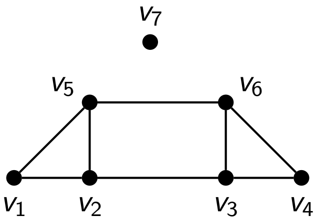

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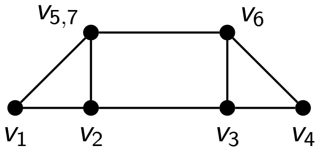
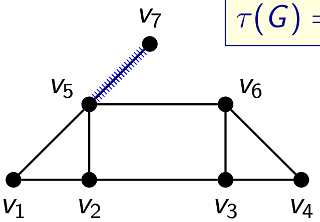


Rješenje

a)

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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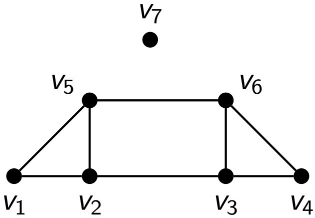

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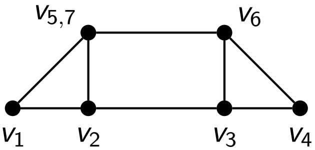

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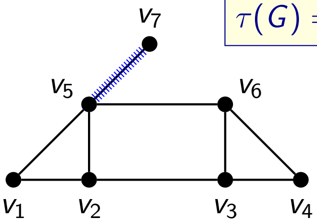
Rješenje

a)

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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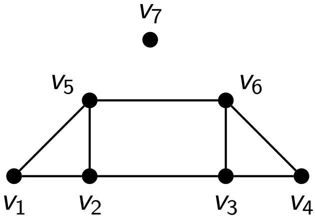

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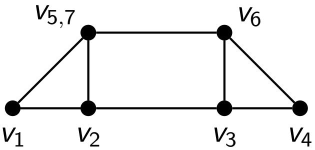

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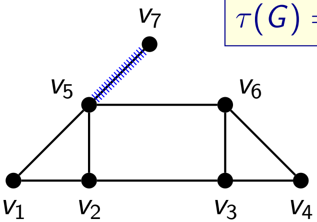
Rješenje

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

a)



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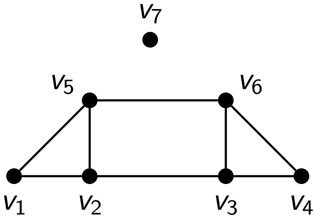

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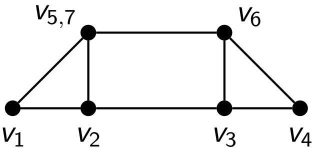

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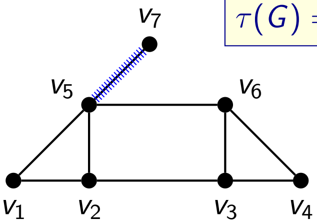
Rješenje

a)

$$\boxed{\tau(G) = \tau(G-e) + \tau(G \cdot e)}$$



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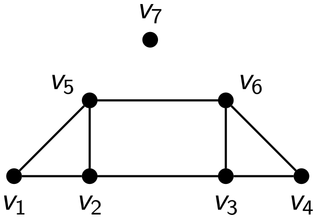

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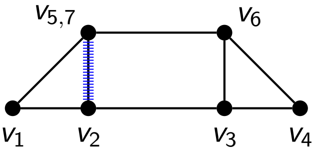

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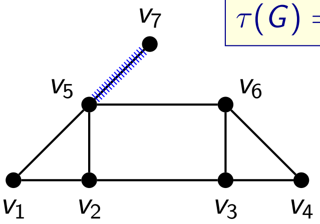
Rješenje

a)



$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

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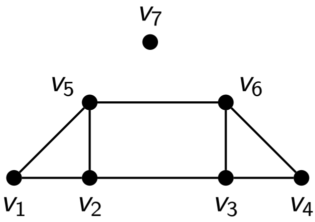

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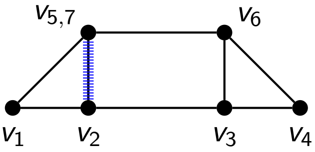

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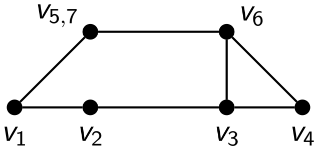
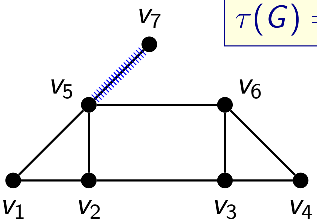


Rješenje

a)

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$



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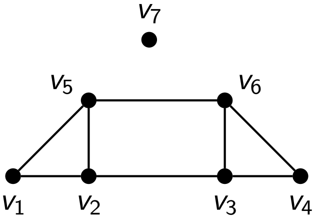

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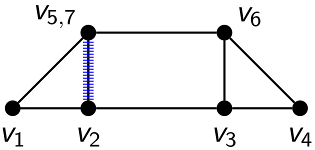

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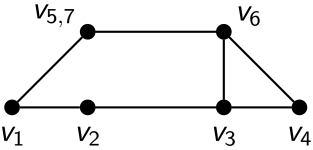

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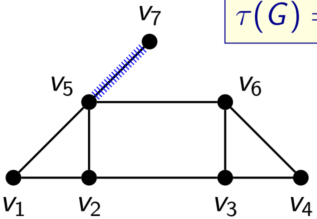
Rješenje

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

a)



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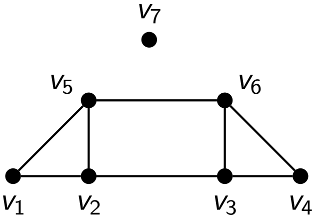

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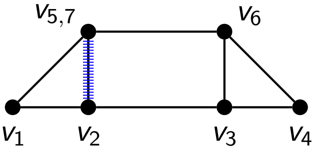

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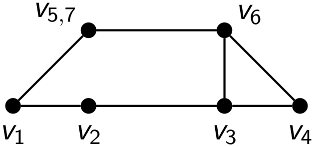

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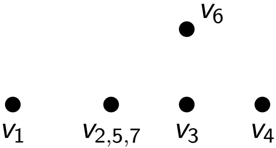

Rješenje

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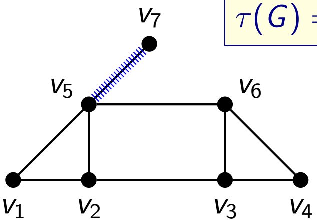

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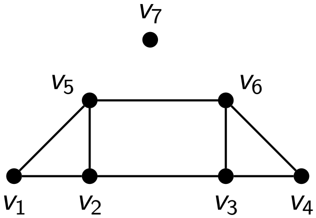

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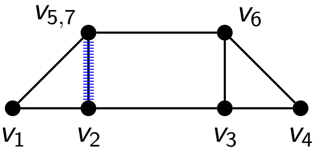

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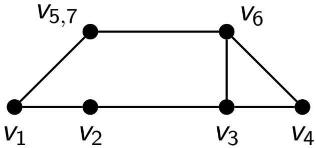

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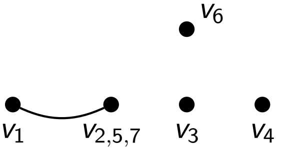

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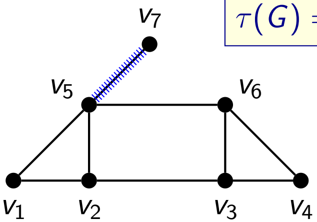

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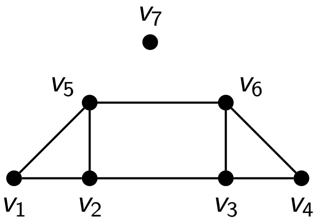

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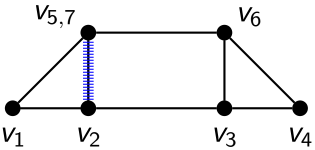

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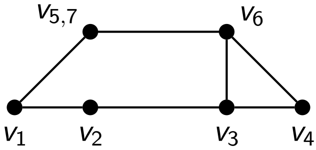

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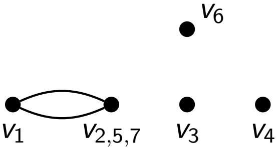

Rješenje

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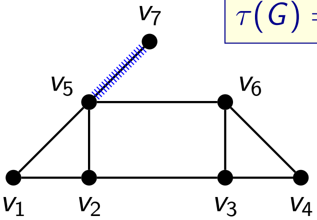

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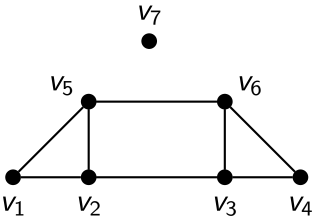

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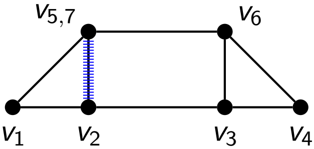

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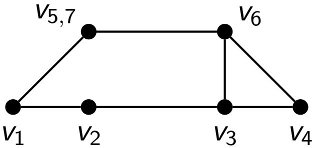

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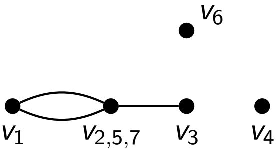

Rješenje

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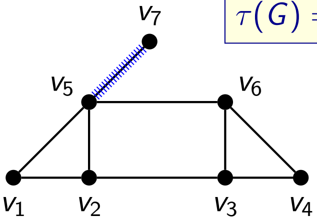
$$\tau(G) = \tau(G - e) + \tau(G \cdot e)$$



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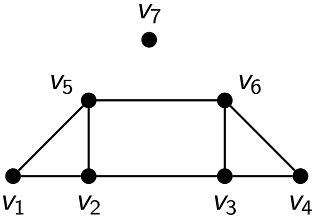

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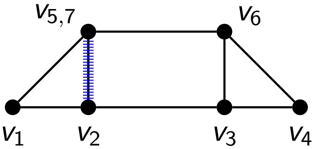

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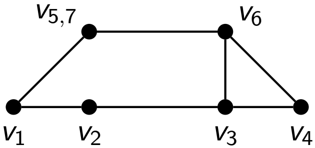

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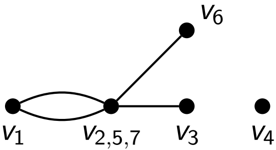

Rješenje

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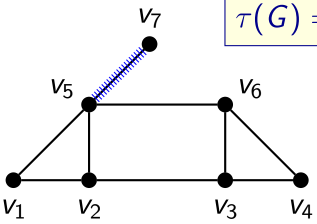
$$\boxed{\tau(G) = \tau(G-e) + \tau(G \cdot e)}$$



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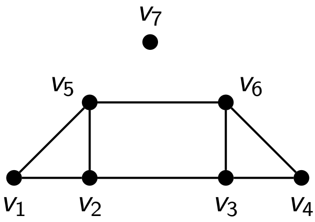

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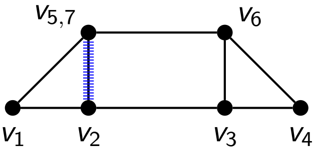

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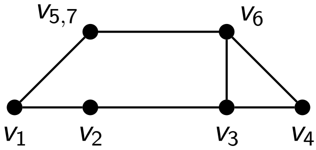

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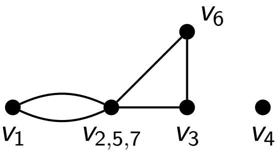
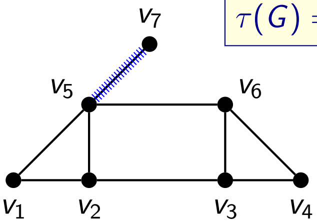


Rješenje

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

a)



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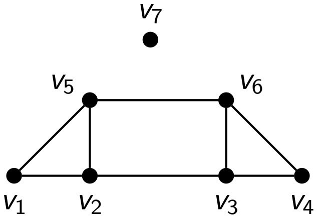

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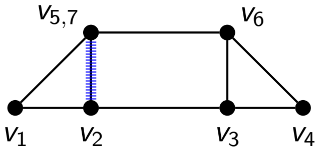

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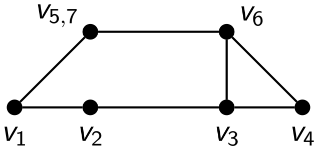

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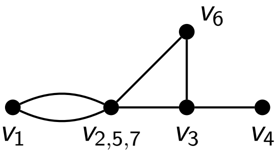

Rješenje

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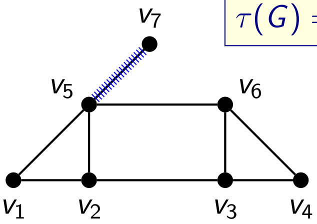

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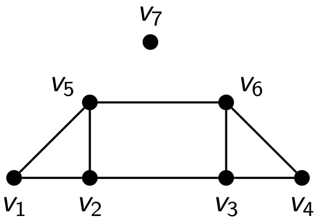

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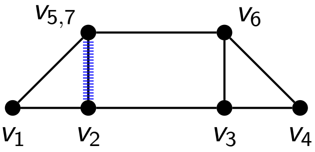

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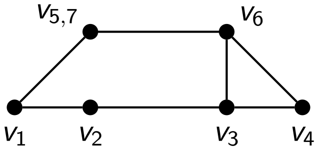

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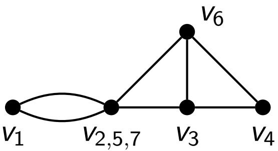

Rješenje

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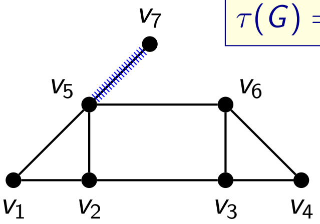

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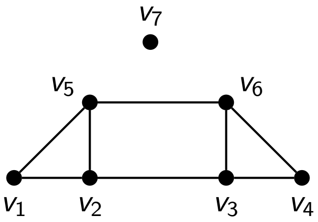

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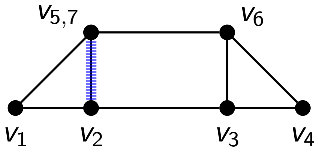

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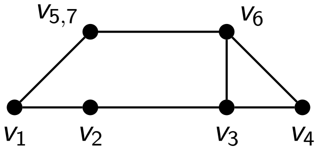

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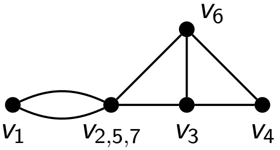

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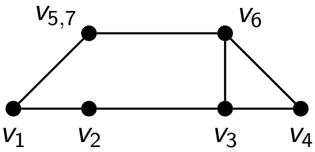

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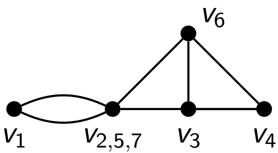
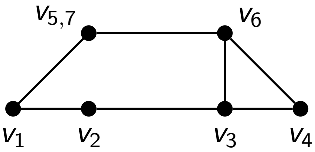
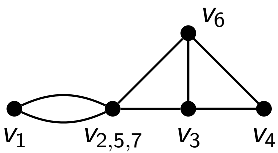


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$ 

$+$ 

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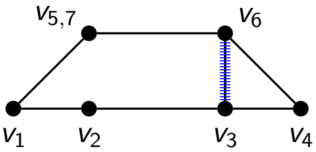

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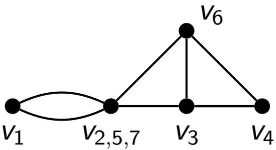

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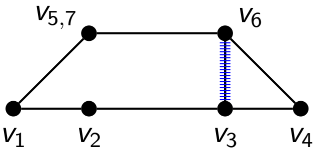

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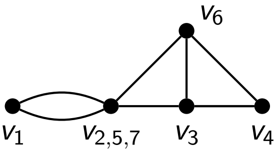

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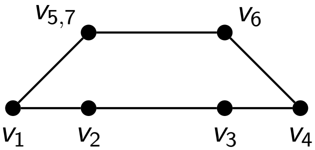

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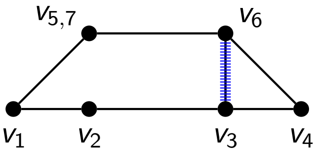
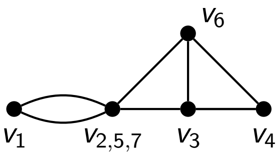
$= 0 +$ 

$+$ 

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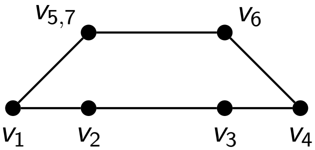
$=$ 

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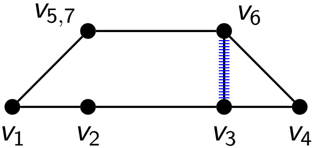
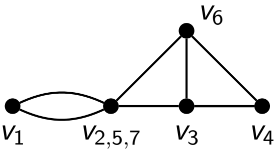
$= 0 +$ 

$+$ 

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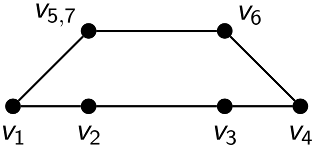
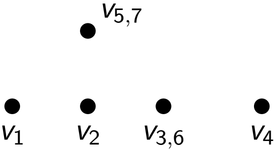
$=$ 

$+$ 

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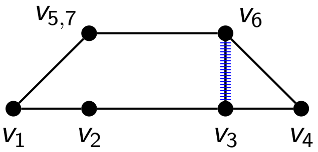

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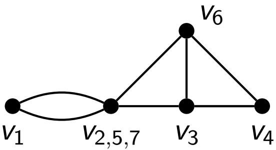

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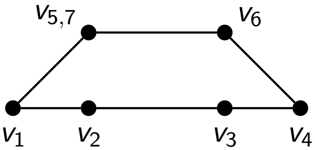

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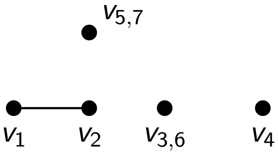
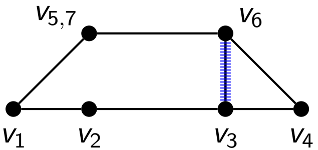


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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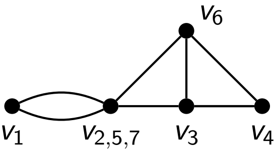

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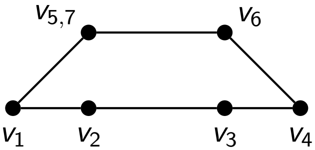

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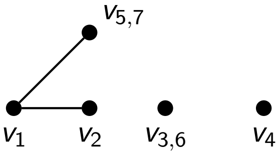

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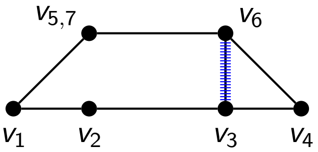

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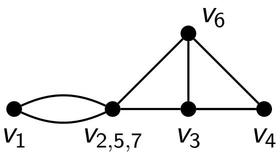

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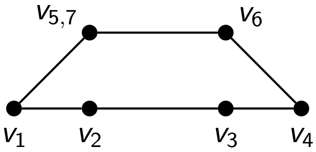

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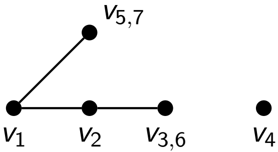

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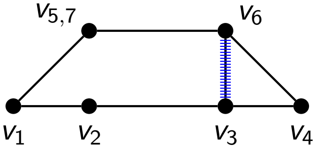

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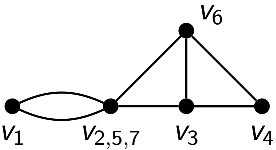

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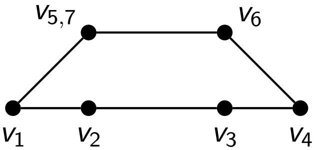

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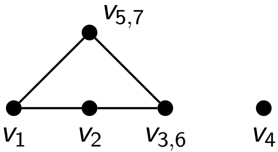

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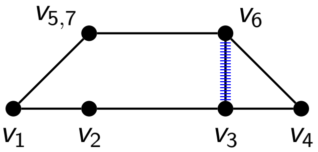
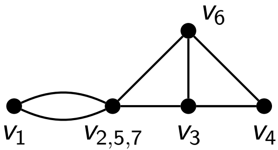
$= 0 +$ 

$+$ 

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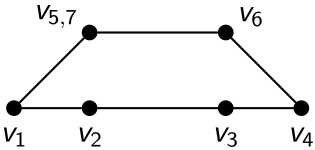
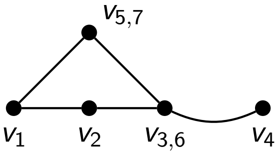
$=$ 

$+$ 

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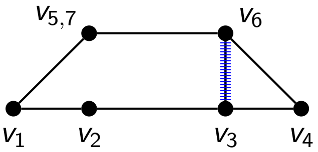
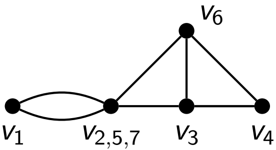
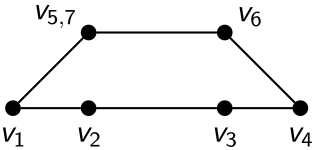
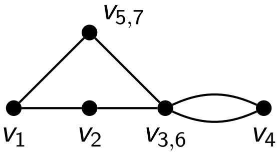
$= 0 +$  $+$  $=$

$=$  $+$ 

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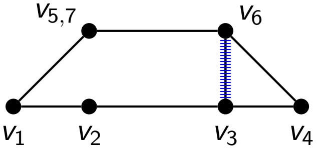
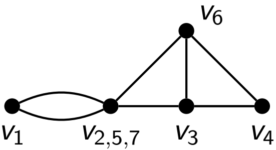
$= 0 +$ 

$+$ 

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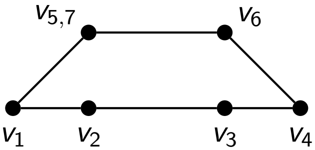
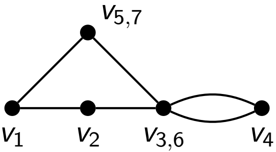
$=$ 

$+$ 

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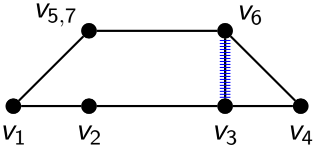
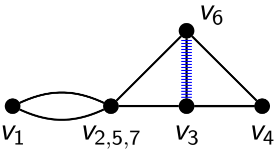
$= 0 +$ 

$+$ 

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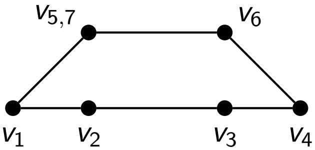
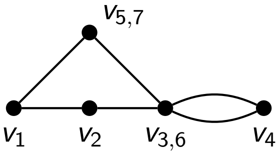
$=$ 

$+$ 

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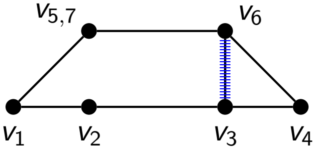

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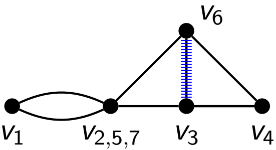

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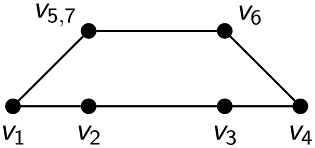

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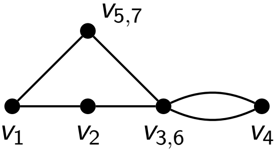

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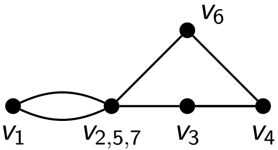

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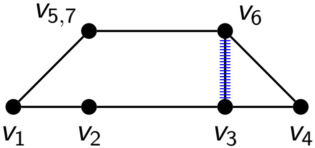

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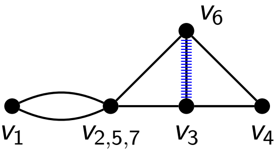

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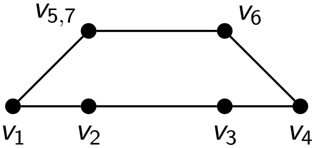

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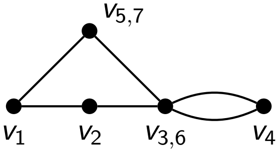

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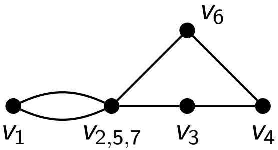

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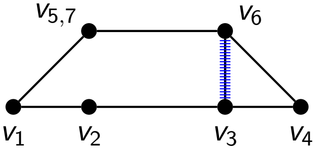

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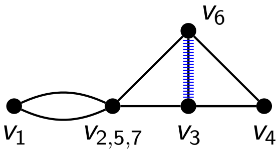

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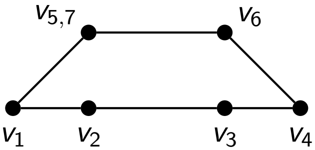

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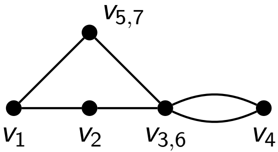

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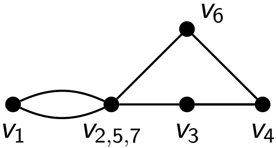

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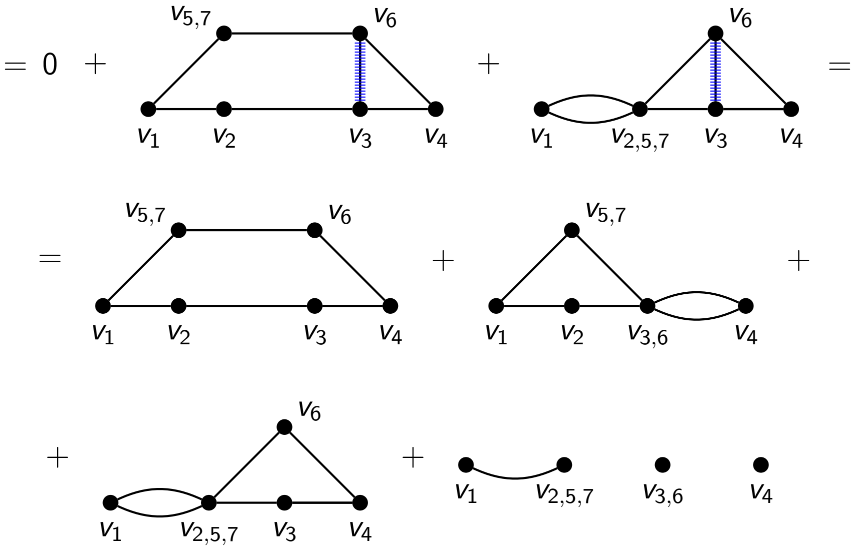

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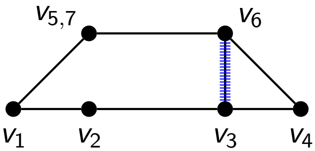
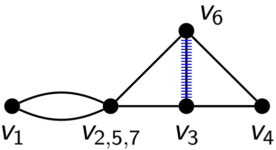
$= 0 +$ 

$+$ 

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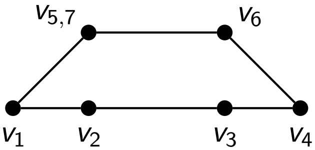
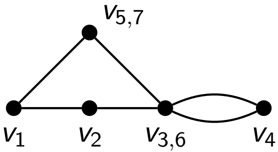
$=$ 

$+$ 

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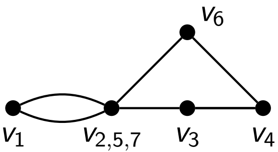
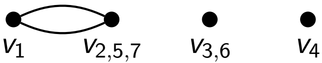
$+$ 

$+$ 

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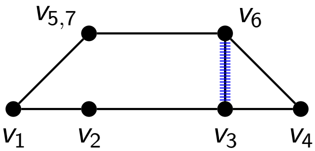

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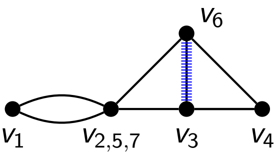

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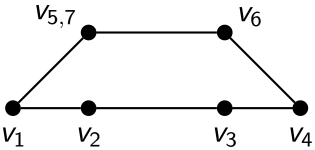

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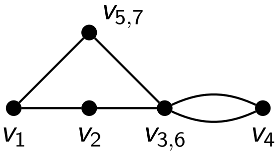

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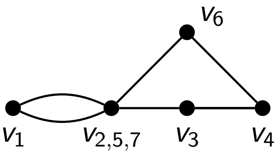

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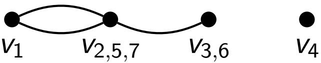

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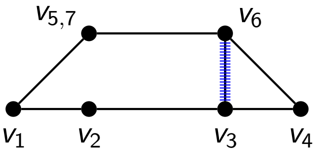

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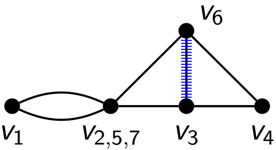

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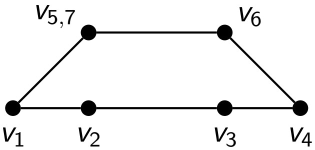

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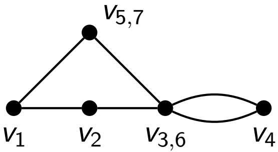

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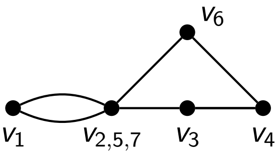

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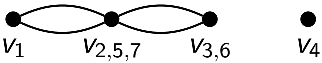

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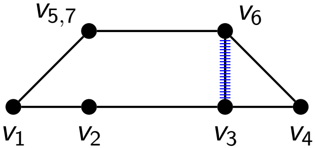

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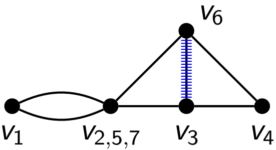

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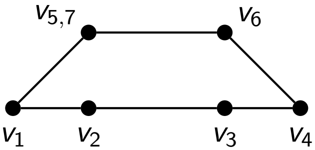

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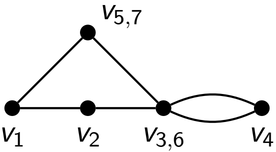

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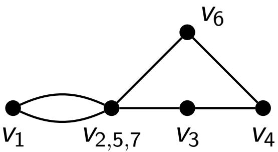

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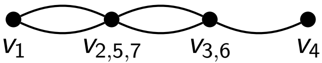
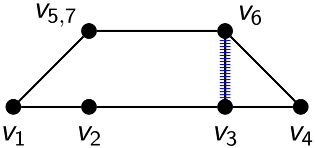


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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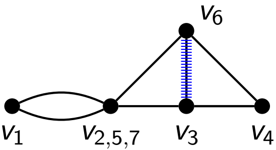

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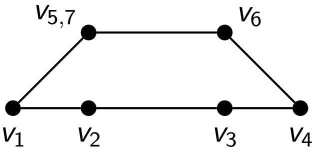

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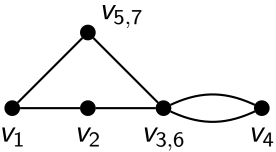

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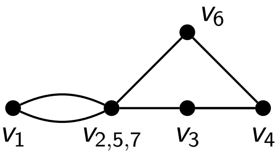

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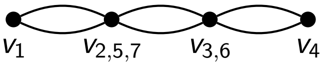
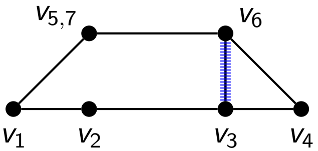


$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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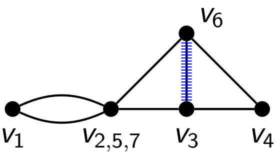

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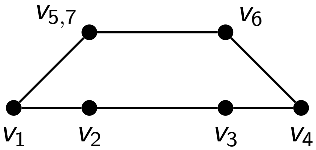

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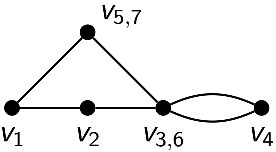

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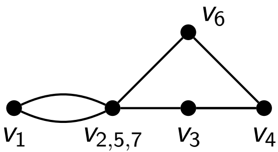

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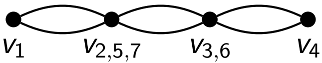

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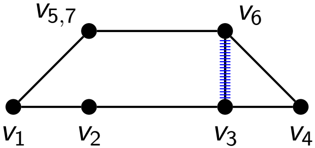
$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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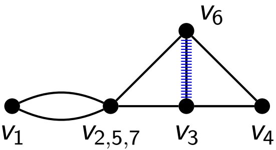

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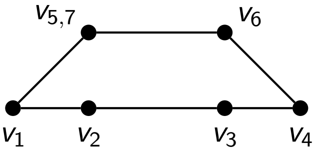

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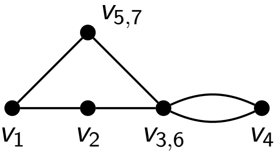

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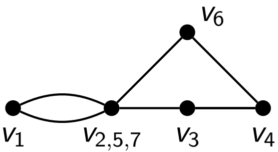

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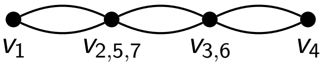

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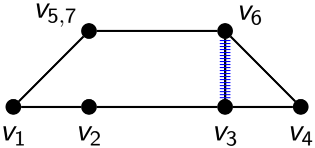
$=$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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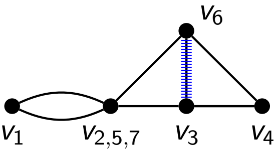

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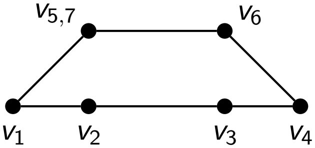

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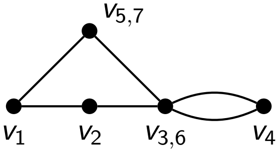

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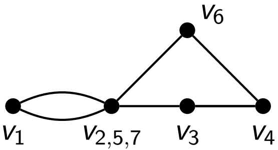

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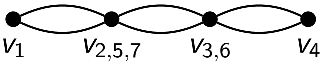
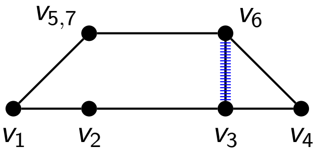


$=$

$= 6$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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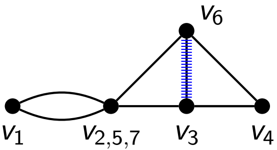

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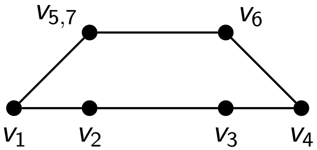

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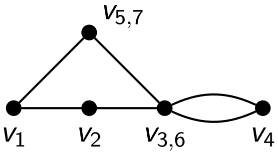

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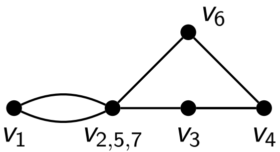

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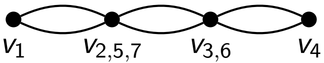
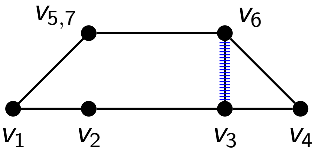


$=$

$= 6 +$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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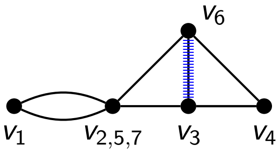

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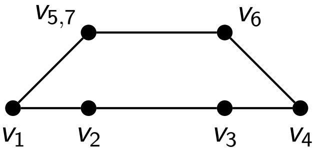

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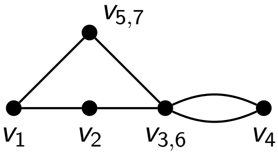

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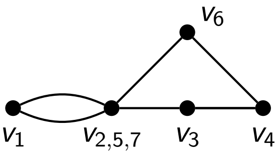

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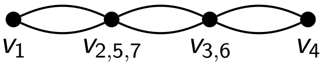
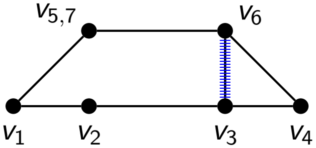


$=$

$= 6 + 4 \cdot 2$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 \; +$



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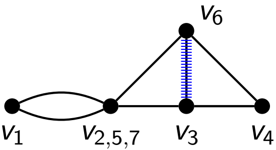

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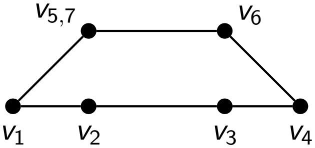

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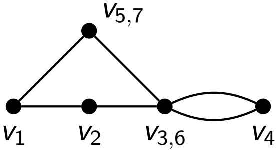

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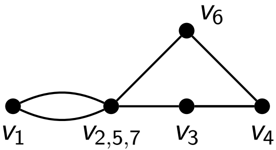

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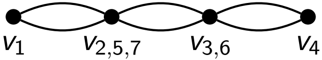
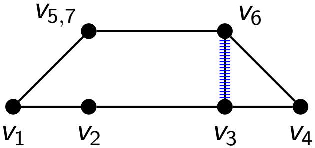


$=$

$= 6 + 4 \cdot 2 +$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 \;+$



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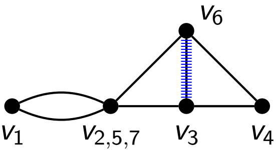

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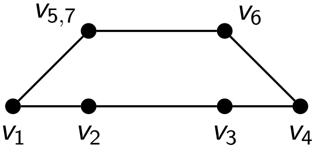

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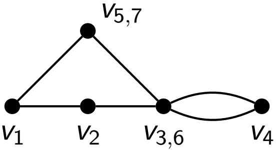

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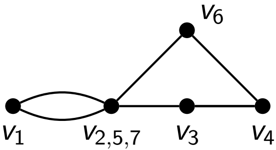

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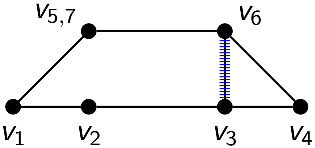
$=$

$$= 6 + 4 \cdot 2 + 2 \cdot 4$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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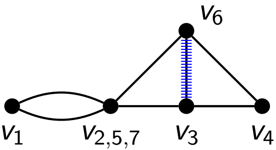

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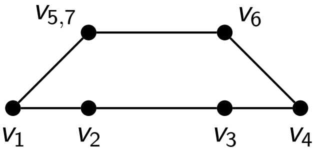

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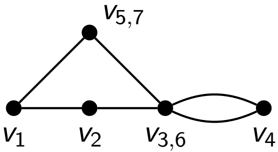

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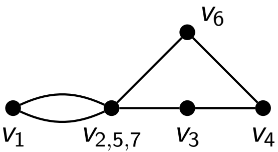

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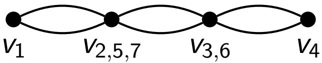
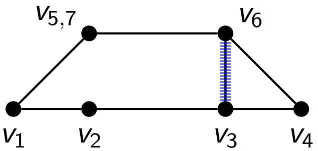


$=$

$= 6 + 4 \cdot 2 + 2 \cdot 4 +$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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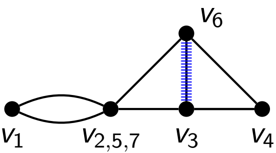

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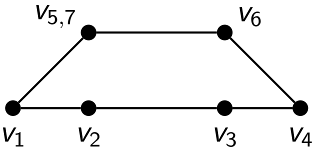

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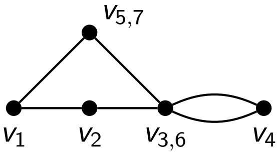

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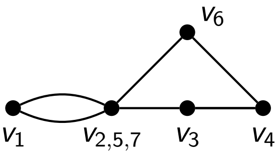

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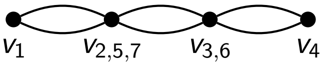
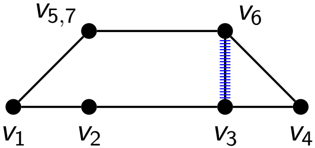


$=$

$$= 6 + 4 \cdot 2 + 2 \cdot 4 + 2 \cdot 2 \cdot 2$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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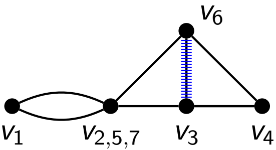

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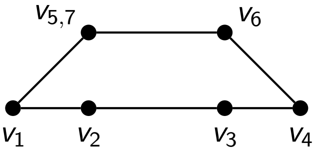

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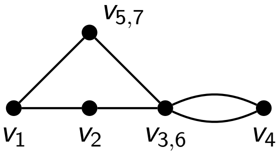

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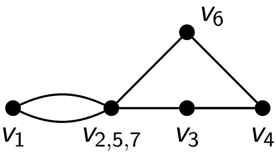

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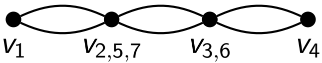
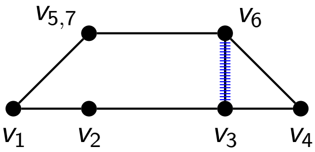


$=$

$$= 6 + 4 \cdot 2 + 2 \cdot 4 + 2 \cdot 2 \cdot 2 = 30$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

$= 0 +$



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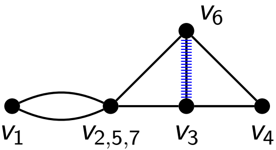

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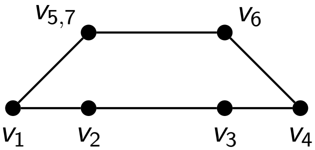

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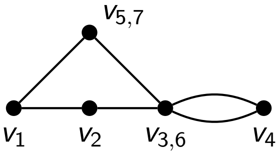
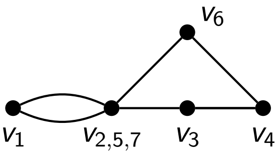
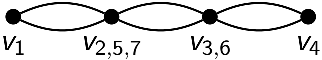


$+$



$+$



$=$

$$= 6 + 4 \cdot 2 + 2 \cdot 4 + 2 \cdot 2 \cdot 2 = 30$$

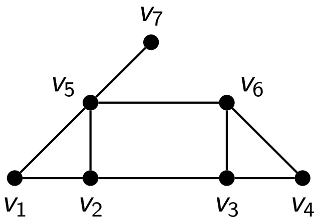
$$\boxed{\tau(G) = 30}$$

$$\boxed{\tau(G) = \tau(G - e) + \tau(G \cdot e)}$$

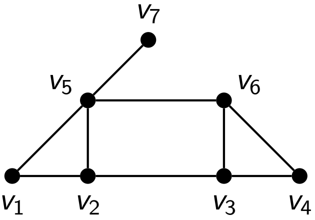
Implementacija BFS algoritma

procedure $\mathrm{BFS}(G, s)$
    for $u \in V(G) \setminus \{s\}$ **do** ▷ inicijalizacija za sve vrhove osim vrha s
        color$[u] \leftarrow$ WHITE ▷ vrhovi su bijele boje
        $d[u] \leftarrow \infty$ ▷ vrhovi su na udaljenosti ∞ od vrha s
        $\pi[u] \leftarrow$ NIL ▷ vrhovi nemaju roditelje
    color$[s] \leftarrow$ GRAY ▷ vrh s je sive boje (vrh je posjećen, ali nije istražen)
    $d[s] \leftarrow 0$ ▷ vrh s je na udaljenosti 0 od samoga sebe
    $\pi[s] \leftarrow$ NIL ▷ vrh s nema roditelja
    $Q \leftarrow \emptyset$ ▷ inicijalizacija reda Q koji će se puniti sa sivim vrhovima
    ENQUEUE(Q, s) ▷ sivi vrh s stavi na kraj reda Q
    while $Q \neq \emptyset$ **do** ▷ sve dok ima sivih vrhova
        $u \leftarrow$ DEQUEUE(Q) ▷ uzmi sivi vrh u koji je prvi u redu Q
        for $v \in Adj[u]$ **do** ▷ za svaki susjedni vrh v od vrha u
            if color$[v] =$ WHITE **then** ▷ ako je vrh v bijele boje
                color$[v] \leftarrow$ GRAY ▷ pridruži vrhu v sivu boju
                $d[v] \leftarrow d[u] + 1$ ▷ udaljenost v od s je za 1 veća od udaljenosti u od s
                $\pi[v] \leftarrow u$ ▷ vrh u je roditelj vrha v
                ENQUEUE(Q, v) ▷ stavi sivi vrh v na kraj reda Q
        color$[u] \leftarrow$ BLACK ▷ pridruži vrhu u crnu boju (vrh u je istražen)

b)

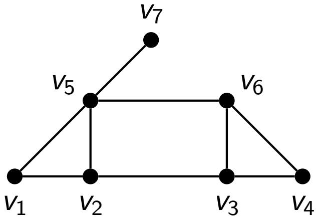


b)



korak	BFS

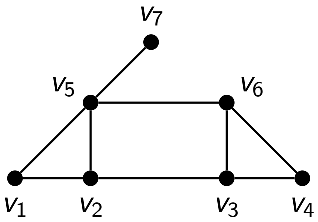
b)



korak	BFS

	$\pi(v)$	$d(v)$
v_1		
v_2		
v_3		
v_4		
v_5		
v_6		
v_7		

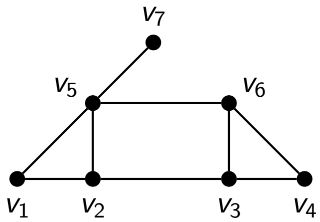
b)



korak	BFS

	$\pi(v)$	$d(v)$
v_1		
v_2		
v_3	—	0
v_4		
v_5		
v_6		
v_7		

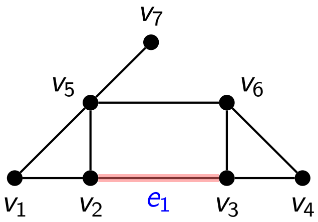
b)



korak	BFS
1	

	$\pi(v)$	$d(v)$
v_1		
v_2		
v_3	—	0
v_4		
v_5		
v_6		
v_7		

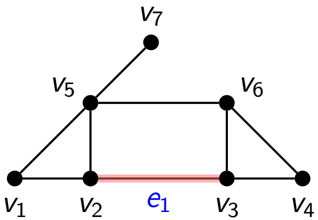
b)



korak	BFS
1	

	$\pi(v)$	$d(v)$
v_1		
v_2		
v_3	—	0
v_4		
v_5		
v_6		
v_7		

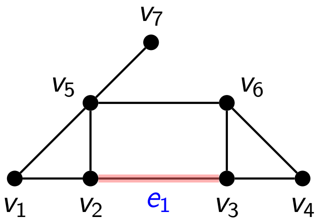
b)

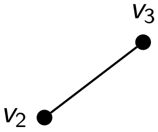


korak	BFS
1	$e_1, \;\; u = v_3$

	$\pi(v)$	$d(v)$
v_1		
v_2		
v_3	—	0
v_4		
v_5		
v_6		
v_7		

b)

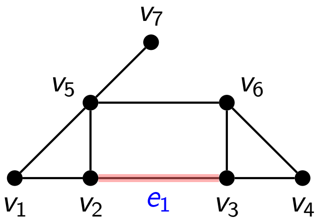


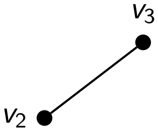


korak	BFS
1	$e_1, \ u = v_3$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	$-$	0
v_4		
v_5		
v_6		
v_7		

b)

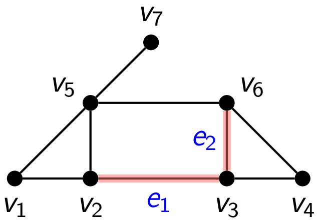


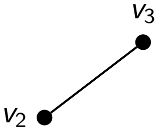


korak	BFS
1	$e_1, \;\; u = v_3$
2	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	$-$	0
v_4		
v_5		
v_6		
v_7		

b)

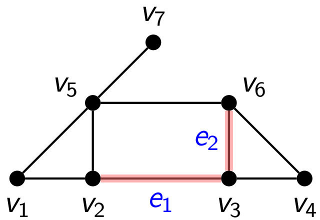


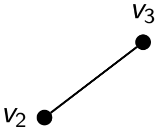


korak	BFS
1	$e_1, \; u = v_3$
2	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4		
v_5		
v_6		
v_7		

b)

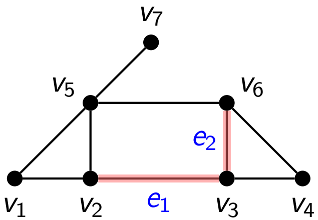


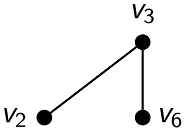


korak	BFS
1	$e_1, \;\; u = v_3$
2	$e_1 e_2, \;\; u = v_3$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4		
v_5		
v_6		
v_7		

b)

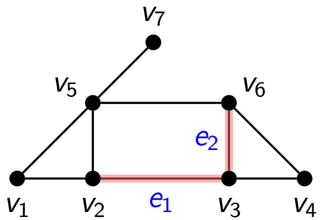


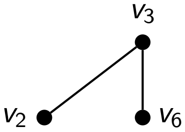


korak	BFS
1	$e_1, \; u = v_3$
2	$e_1 e_2, \; u = v_3$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4		
v_5		
v_6	v_3	1
v_7		

b)

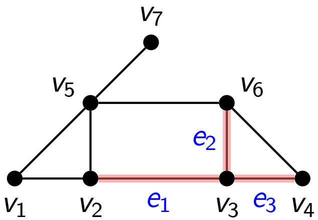


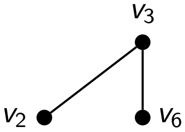


korak	BFS
1	$e_1, \; u = v_3$
2	$e_1 e_2, \; u = v_3$
3	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	–	0
v_4		
v_5		
v_6	v_3	1
v_7		

b)

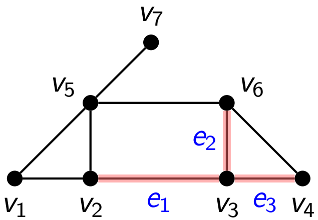


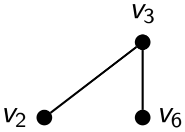


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4		
v_5		
v_6	v_3	1
v_7		

b)

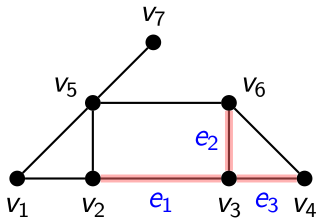


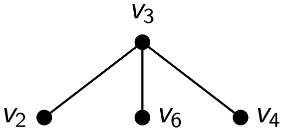


korak	BFS
1	$e_1, \; u = v_3$
2	$e_1 e_2, \; u = v_3$
3	$e_1 e_2 e_3, \; u = v_3$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	$-$	0
v_4		
v_5		
v_6	v_3	1
v_7		

b)

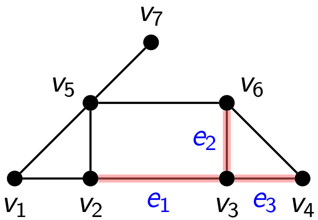


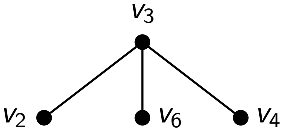


korak	BFS
1	$e_1, \; u = v_3$
2	$e_1 e_2, \; u = v_3$
3	$e_1 e_2 e_3, \; u = v_3$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5		
v_6	v_3	1
v_7		

b)

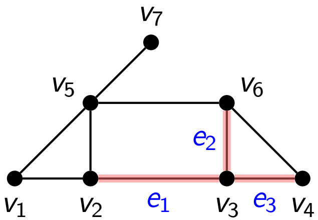


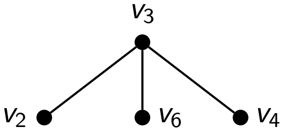


korak	BFS
1	$e_1,\ \ u = v_3$
2	$e_1 e_2,\ \ u = v_3$
3	$e_1 e_2 e_3,\ \ u = v_3$
4	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5		
v_6	v_3	1
v_7		

b)

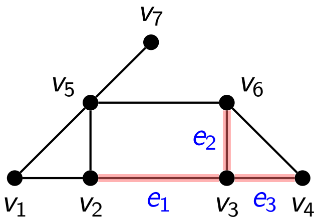


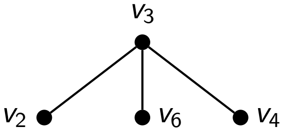


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5		
v_6	v_3	1
v_7		

b)

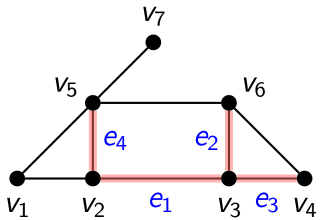


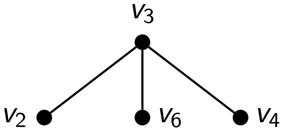


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	$-$	0
v_4	v_3	1
v_5		
v_6	v_3	1
v_7		

b)

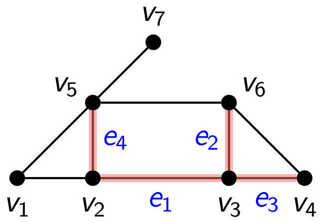


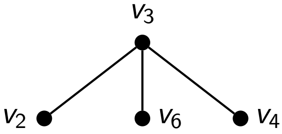


korak	BFS
1	$e_1, \; u = v_3$
2	$e_1 e_2, \; u = v_3$
3	$e_1 e_2 e_3, \; u = v_3$
4	$e_1 e_2 e_3, \; u = v_2$
5	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5		
v_6	v_3	1
v_7		

b)

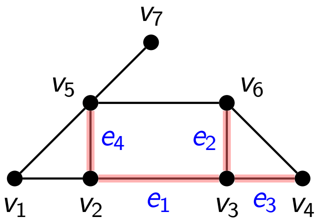


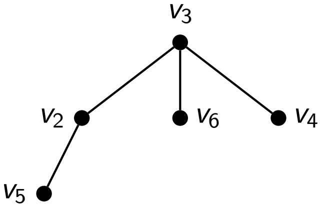


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5		
v_6	v_3	1
v_7		

b)

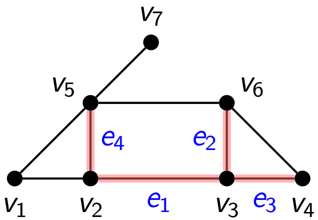


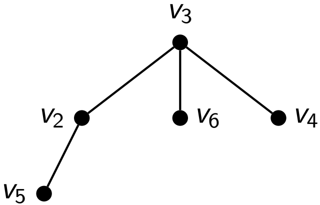


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

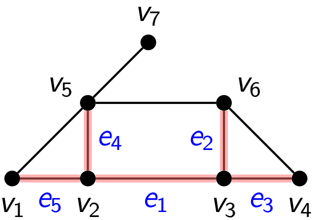


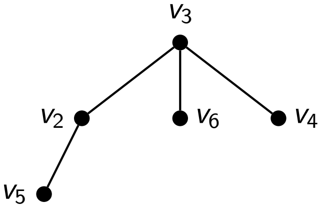


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

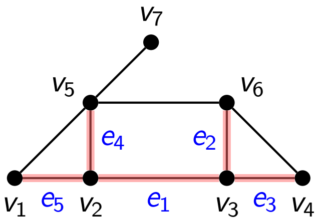


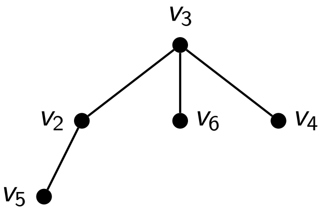


korak	BFS
1	$e_1, \;\; u = v_3$
2	$e_1 e_2, \;\; u = v_3$
3	$e_1 e_2 e_3, \;\; u = v_3$
4	$e_1 e_2 e_3, \;\; u = v_2$
5	$e_1 e_2 e_3 e_4, \;\; u = v_2$
6	

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

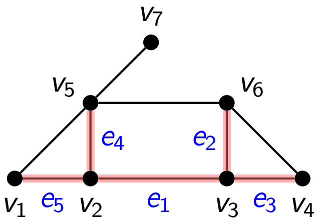


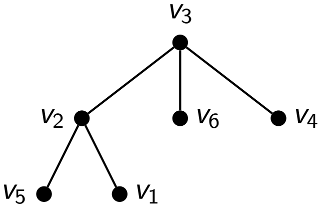


korak	BFS
1	$e_1, \; u = v_3$
2	$e_1 e_2, \; u = v_3$
3	$e_1 e_2 e_3, \; u = v_3$
4	$e_1 e_2 e_3, \; u = v_2$
5	$e_1 e_2 e_3 e_4, \; u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_2$

	$\pi(v)$	$d(v)$
v_1		
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

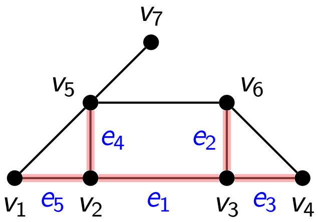


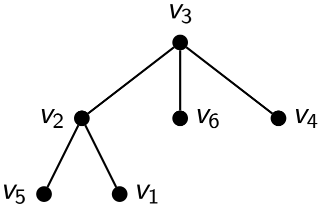


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_2$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)





korak	BFS
1	$e_1, \ u = v_3$
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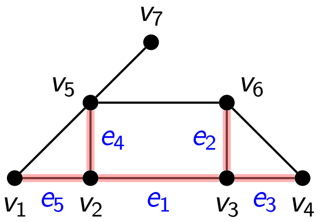
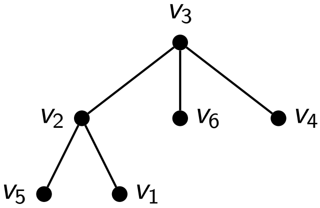
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_2$
7	

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	$-$	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

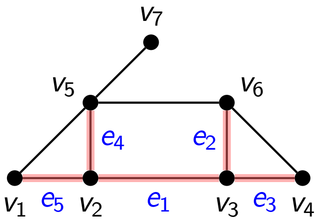


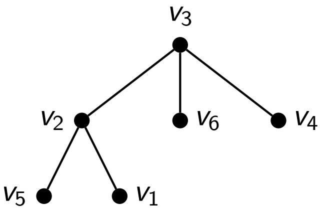


korak	BFS
1	$e_1,\ \ u = v_3$
2	$e_1 e_2,\ \ u = v_3$
3	$e_1 e_2 e_3,\ \ u = v_3$
4	$e_1 e_2 e_3,\ \ u = v_2$
5	$e_1 e_2 e_3 e_4,\ \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5,\ \ u = v_2$
7	$e_1 e_2 e_3 e_4 e_5,\ \ u = v_6$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

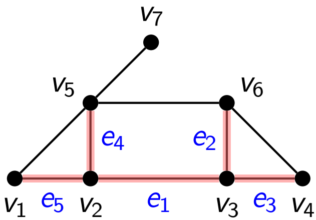


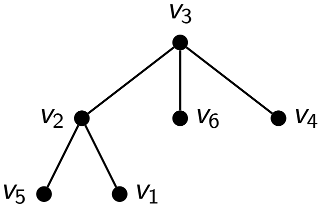


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	$-$	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

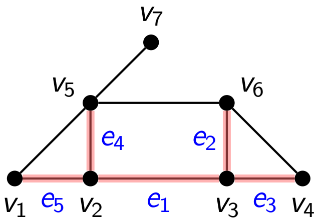


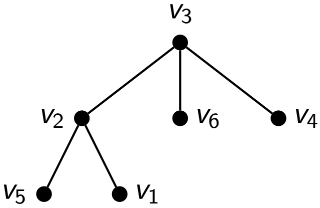


korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	$-$	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)

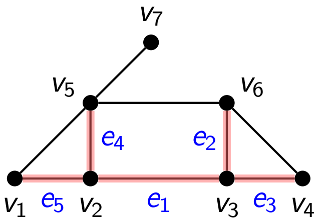


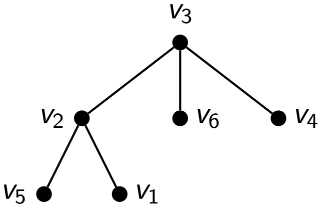


korak	BFS
1	$e_1, \;\; u = v_3$
2	$e_1 e_2, \;\; u = v_3$
3	$e_1 e_2 e_3, \;\; u = v_3$
4	$e_1 e_2 e_3, \;\; u = v_2$
5	$e_1 e_2 e_3 e_4, \;\; u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_4$
9	

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)





korak	BFS
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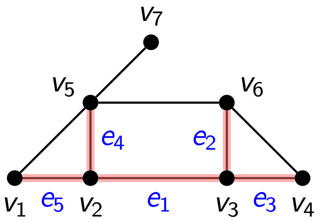
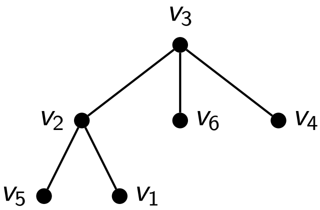
5	$e_1 e_2 e_3 e_4, \;\; u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_5$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	$-$	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)





korak	BFS
1	$e_1, \ u = v_3$
2	$e_1 e_2, \ u = v_3$
3	$e_1 e_2 e_3, \ u = v_3$
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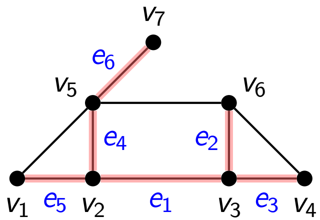
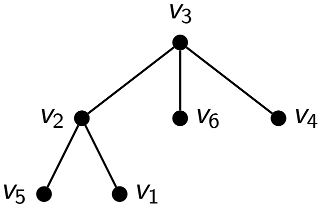
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
10	

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)





korak	BFS
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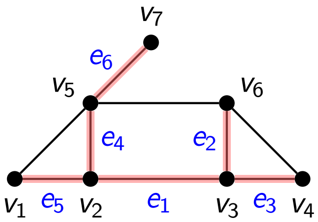
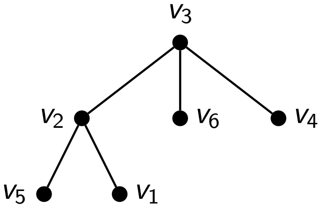
3	$e_1 e_2 e_3, \;\; u = v_3$
4	$e_1 e_2 e_3, \;\; u = v_2$
5	$e_1 e_2 e_3 e_4, \;\; u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_5$
10	

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)





korak	BFS
1	$e_1, \ u = v_3$
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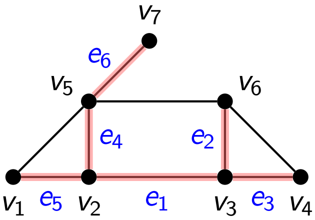
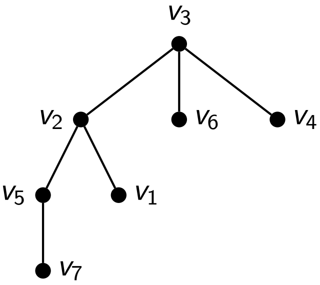
4	$e_1 e_2 e_3, \ u = v_2$
5	$e_1 e_2 e_3 e_4, \ u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
10	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_5$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7		

b)





korak	BFS
1	$e_1, \;\; u = v_3$
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6	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_2$
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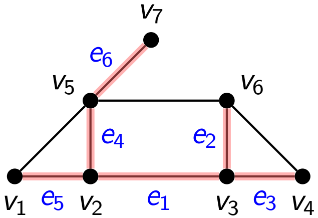
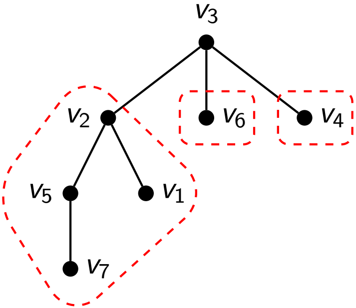
8	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_5$
10	$e_1 e_2 e_3 e_4 e_5 e_6, \;\; u = v_5$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	$-$	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7	v_5	3

b)

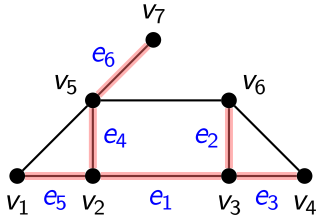


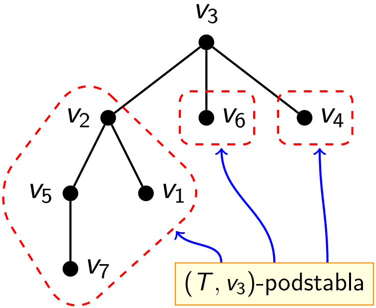


korak	BFS
1	$e_1, \quad u = v_3$
2	$e_1 e_2, \quad u = v_3$
3	$e_1 e_2 e_3, \quad u = v_3$
4	$e_1 e_2 e_3, \quad u = v_2$
5	$e_1 e_2 e_3 e_4, \quad u = v_2$
6	$e_1 e_2 e_3 e_4 e_5, \quad u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \quad u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \quad u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \quad u = v_5$
10	$e_1 e_2 e_3 e_4 e_5 e_6, \quad u = v_5$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7	v_5	3

b)





korak	BFS
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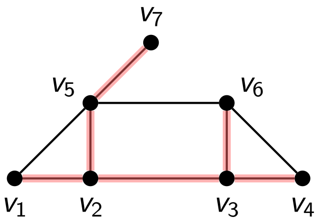
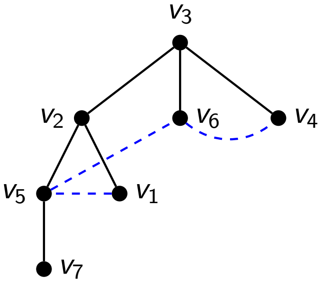
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_2$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
9	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
10	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_5$

	$\pi(v)$	$d(v)$
v_1	v_2	2
v_2	v_3	1
v_3	—	0
v_4	v_3	1
v_5	v_2	2
v_6	v_3	1
v_7	v_5	3

Neka svojstva BFS algoritma





Neka je G povezani graf i (T, v) korijensko stablo dobiveno BFS algoritmom započetom u vrhu $v \in V(G)$.

- Krajevi svakog brida povezanog grafa G koji ne pripada BFS stablu (T, v) nalaze se na istom ili susjednim nivoima stabla (T, v).

Neka svojstva BFS algoritma

- U povezanom grafu G postoji ciklus neparne duljine akko postoji brid iz $E(G) \setminus E(T)$ koji ima oba kraja na istom nivou BFS stabla (T, v).
- U povezanom grafu G postoji ciklus koji sadrži vrh $v \in V(G)$ akko postoji brid iz $E(G) \setminus E(T)$ čiji krajevi pripadaju različitim (T, v)-podstablima BFS stabla (T, v).
- U povezanom grafu G postoji ciklus koji sadrži brid $\{v, w\}$ akko postoji brid iz $E(G) \setminus E(T)$ kojemu jedan kraj pripada (T, v)-podstablu s korijenom w, a drugi kraj pripada nekom drugom (T, v)-podstablu.

Modifikacije BFS algoritma

Nivoi u BFS stablu

Standardni BFS algoritam pokrenut u vrhu $v \in V(G)$ daje udaljenosti $d(y)$ vrha v do svakog vrha $y \in V(G)$.

Pomoću vrijednosti $\pi(y)$ rekonstruiraju se najkraći putovi od vrha v do svih preostalih vrhova u grafu G.

Za svaki brid $\{x, y\}$ koji ne pripada BFS stablu (T, v), ispitivanjem uvjeta $d(x) = d(y)$ dobivamo algoritam koji daje odgovor na pitanje "*Postoji li u grafu G ciklus neparne duljine?*".

Modifikacije BFS algoritma

BFS podstabla

Pretpostavimo da unutar BFS algoritma pokrenutog iz vrha $v \in V(G)$ za svaki vrh $y \in V(G)$ spremamo informaciju $\alpha(y)$ kojem (T, v)-podstablu vrh y pripada.

Za svaki brid $\{x, y\}$ koji ne pripada BFS stablu (T, v), istraživanjem vrijednosti $\alpha(x)$ i $\alpha(y)$ dobivamo

- algoritam koji određuje postoji li u grafu G ciklus koji sadrži vrh v,
- algoritam koji određuje postoji li u grafu G ciklus koji sadrži brid $\{v, w\}$.

Modifikacije BFS algoritma

BFS nivoi i BFS podstabla

Pretpostavimo da unutar BFS algoritma pokrenutog iz vrha $v \in V(G)$ za svaki vrh $y \in V(G)$ spremamo informacije $d(y)$ i $\alpha(y)$.

Za svaki brid $\{x, y\}$ koji ne pripada BFS stablu (T, v), istraživanjem vrijednosti $\alpha(x)$ i $\alpha(y)$ dobivamo

- algoritam koji određuje duljinu najkraćeg ciklusa u grafu G koji sadrži vrh v,
- algoritam koji određuje duljinu najkraćeg ciklusa u grafu G koji sadrži brid $\{v, w\}$.

Modifikacije BFS algoritma

Nakon što prvi put pronađemo brid $\{x, y\} \in E(G) \setminus E(T)$ za koji je $\alpha(x) \neq \alpha(y)$, tada uspoređujemo vrijednosti $d(x)$ i $d(y)$.

Ako je $d(x) = d(y)$, tada algoritam završava.

Ako je $d(x) \neq d(y)$, npr. $d(x) < d(y)$, tada moramo nastaviti s istraživanjem vrhova koji se nalaze na nivou $d(x)$ kako bismo saznali postoji li brid koji ima oba kraja na nivou $d(x)$ koji pripadaju različitim (T, v)-podstablima.

Dakle, ako postoji ciklus u grafu G koji sadrži vrh v, duljina najkraćeg takvog ciklusa jednaka je $2d(x) + 1$ ili $2d(x) + 2$.

Modifikacije BFS algoritma

Određivanje struka grafa

Ako primijenimo algoritam za određivanje najkraćeg ciklusa u grafu G koji sadrži vrh v na svaki vrh $v \in V(G)$, dobivamo algoritam za određivanje struka grafa koji ima složenost $O(\nu\varepsilon)$.

Ako pritom još za svaki vrh $v \in V(G)$ spremamo informaciju $\pi(v)$, možemo rekonstruirati jedan takav najkraći ciklus.

Neki problemi su ipak teški

Odrediti ciklus neparne duljine u grafu G koji sadrži vrh v.

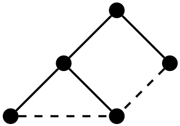
- Ovaj problem je u općenitom slučaju jednako težak kao i problem ispitivanja je li graf Hamiltonov.
- Problem je težak jer nemamo općenitu tvrdnju koja bi davala nužne i dovoljne uvjete za postojanje neparnog ciklusa u grafu G koji sadrži zadani vrh v.

Neki problemi su ipak teški

Tvrdnja 1.

Ako postoji brid $e \in E(G) \setminus E(T)$ *koji ima oba kraja na istom nivou BFS stabla* (T, v) *i u različitim* (T, v)*-podstablima, tada u grafu* G *postoji neparni ciklus koji sadrži vrh* v*.*

Gornja tvrdnja daje samo dovoljan, ali ne i nužan uvjet za postojanje neparnog ciklusa koji sadrži zadani vrh u grafu.

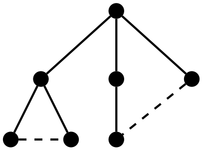


Neki problemi su ipak teški

Tvrdnja 2.

Ako u grafu G postoji neparni ciklus koji sadrži vrh v, tada postoji brid $e_1 \in E(G) \setminus E(T)$ koji ima oba kraja na istom nivou u BFS stablu (T, v) i postoji brid $e_2 \in E(G) \setminus E(T)$ čiji krajevi pripadaju različitim (T, v)-podstablima.

Gornja tvrdnja daje samo nužan, ali ne i dovoljan uvjet za postojanje neparnog ciklusa koji sadrži zadani vrh u grafu.



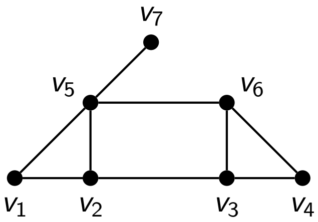
Rekurzivna implementacija DFS algoritma

```
procedure DFS(G)
    for u ∈ V(G) do                         ▷ inicijalizacija za sve vrhove grafa G
        color[u] ← WHITE                    ▷ vrhovi su bijele boje
        π[u] ← NIL                          ▷ vrhovi nemaju roditelje
    time ← 0                                ▷ globalna varijabla koja mjeri vrijeme
    for u ∈ V(G) do                         ▷ za svaki vrh u grafa G
        if color[u] = WHITE then            ▷ ako je vrh u bijele boje
            DFS_VISIT(u)                    ▷ posjeti vrh u s DFS_VISIT procedurom
procedure DFS_VISIT(u)
    color[u] ← GRAY                         ▷ pridruži vrhu u sivu boju
    time ← time + 1                         ▷ povećaj vrijeme za 1
    d[u] ← time                             ▷ spremanje trenutka kada je vrh u otkriven
    for v ∈ Adj[u] do                       ▷ za svaki susjedni vrh v od vrha u
        if color[v] = WHITE then            ▷ ako je vrh v bijele boje
            π[v] ← u                        ▷ vrh u je roditelj vrha v
            DFS_VISIT(v)                    ▷ posjeti vrh v s DFS_VISIT procedurom
    color[u] ← BLACK                        ▷ pridruži vrhu u crnu boju (vrh u je istražen)
    time ← time + 1                         ▷ povećaj vrijeme za 1
    f[u] ← time                             ▷ spremanje trenutka kada je vrh u istražen
```

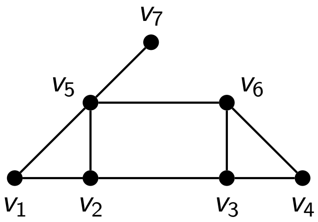
Implementacija DFS algoritma pomoću stoga

procedure DFS_VISIT(u)
    $S \leftarrow \emptyset$ ▷ inicijalizacija stoga koji će se puniti s vrhovima grafa G
    PUSH(S, u) ▷ stavi vrh u na stog S
    while $S \neq \emptyset$ **do** ▷ sve dok na stogu S ima vrhova
        $x \leftarrow$ POP(S) ▷ uzmi vrh sa stoga S i spremi ga u varijablu x
        if color[x] = WHITE **then** ▷ ako je vrh x bijele boje
            time ← time + 1 ▷ povećaj vrijeme za 1
            $d[x] \leftarrow$ time ▷ spremanje trenutka kada je vrh x otkriven
            color[x] ← GRAY ▷ pridruži vrhu x sivu boju
            PUSH(S, x) ▷ stavi ponovo vrh x na stog S
            for $v \in Adj[x]$ **do** ▷ za svaki susjedni vrh v od vrha x
                if color[v] = WHITE **then** ▷ ako je vrh v bijele boje
                    $\pi[v] \leftarrow x$ ▷ stavi da je vrh x roditelj vrha v
                    PUSH(S, v) ▷ stavi vrh v na stog S
        else if color[x] = GRAY **then** ▷ ako vrh x nije bijele boje, ali je sive boje
            time ← time + 1 ▷ povećaj vrijeme za 1
            $f[x] \leftarrow$ time ▷ spremanje trenutka kada je vrh x istražen
            color[x] ← BLACK ▷ pridruži vrhu x crnu boju (vrh x je istražen)

c)

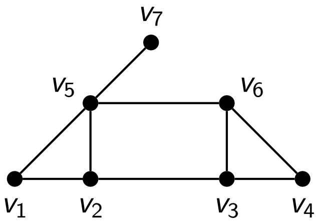


c)



korak	DFS

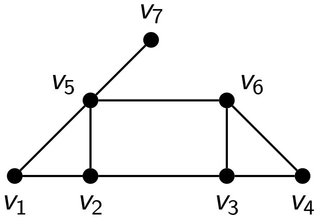
c)



korak	DFS

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3			
v_4			
v_5			
v_6			
v_7			

c)

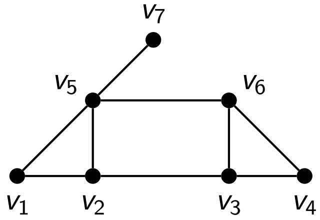


time $\leftarrow 0$

korak	DFS

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3			
v_4			
v_5			
v_6			
v_7			

c)



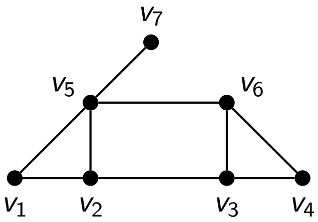
v_3

time $\leftarrow 0$

korak	DFS

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3			
v_4			
v_5			
v_6			
v_7			

c)

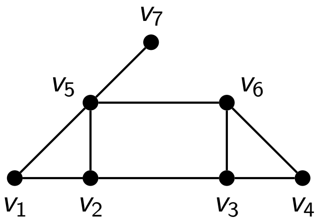


v_3

time $\leftarrow 1$

korak	DFS

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3			
v_4			
v_5			
v_6			
v_7			

c) 

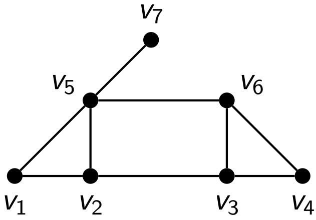
v_3

time $\leftarrow 1$

korak	DFS

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



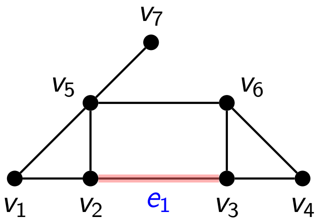
$\bullet\ v_3$

time $\leftarrow 1$

korak	DFS
1	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



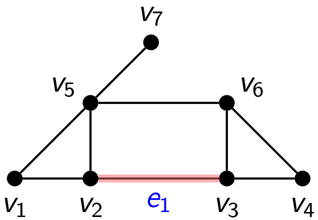
$\bullet\ v_3$

time $\leftarrow 1$

korak	DFS
1	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3	$-$	1	
v_4			
v_5			
v_6			
v_7			

c)



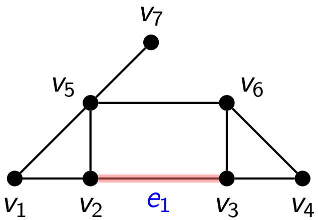
● v_3

time $\leftarrow 1$

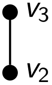
korak	DFS
1	$e_1, \ u = v_2$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3	$-$	1	
v_4			
v_5			
v_6			
v_7			

c)



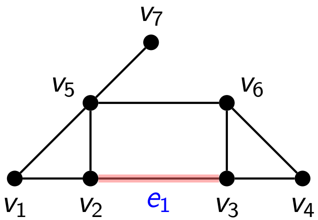
time $\leftarrow 1$



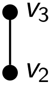
korak	DFS
1	$e_1, \ u = v_2$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



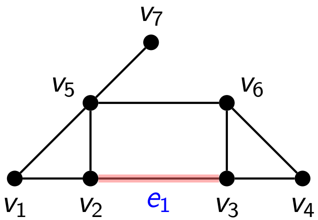
time $\leftarrow$ 2



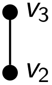
korak	DFS
1	$e_1,\ u = v_2$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2			
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



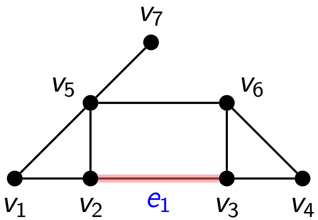
time $\leftarrow 2$



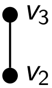
korak	DFS
1	$e_1, \ u = v_2$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



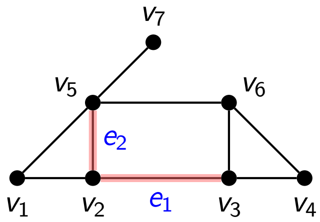
time $\leftarrow$ 2



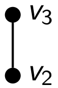
korak	DFS
1	$e_1, \ u = v_2$
2	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



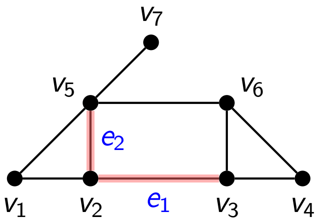
time $\leftarrow$ 2



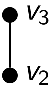
korak	DFS
1	$e_1, \ u = v_2$
2	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



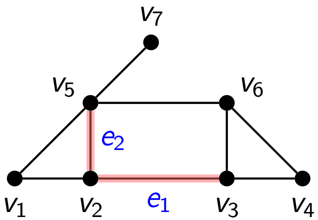
time $\leftarrow 2$



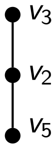
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



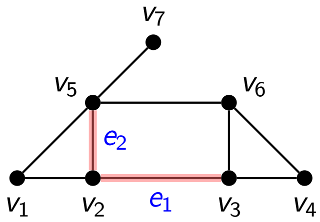
time $\leftarrow 2$



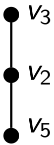
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



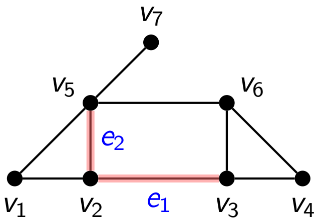
time $\leftarrow 3$



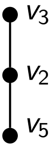
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5			
v_6			
v_7			

c)



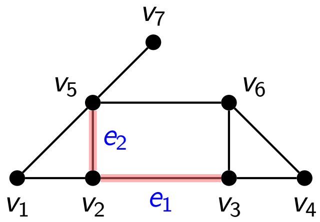
time $\leftarrow 3$



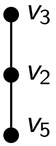
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7			

c)



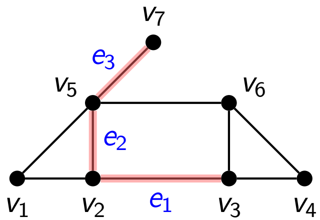
time $\leftarrow 3$



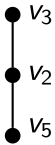
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7			

c)



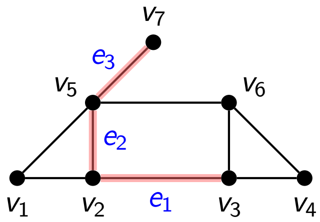
time $\leftarrow 3$



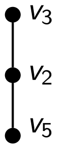
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7			

c)



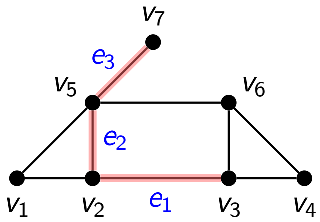
time $\leftarrow 3$



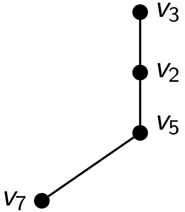
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7			

c)



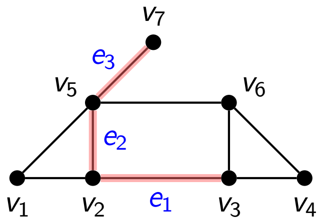
time $\leftarrow 3$



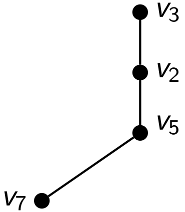
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7			

c)



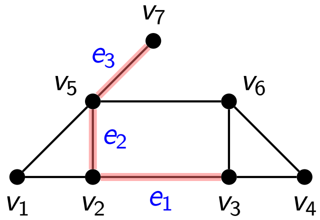
time $\leftarrow 4$



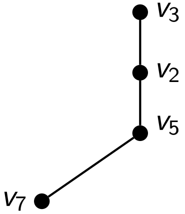
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7			

c)



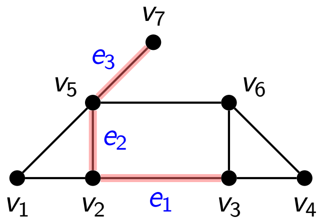
time $\leftarrow 4$



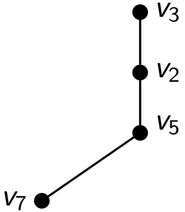
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	

c)



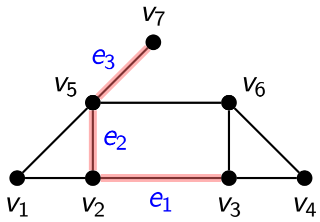
time $\leftarrow$ 4



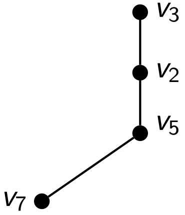
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$
3	$e_1 e_2 e_3, \;\; u = v_7$
4	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	

c)



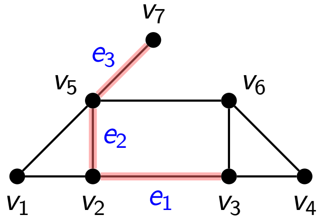
time $\leftarrow$ 4



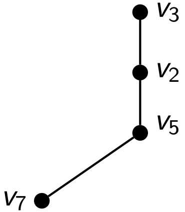
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	

c)



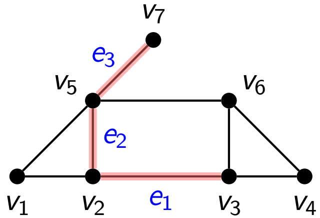
time $\leftarrow 5$



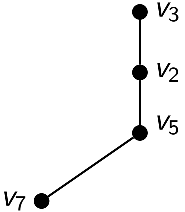
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	

c)



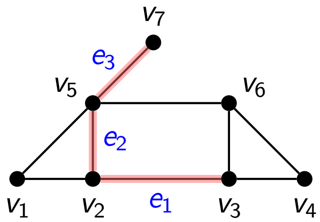
time $\leftarrow$ 5



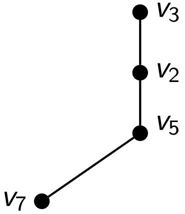
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$
3	$e_1 e_2 e_3, \;\; u = v_7$
4	$e_1 e_2 e_3, \;\; u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	5

c)



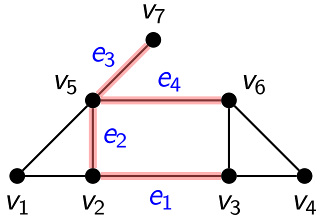
time $\leftarrow$ 5



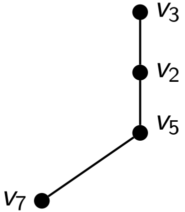
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	5

c)



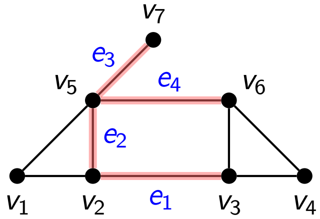
time $\leftarrow$ 5



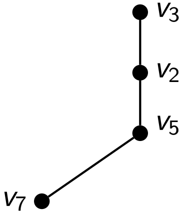
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	5

c)



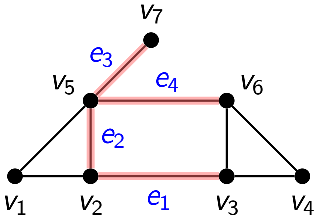
time $\leftarrow 5$



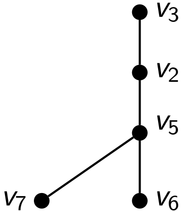
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	$-$	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	5

c)



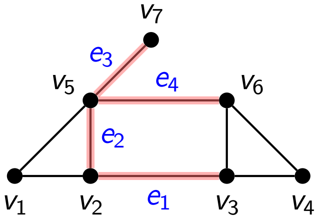
time $\leftarrow 5$



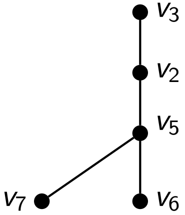
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	5

c)



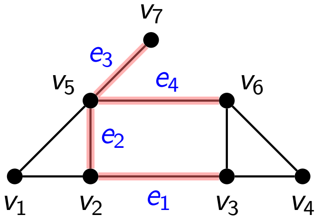
time $\leftarrow 6$



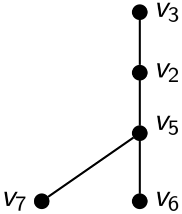
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6			
v_7	v_5	4	5

c)



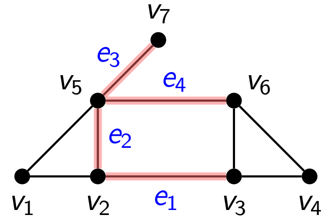
time $\leftarrow 6$



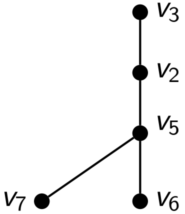
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



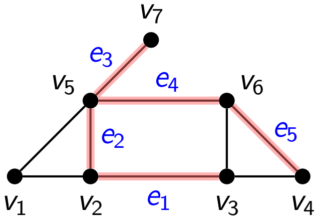
time $\leftarrow$ 6



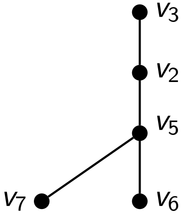
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$
3	$e_1 e_2 e_3, \;\; u = v_7$
4	$e_1 e_2 e_3, \;\; u = v_5$
5	$e_1 e_2 e_3 e_4, \;\; u = v_6$
6	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



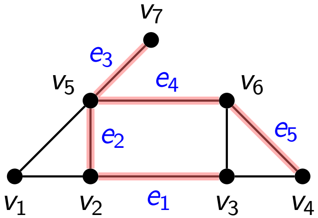
time $\leftarrow 6$



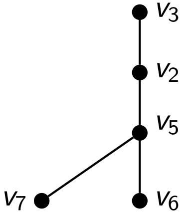
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



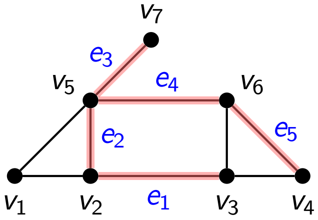
time $\leftarrow 6$



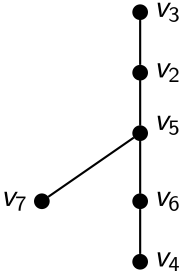
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4			
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



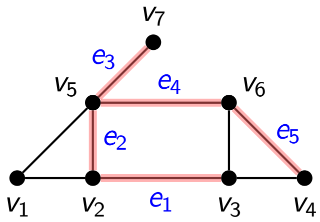
time $\leftarrow$ 6



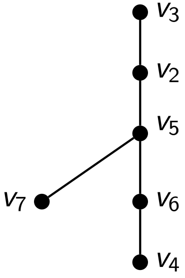
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$
3	$e_1 e_2 e_3, \;\; u = v_7$
4	$e_1 e_2 e_3, \;\; u = v_5$
5	$e_1 e_2 e_3 e_4, \;\; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_4$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	$-$	1	
v_4			
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



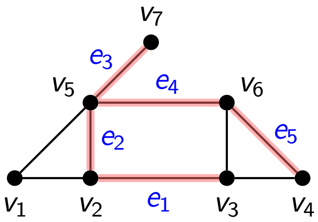
time $\leftarrow 7$



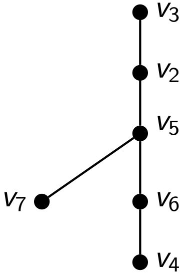
korak	DFS
1	$e_1,\ u = v_2$
2	$e_1 e_2,\ u = v_5$
3	$e_1 e_2 e_3,\ u = v_7$
4	$e_1 e_2 e_3,\ u = v_5$
5	$e_1 e_2 e_3 e_4,\ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5,\ u = v_4$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	$-$	1	
v_4			
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



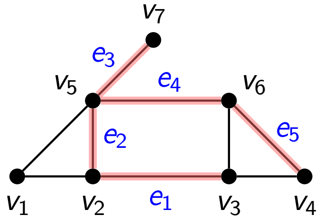
time $\leftarrow 7$



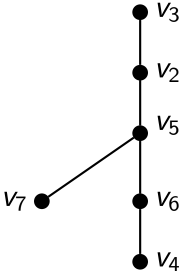
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	$-$	1	
v_4	v_6	7	
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



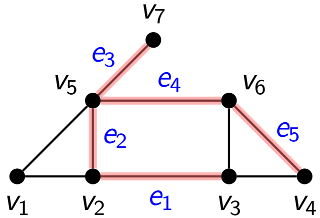
time $\leftarrow$ 7



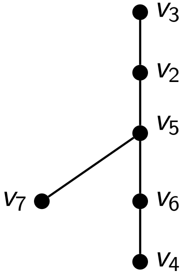
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



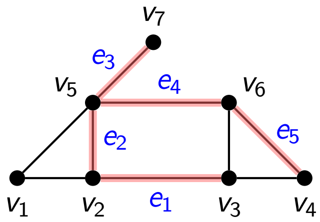
time $\leftarrow 7$



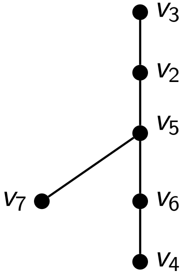
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_5$
3	$e_1 e_2 e_3, \;\; u = v_7$
4	$e_1 e_2 e_3, \;\; u = v_5$
5	$e_1 e_2 e_3 e_4, \;\; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \;\; u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



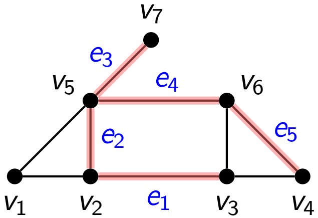
time $\leftarrow$ 8



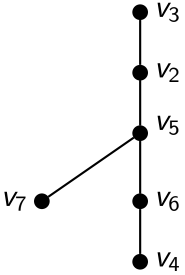
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



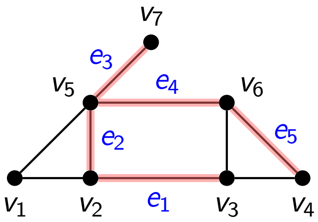
time $\leftarrow$ 8



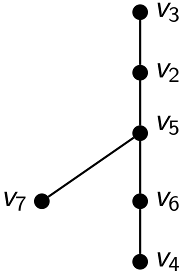
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



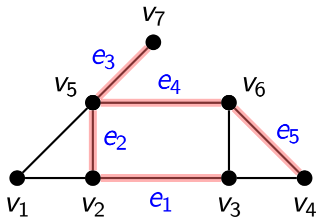
time $\leftarrow 8$



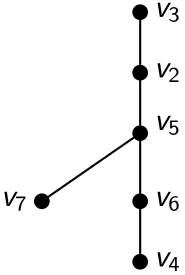
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



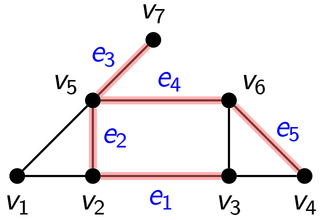
time $\leftarrow$ 8



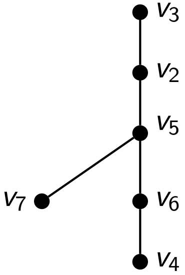
korak	DFS
1	$e_1, \quad u = v_2$
2	$e_1 e_2, \quad u = v_5$
3	$e_1 e_2 e_3, \quad u = v_7$
4	$e_1 e_2 e_3, \quad u = v_5$
5	$e_1 e_2 e_3 e_4, \quad u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \quad u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \quad u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \quad u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	$-$	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



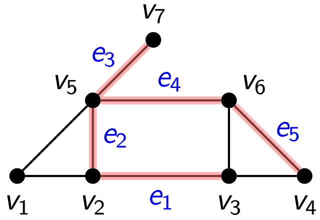
time $\leftarrow 9$



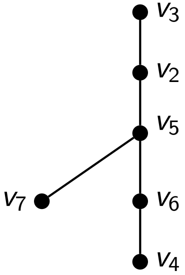
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	
v_7	v_5	4	5

c)



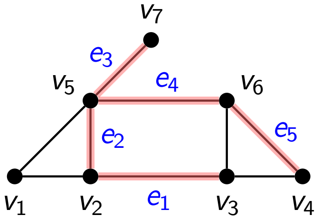
time $\leftarrow 9$



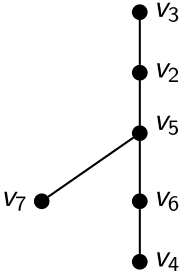
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



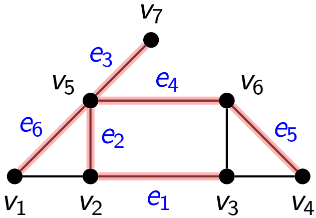
time $\leftarrow 9$



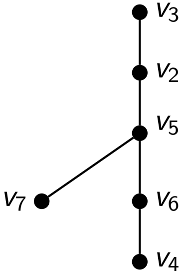
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



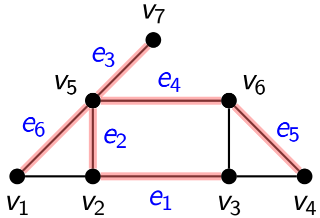
time $\leftarrow$ 9



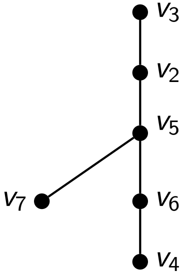
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	$-$	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



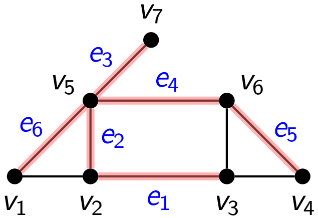
time $\leftarrow$ 9



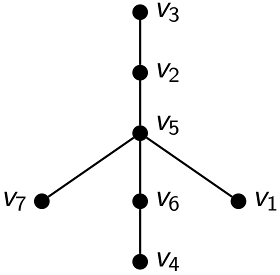
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



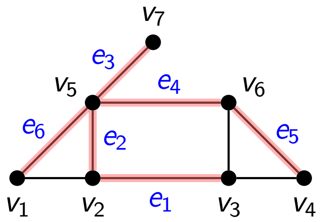
time $\leftarrow$ 9



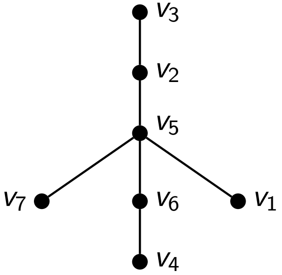
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



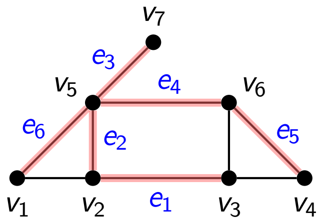
time $\leftarrow$ 10



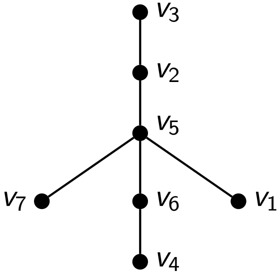
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$

	$\pi(v)$	$d(v)$	$f(v)$
v_1			
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)

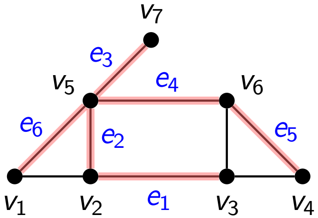


time $\leftarrow 10$

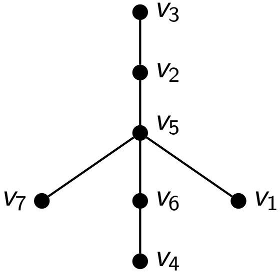


korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c) 

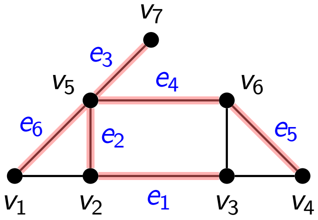
time $\leftarrow$ 10



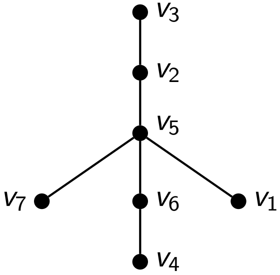
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_1$
	$u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



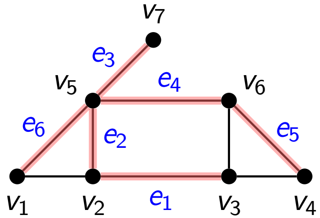
time $\leftarrow$ 11



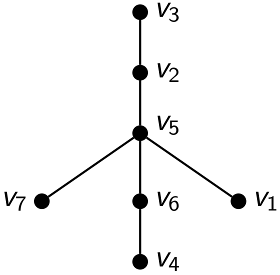
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_1$
	$u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



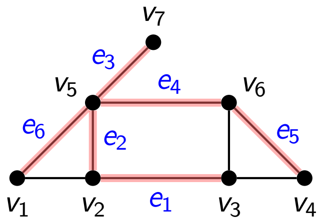
time $\leftarrow$ 11



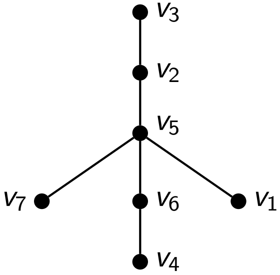
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_1$
	$u = v_5$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



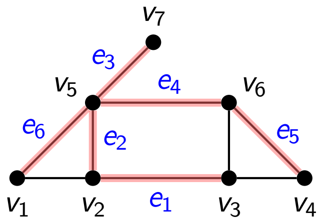
time $\leftarrow$ 11



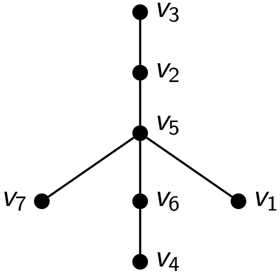
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



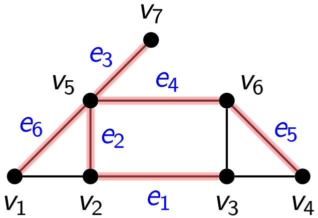
time $\leftarrow$ 12



korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2$

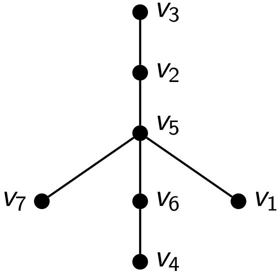
	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	
v_6	v_5	6	9
v_7	v_5	4	5

c)



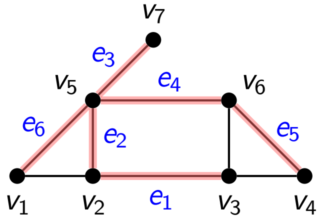
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2$

time $\leftarrow$ 12

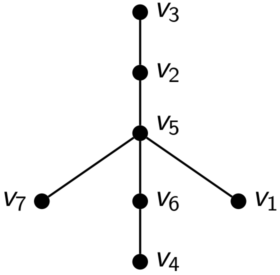


	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



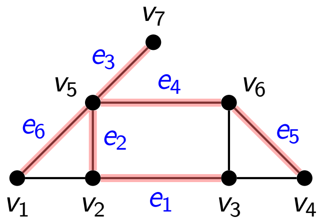
time $\leftarrow$ 12



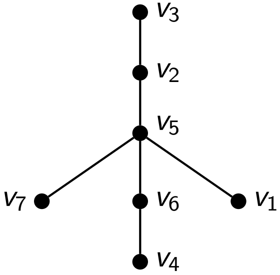
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2, \ u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



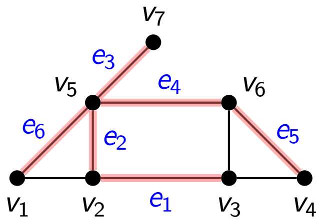
time $\leftarrow$ 13



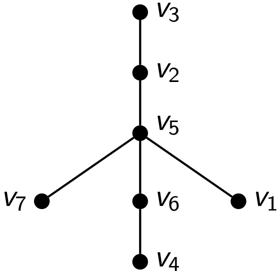
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2, \ u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



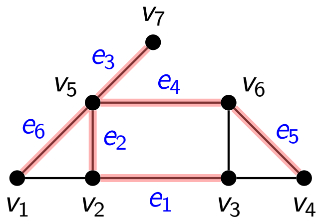
time $\leftarrow$ 13



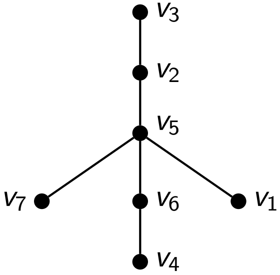
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_1$
	$u = v_5, \; u = v_2, \; u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	13
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



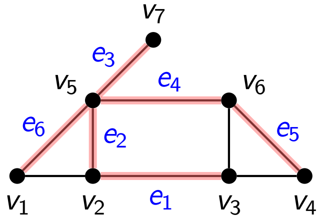
time $\leftarrow$ 14



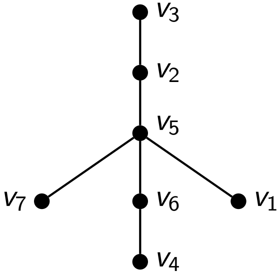
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2, \ u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	13
v_3	—	1	
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



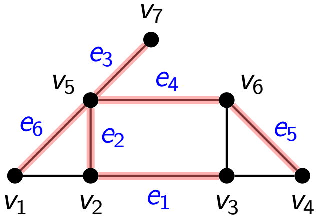
time $\leftarrow$ 14



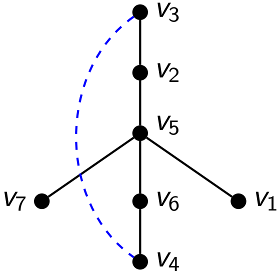
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2, \ u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	13
v_3	—	1	14
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



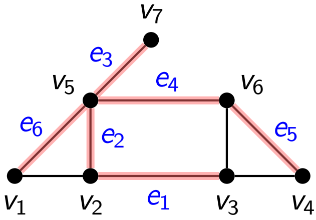
time $\leftarrow$ 14



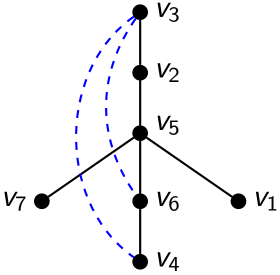
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_5$
3	$e_1 e_2 e_3, \ u = v_7$
4	$e_1 e_2 e_3, \ u = v_5$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \ u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \ u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_1$
	$u = v_5, \ u = v_2, \ u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	13
v_3	—	1	14
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



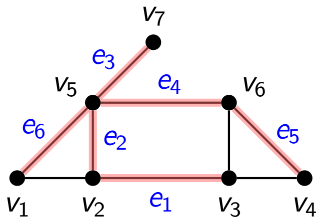
time $\leftarrow$ 14



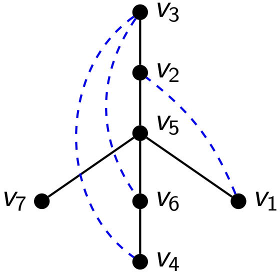
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_1$
	$u = v_5, \; u = v_2, \; u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	13
v_3	—	1	14
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

c)



time $\leftarrow$ 14



korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_5$
3	$e_1 e_2 e_3, \; u = v_7$
4	$e_1 e_2 e_3, \; u = v_5$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_4$
7	$e_1 e_2 e_3 e_4 e_5, \; u = v_6$
8	$e_1 e_2 e_3 e_4 e_5, \; u = v_5$
9	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_1$
	$u = v_5, \; u = v_2, \; u = v_3$

	$\pi(v)$	$d(v)$	$f(v)$
v_1	v_5	10	11
v_2	v_3	2	13
v_3	—	1	14
v_4	v_6	7	8
v_5	v_2	3	12
v_6	v_5	6	9
v_7	v_5	4	5

Teorem

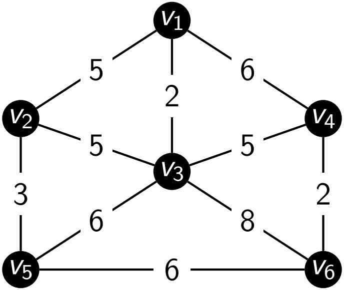
Ako primijenimo DFS algoritam na neusmjereni ili usmjereni graf G, tada za svaka dva vrha $v_1, v_2 \in V(G)$ vrijedi točno jedan od sljedeća tri uvjeta:

- *Intervali $\left[d(v_1), f(v_1)\right]$ i $\left[d(v_2), f(v_2)\right]$ su disjunktni. Vrh v_1 nije potomak vrha v_2 i vrh v_2 nije potomak vrha v_1.*
- *Interval $\left[d(v_1), f(v_1)\right]$ je sadržan unutar intervala $\left[d(v_2), f(v_2)\right]$. Vrh v_1 je potomak vrha v_2 u pripadnoj DFS šumi.*
- *Interval $\left[d(v_2), f(v_2)\right]$ je sadržan unutar intervala $\left[d(v_1), f(v_1)\right]$. Vrh v_2 je potomak vrha v_1 u pripadnoj DFS šumi.*

četvrti zadatak

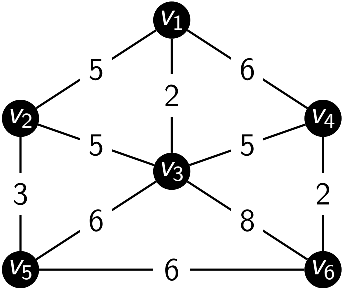
Zadatak 4

Pomoću Kruskalovog i Primovog algoritma pronađite minimalno razapinjuće stablo u težinskom grafu G.



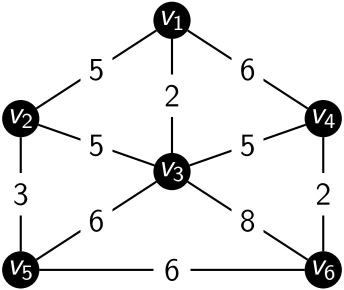
Rješenje

Kruskal



Rješenje

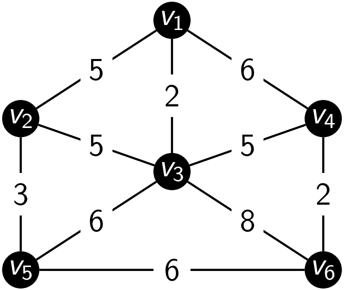
Kruskal



korak					
brid					
težina					

Rješenje

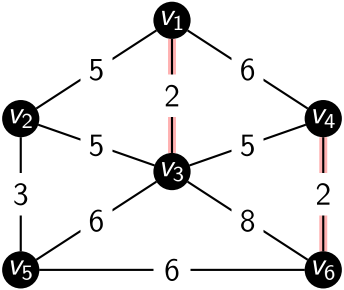
Kruskal



korak	1				
brid					
težina					

Rješenje

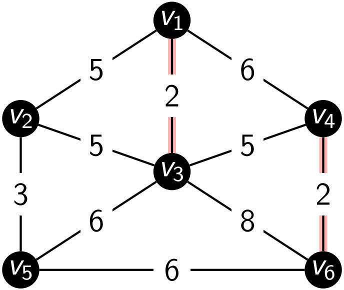
Kruskal



korak	1				
brid					
težina					

Rješenje

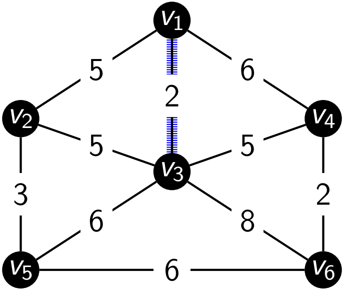
Kruskal



korak	1				
brid	$v_1 v_3$				
težina	2				

Rješenje

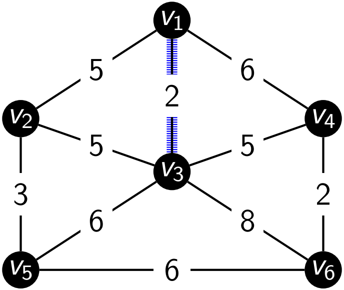
Kruskal



korak	1				
brid	$v_1 v_3$				
težina	2				

Rješenje

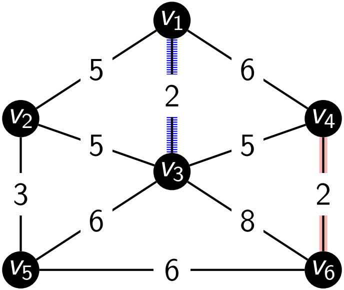
Kruskal



korak	1	2			
brid	$v_1 v_3$				
težina	2				

Rješenje

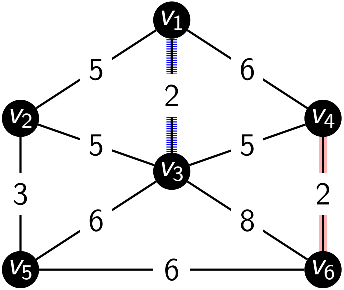
Kruskal



korak	1	2			
brid	$v_1 v_3$				
težina	2				

Rješenje

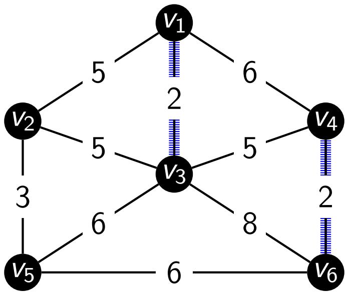
Kruskal



korak	1	2			
brid	$v_1 v_3$	$v_4 v_6$			
težina	2	2			

Rješenje

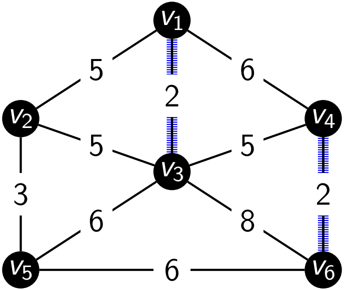
Kruskal



korak	1	2			
brid	$v_1 v_3$	$v_4 v_6$			
težina	2	2			

Rješenje

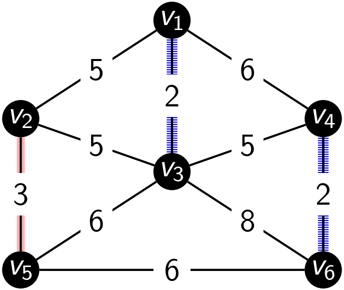
Kruskal



korak	1	2	3		
brid	$v_1 v_3$	$v_4 v_6$			
težina	2	2			

Rješenje

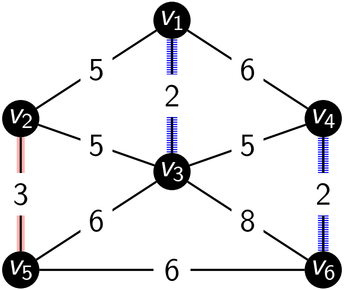
Kruskal



korak	1	2	3		
brid	$v_1 v_3$	$v_4 v_6$			
težina	2	2			

Rješenje

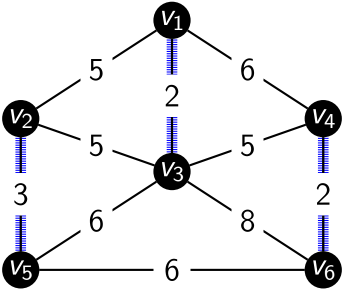
Kruskal



korak	1	2	3		
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$		
težina	2	2	3		

Rješenje

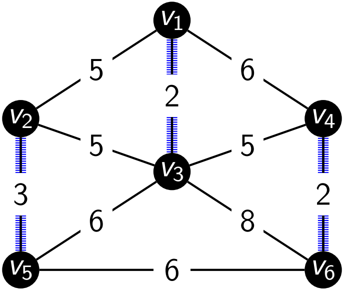
Kruskal



korak	1	2	3		
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$		
težina	2	2	3		

Rješenje

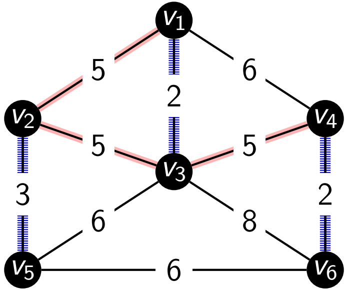
Kruskal



korak	1	2	3	4	
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$		
težina	2	2	3		

Rješenje

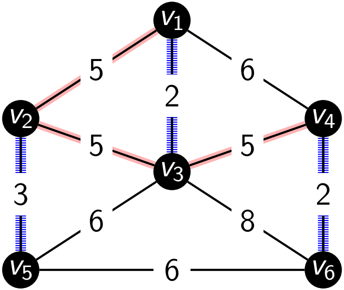
Kruskal



korak	1	2	3	4	
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$		
težina	2	2	3		

Rješenje

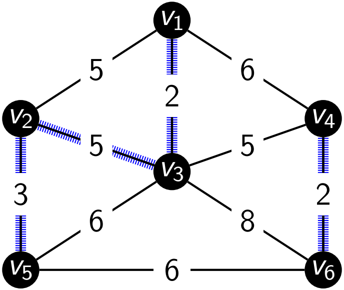
Kruskal



korak	1	2	3	4	
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	
težina	2	2	3	5	

Rješenje

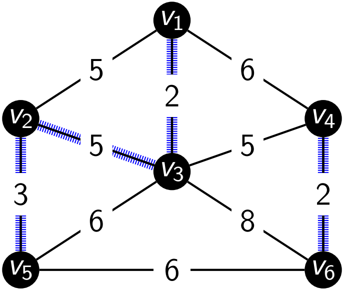
Kruskal



korak	1	2	3	4	
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	
težina	2	2	3	5	

Rješenje

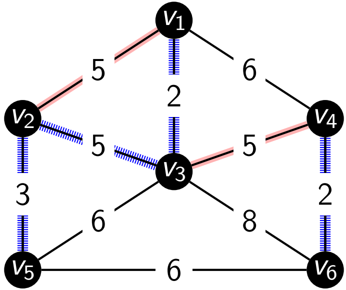
Kruskal



korak	1	2	3	4	5
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	
težina	2	2	3	5	

Rješenje

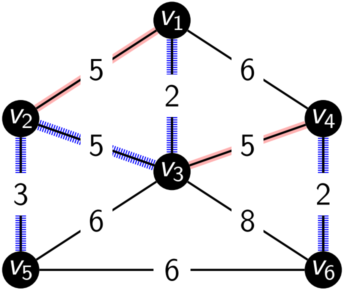
Kruskal



korak	1	2	3	4	5
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	
težina	2	2	3	5	

Rješenje

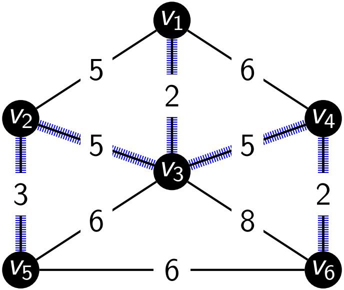
Kruskal



korak	1	2	3	4	5
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	$v_3 v_4$
težina	2	2	3	5	5

Rješenje

Kruskal

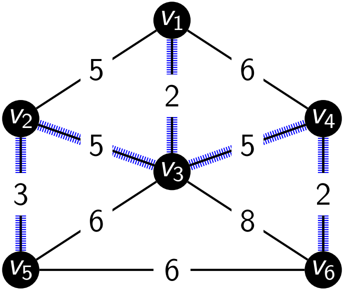


korak	1	2	3	4	5
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	$v_3 v_4$
težina	2	2	3	5	5

Rješenje

Kruskal

težina stabla:

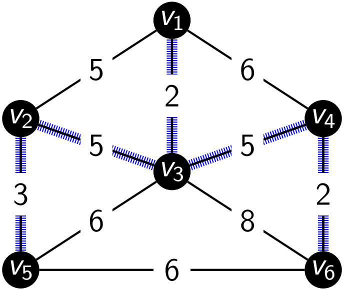


korak	1	2	3	4	5
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	$v_3 v_4$
težina	2	2	3	5	5

Rješenje

Kruskal

težina stabla: $2+2+3+5+5=17$

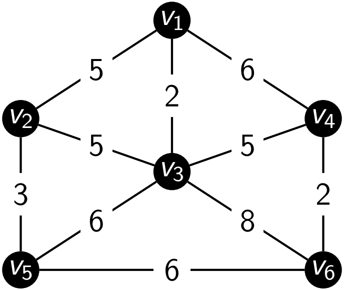


korak	1	2	3	4	5
brid	$v_1 v_3$	$v_4 v_6$	$v_2 v_5$	$v_2 v_3$	$v_3 v_4$
težina	2	2	3	5	5

Rješenje

Prim

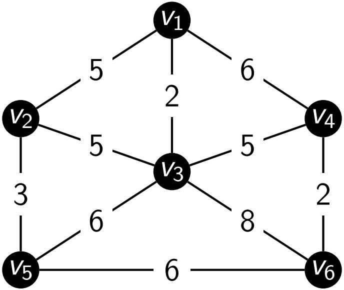
početni vrh: v_5



Rješenje

Prim

početni vrh: v_5

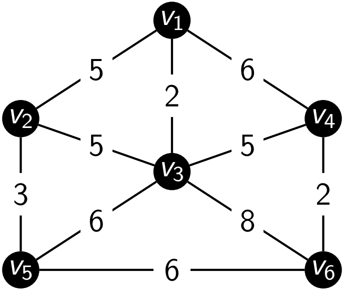


korak					
brid					
težina					

Rješenje

Prim

početni vrh: v_5

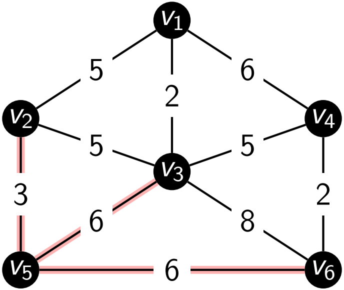


korak	1				
brid					
težina					

Rješenje

Prim

početni vrh: v_5

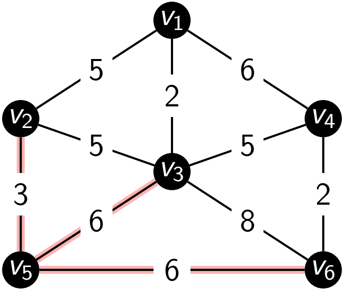


korak	1				
brid					
težina					

Rješenje

Prim

početni vrh: v_5

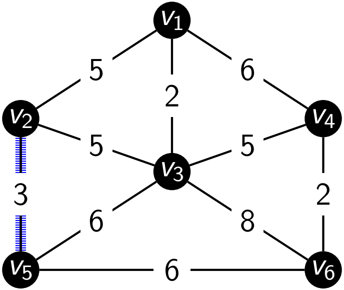


korak	1				
brid	$v_2 v_5$				
težina	3				

Rješenje

Prim

početni vrh: v_5

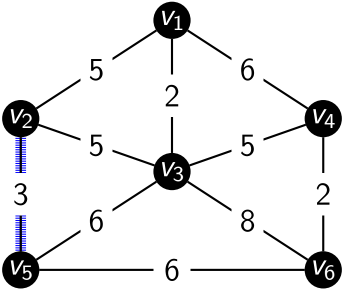


korak	1				
brid	$v_2 v_5$				
težina	3				

Rješenje

Prim

početni vrh: v_5

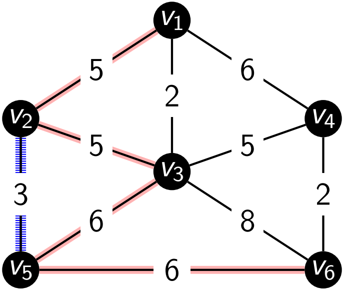


korak	1	2			
brid	$v_2 v_5$				
težina	3				

Rješenje

Prim

početni vrh: v_5

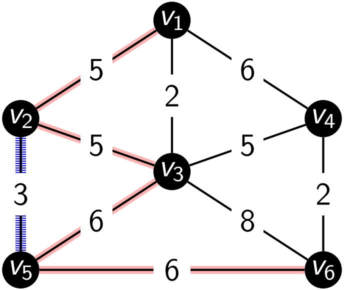


korak	1	2			
brid	$v_2 v_5$				
težina	3				

Rješenje

Prim

početni vrh: v_5

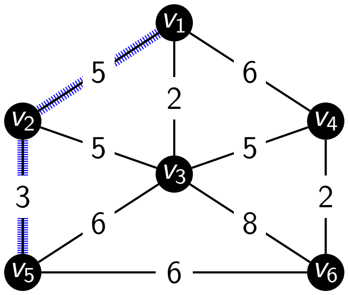


korak	1	2			
brid	$v_2 v_5$	$v_1 v_2$			
težina	3	5			

Rješenje

Prim

početni vrh: v_5

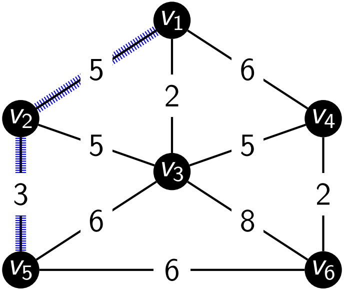


korak	1	2			
brid	$v_2 v_5$	$v_1 v_2$			
težina	3	5			

Rješenje

Prim

početni vrh: v_5

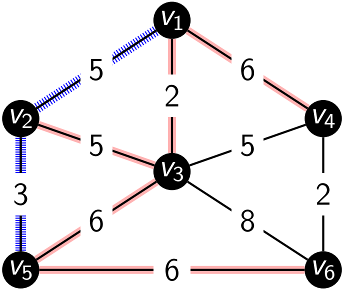


korak	1	2	3		
brid	$v_2 v_5$	$v_1 v_2$			
težina	3	5			

Rješenje

Prim

početni vrh: v_5

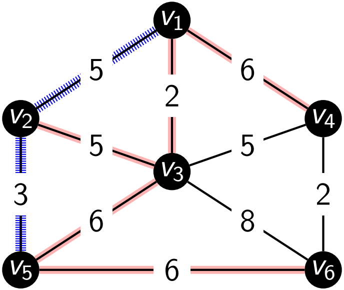


korak	1	2	3		
brid	$v_2 v_5$	$v_1 v_2$			
težina	3	5			

Rješenje

Prim

početni vrh: v_5

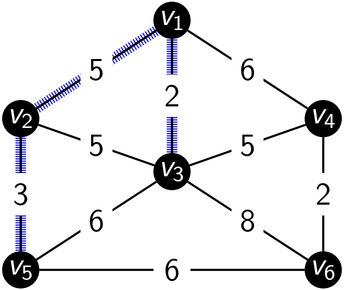


korak	1	2	3		
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$		
težina	3	5	2		

Rješenje

Prim

početni vrh: v_5

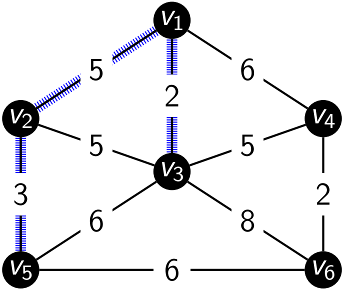


korak	1	2	3		
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$		
težina	3	5	2		

Rješenje

Prim

početni vrh: v_5

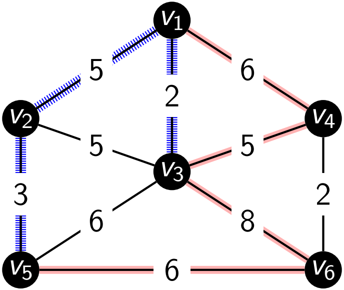


korak	1	2	3	4	
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$		
težina	3	5	2		

Rješenje

Prim

početni vrh: v_5

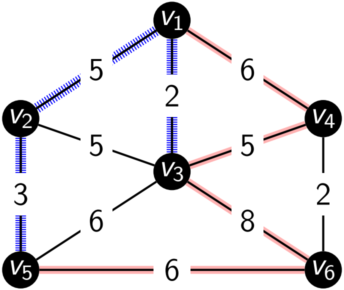


korak	1	2	3	4	
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$		
težina	3	5	2		

Rješenje

Prim

početni vrh: v_5

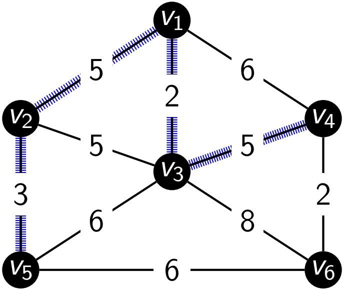


korak	1	2	3	4	
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$	$v_3 v_4$	
težina	3	5	2	5	

Rješenje

Prim

početni vrh: v_5

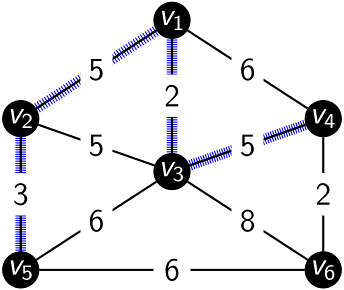


korak	1	2	3	4	
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$	$v_3 v_4$	
težina	3	5	2	5	

Rješenje

Prim

početni vrh: v_5

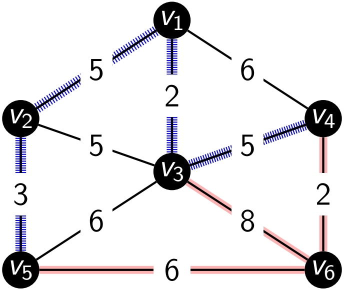


korak	1	2	3	4	5
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$	$v_3 v_4$	
težina	3	5	2	5	

Rješenje

Prim

početni vrh: v_5

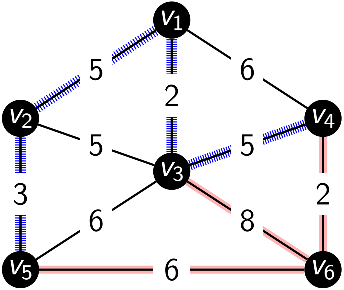


korak	1	2	3	4	5
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$	$v_3 v_4$	
težina	3	5	2	5	

Rješenje

Prim

početni vrh: v_5

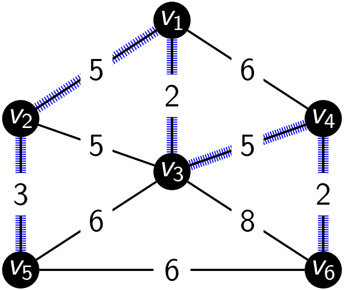


korak	1	2	3	4	5
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$	$v_3 v_4$	$v_4 v_6$
težina	3	5	2	5	2

Rješenje

Prim

početni vrh: v_5



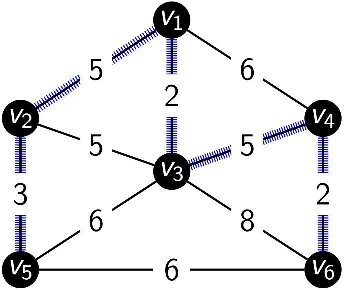
korak	1	2	3	4	5
brid	$v_2 v_5$	$v_1 v_2$	$v_1 v_3$	$v_3 v_4$	$v_4 v_6$
težina	3	5	2	5	2

Rješenje

Prim

težina stabla:

početni vrh: v_5



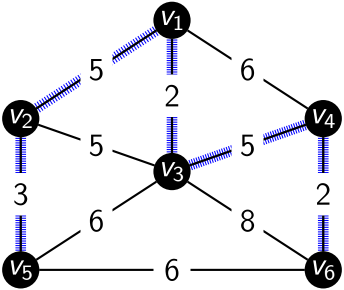
korak	1	2	3	4	5
brid	$v_2\,v_5$	$v_1\,v_2$	$v_1\,v_3$	$v_3\,v_4$	$v_4\,v_6$
težina	3	5	2	5	2

Rješenje

Prim

težina stabla: $3+5+2+5+2=17$

početni vrh: v_5

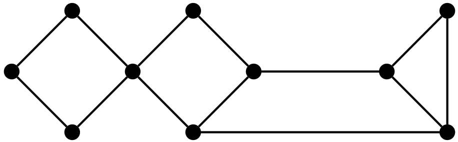


korak	1	2	3	4	5
brid	$v_2\,v_5$	$v_1\,v_2$	$v_1\,v_3$	$v_3\,v_4$	$v_4\,v_6$
težina	3	5	2	5	2

peti zadatak

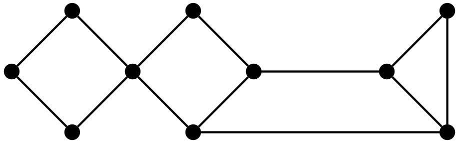
Zadatak 5

Odredite jednu jaku orijentaciju na grafu G ukoliko ona postoji.



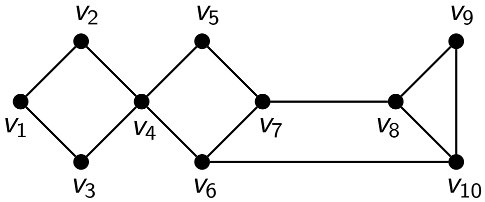
Zadatak 5

Odredite jednu jaku orijentaciju na grafu G ukoliko ona postoji.

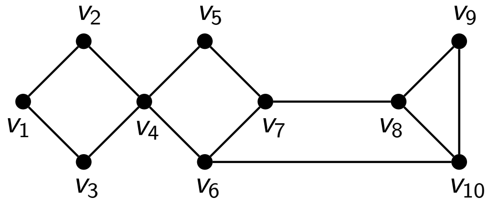


Rješenje

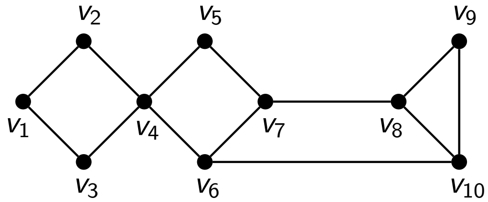
Jaka orijentacija na grafu G postoji jer graf G nema reznih bridova.



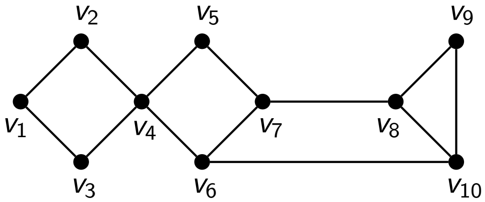
v_2
v_5
v_9
v_1
v_4
v_7
v_8
v_3
v_6
v_{10}



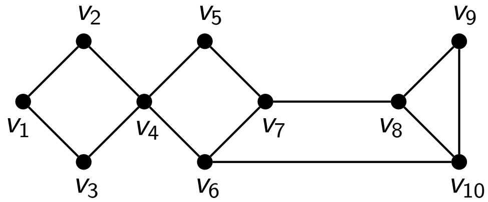
korak	DFS



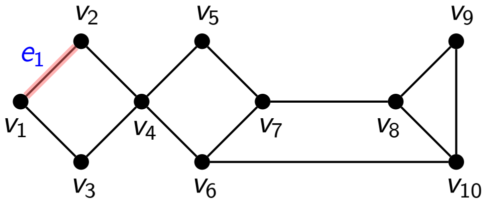
korak	DFS



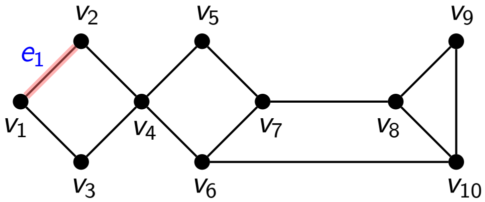
korak	DFS



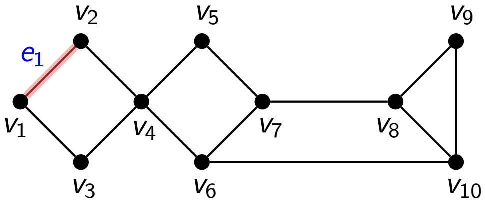
korak	DFS
1	

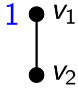


korak	DFS
1	

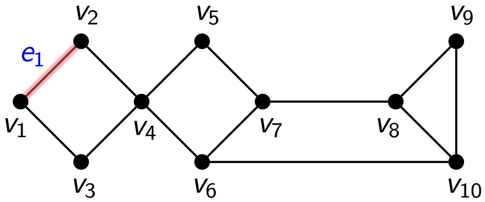


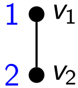
korak	DFS
1	$e_1, \;\; u = v_2$



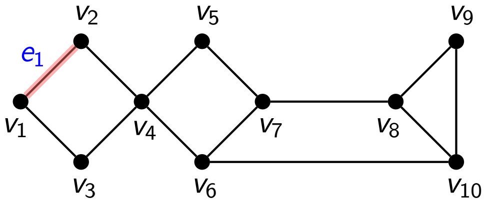


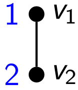
korak	DFS
1	$e_1, \;\; u = v_2$





korak	DFS
1	$e_1, \;\; u = v_2$





korak	DFS
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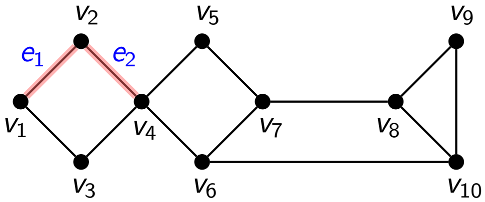
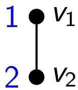
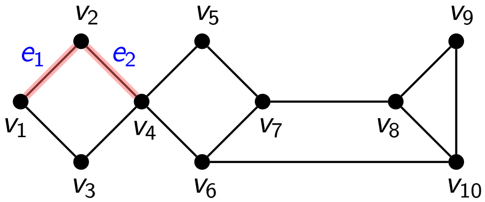
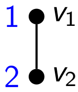
2	



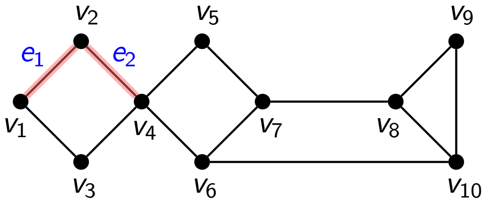


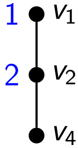
korak	DFS
1	$e_1, \;\; u = v_2$
2	



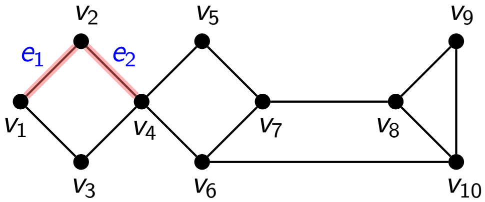


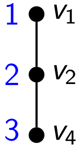
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_4$



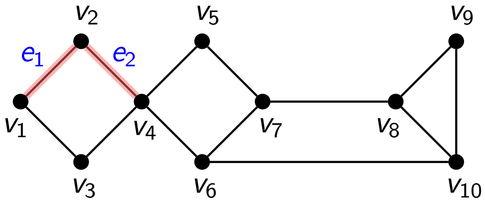


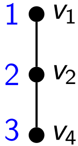
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$



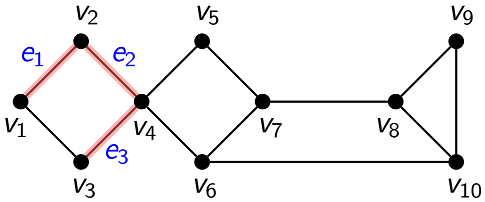


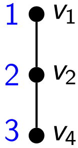
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_4$



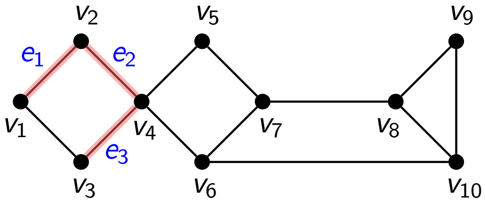


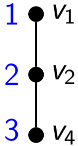
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	



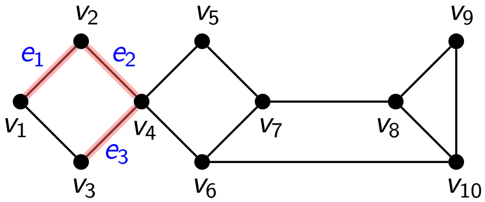


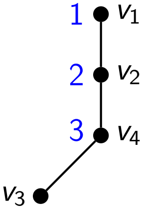
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	





korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$





korak	DFS
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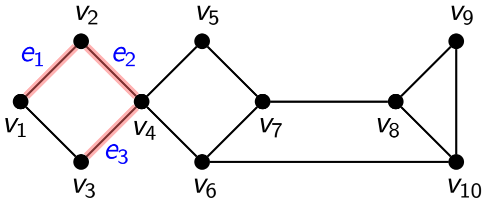
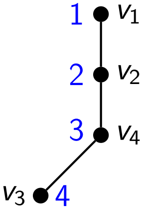
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$





korak	DFS
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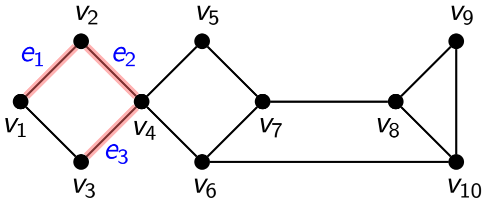
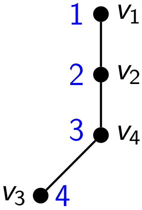
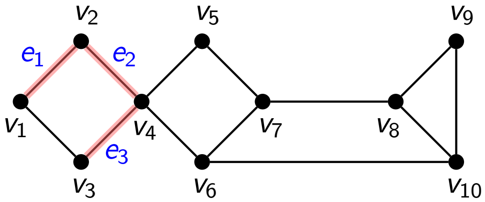
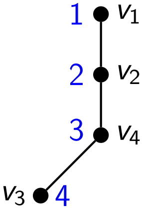
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$



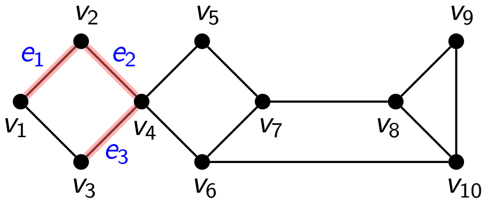


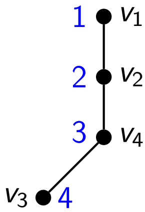
korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_4$
3	$e_1 e_2 e_3, \;\; u = v_3$
4	



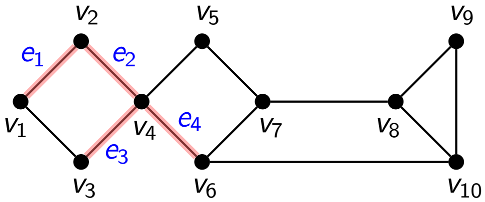


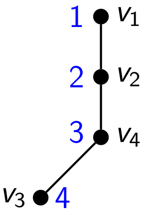
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$





korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_4$
3	$e_1 e_2 e_3, \; u = v_3$
4	$e_1 e_2 e_3, \; u = v_4$
5	





korak	DFS
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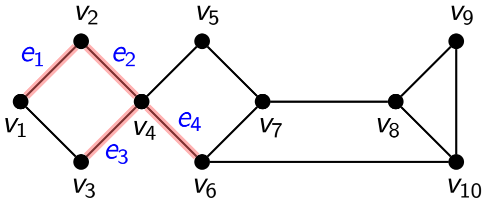
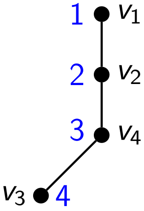
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	





korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_4$
3	$e_1 e_2 e_3, \;\; u = v_3$
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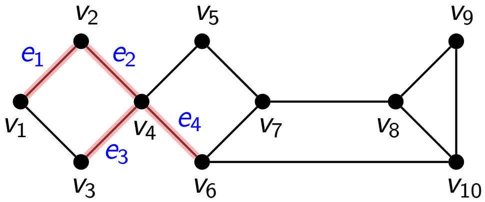
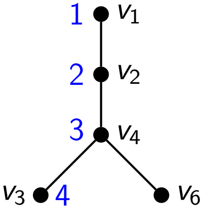
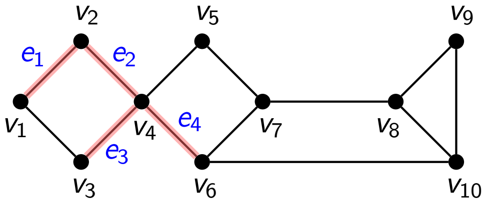
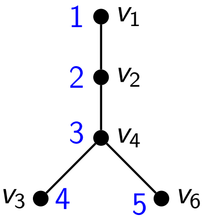
5	$e_1 e_2 e_3 e_4, \;\; u = v_6$



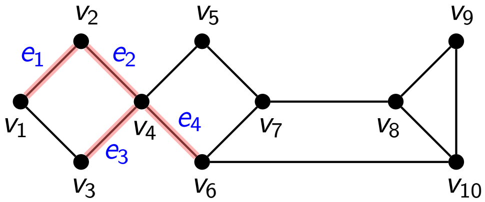


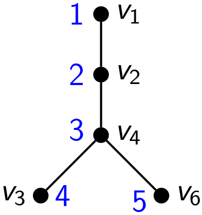
korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_4$
3	$e_1 e_2 e_3, \; u = v_3$
4	$e_1 e_2 e_3, \; u = v_4$
5	$e_1 e_2 e_3 e_4, \; u = v_6$





korak	DFS
1	$e_1, \;\; u = v_2$
2	$e_1 e_2, \;\; u = v_4$
3	$e_1 e_2 e_3, \;\; u = v_3$
4	$e_1 e_2 e_3, \;\; u = v_4$
5	$e_1 e_2 e_3 e_4, \;\; u = v_6$





korak	DFS
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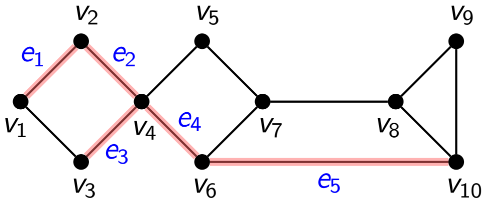
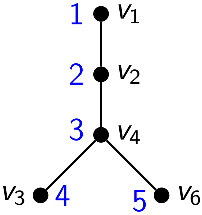
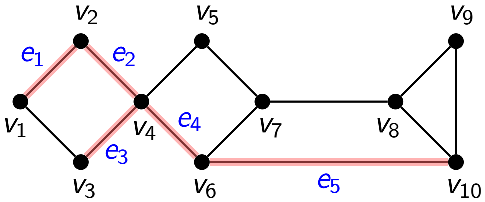
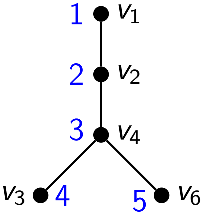
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	



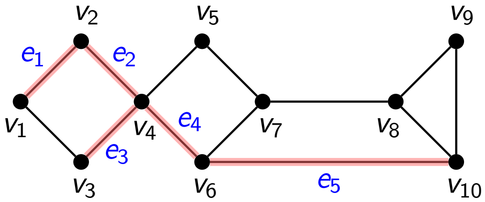


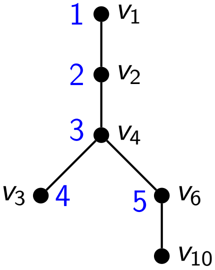
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	



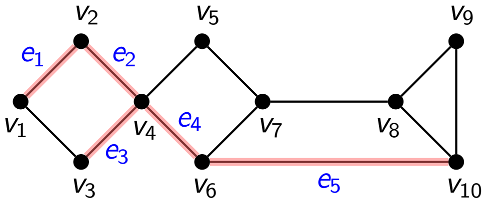


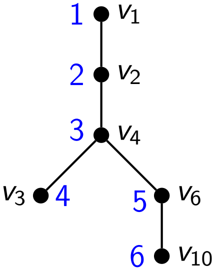
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$



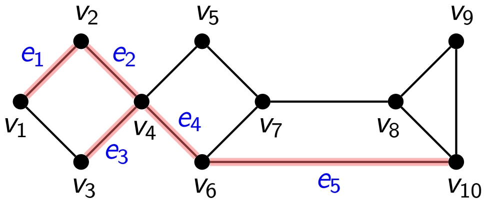


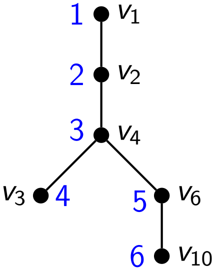
korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$





korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$





korak	DFS
1	$e_1, \ u = v_2$
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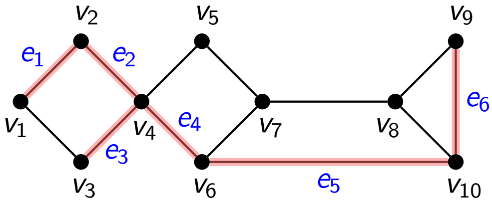
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$
7	



korak	DFS
1	$e_1,\ u = v_2$
2	$e_1 e_2,\ u = v_4$
3	$e_1 e_2 e_3,\ u = v_3$
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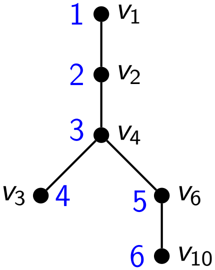
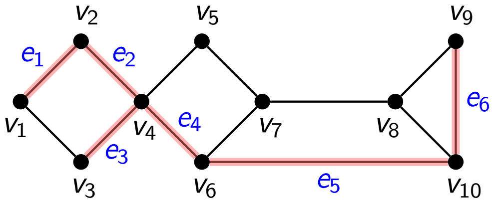
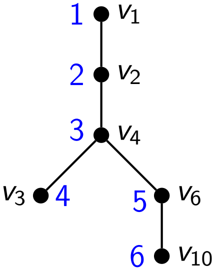
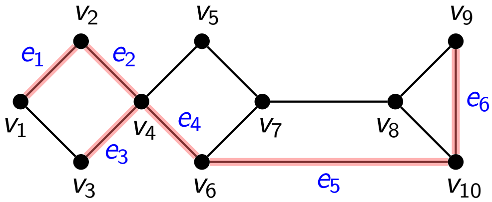
5	$e_1 e_2 e_3 e_4,\ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5,\ u = v_{10}$
7	

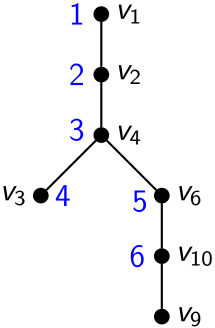




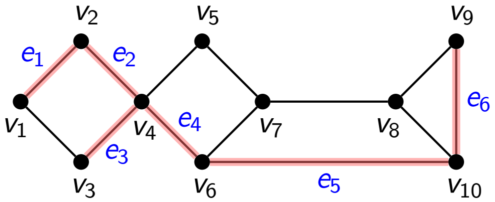


korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_9$

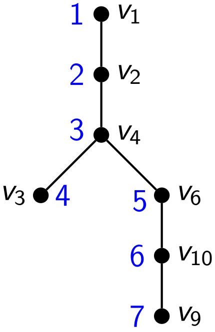


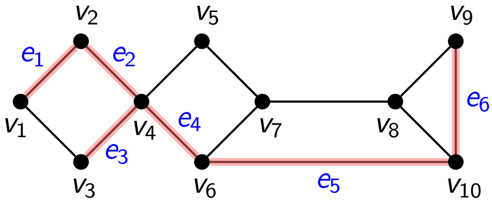


korak	DFS
1	$e_1, \ u = v_2$
2	$e_1 e_2, \ u = v_4$
3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_9$



korak	DFS
1	$e_1, \; u = v_2$
2	$e_1 e_2, \; u = v_4$
3	$e_1 e_2 e_3, \; u = v_3$
4	$e_1 e_2 e_3, \; u = v_4$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_9$





korak	DFS
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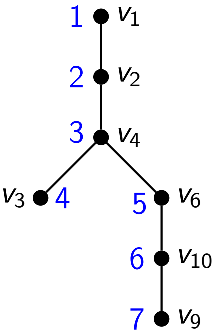
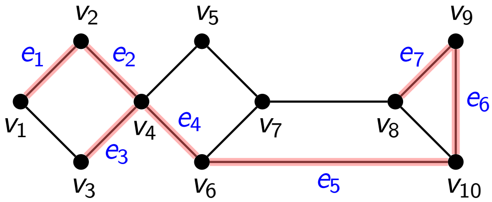
3	$e_1 e_2 e_3, \; u = v_3$
4	$e_1 e_2 e_3, \; u = v_4$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_9$
8	





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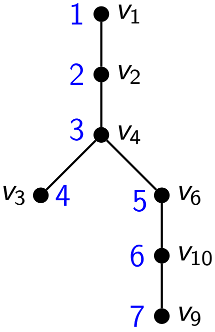
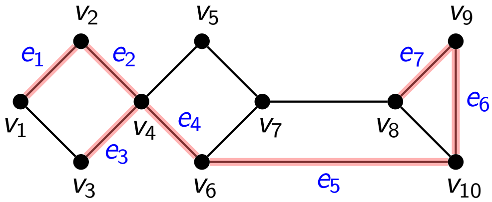
4	$e_1 e_2 e_3, \; u = v_4$
5	$e_1 e_2 e_3 e_4, \; u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_9$
8	





korak	DFS
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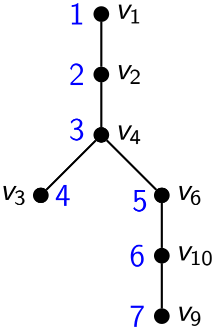
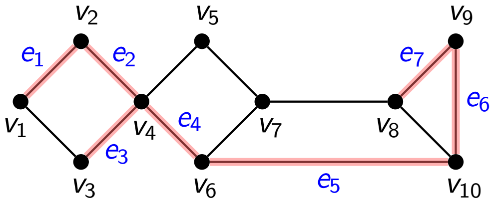
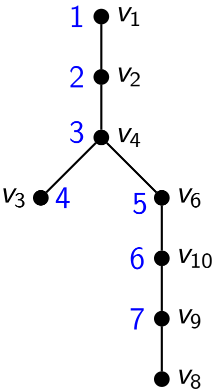
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_9$
8	$e_1 e_2 e_3 e_4 e_5 e_6 e_7, \ u = v_8$







korak	DFS
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2	$e_1 e_2, \ u = v_4$
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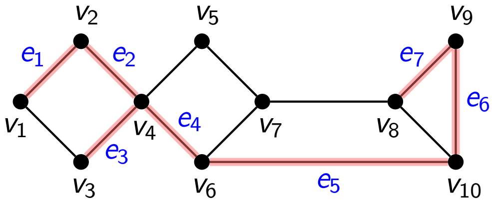
4	$e_1 e_2 e_3, \ u = v_4$
5	$e_1 e_2 e_3 e_4, \ u = v_6$
6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_9$
8	$e_1 e_2 e_3 e_4 e_5 e_6 e_7, \ u = v_8$



korak	DFS
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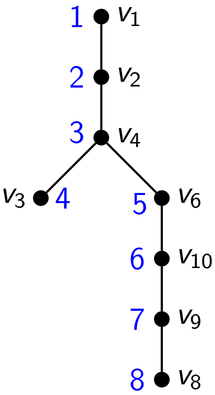
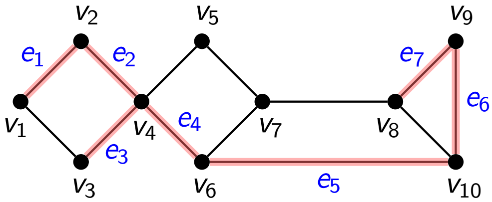
7	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_9$
8	$e_1 e_2 e_3 e_4 e_5 e_6 e_7, \ u = v_8$





korak	DFS
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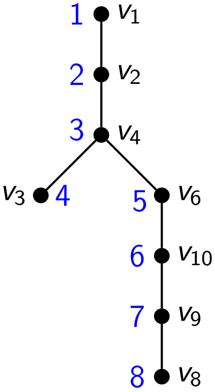
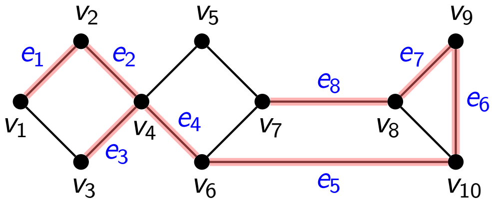
6	$e_1 e_2 e_3 e_4 e_5, \; u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_9$
8	$e_1 e_2 e_3 e_4 e_5 e_6 e_7, \; u = v_8$
9	





korak	DFS
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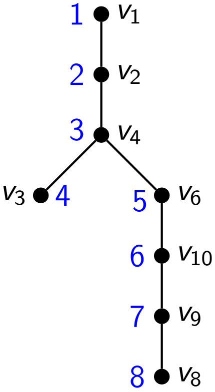
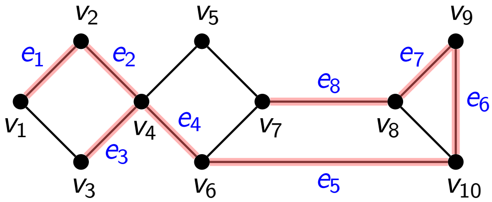
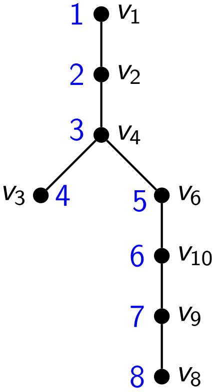
9	







korak	DFS
1	$e_1,\ u = v_2$
2	$e_1 e_2,\ u = v_4$
3	$e_1 e_2 e_3,\ u = v_3$
4	$e_1 e_2 e_3,\ u = v_4$
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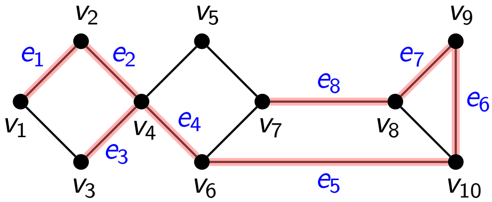
9	$e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8,\ u = v_7$



korak	DFS
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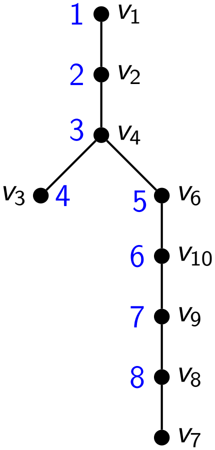

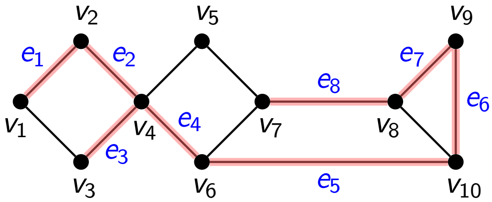

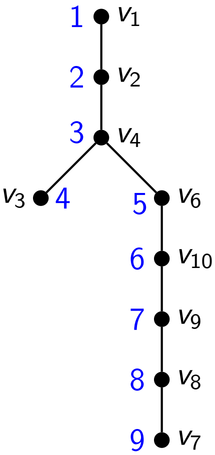

korak	DFS
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2	$e_1 e_2, \;\; u = v_4$
3	$e_1 e_2 e_3, \;\; u = v_3$
4	$e_1 e_2 e_3, \;\; u = v_4$
5	$e_1 e_2 e_3 e_4, \;\; u = v_6$
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7	$e_1 e_2 e_3 e_4 e_5 e_6, \;\; u = v_9$
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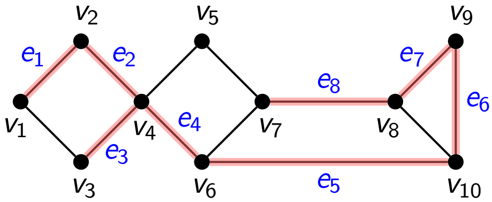
9	$e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8, \;\; u = v_7$



korak	DFS
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2	$e_1 e_2, \; u = v_4$
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7	$e_1 e_2 e_3 e_4 e_5 e_6, \; u = v_9$
8	$e_1 e_2 e_3 e_4 e_5 e_6 e_7, \; u = v_8$
9	$e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8, \; u = v_7$
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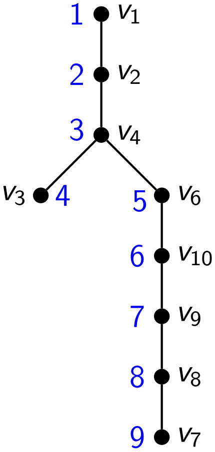

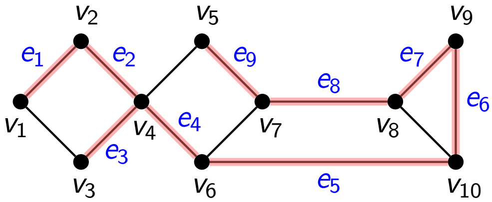

korak	DFS
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3	$e_1 e_2 e_3, \ u = v_3$
4	$e_1 e_2 e_3, \ u = v_4$
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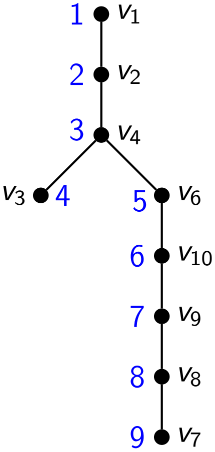
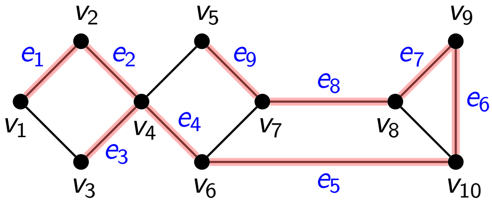
8	$e_1 e_2 e_3 e_4 e_5 e_6 e_7, \ u = v_8$
9	$e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8, \ u = v_7$
10	





korak	DFS
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6	$e_1 e_2 e_3 e_4 e_5, \ u = v_{10}$
7	$e_1 e_2 e_3 e_4 e_5 e_6, \ u = v_9$
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9	$e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8, \ u = v_7$
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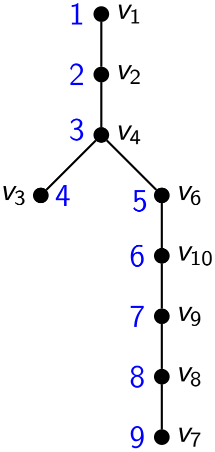

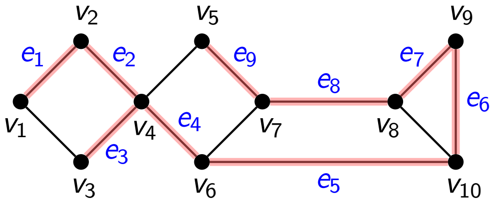

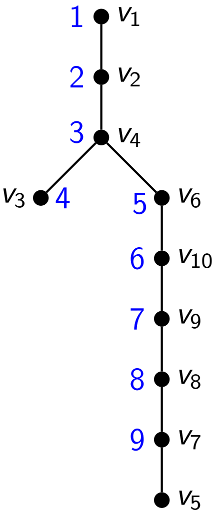

korak	DFS
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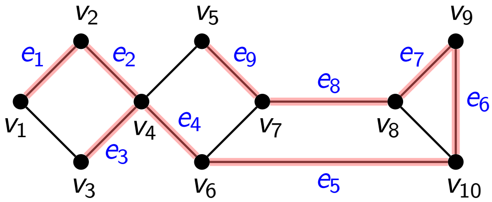

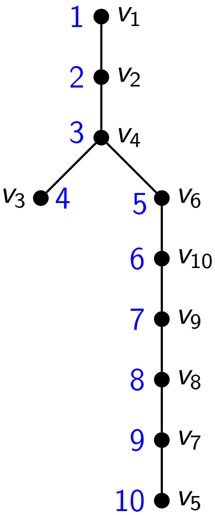

korak	DFS
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9	$e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8, \; u = v_7$
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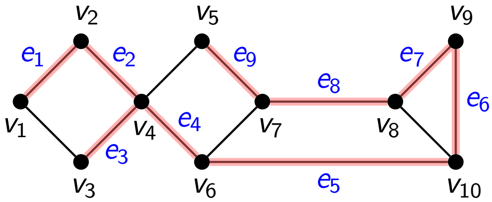

v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
v_{10}
e_1
e_2
e_3
e_4
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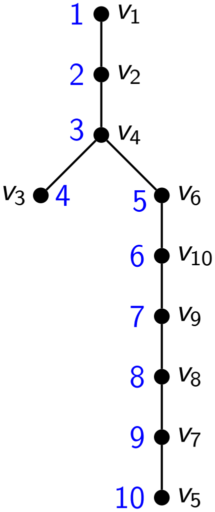
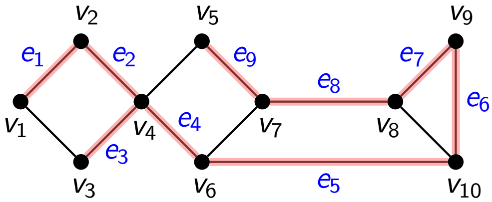
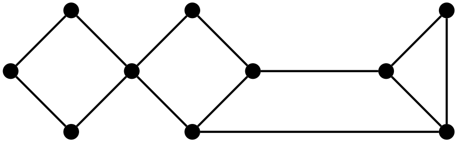
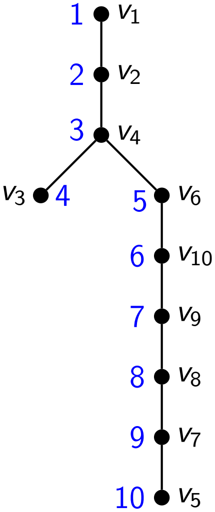
e_7
e_8
e_9



1 v_1
2 v_2
3 v_4
v_3 4
5 v_6
6 v_{10}
7 v_9
8 v_8
9 v_7
10 v_5


v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10}
e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9




1 v_1
2 v_2
3 v_4
v_3 4
5 v_6
6 v_{10}
7 v_9
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9 v_7
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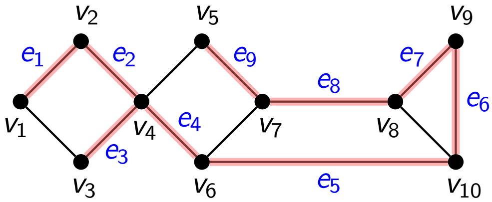
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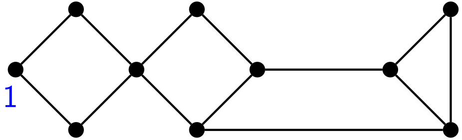
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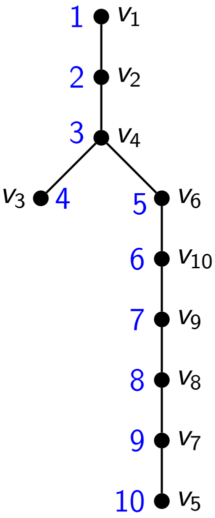
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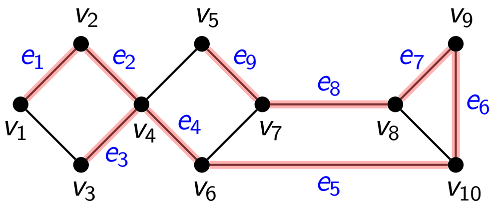

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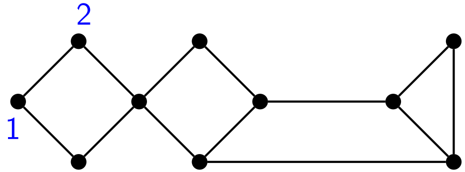

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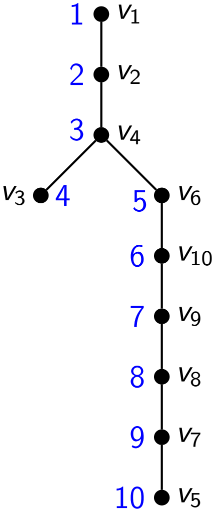

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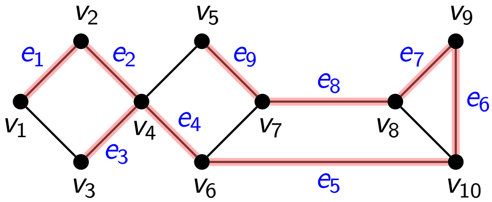
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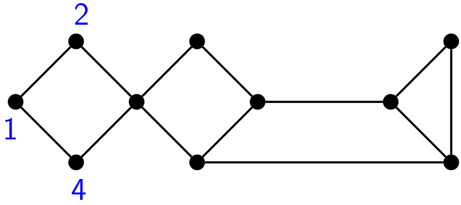
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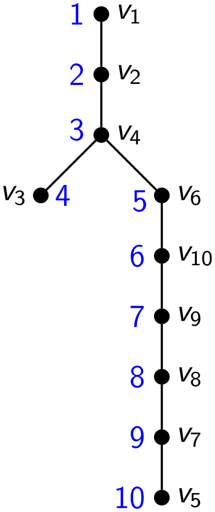
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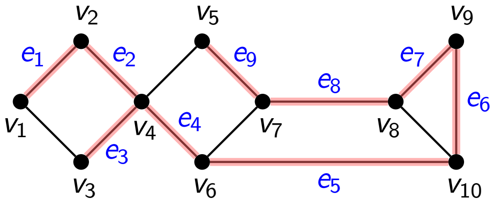
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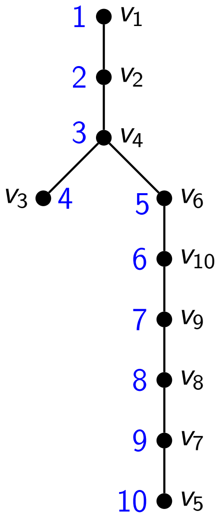
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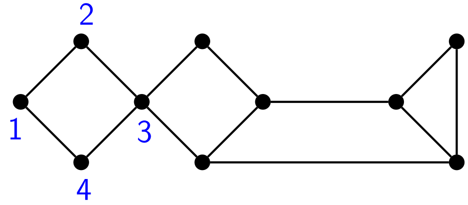
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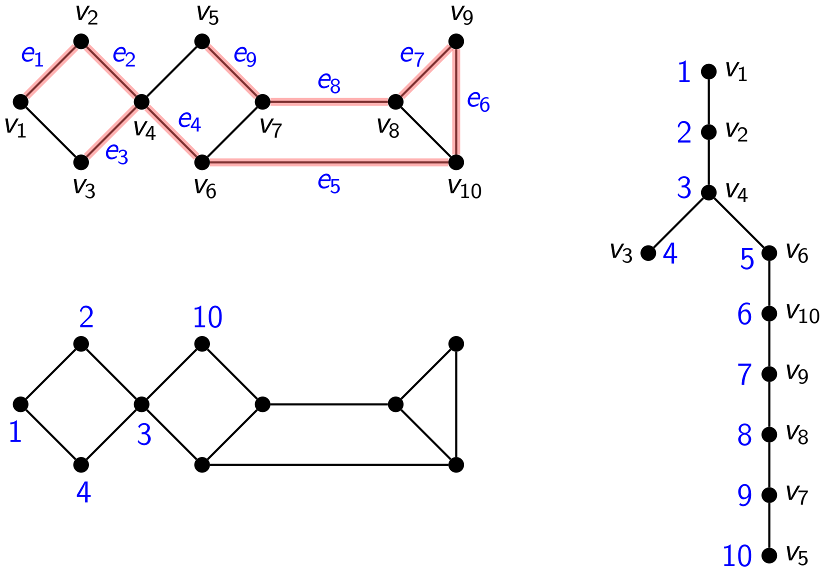
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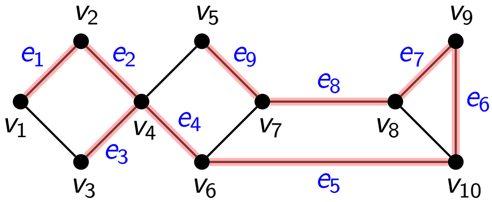
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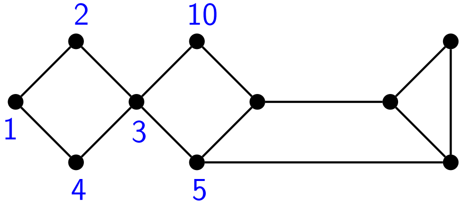
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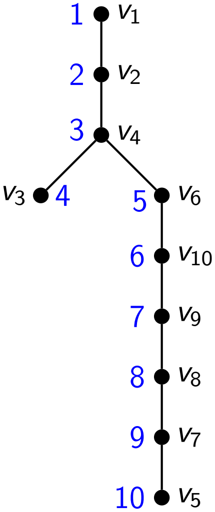
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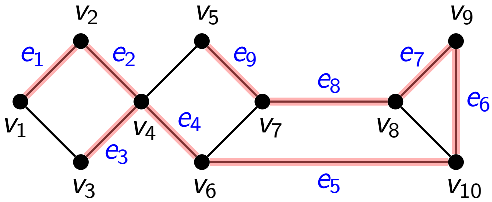
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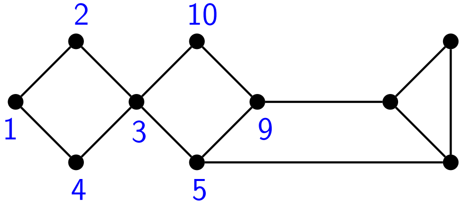
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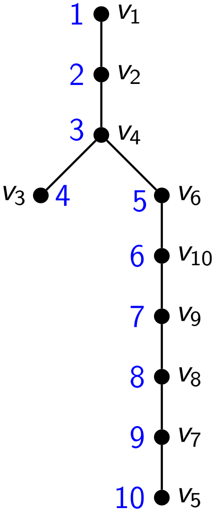
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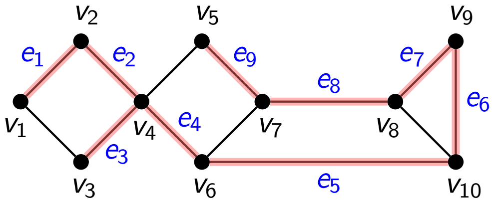
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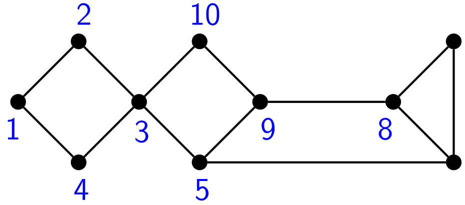
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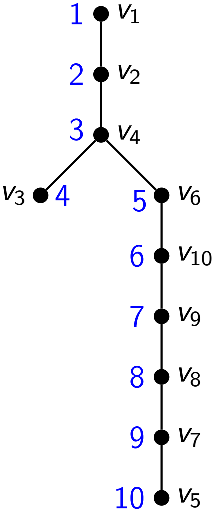
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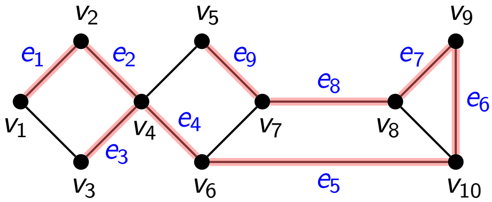
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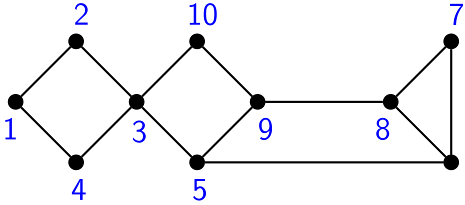
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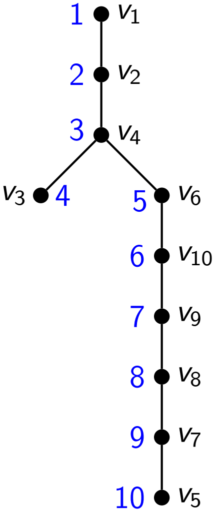
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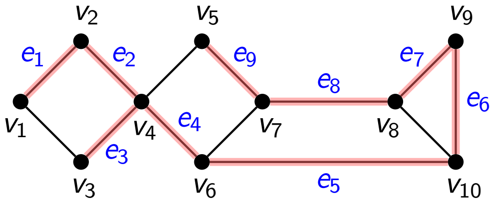
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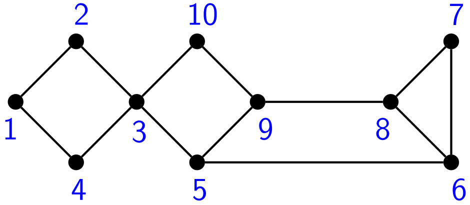
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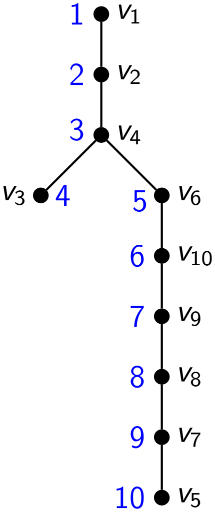
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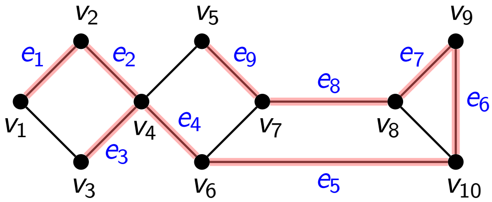
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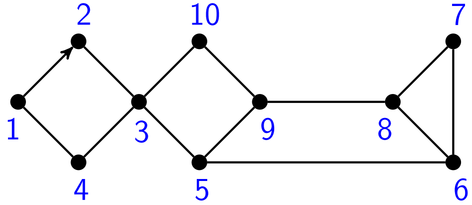
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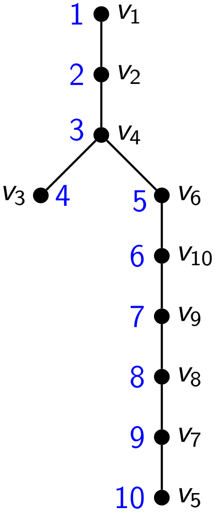
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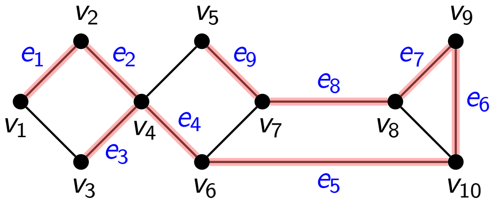
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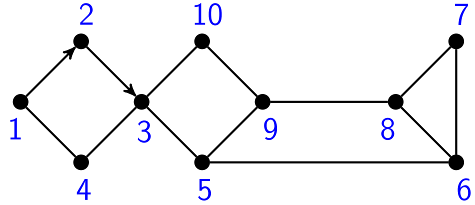
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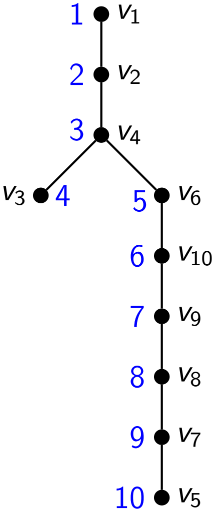
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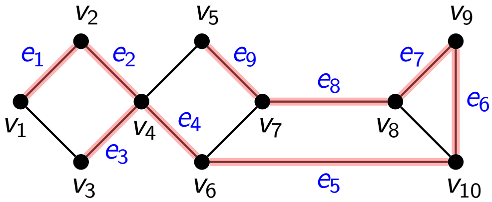
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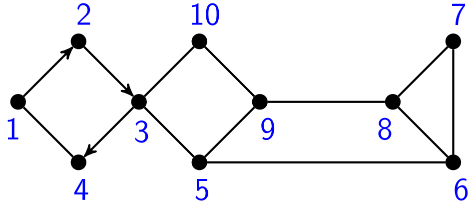
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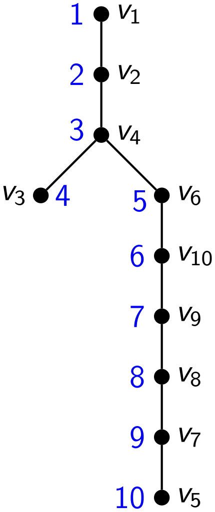
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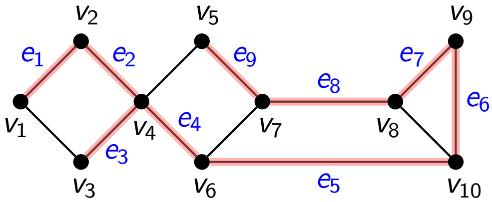
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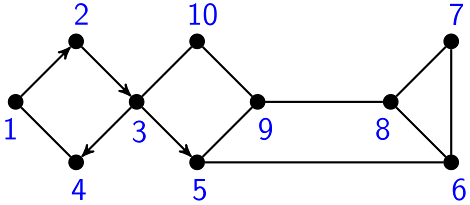
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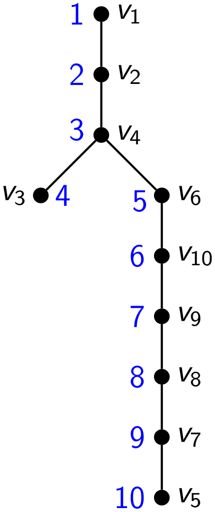
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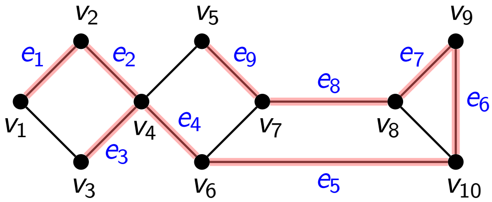
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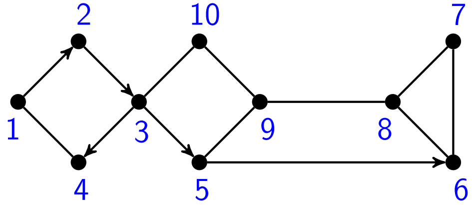
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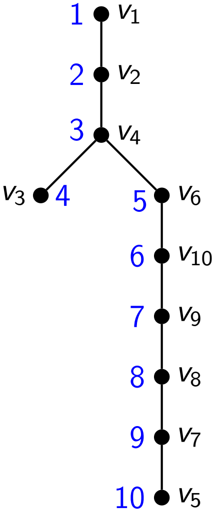
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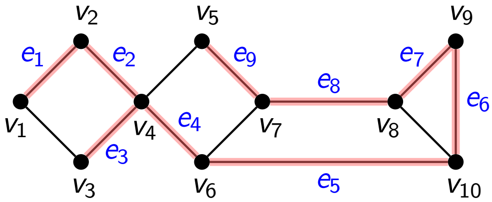
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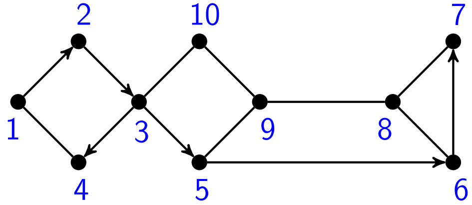
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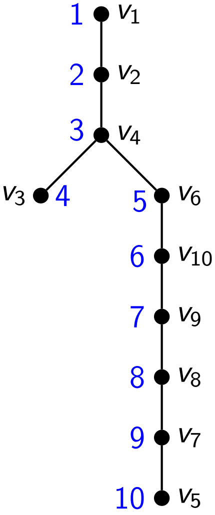
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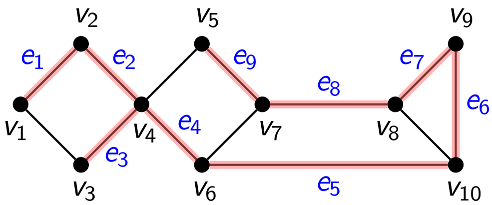
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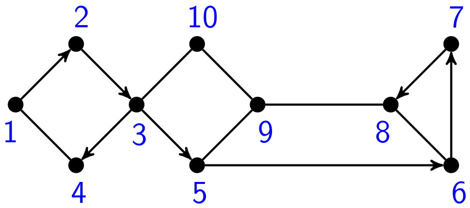
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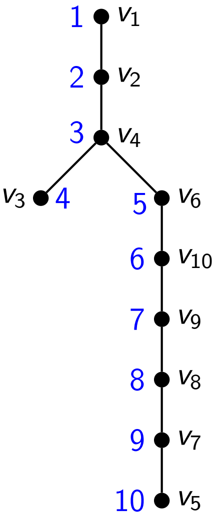
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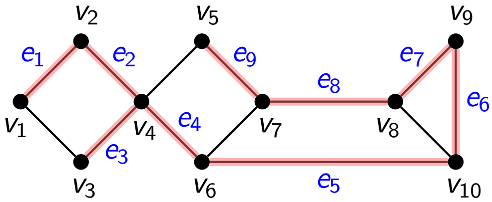
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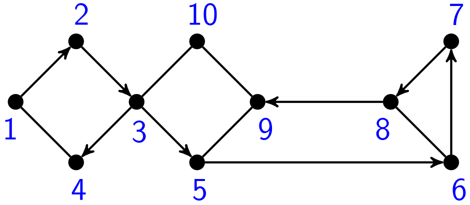
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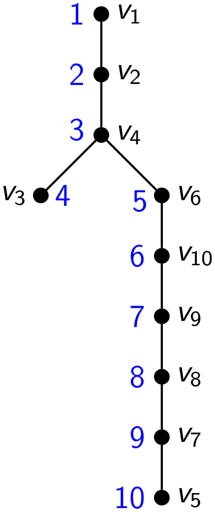
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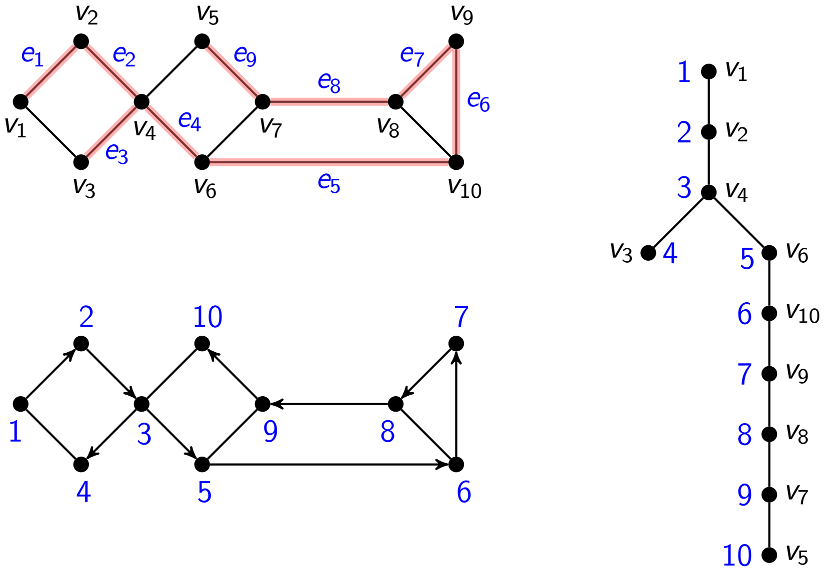

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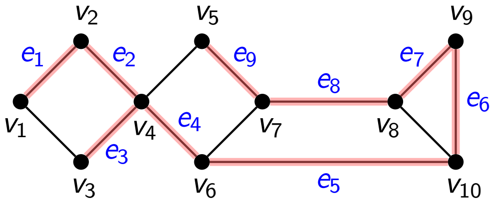
9 v_7
10 v_5


v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
v_{10}
e_1
e_2
e_3
e_4
e_5
e_6
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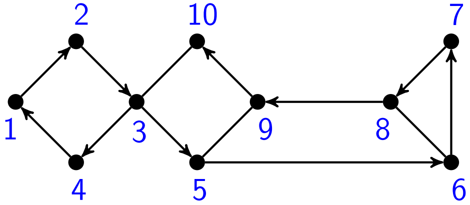
e_9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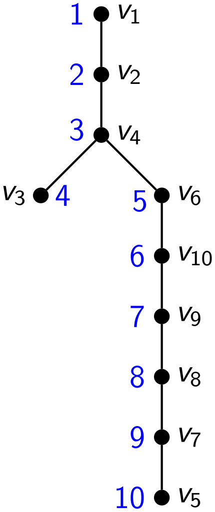
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v_3 4
5 v_6
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7 v_9
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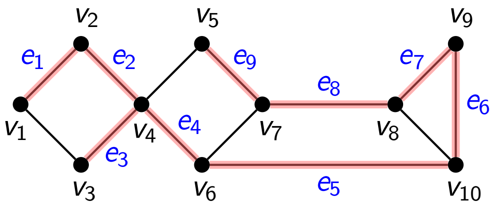
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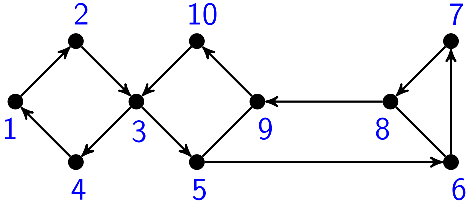
e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9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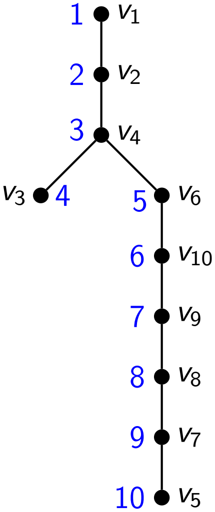
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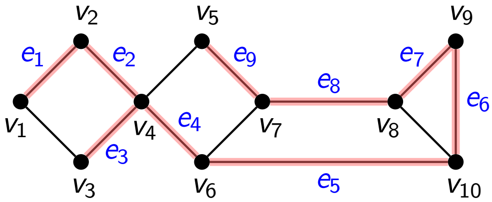
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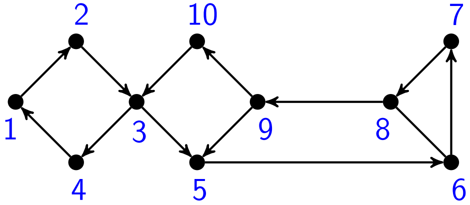
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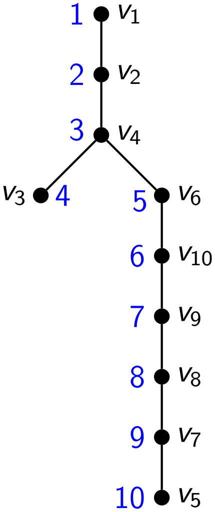
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v_3 4
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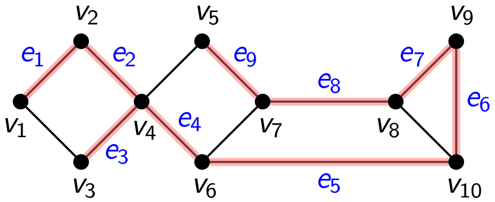
v_1
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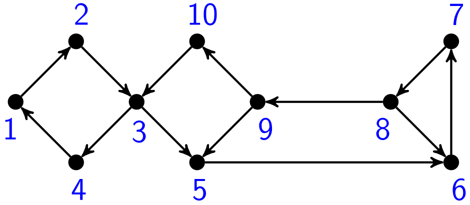
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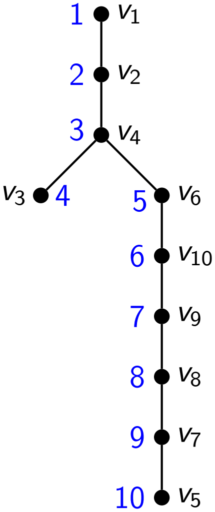
1 v_1
2 v_2
3 v_4
v_3 4
5 v_6
6 v_{10}
7 v_9
8 v_8
9 v_7
10 v_5