Šetnje u grafu. Težinski grafovi

Diskretne strukture s teorijom grafova

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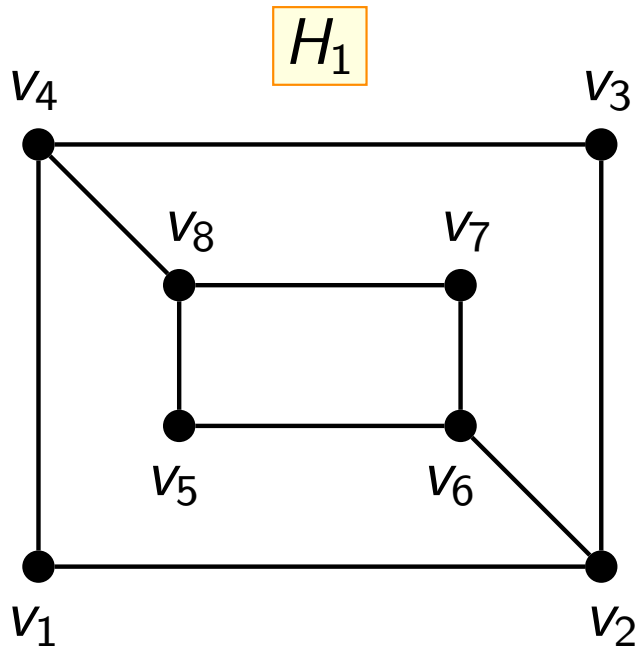
FOI, Varaždin

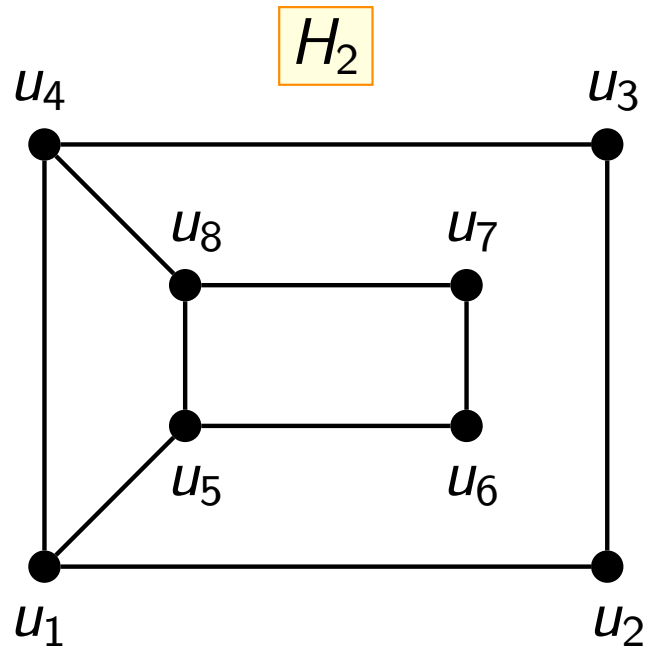
Sadržaj

prvi zadatak

Zadatak 1

Zadani su grafovi H_1 i H_2.





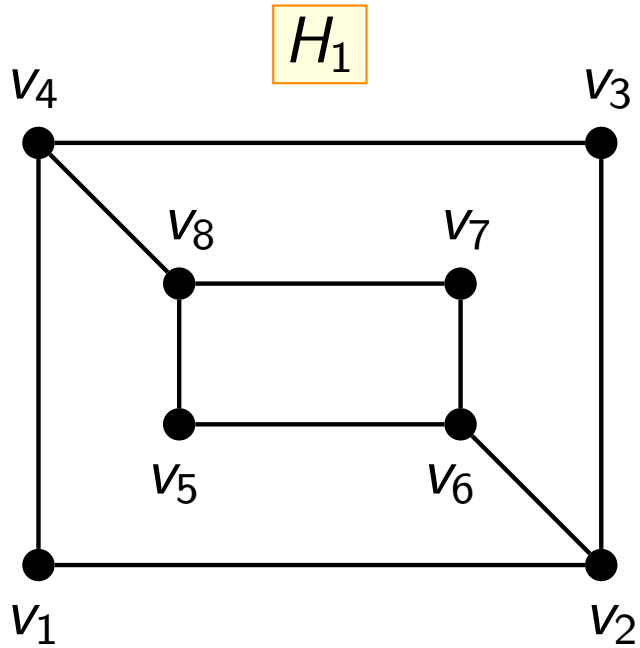
a) *Ispitajte jesu li grafovi H_1 i H_2 izomorfni.*

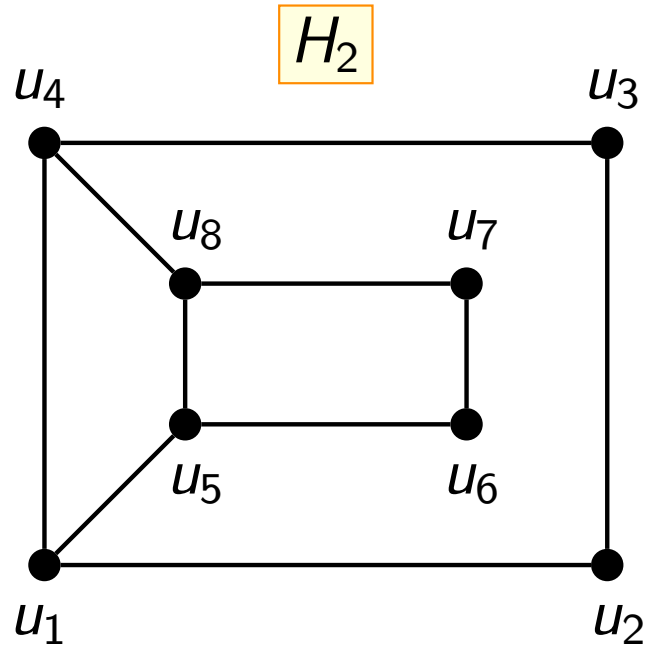
b) *Napišite jednu (v_2, v_4)-šetnju duljine 8 u grafu H_1 koja nije staza.*

c) *Napišite jednu (v_2, v_4)-stazu duljine 8 u grafu H_1.*

d) *Napišite tri (u_2, u_4)-puta različitih duljina u grafu H_2.*

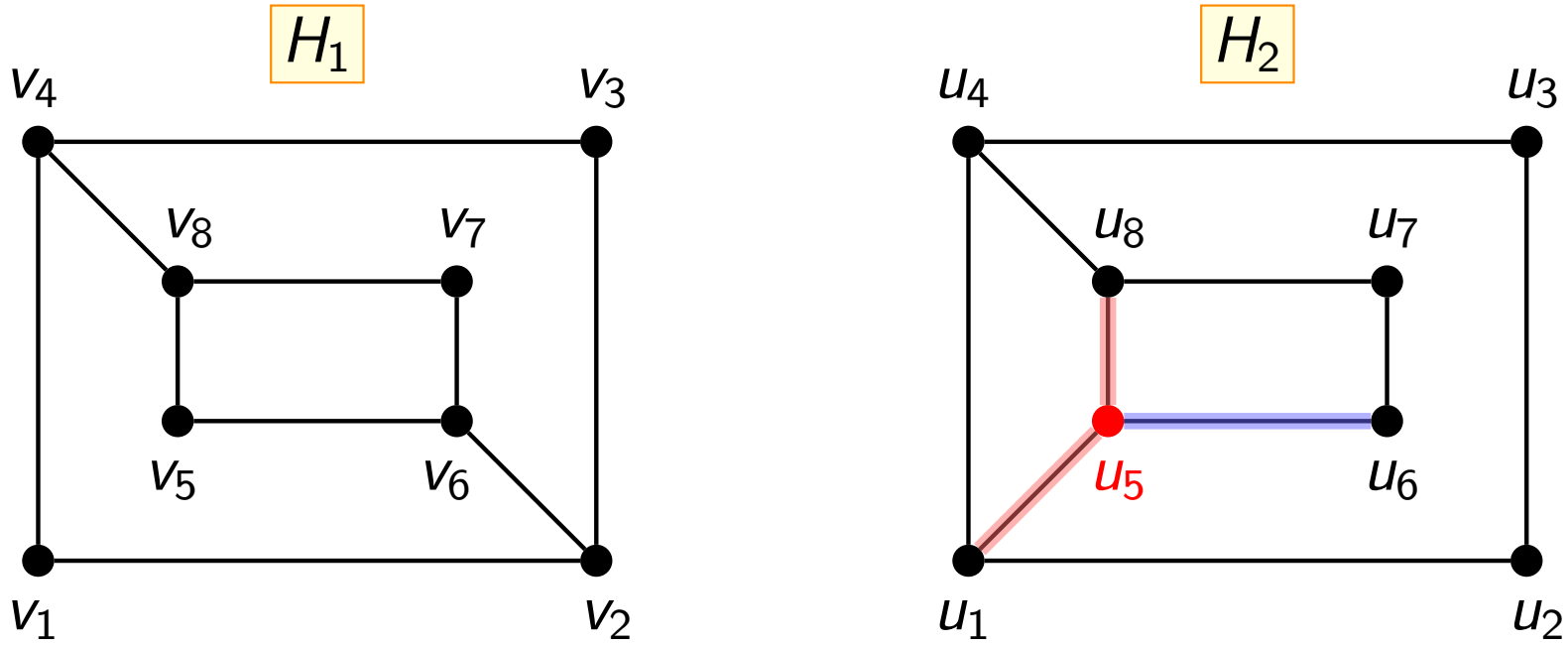
Rješenje





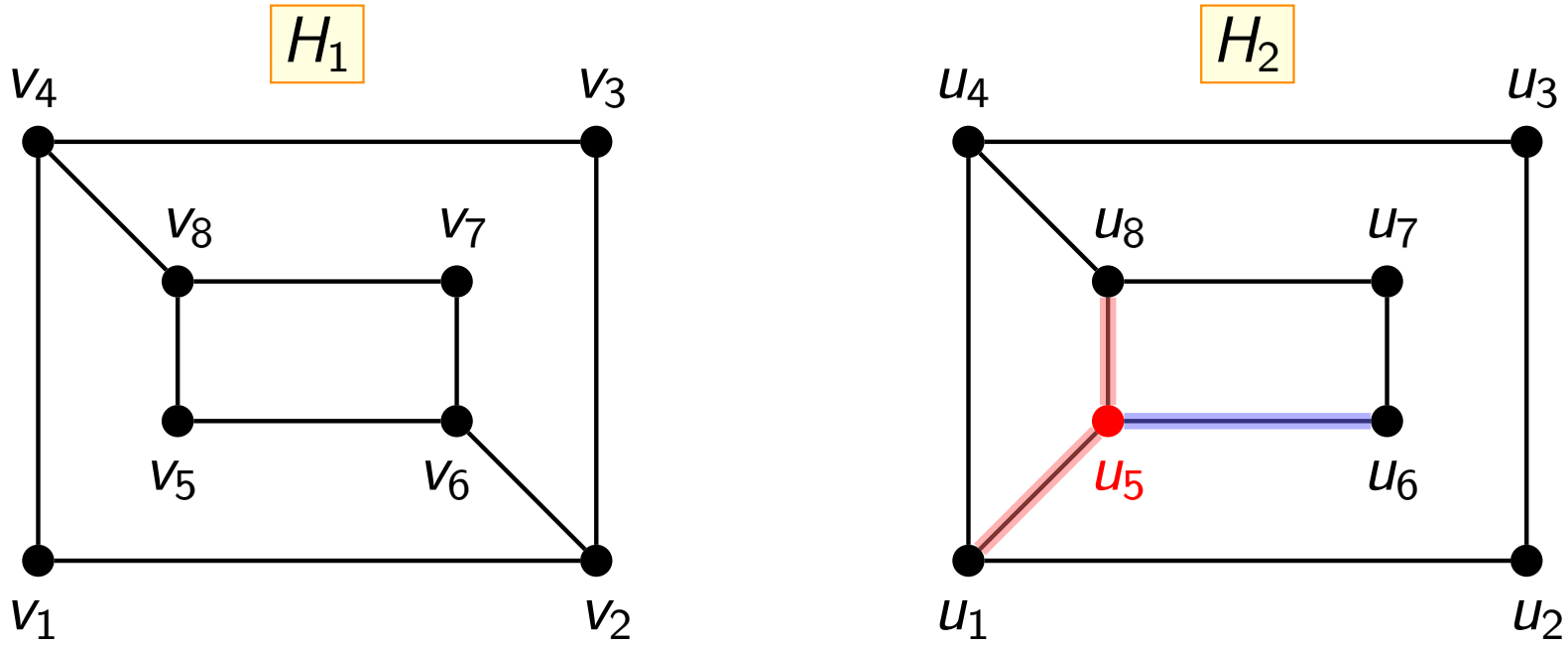
a)

Rješenje



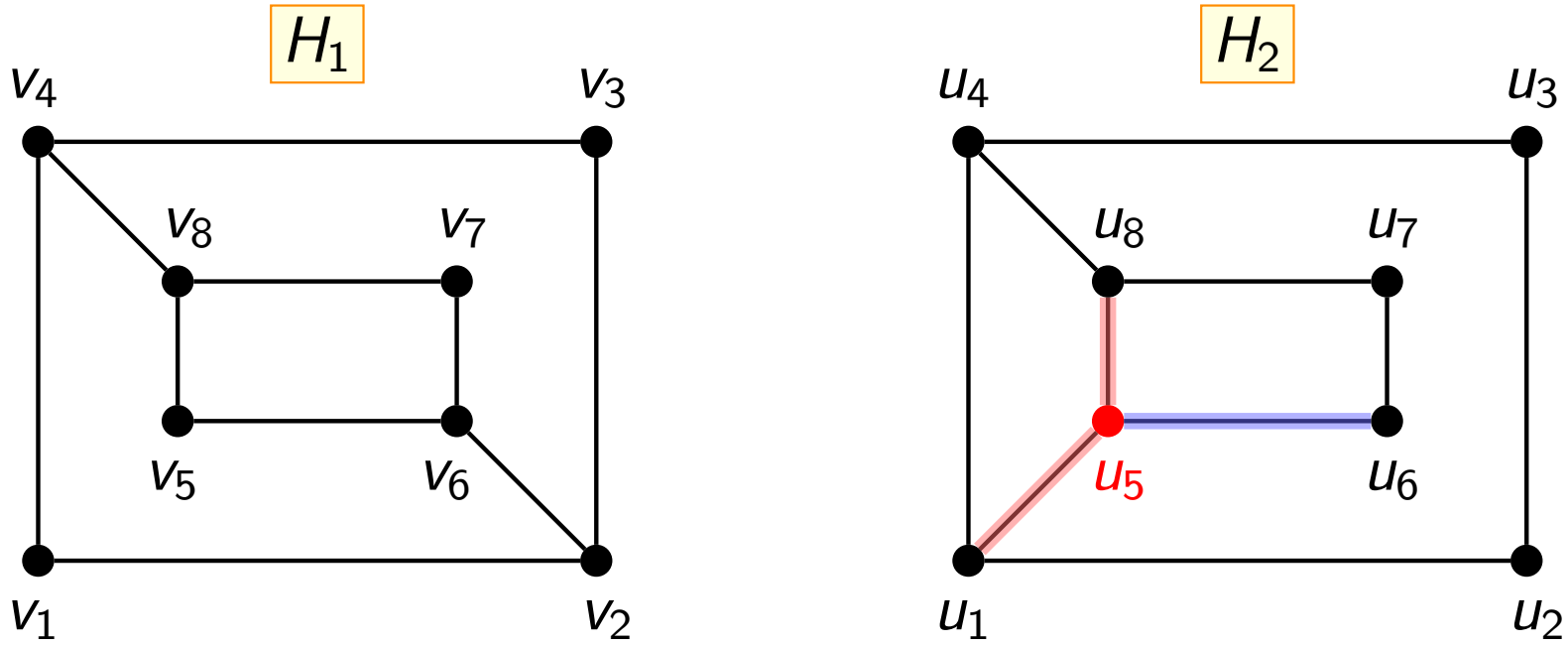
a) U grafu H_2 vrh u_5 stupnja 3 susjedan je s dva vrha stupnja 3 i jednim vrhom stupnja 2.

Rješenje



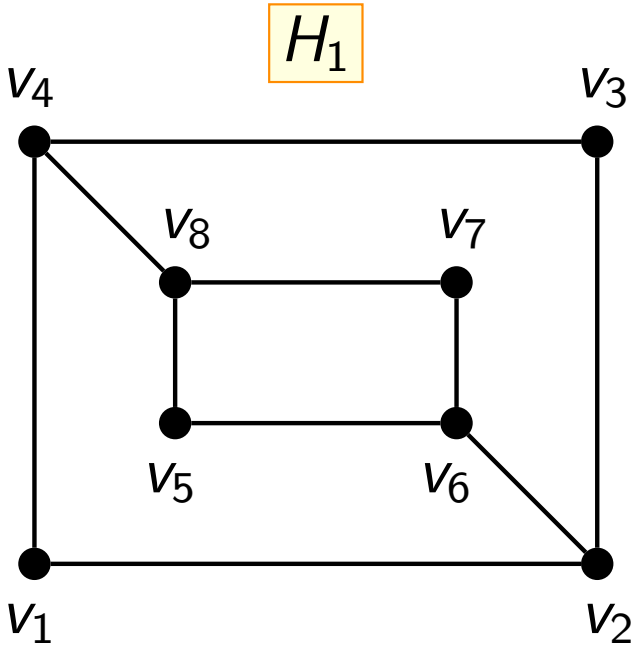
a) U grafu H_2 vrh u_5 stupnja 3 susjedan je s dva vrha stupnja 3 i jednim vrhom stupnja 2. U grafu H_1 takav vrh stupnja 3 ne postoji.

Rješenje



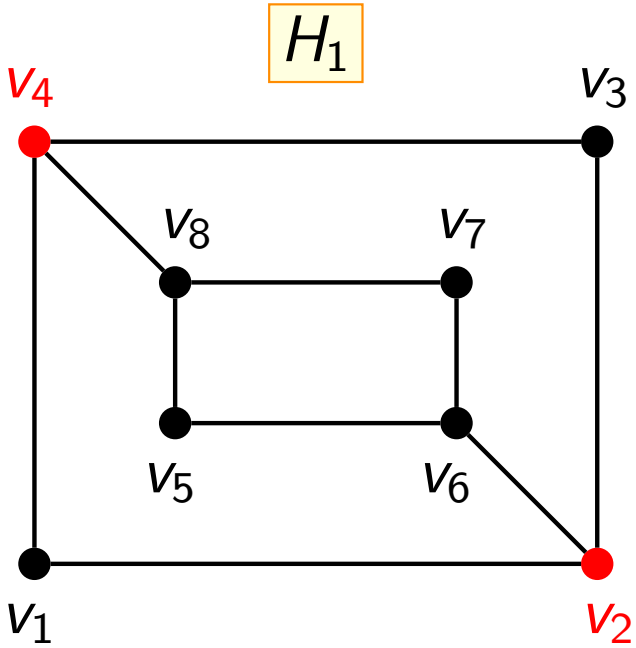
a) U grafu H_2 vrh u_5 stupnja 3 susjedan je s dva vrha stupnja 3 i jednim vrhom stupnja 2. U grafu H_1 takav vrh stupnja 3 ne postoji. Stoga H_1 i H_2 nisu izomorfni grafovi.

b)



(v_2, v_4)-šetnja duljine 8

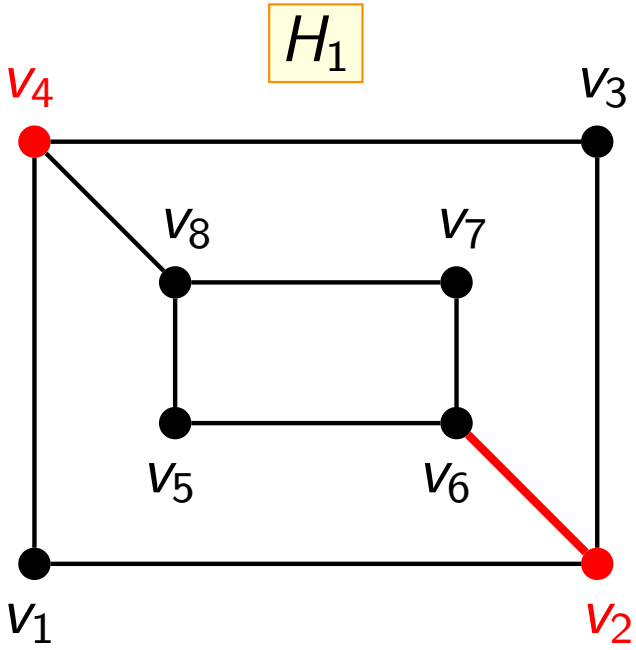
b)



(v_2, v_4)-šetnja duljine 8

v_2

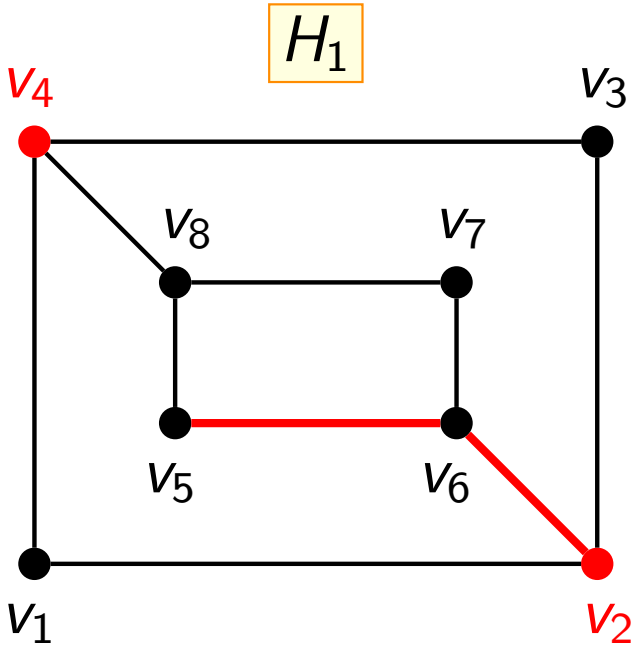
b)



(v_2, v_4)-šetnja duljine 8

$v_2 v_6$

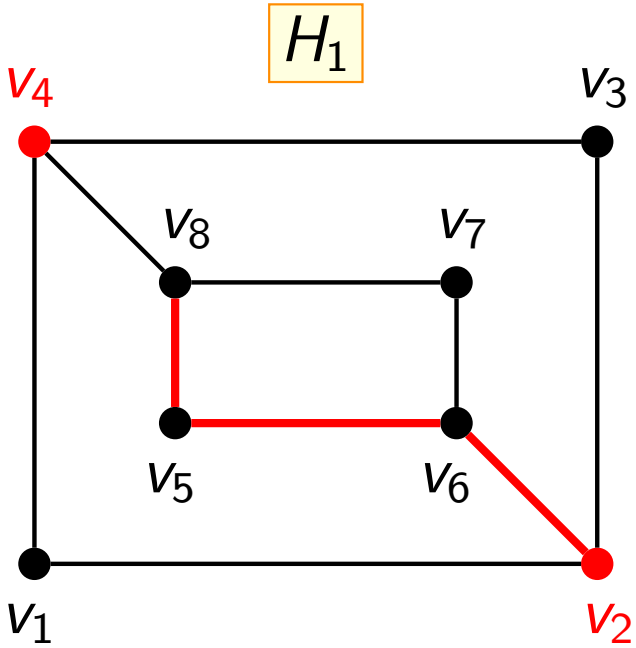
b)



(v_2, v_4)-šetnja duljine 8

$v_2 v_6 v_5$

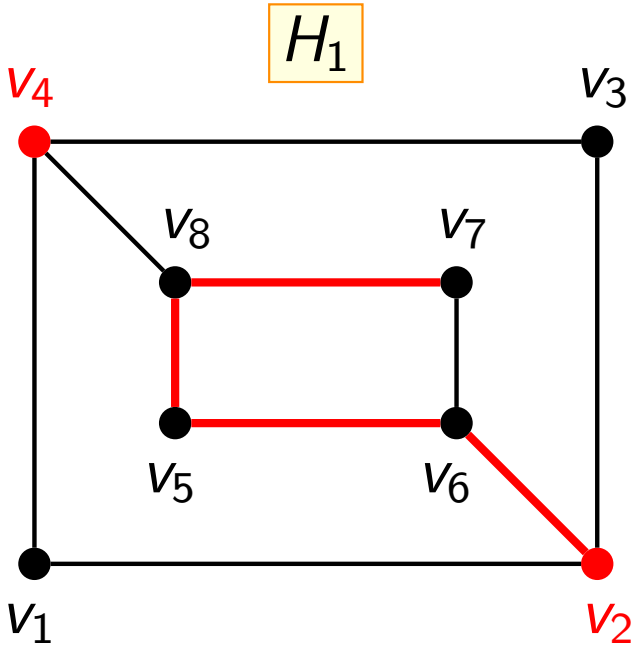
b)



(v_2, v_4)-šetnja duljine 8

$v_2 v_6 v_5 v_8$

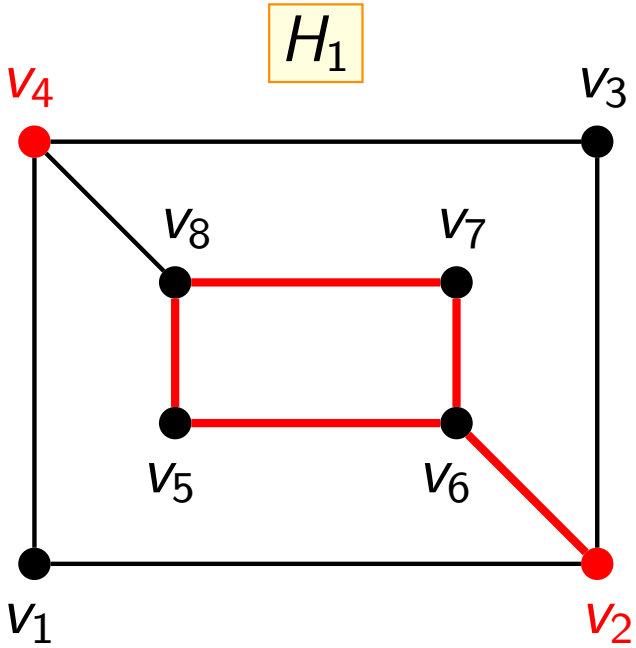
b)



(v_2, v_4)-šetnja duljine 8

$v_2 v_6 v_5 v_8 v_7$

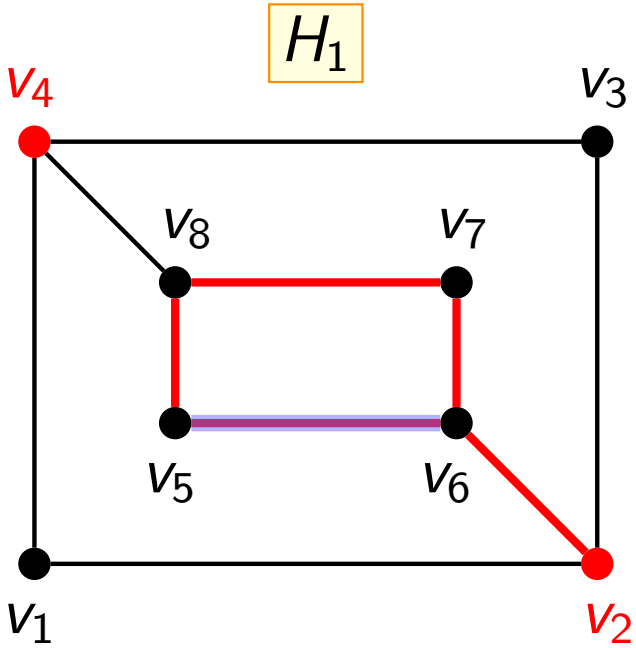
b)



(v_2, v_4)-šetnja duljine 8

$v_2 v_6 v_5 v_8 v_7 v_6$

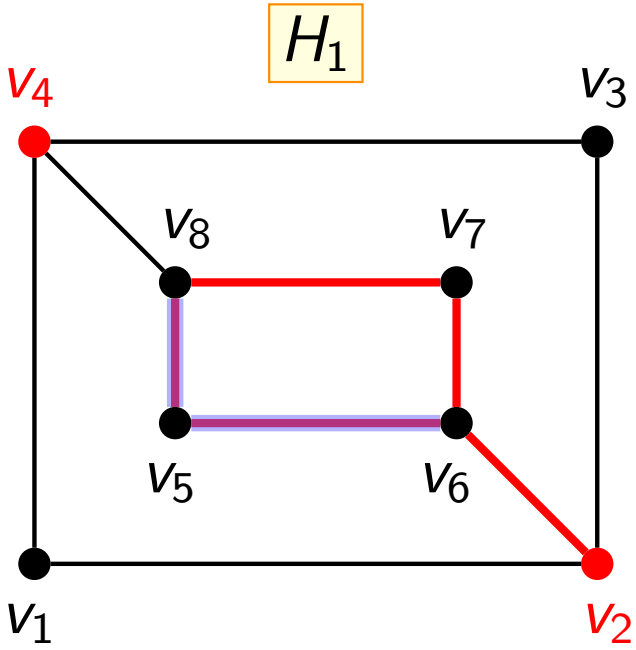
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5$

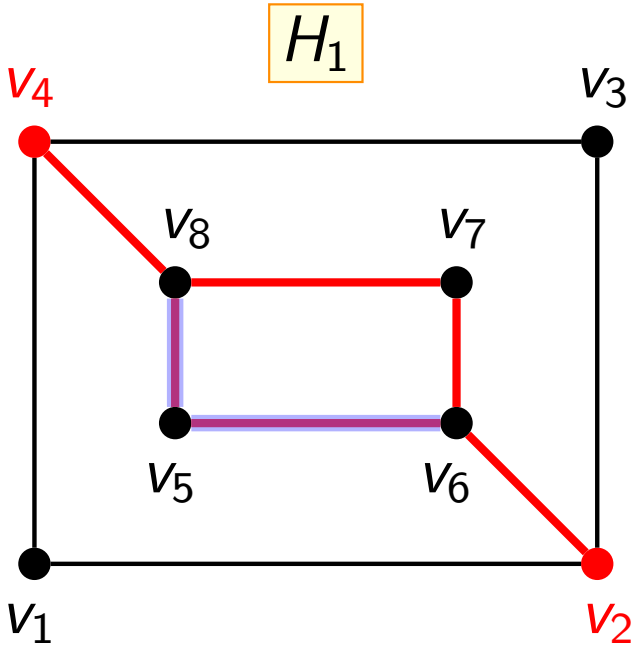
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8$

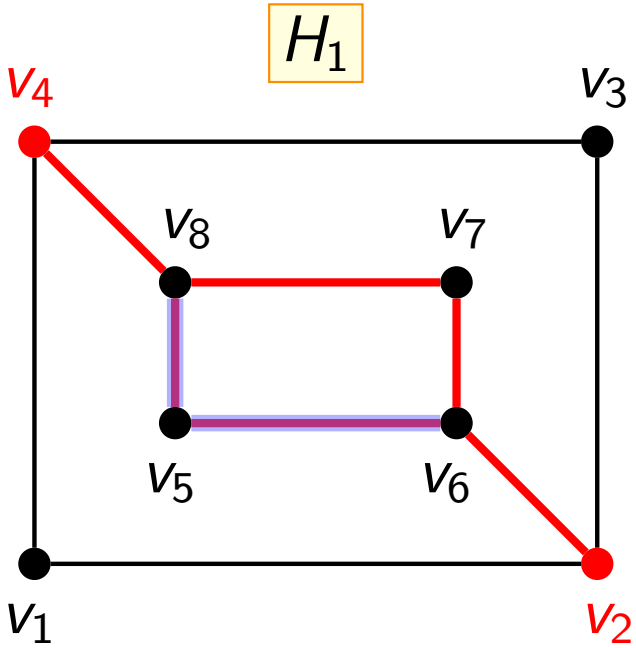
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

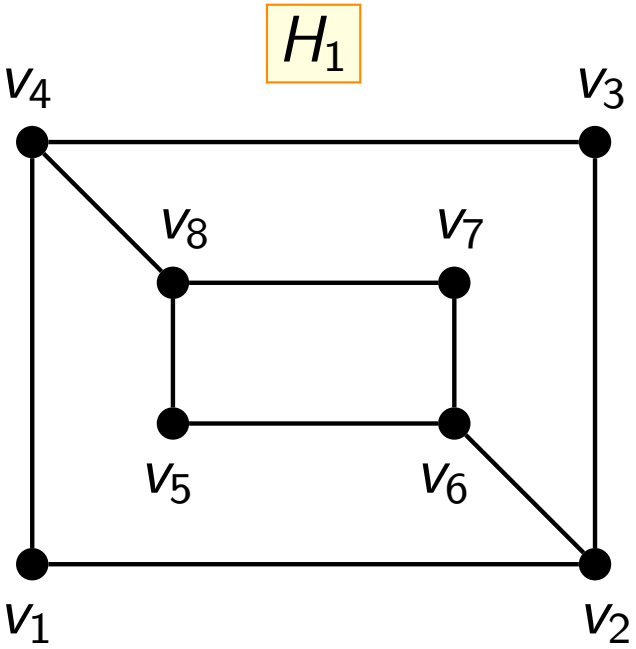
b)



(v_2, v_4)-šetnja duljine 8

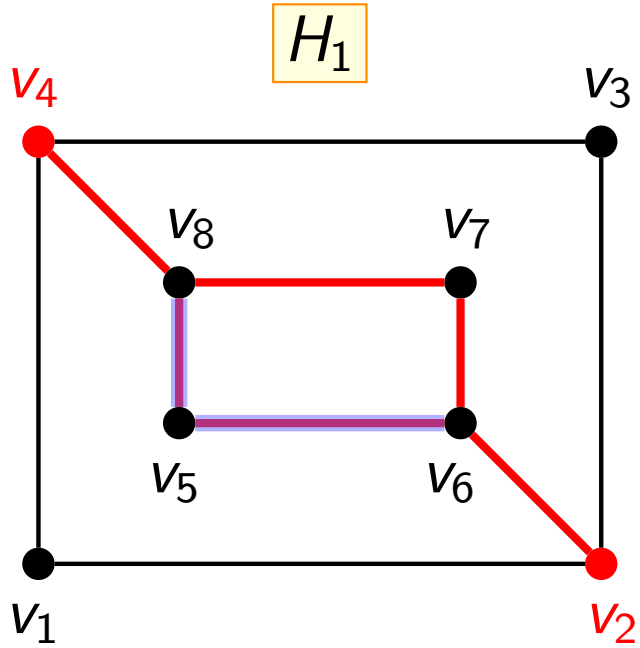
$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

c)



(v_2, v_4)-staza duljine 8

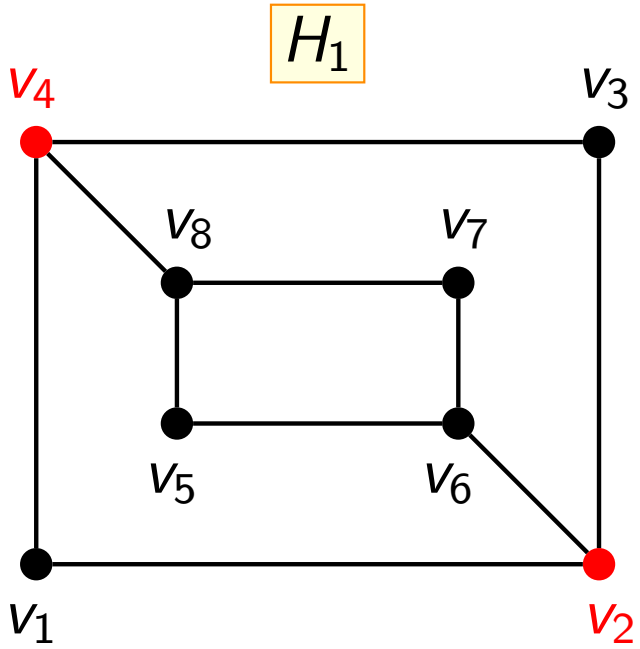
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

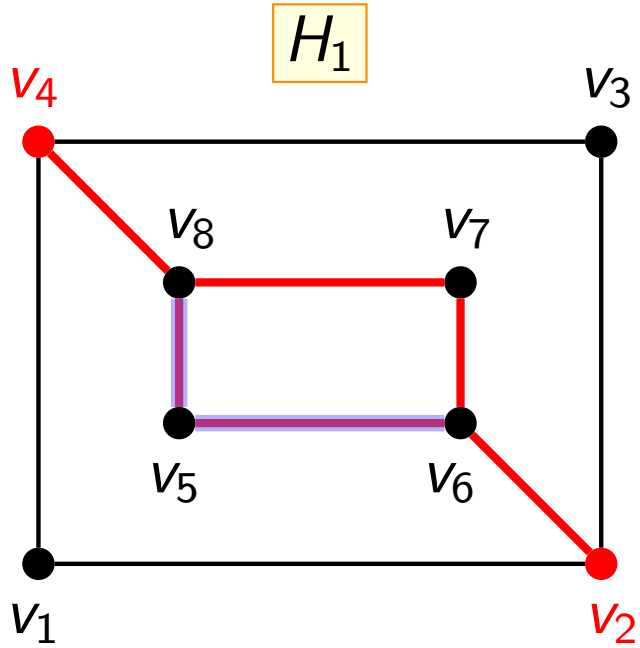
c)



(v_2, v_4)-staza duljine 8

v_2

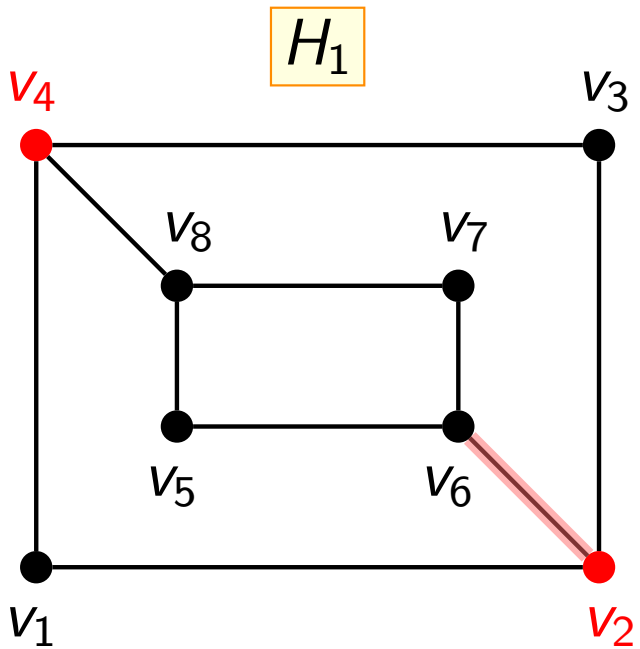
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

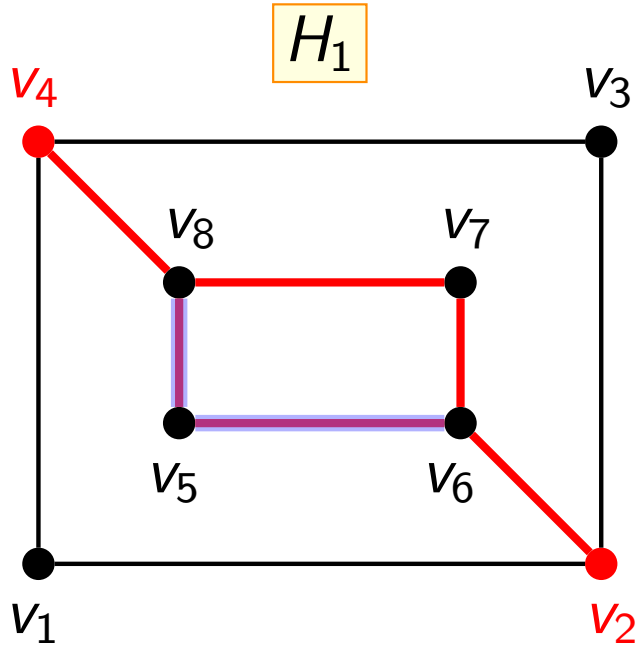
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6$

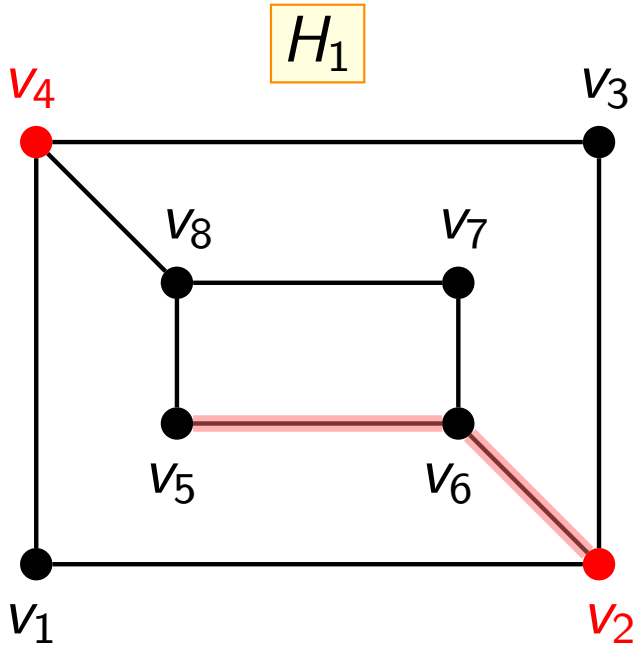
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

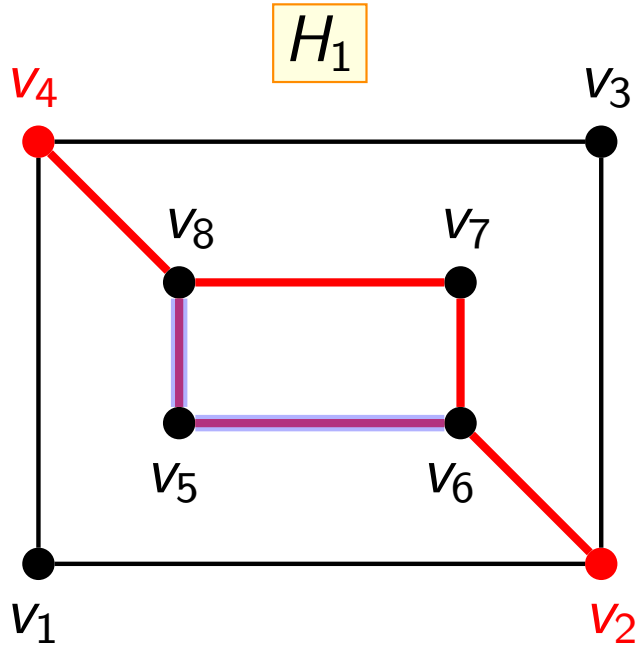
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5$

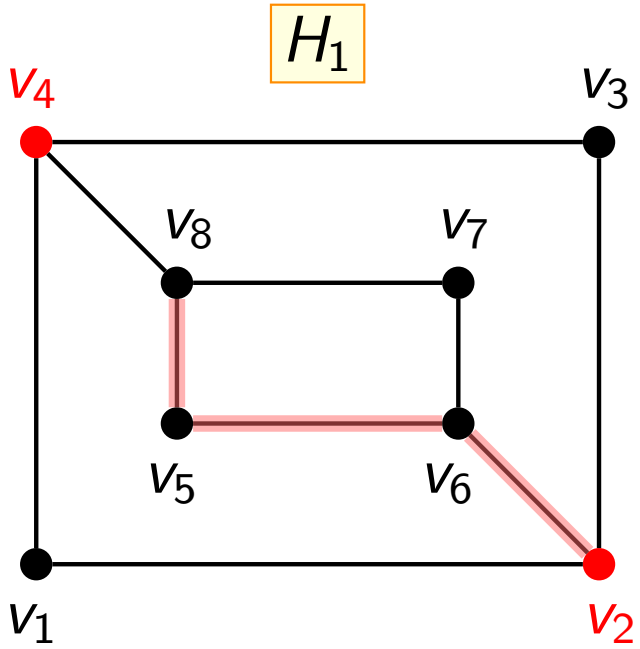
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

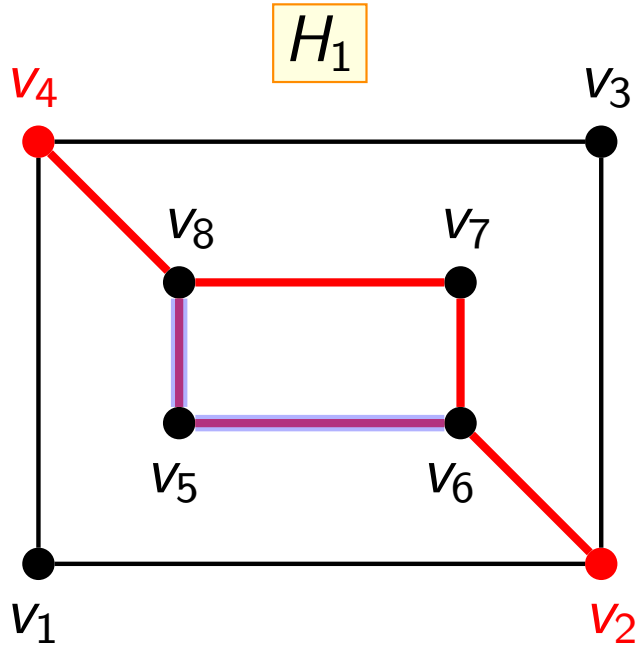
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5\, v_8$

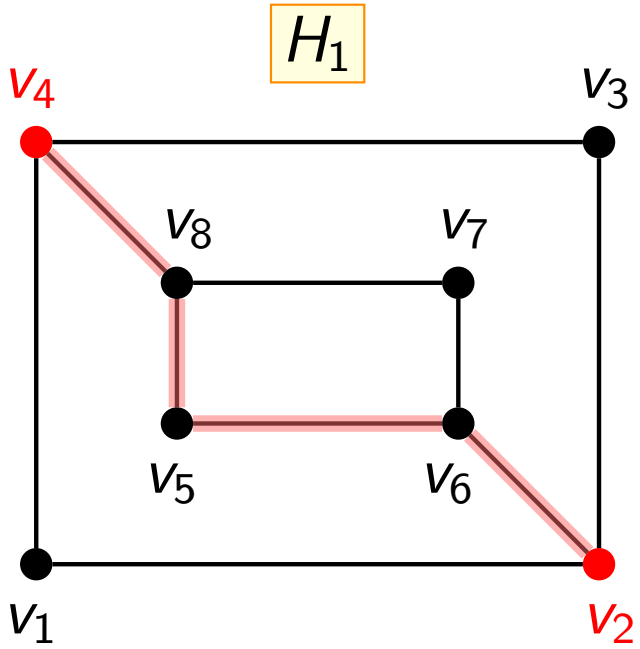
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5\, v_8\, v_4$

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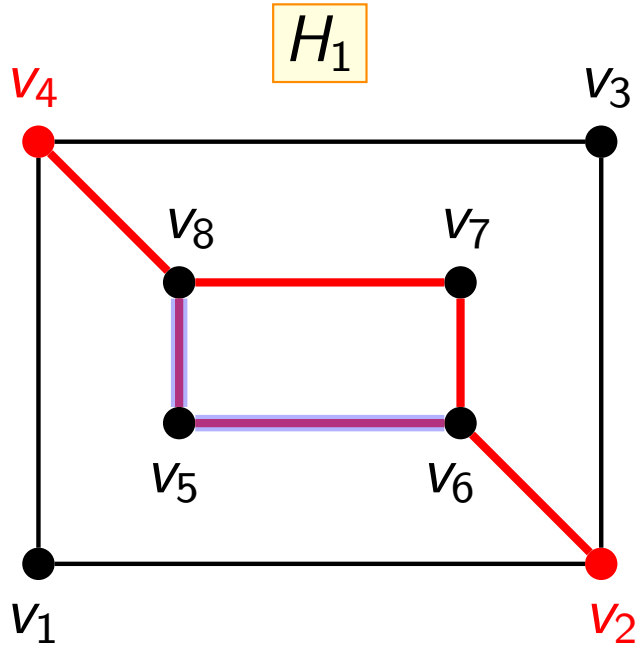
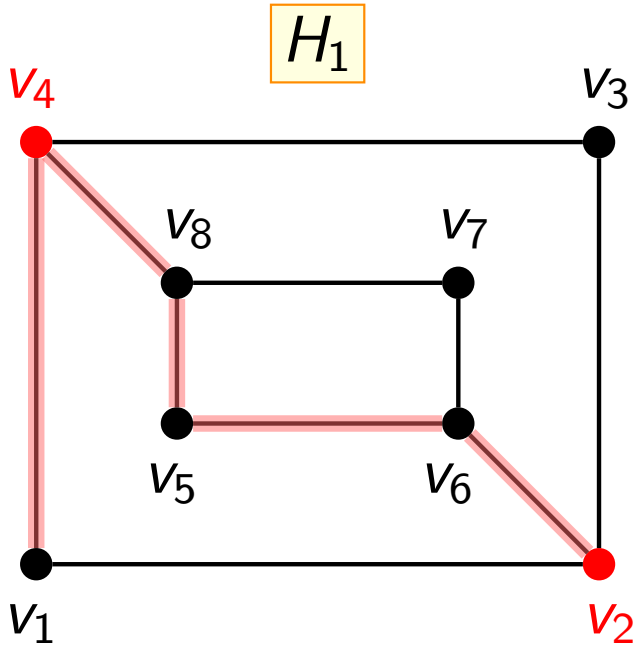


(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

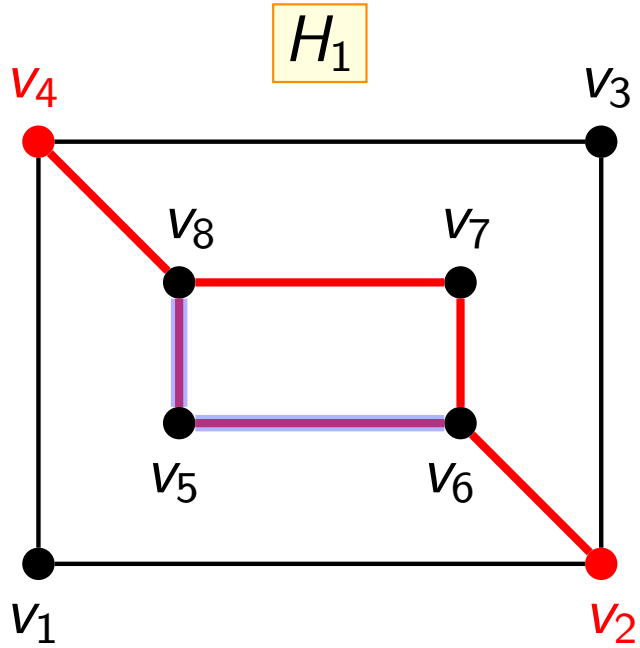
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5\, v_8\, v_4\, v_1$

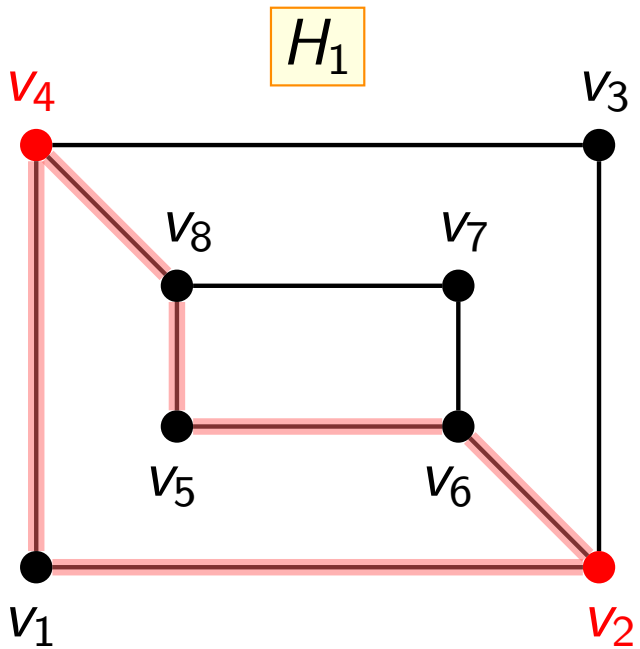
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

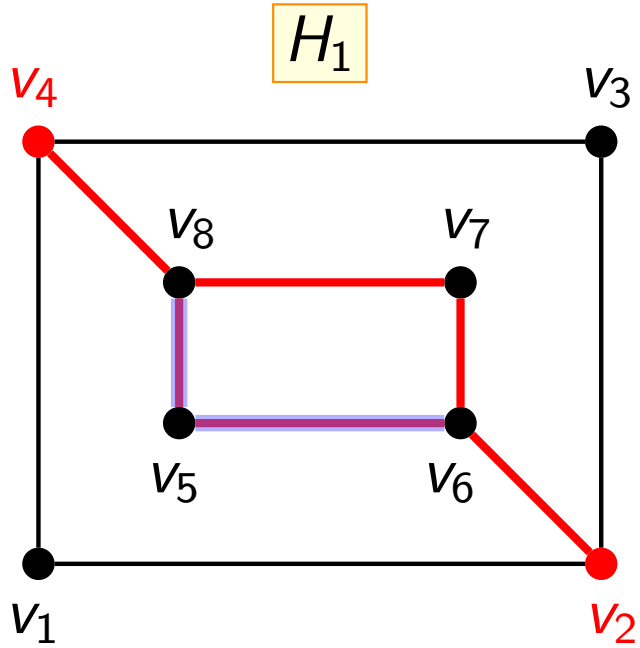
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5\, v_8\, v_4\, v_1\, v_2$

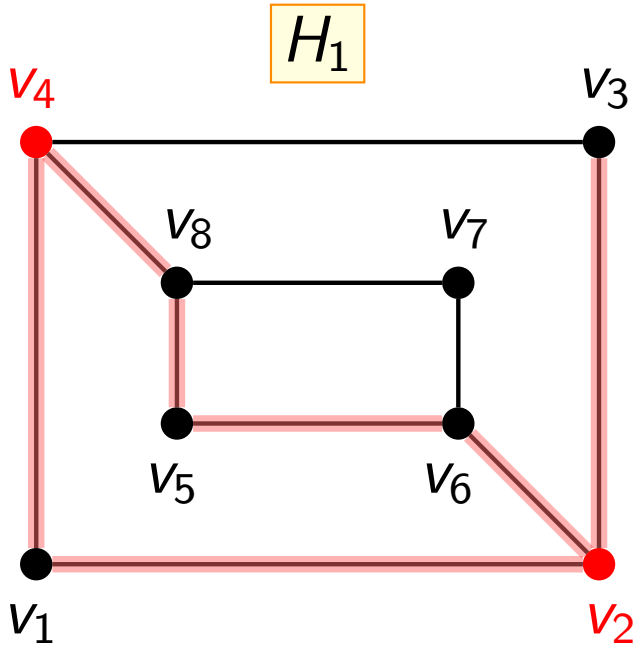
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

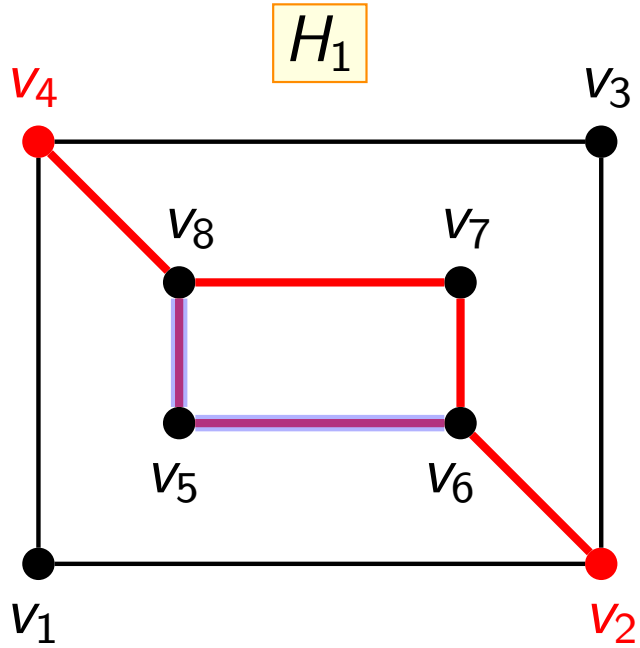
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5\, v_8\, v_4\, v_1\, v_2\, v_3$

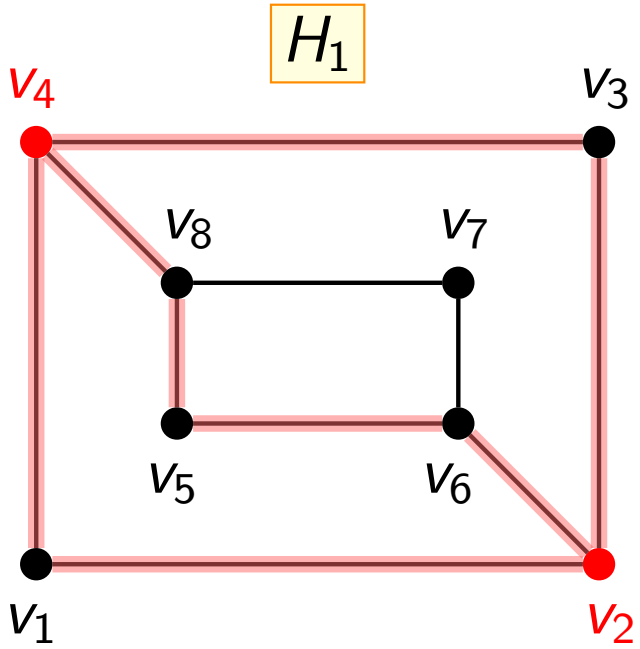
b)



(v_2, v_4)-šetnja duljine 8

$v_2\, v_6\, v_5\, v_8\, v_7\, v_6\, v_5\, v_8\, v_4$

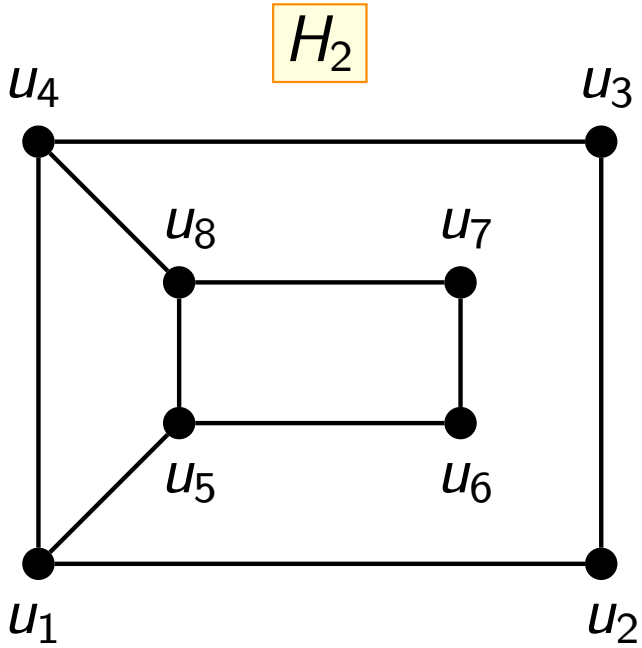
c)



(v_2, v_4)-staza duljine 8

$v_2\, v_6\, v_5\, v_8\, v_4\, v_1\, v_2\, v_3\, v_4$

d)



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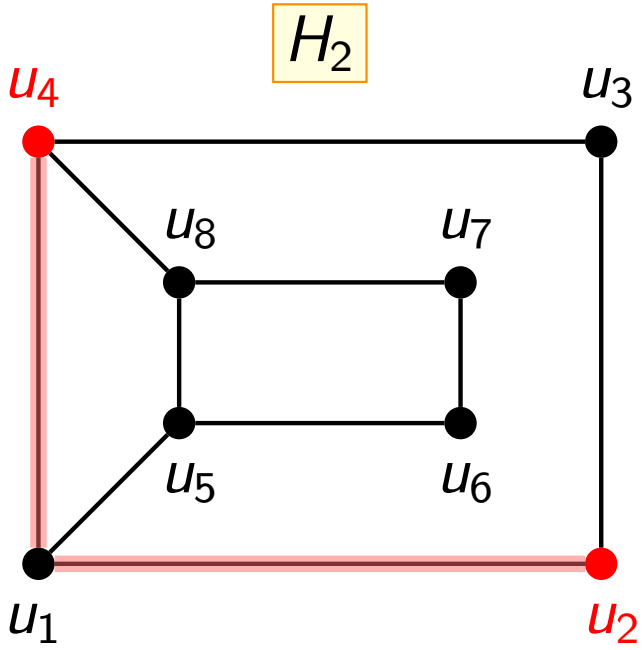
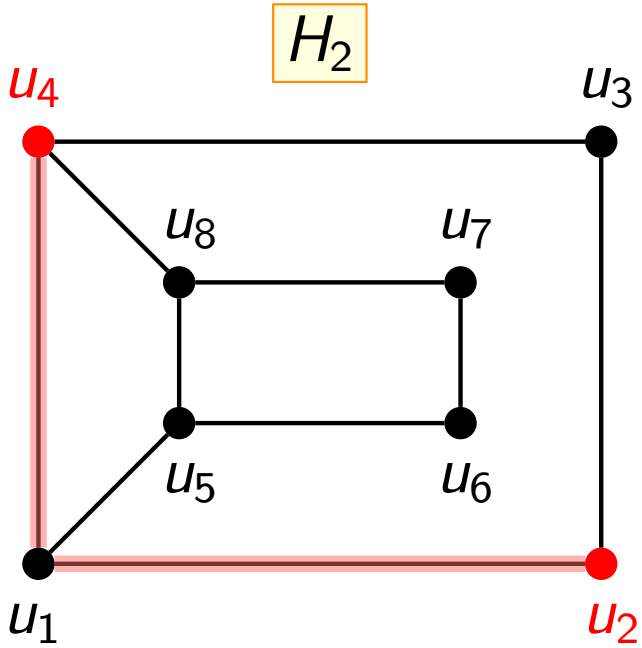
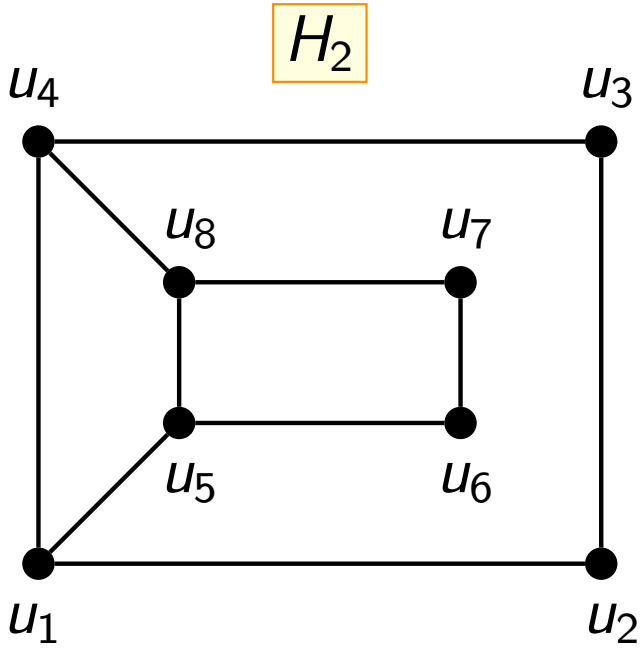


(u_2, u_4)-put duljine 2 $u_2 u_1 u_4$

d)



(u_2, u_4)-put duljine 2 $u_2 u_1 u_4$



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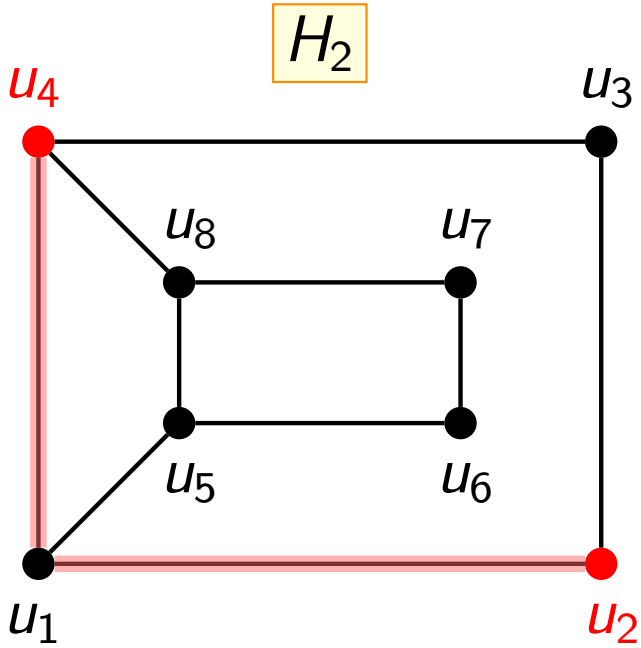
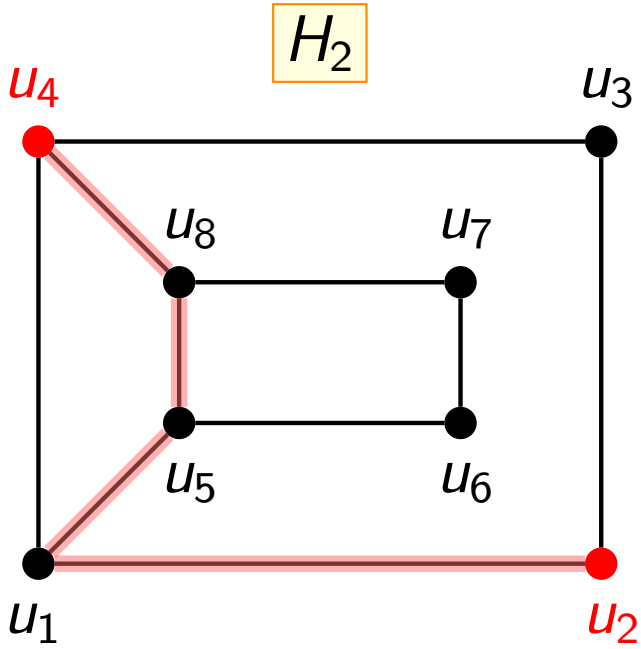


(u_2, u_4)-put duljine 2 $u_2\,u_1\,u_4$

(u_2, u_4)-put duljine 4 $u_2\,u_1\,u_5\,u_8\,u_4$



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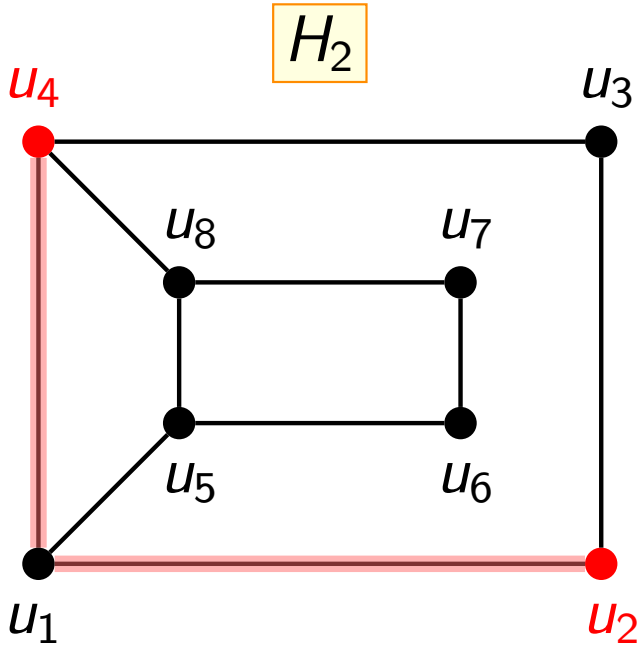
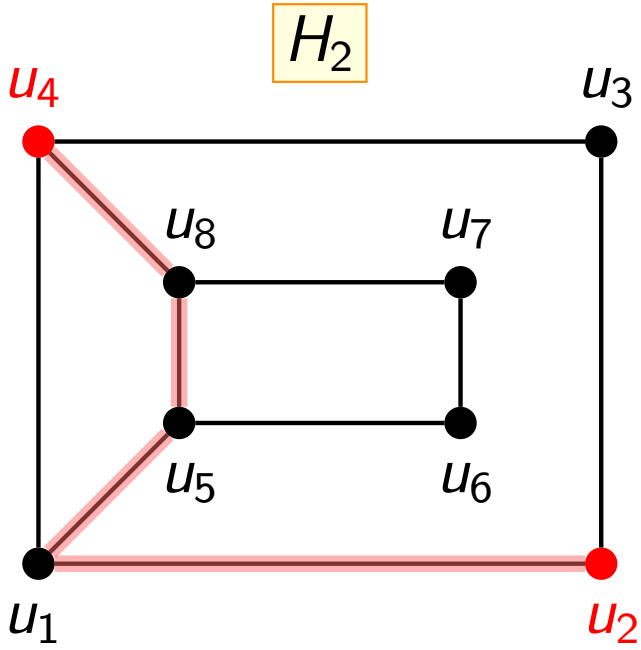
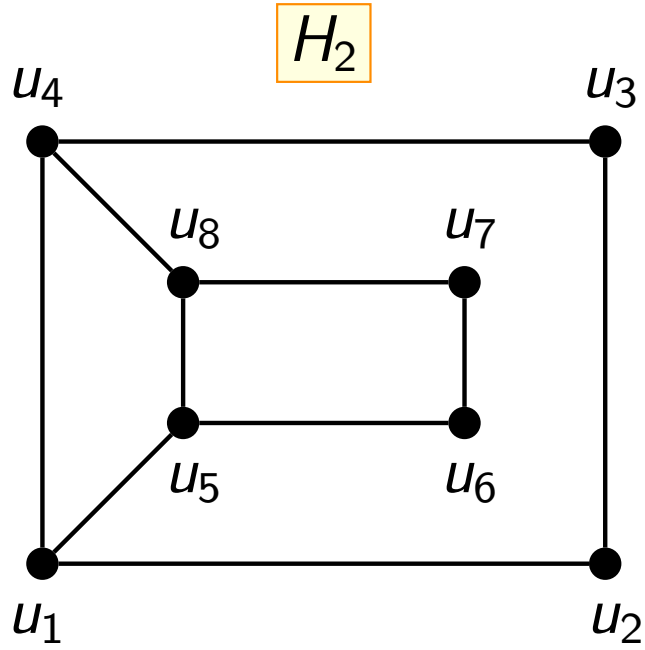


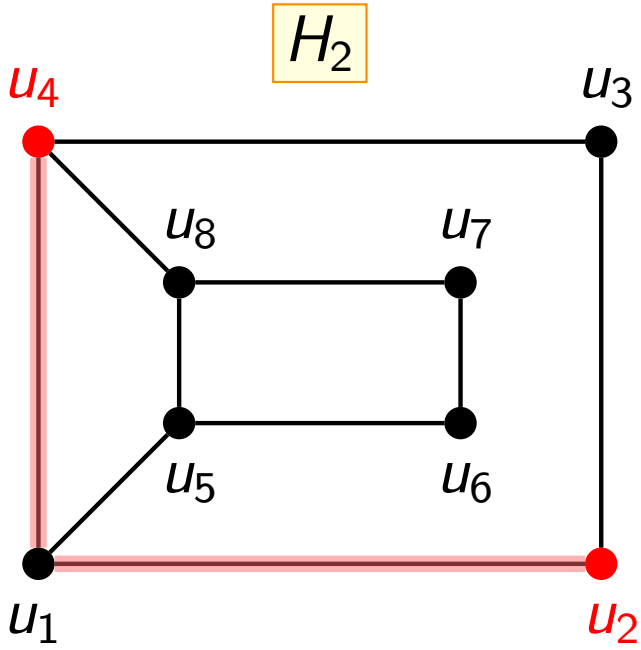
(u_2, u_4)-put duljine 2 $u_2\,u_1\,u_4$

(u_2, u_4)-put duljine 4 $u_2\,u_1\,u_5\,u_8\,u_4$





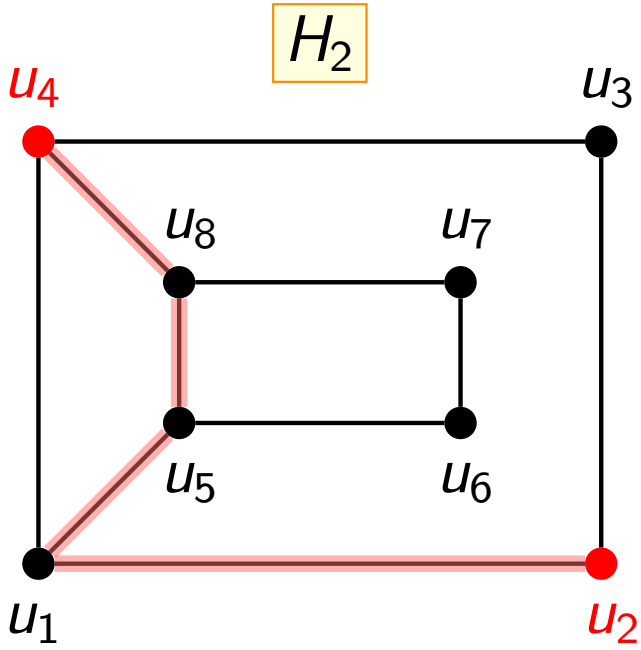
d)

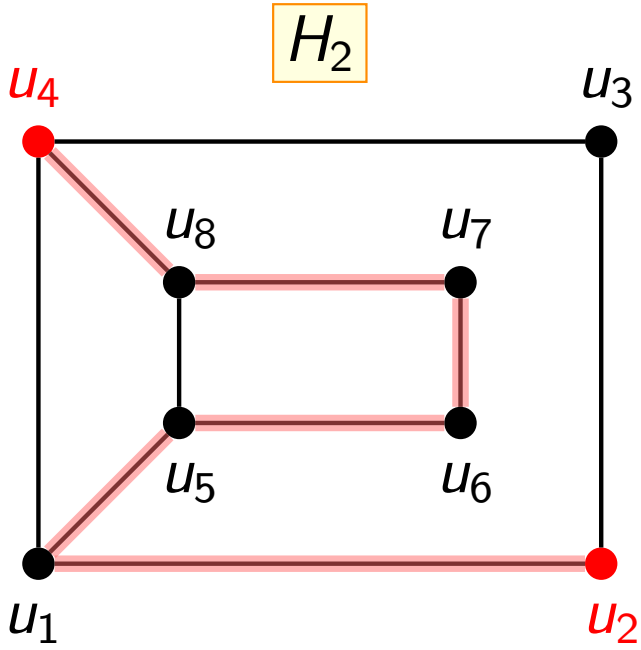


(u_2, u_4)-put duljine 2 $u_2\,u_1\,u_4$

(u_2, u_4)-put duljine 4 $u_2\,u_1\,u_5\,u_8\,u_4$

(u_2, u_4)-put duljine 6 $u_2\,u_1\,u_5\,u_6\,u_7\,u_8\,u_4$





drugi zadatak

Propozicija

Neka je $A = A(G) = \left[a_{ij}\right]$ matrica susjedstva grafa G. Tada je (i,j)-ti element matrice A^k jednak broju $\left(v_i, v_j\right)$-šetnji duljine k u grafu G. Stoga je broj svih šetnji duljine k u grafu G jednak sumi svih elemenata od A^k.

Zadatak 2

Zadan je graf G matricom susjedstva

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

pri čemu i-tom retku pripada vrh v_i.

a) *Pomoću potencija matrice A ispitajte je li G povezani graf.*

b) *Nacrtajte graf G i njegov linijski graf $L(G)$.*

c) *Odredite struk grafa G i njegovog linijskog grafa $L(G)$.*

d) *Odredite ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G. Jesu li neke od tih šetnji ujedno i putovi?*

e) *Odredite ukupni broj svih šetnji duljine 3 u grafu G.*

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 =$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix}$$

$$A + A^2 =$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix}$$

$$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix} \quad A^3 =$$

$$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix} \quad A^3 = \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$

$$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix}$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix} \quad A^3 = \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$

$$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix} \quad A + A^2 + A^3 =$$

Rješenje

a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix} \quad A^3 = \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$

$$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix} \quad A + A^2 + A^3 = \begin{bmatrix} 2 & 6 & 1 & 2 & 5 \\ 6 & 3 & 4 & 6 & 2 \\ 1 & 4 & 1 & 1 & 2 \\ 2 & 6 & 1 & 2 & 5 \\ 5 & 2 & 2 & 5 & 2 \end{bmatrix}$$

Rješenje

a) Kako su svi elementi matrice $A + A^2 + A^3$ različiti od nule, zaključujemo da je G povezani graf.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{bmatrix} \quad A^3 = \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$

$$A + A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix} \quad A + A^2 + A^3 = \begin{bmatrix} 2 & 6 & 1 & 2 & 5 \\ 6 & 3 & 4 & 6 & 2 \\ 1 & 4 & 1 & 1 & 2 \\ 2 & 6 & 1 & 2 & 5 \\ 5 & 2 & 2 & 5 & 2 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

b)

$$A = \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

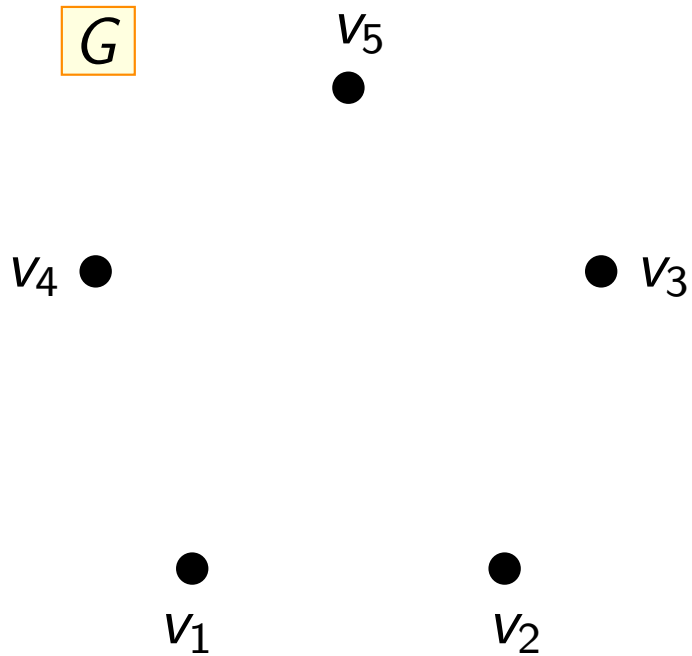
b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

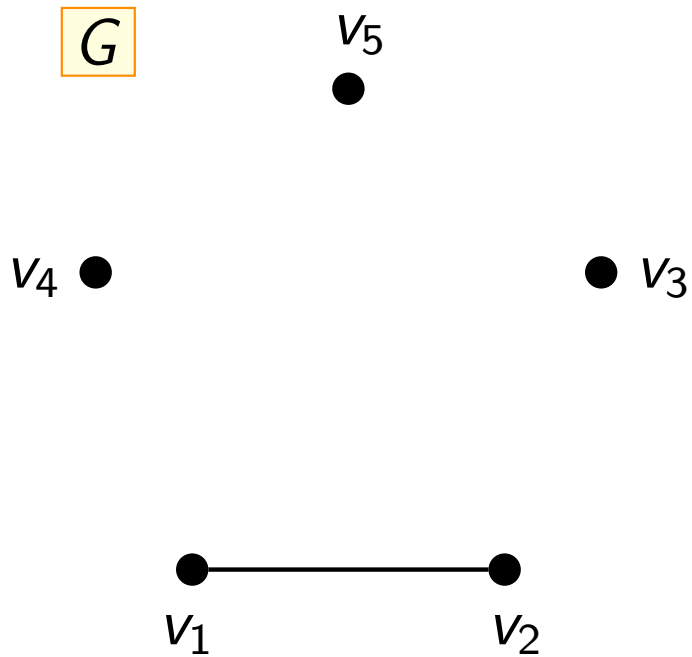
G



b)

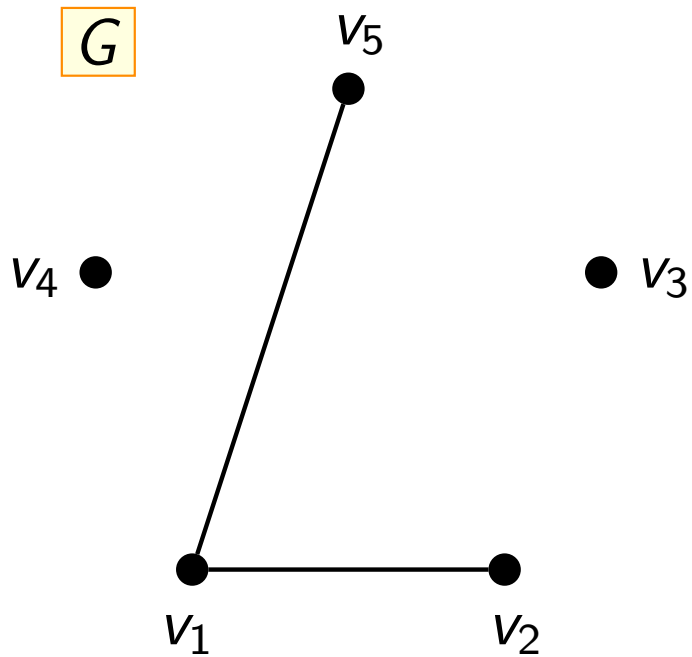
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G



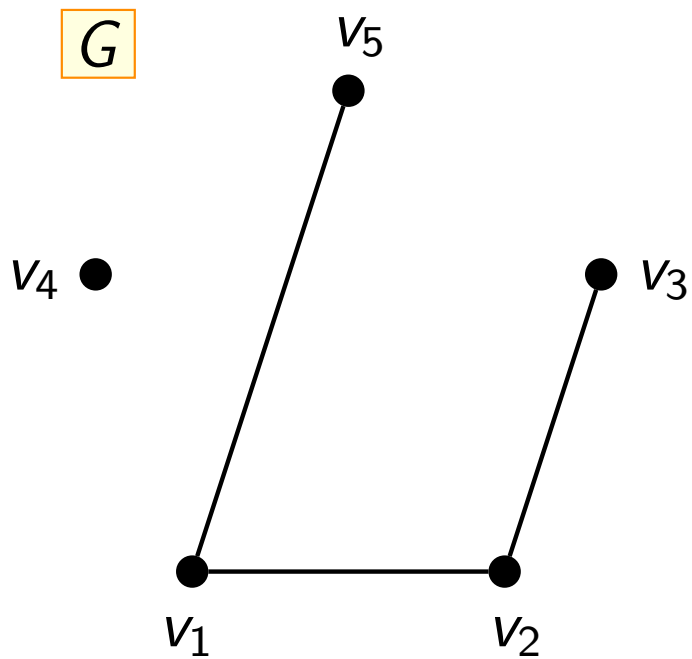
b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



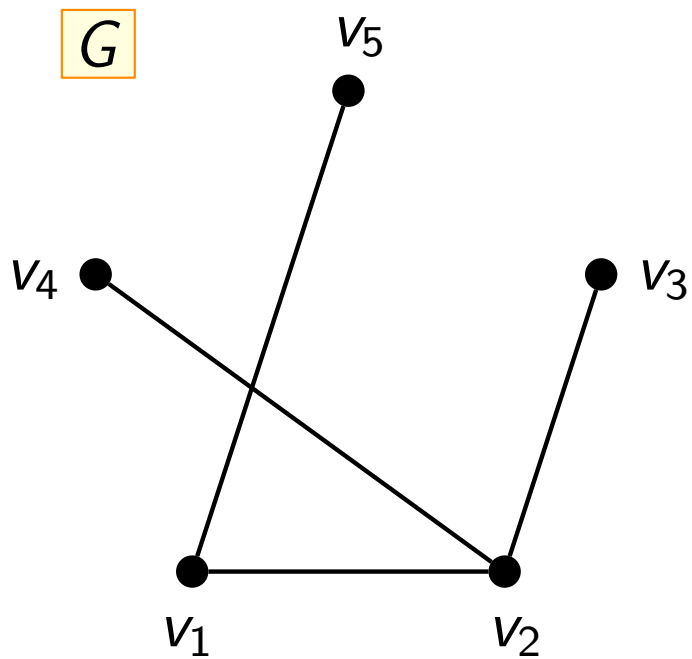
b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



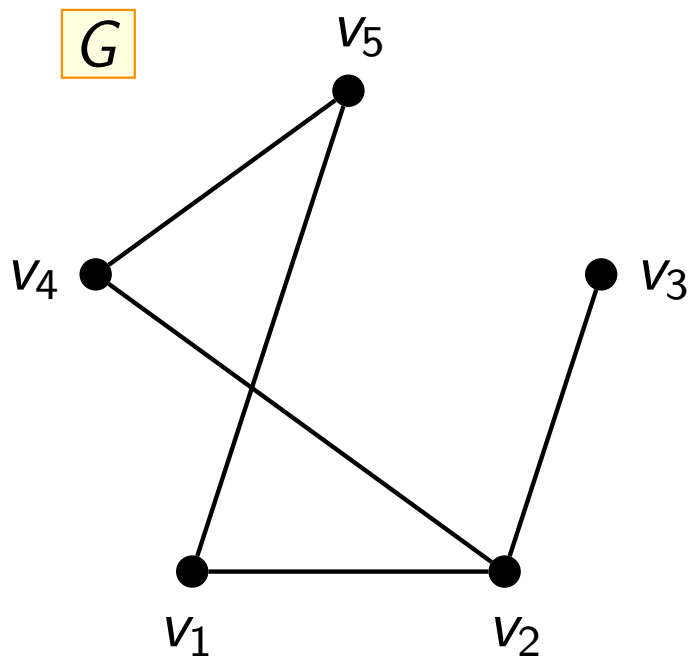
b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



b)

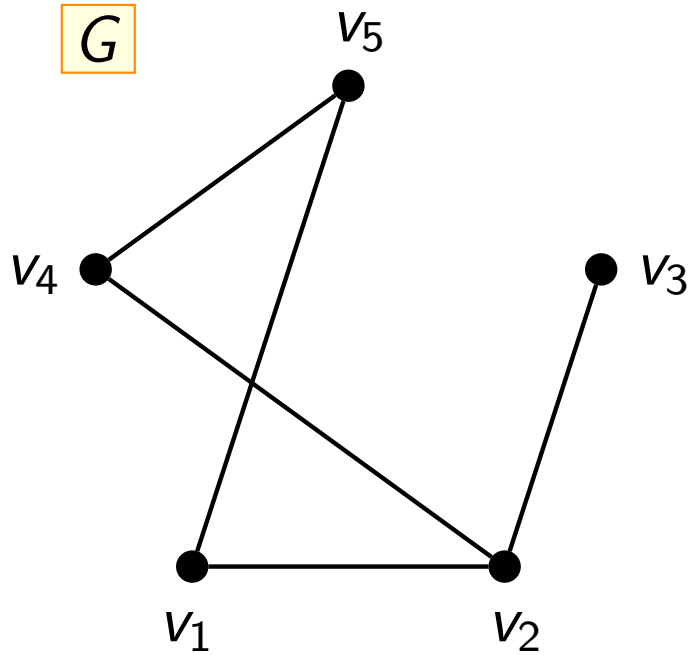
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G

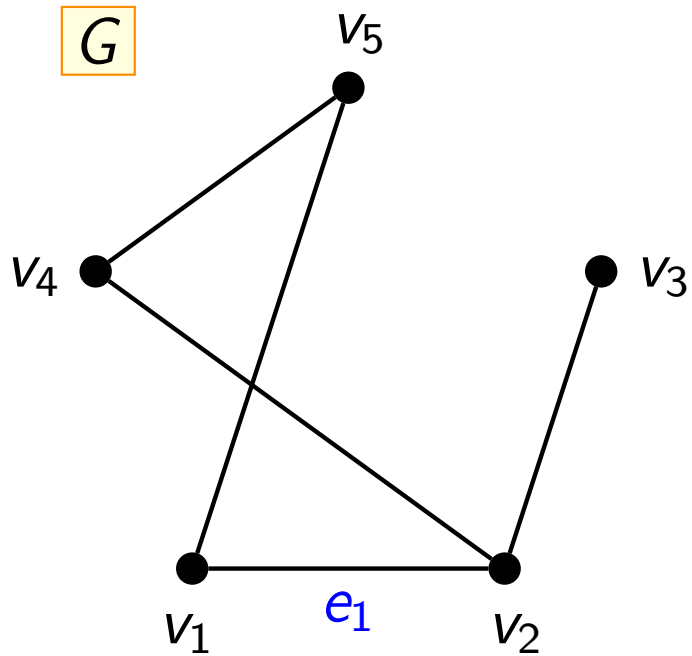


$L(G)$

b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G

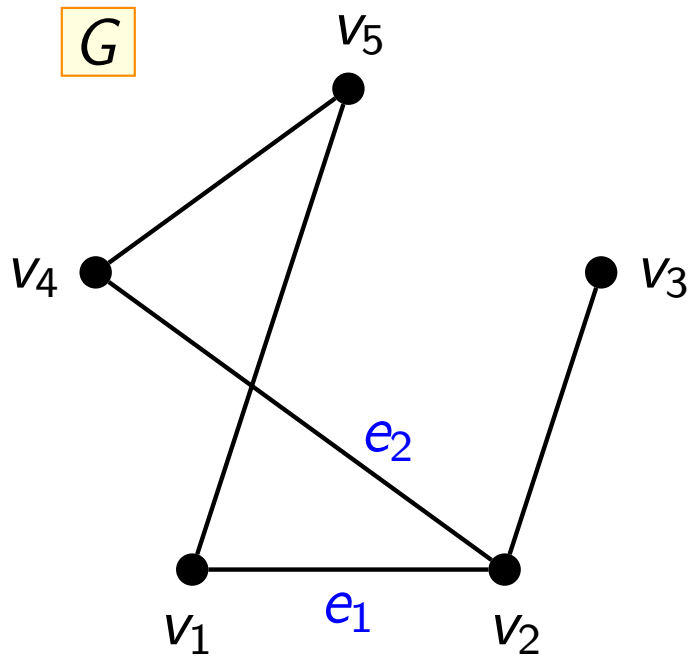


$L(G)$

b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G

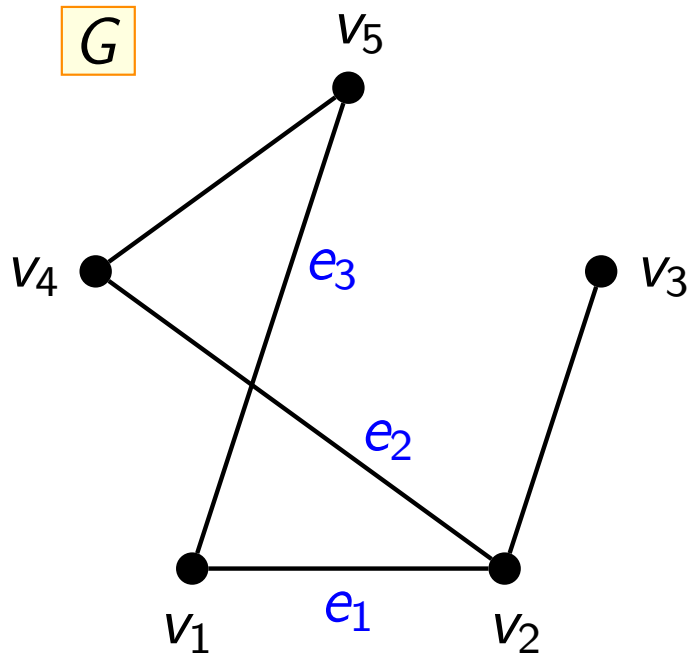


$L(G)$

b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G

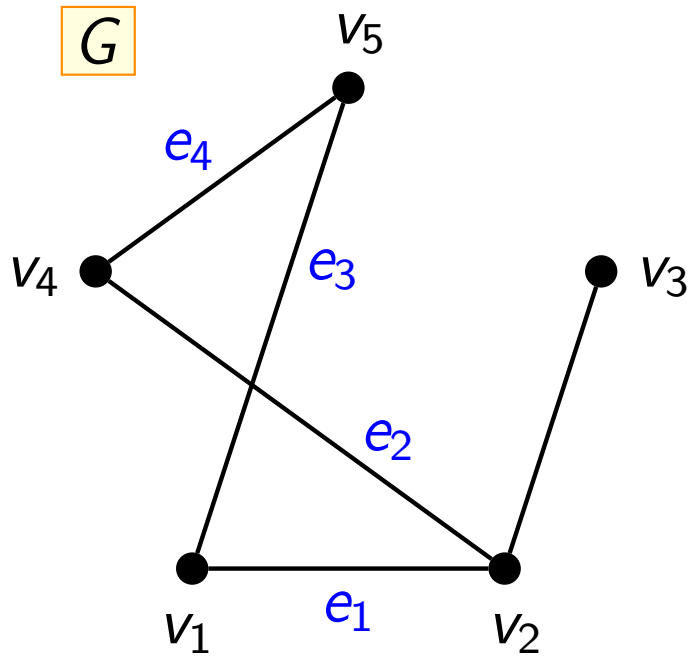


$L(G)$

b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G

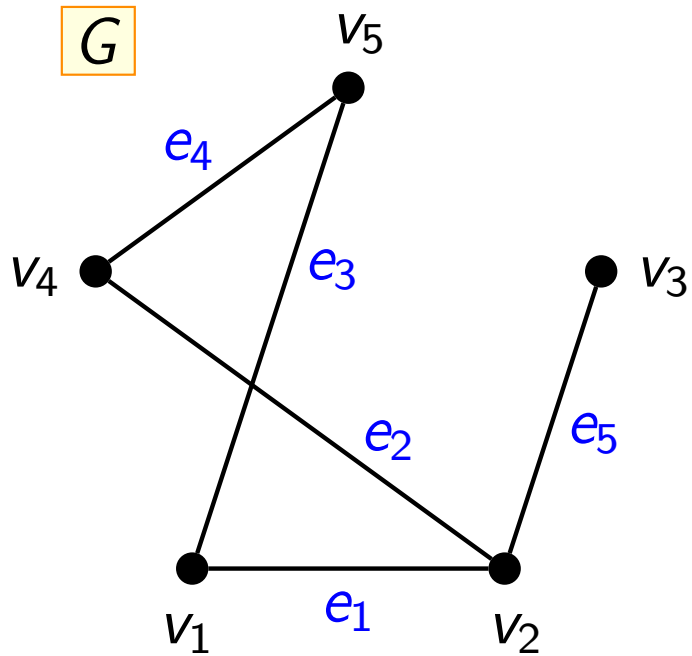


$L(G)$

b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

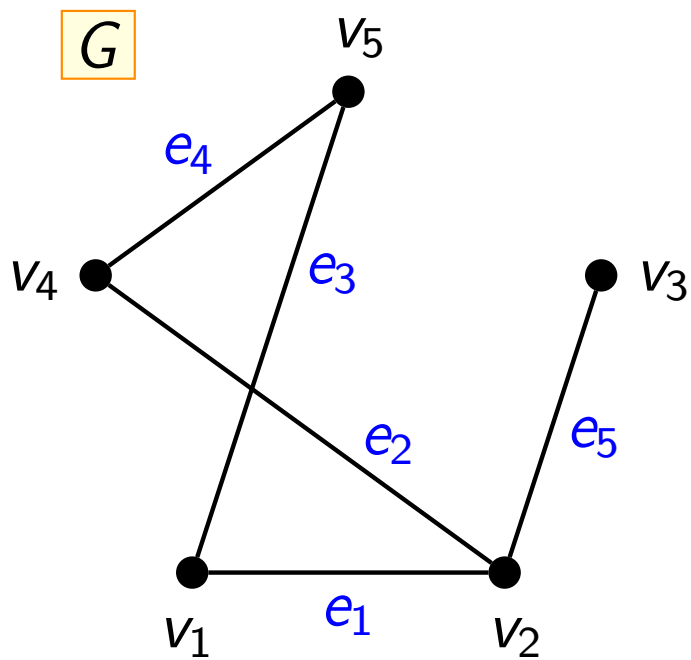
G

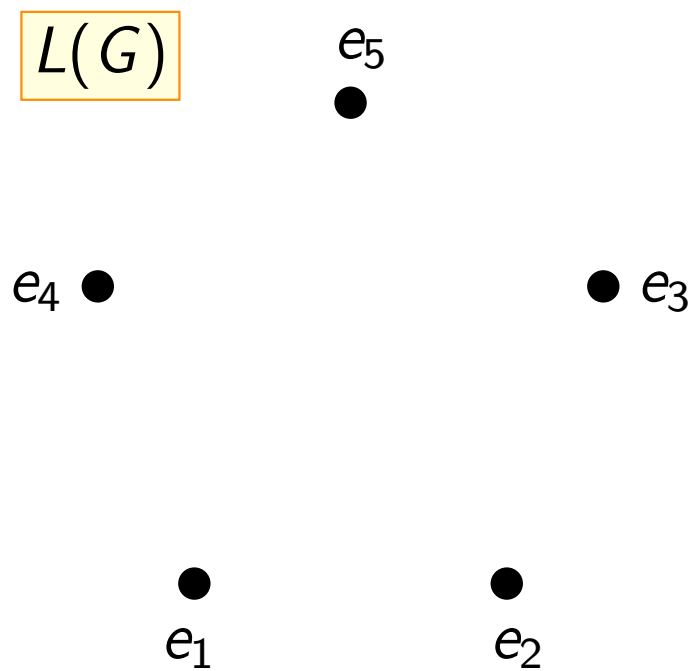


$L(G)$

b)

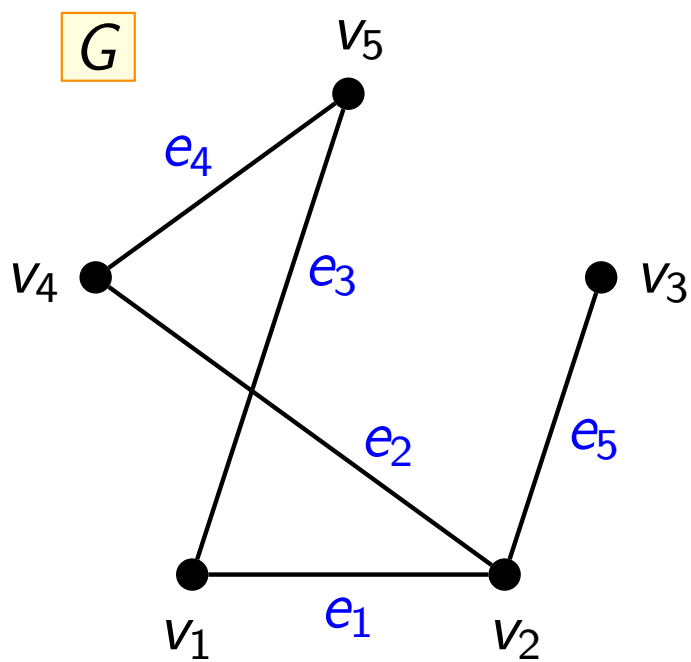
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

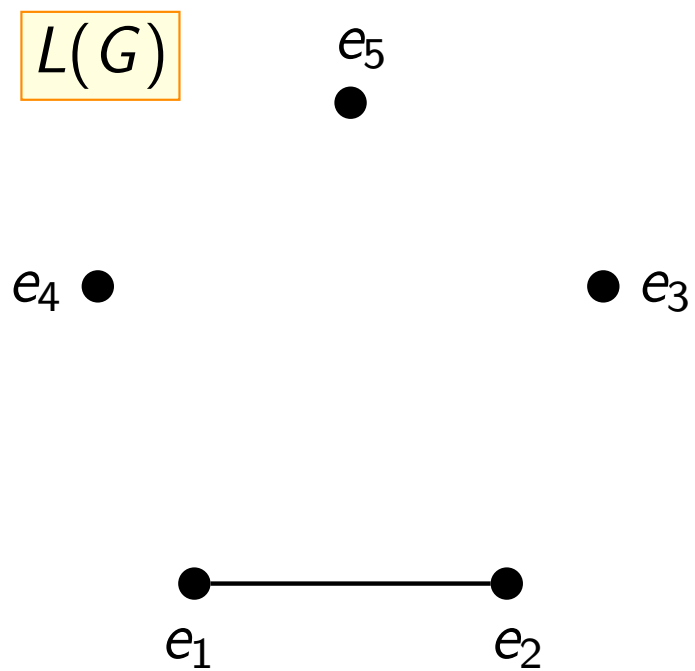




b)

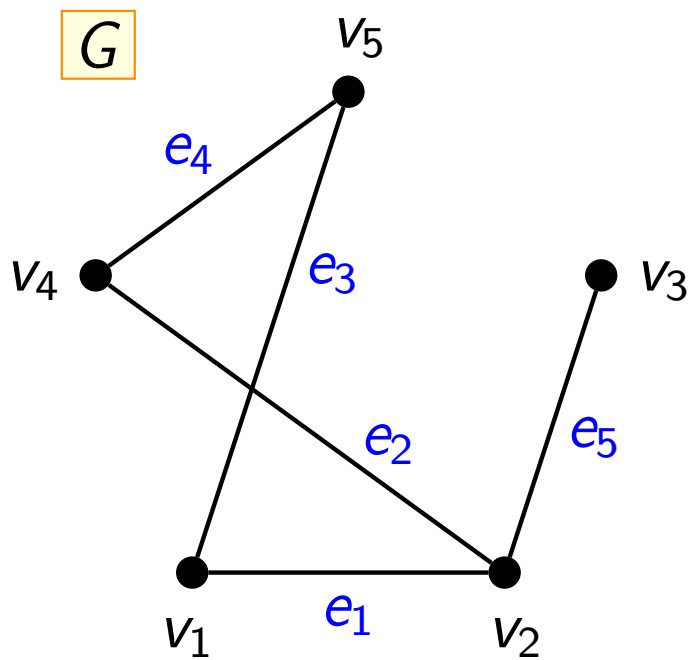
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

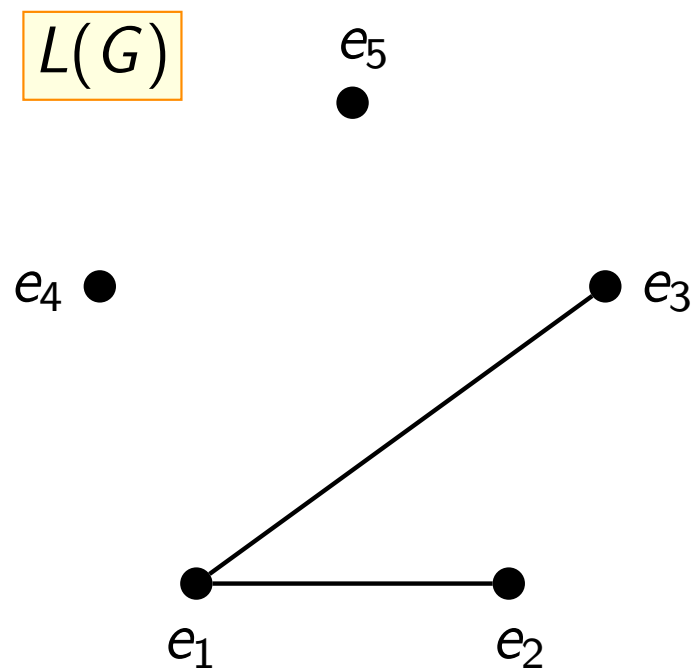




b)

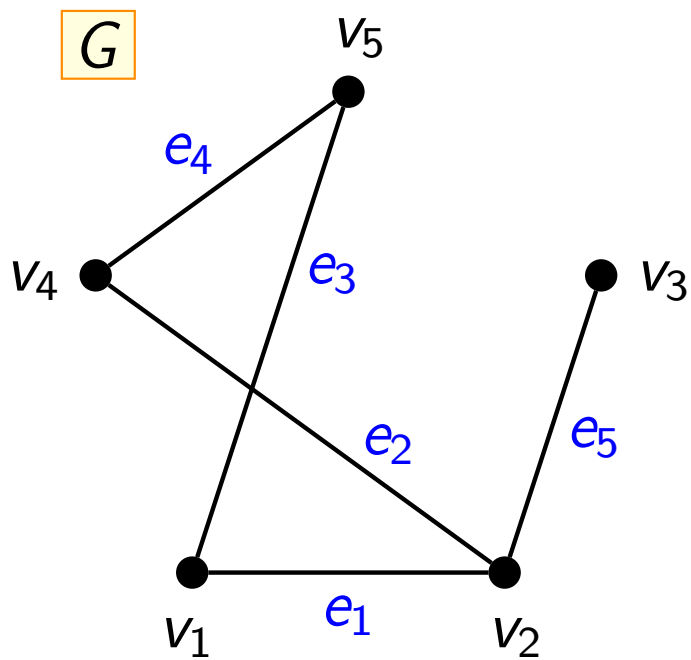
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

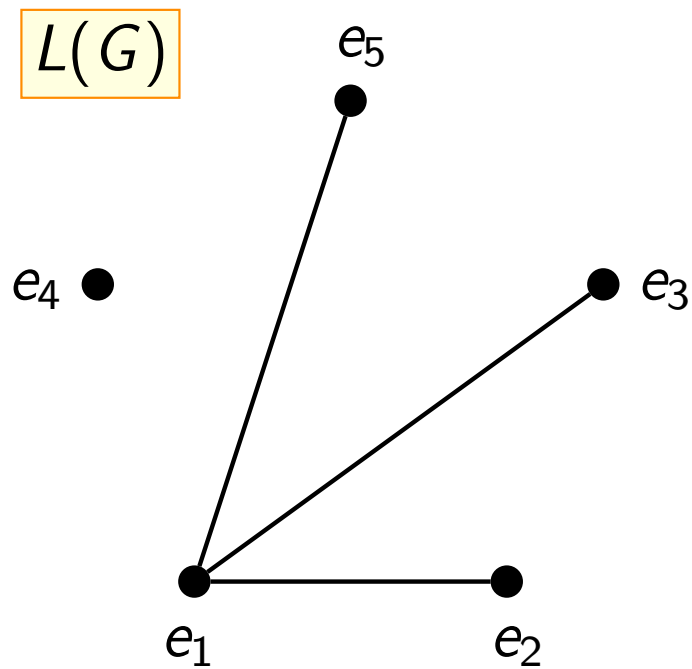




b)

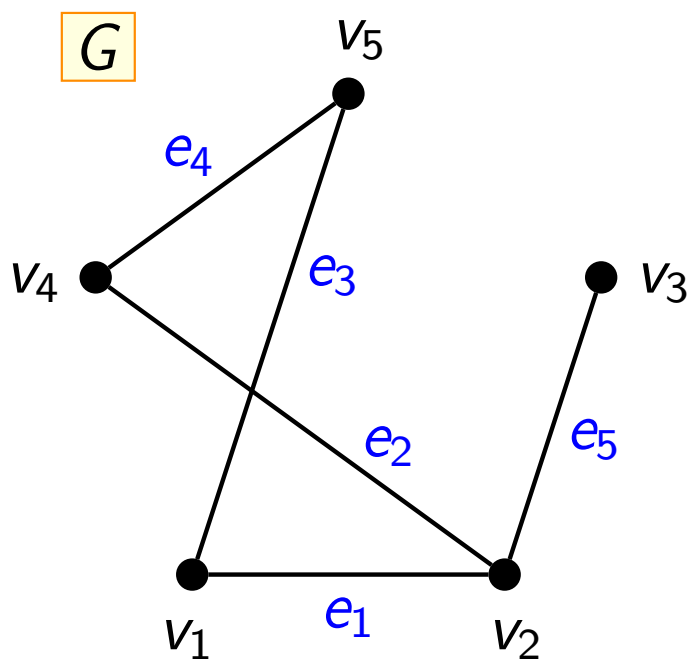
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

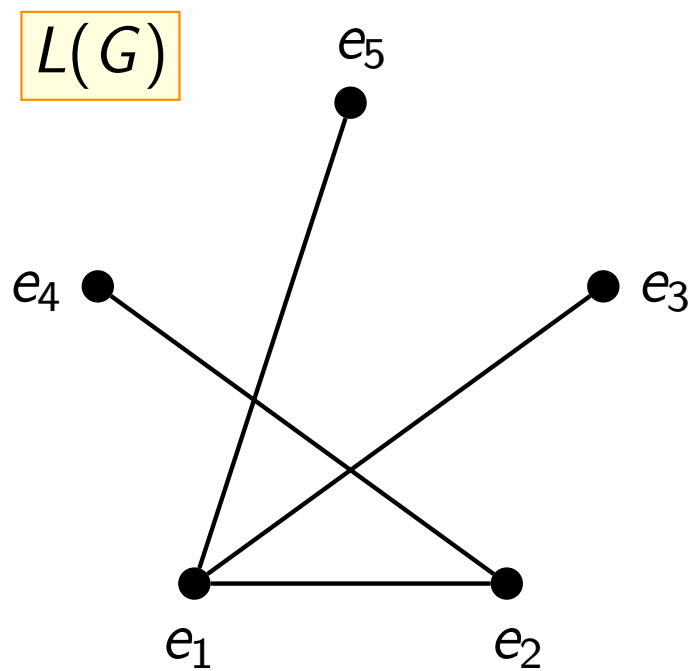




b)

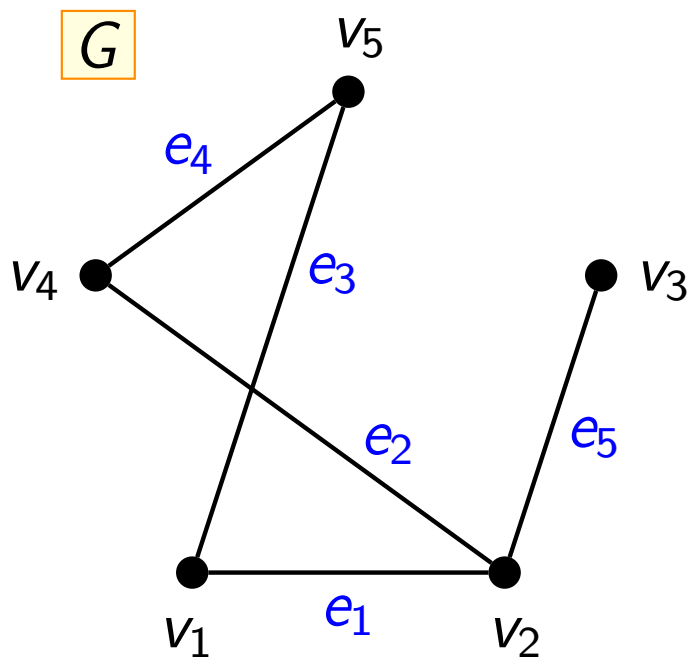
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

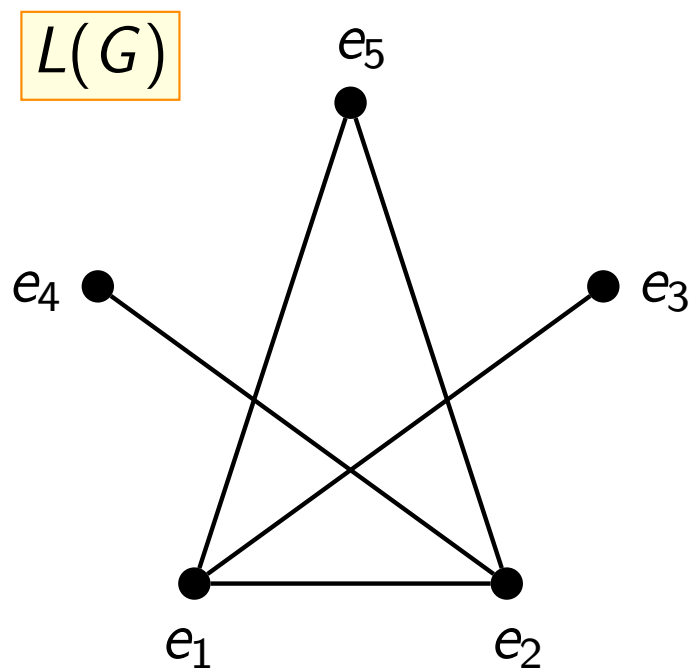




b)

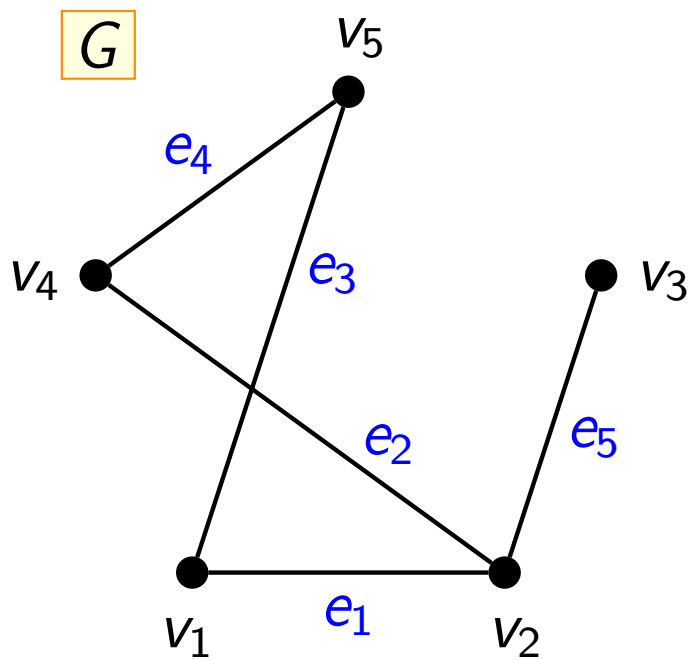
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

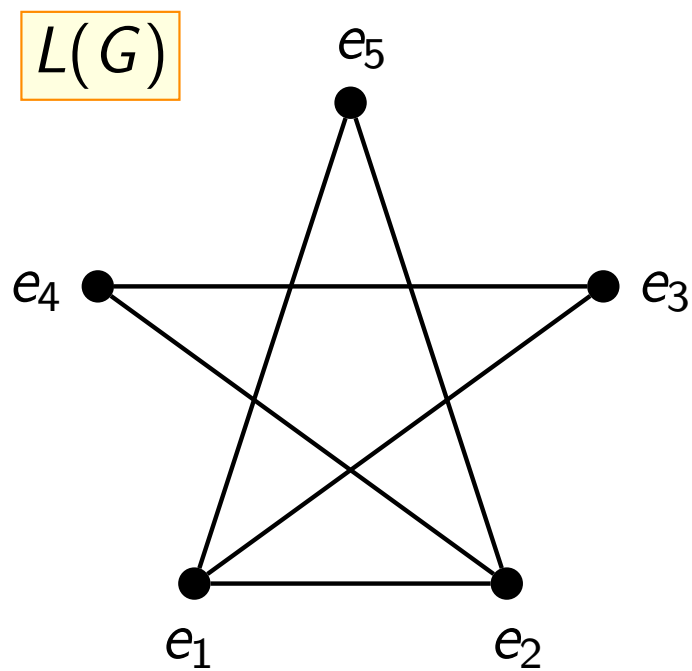




b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

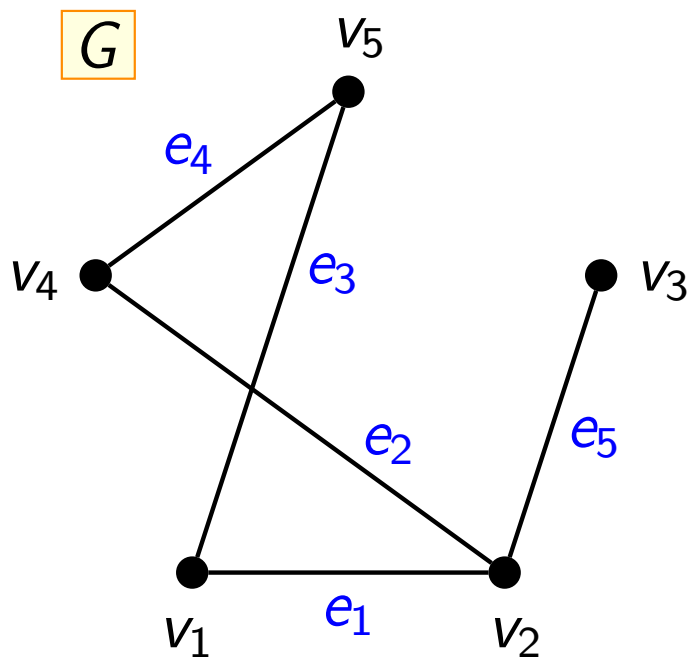


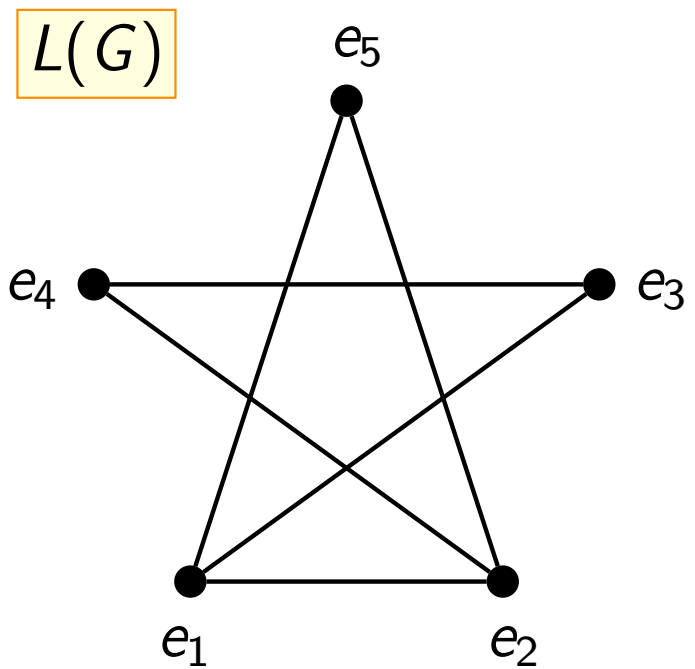


b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

c)

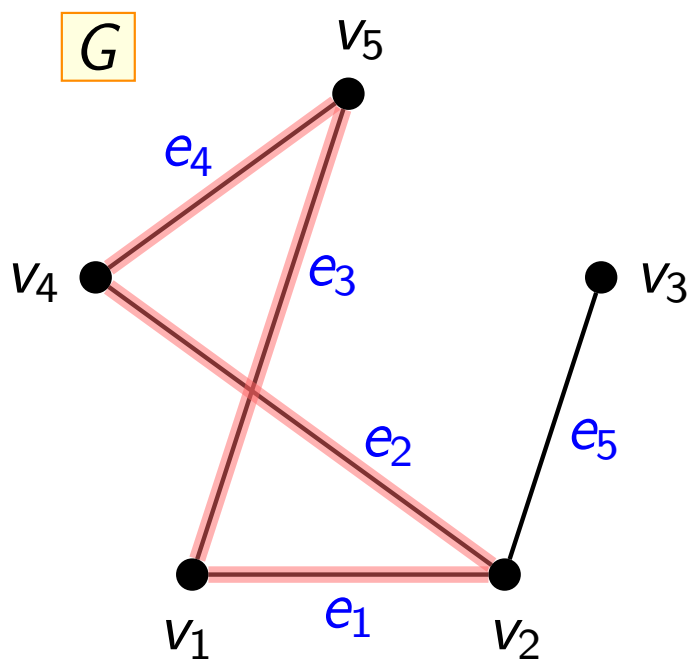


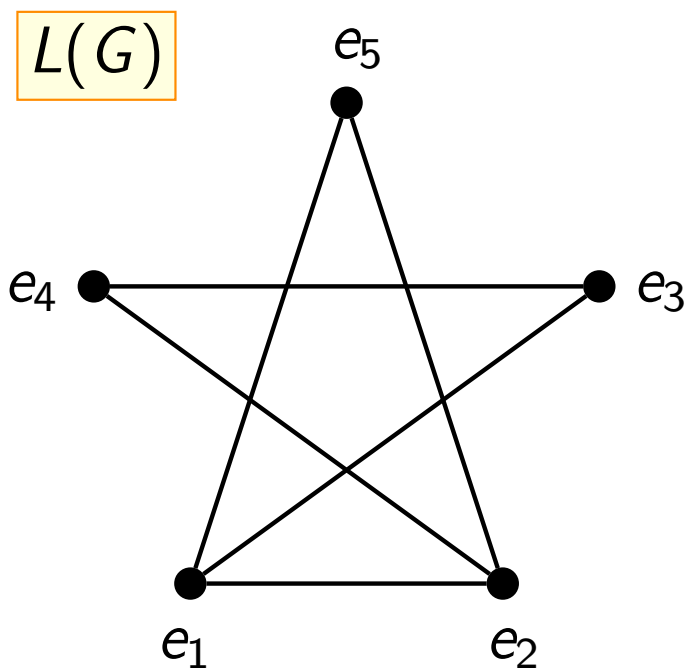


b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

c) Struk grafa G jednak je 4 jer je $v_1 v_2 v_4 v_5 v_1$ ciklus najmanje duljine u grafu G.



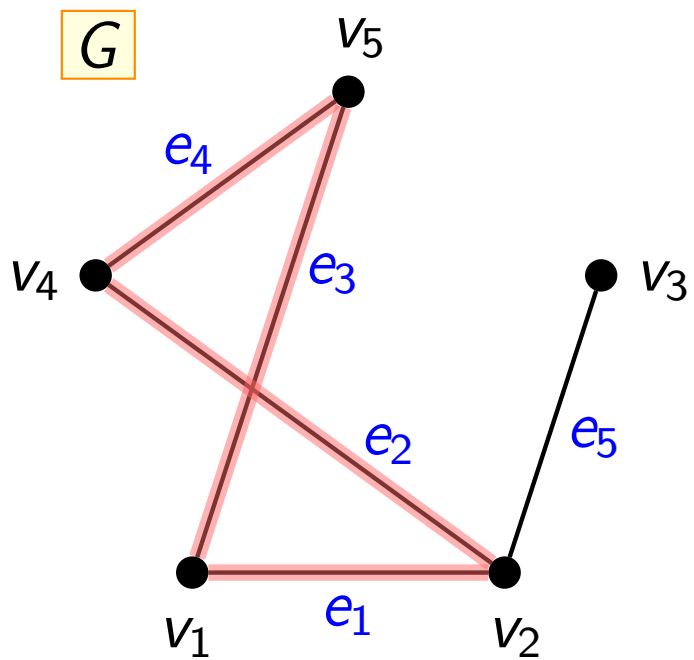


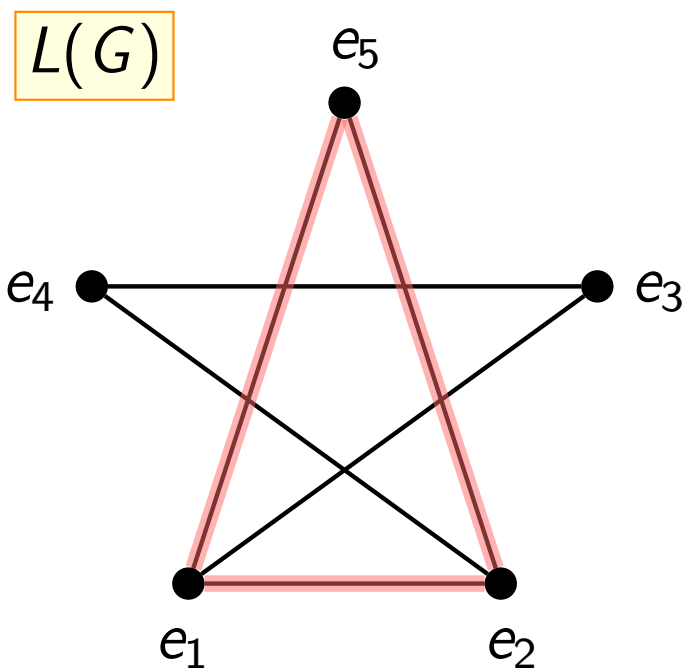
b)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

c) Struk grafa G jednak je 4 jer je $v_1 v_2 v_4 v_5 v_1$ ciklus najmanje duljine u grafu G.

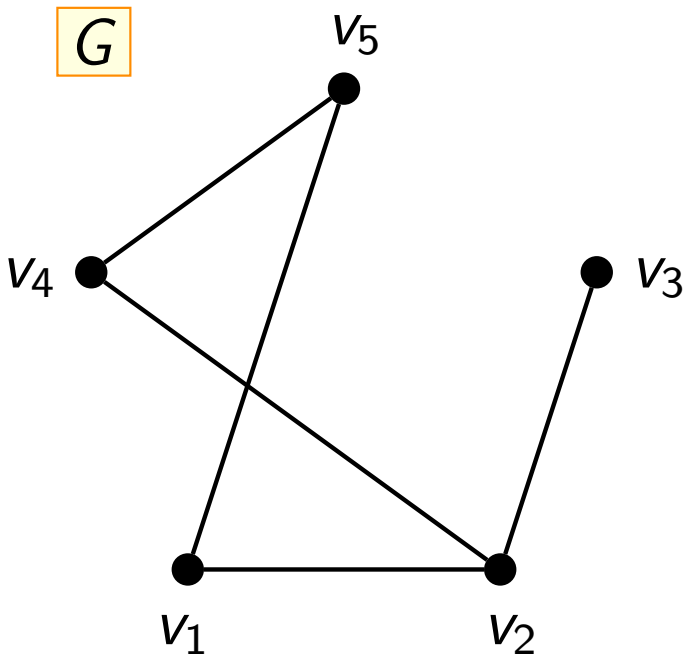
Struk grafa $L(G)$ jednak je 3 jer je $e_1 e_2 e_5 e_1$ ciklus najmanje duljine u grafu $L(G)$.





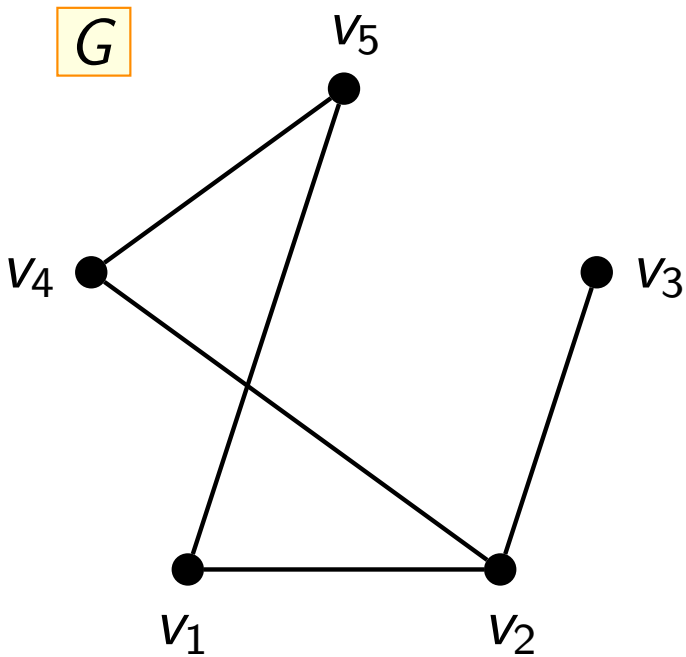
d)

$$A^3 = \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix}$$



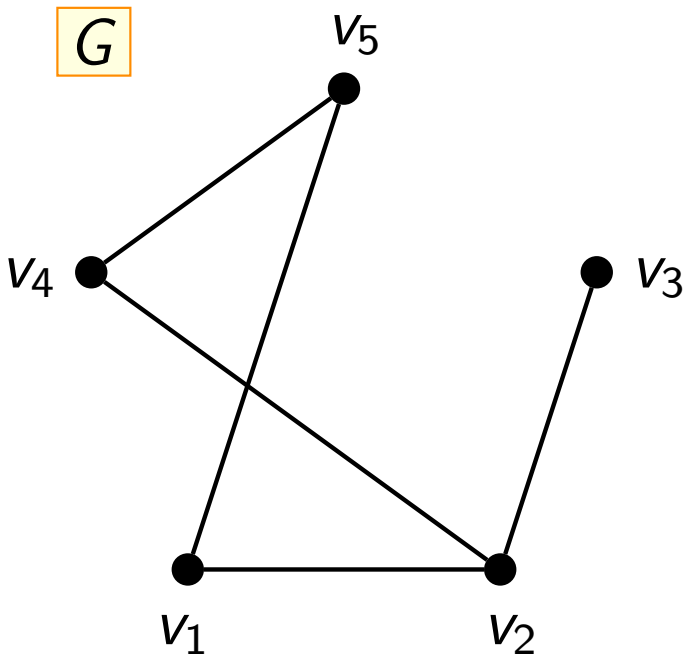
d)

$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$



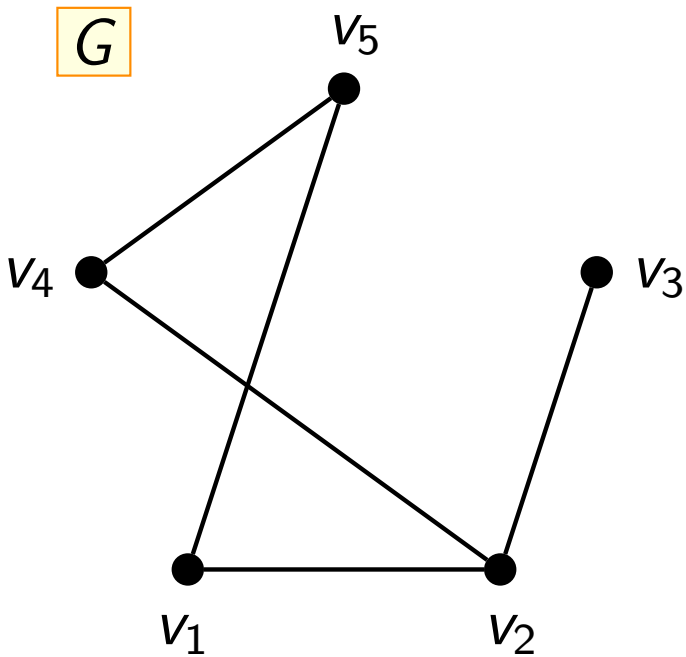
d)

$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$



d)

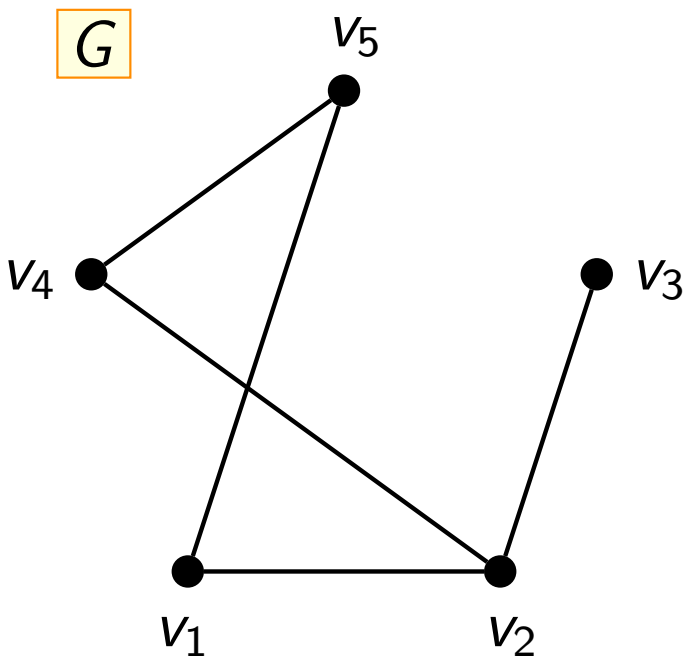
$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$



d)

$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

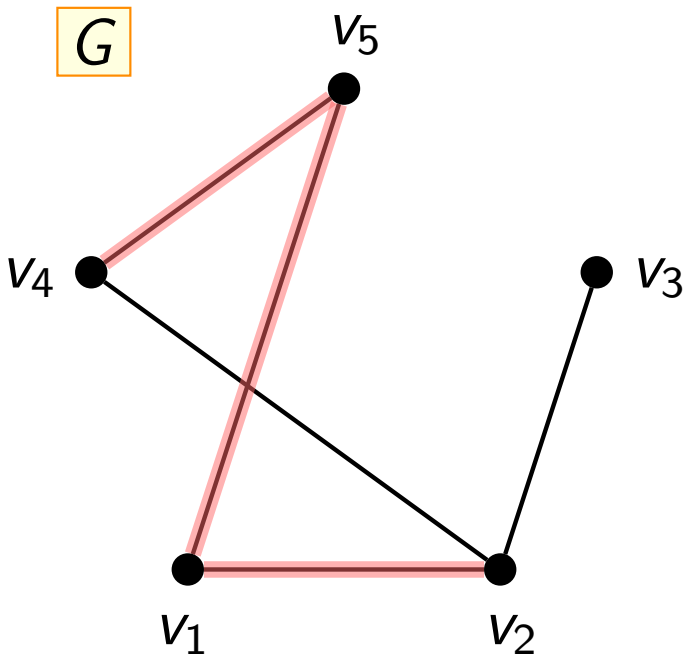


d)

$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

Šetnja $v_2 v_1 v_5 v_4$ je ujedno i put.



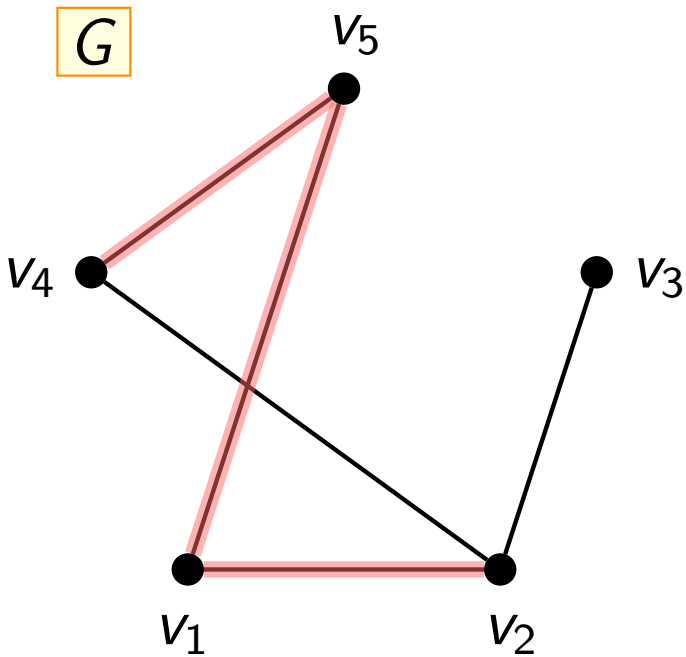
d)

$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

Šetnja $v_2 v_1 v_5 v_4$ je ujedno i put.

Preostale četiri šetnje:



d)

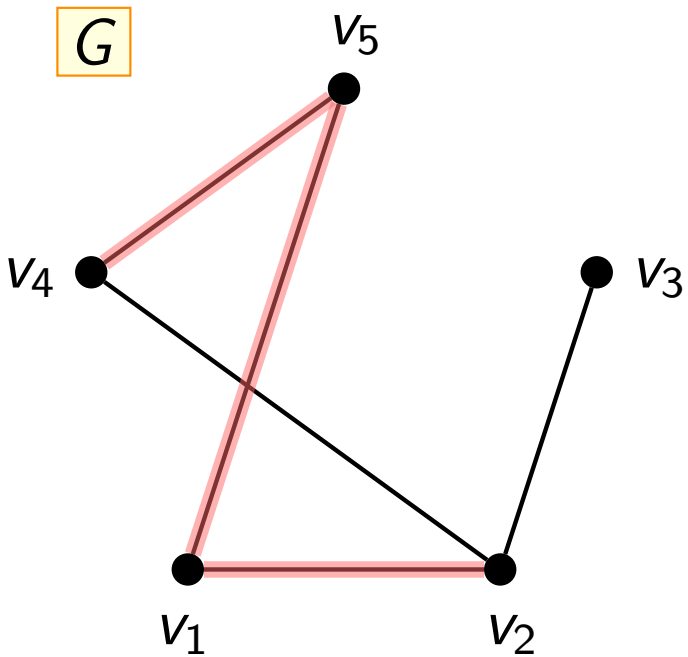
$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

Šetnja $v_2v_1v_5v_4$ je ujedno i put.

Preostale četiri šetnje:

$v_2v_3v_2v_4$, $v_2v_1v_2v_4$, $v_2v_4v_2v_4$, $v_2v_4v_5v_4$



d)

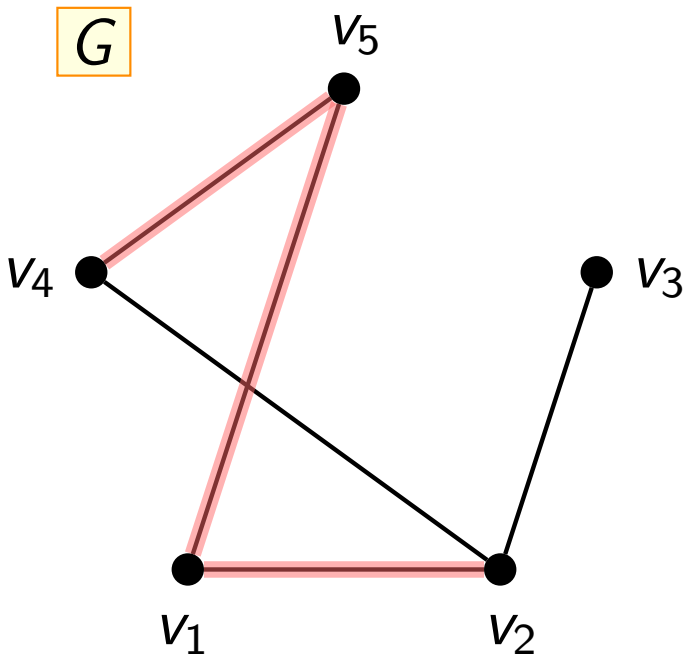
$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

Šetnja $v_2v_1v_5v_4$ je ujedno i put.

Preostale četiri šetnje:

$v_2v_3v_2v_4$, $v_2v_1v_2v_4$, $v_2v_4v_2v_4$, $v_2v_4v_5v_4$



e)

d)

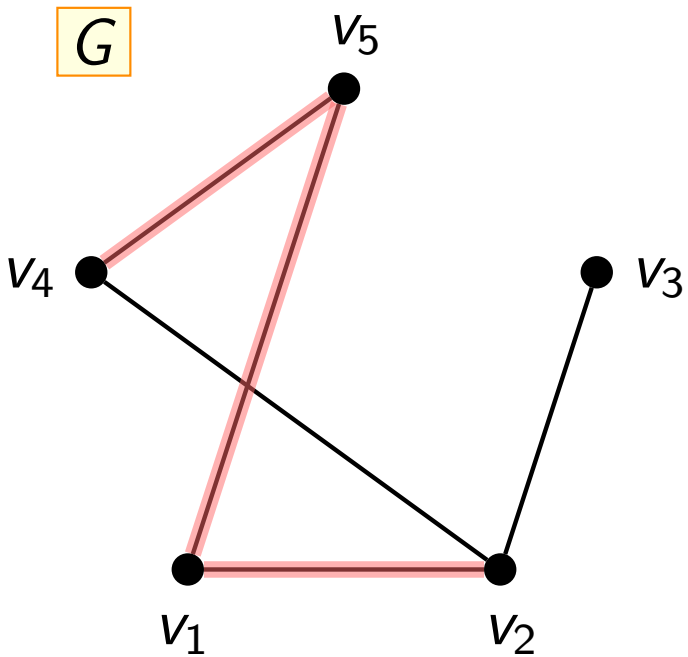
$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

Šetnja $v_2v_1v_5v_4$ je ujedno i put.

Preostale četiri šetnje:

$v_2v_3v_2v_4$, $v_2v_1v_2v_4$, $v_2v_4v_2v_4$, $v_2v_4v_5v_4$



e) Ukupni broj svih šetnji duljine 3 u grafu G jednak je sumi svih elemenata matrice A^3.

d)

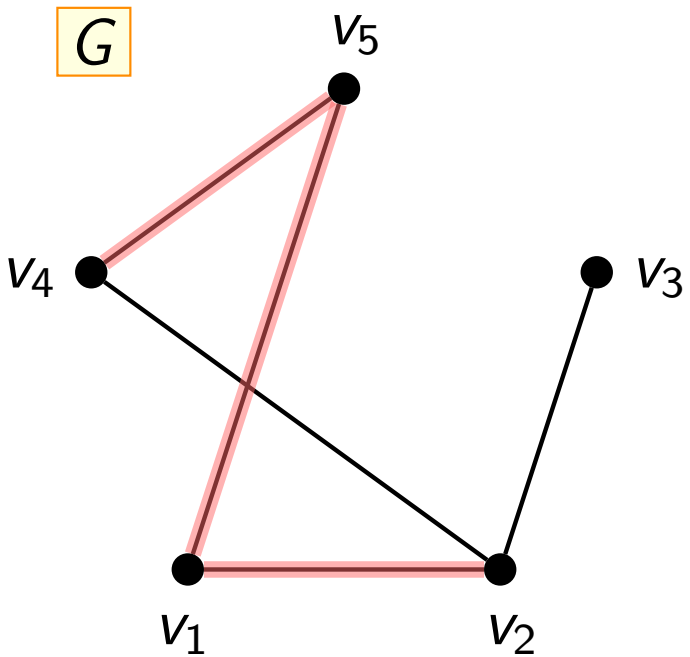
$$A^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 3 & \textcircled{5} & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 5 & 0 & 0 & 4 \\ 4 & 0 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Ukupni broj (v_2, v_4)-šetnji duljine 3 u grafu G jednak je 5.

Šetnja $v_2 v_1 v_5 v_4$ je ujedno i put.

Preostale četiri šetnje:

$v_2 v_3 v_2 v_4$, $v_2 v_1 v_2 v_4$, $v_2 v_4 v_2 v_4$, $v_2 v_4 v_5 v_4$



e) Ukupni broj svih šetnji duljine 3 u grafu G jednak je sumi svih elemenata matrice A^3.

Svih šetnji duljine 3 u grafu G ima ukupno 46.

Napomena

- Ispitivanje povezanosti grafa preko potencija matrice susjedstva općenito nije efikasni algoritam.
- Efikasni algoritam za ispitivanje povezanosti grafa temelji se na DFS algoritmu ili BFS algoritmu.
- DFS i BFS algoritam omogućuju računalu da samostalno pretražuje po grafu.
- DFS i BFS algoritam su dva temeljna algoritma koji omogućuju računalu da samostalno riješi mnoge probleme iz teorije grafova: određivanje struka grafa, pronalaženje najkraćeg puta između dva vrha u grafu, pronalaženje ciklusa u grafu, ispitivanje povezanosti grafa i određivanje komponenata povezanosti, određivanje jake orijentacije na grafu, ...

Strpite se. DFS i BFS algoritam jesu dva zaista vrlo simpatična algoritma i oba ćemo detaljno obraditi kasnije kod stabala.

treći zadatak

Zadatak 3

Neka su G_1 i G_2 dva grafa, a $L(G_1)$ i $L(G_2)$ njihovi linijski grafovi. Dokažite ili opovrgnite sljedeću tvrdnju:

> *Ako su $L(G_1)$ i $L(G_2)$ izomorfni grafovi, tada su G_1 i G_2 izomorfni grafovi.*

Zadatak 3

Neka su G_1 i G_2 dva grafa, a $L(G_1)$ i $L(G_2)$ njihovi linijski grafovi. Dokažite ili opovrgnite sljedeću tvrdnju:

> *Ako su $L(G_1)$ i $L(G_2)$ izomorfni grafovi, tada su G_1 i G_2 izomorfni grafovi.*

Rješenje

Tvrdnja općenito ne vrijedi.

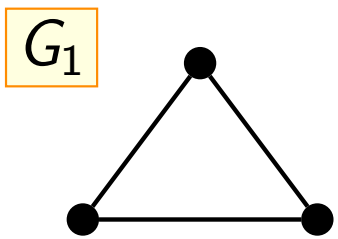
Zadatak 3

Neka su G_1 i G_2 dva grafa, a $L(G_1)$ i $L(G_2)$ njihovi linijski grafovi. Dokažite ili opovrgnite sljedeću tvrdnju:

> *Ako su $L(G_1)$ i $L(G_2)$ izomorfni grafovi, tada su G_1 i G_2 izomorfni grafovi.*

Rješenje

Tvrdnja općenito ne vrijedi.



Zadatak 3

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Rješenje

Tvrdnja općenito ne vrijedi.

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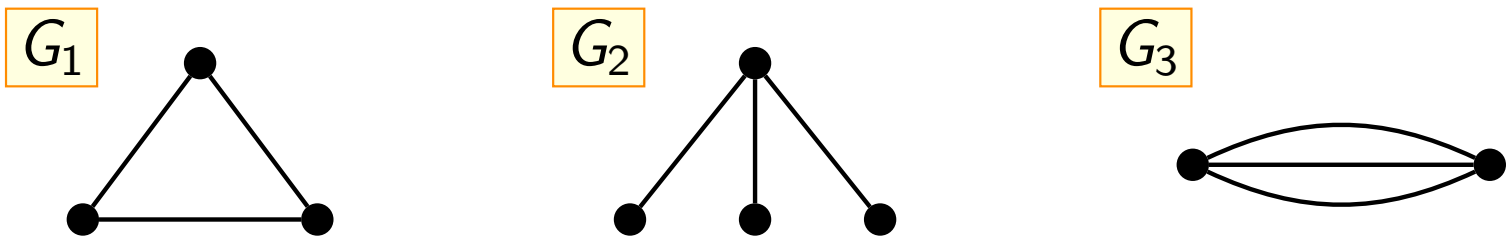
Rješenje

Tvrdnja općenito ne vrijedi.



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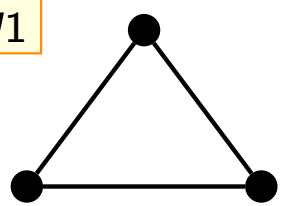
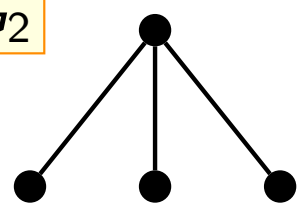
Rješenje

Tvrdnja općenito ne vrijedi.

G_1



G_2



G_3

$L(G_1)$

Zadatak 3

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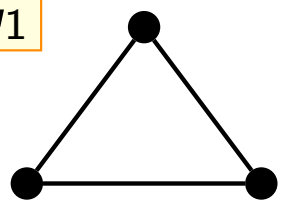
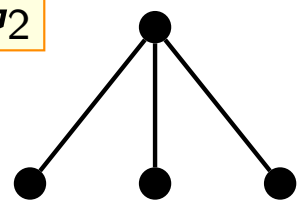
Rješenje

Tvrdnja općenito ne vrijedi.

G_1



G_2



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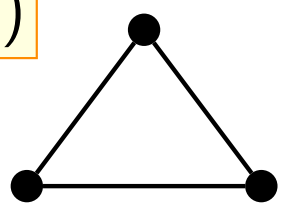

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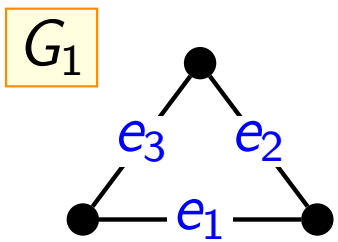
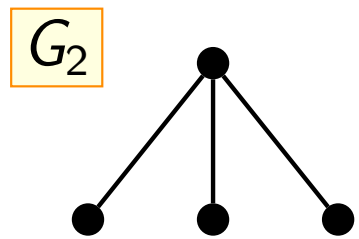
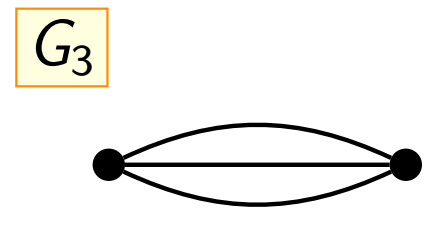
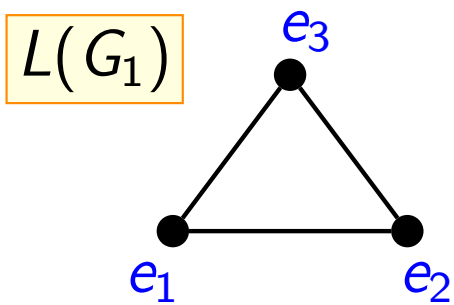
Rješenje

Tvrdnja općenito ne vrijedi.









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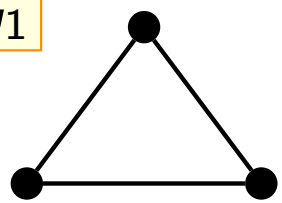
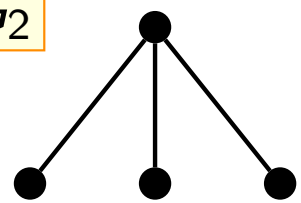
Rješenje

Tvrdnja općenito ne vrijedi.

G_1

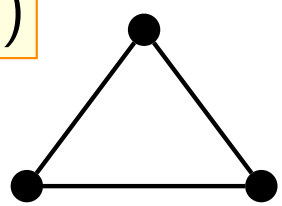


G_2



G_3

$L(G_1)$



$L(G_2)$

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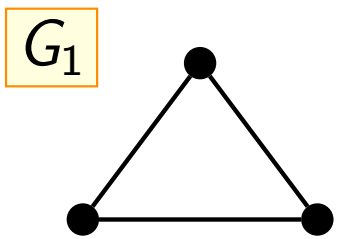

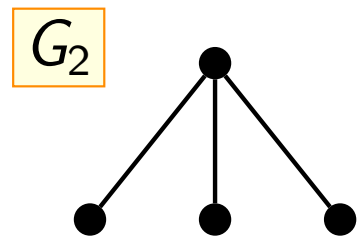

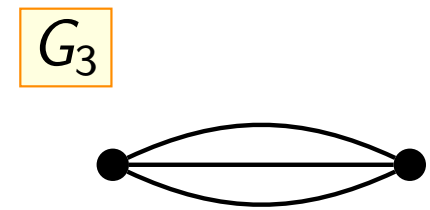

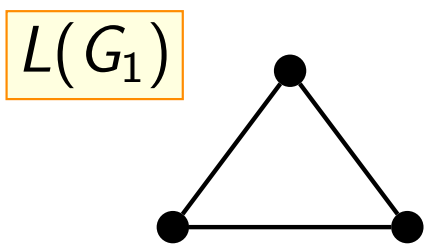

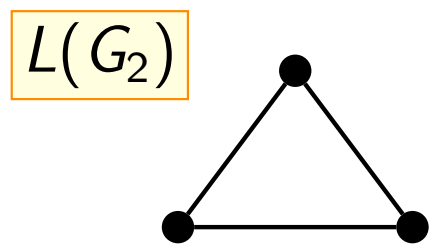

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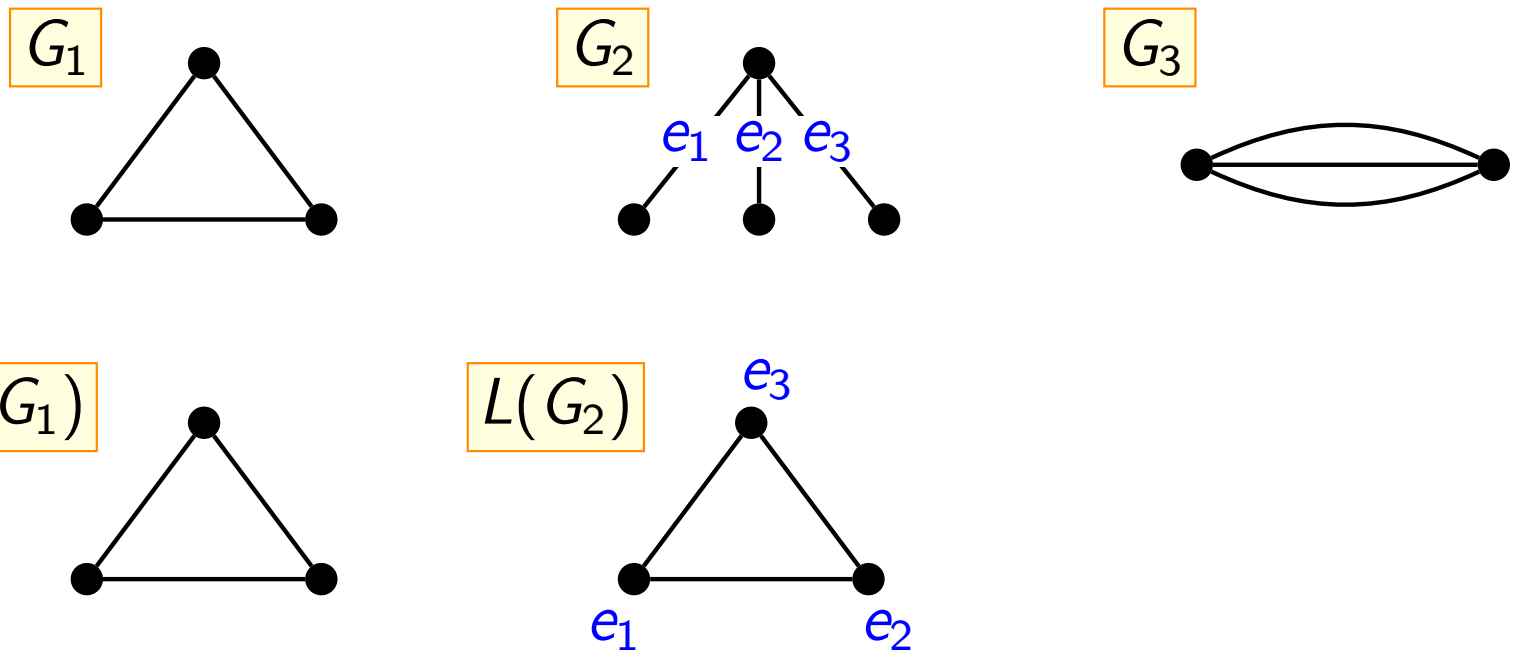

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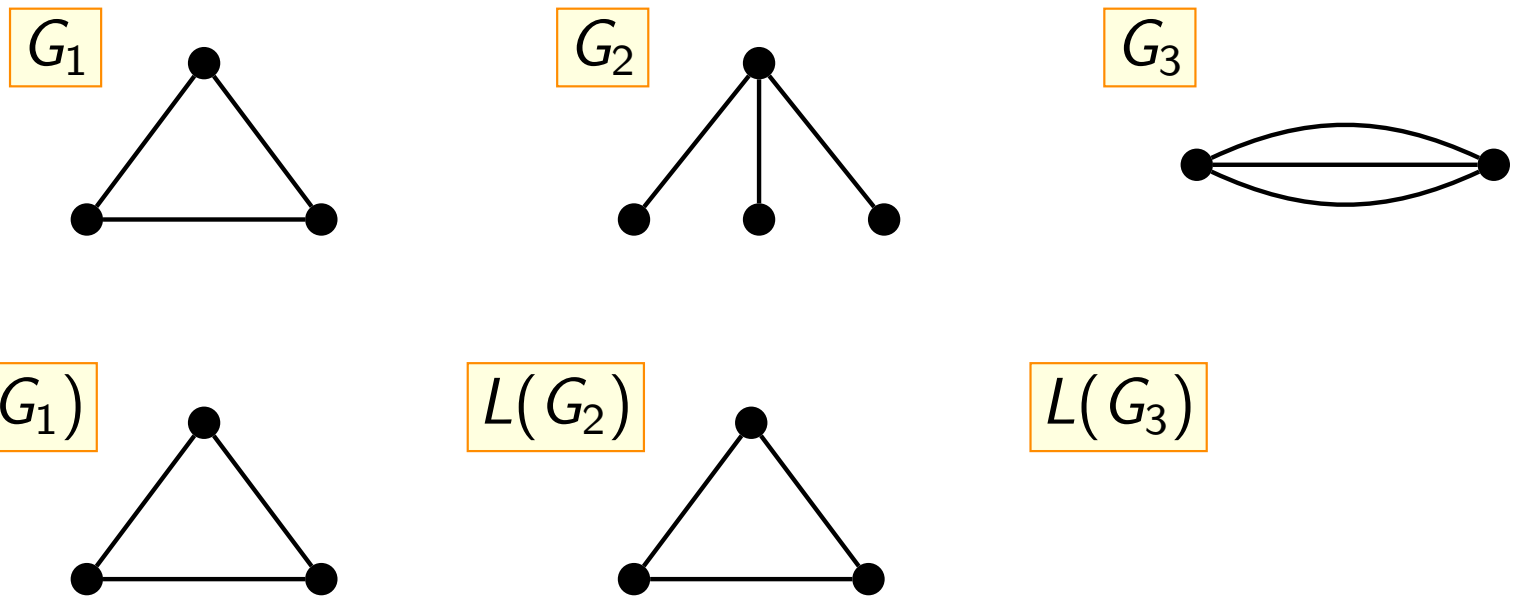

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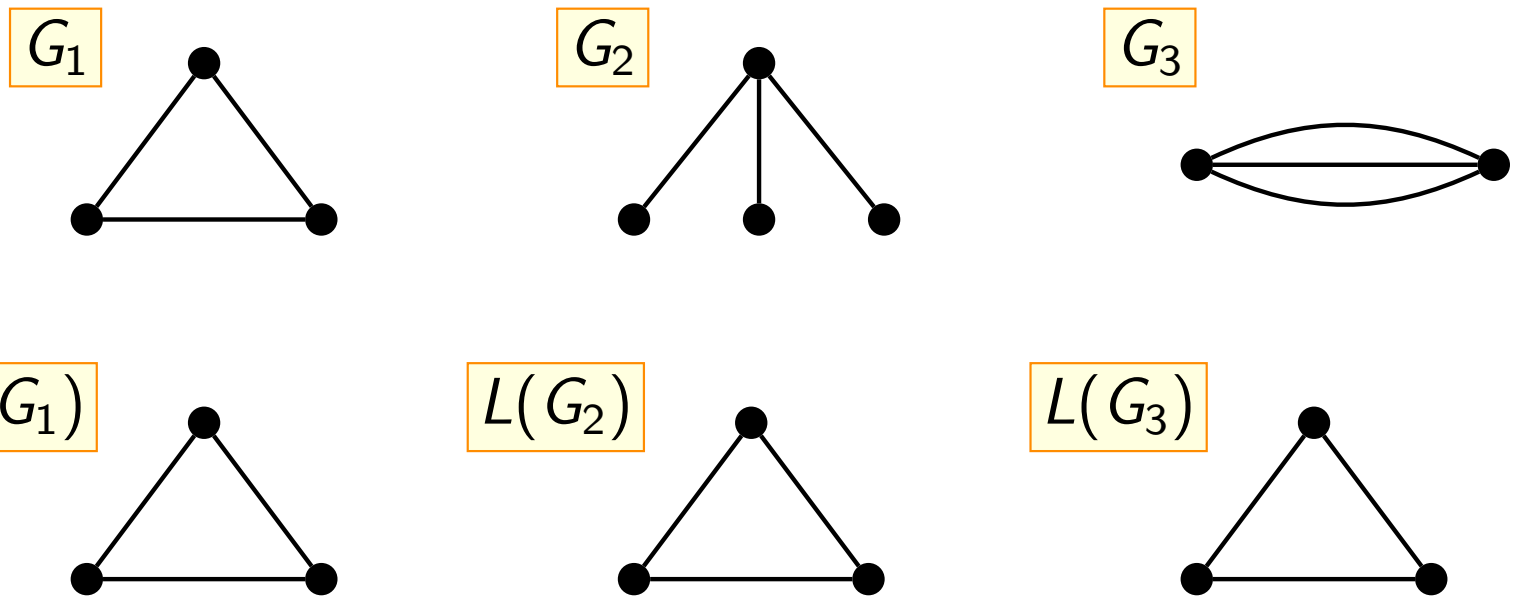

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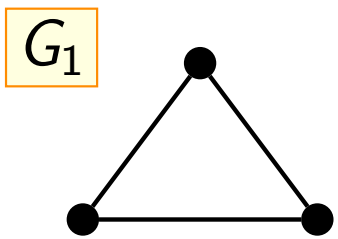

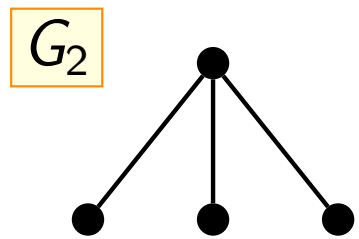

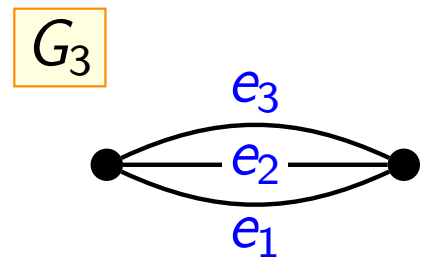

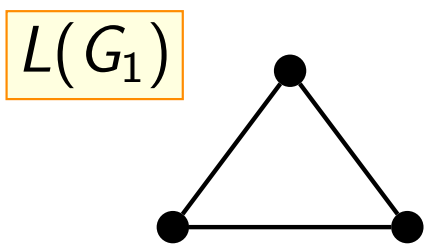

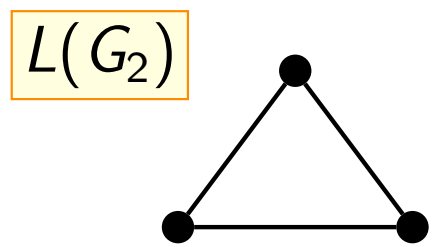

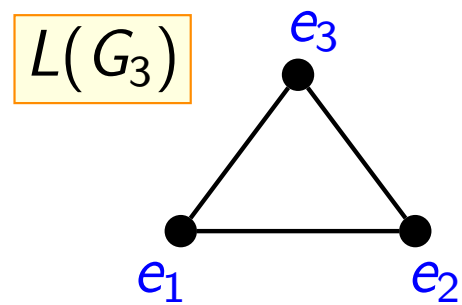

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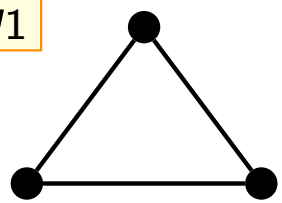
Rješenje

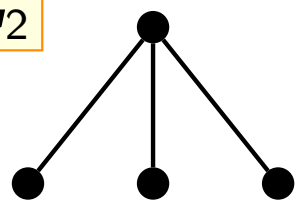
Tvrdnja općenito ne vrijedi.

$L(G_1)$, $L(G_2)$ i $L(G_3)$ su izomorfni, ali G_1, G_2 i G_3 nisu izomorfni.

G_1

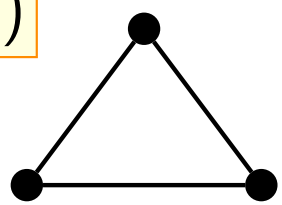


G_2

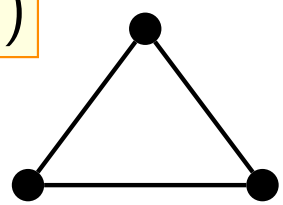


G_3

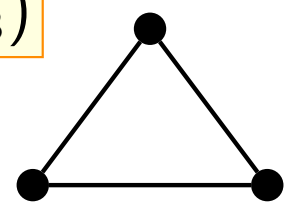
$L(G_1)$



$L(G_2)$



$L(G_3)$



Teorem (Whitney)

Neka su G_1 i G_2 povezani jednostavni grafovi s izomorfnim linijskim grafovima. Tada su G_1 i G_2 također izomorfni grafovi osim u slučaju ako je jedan od njih K_3, a drugi $K_{1,3}$.

četvrti zadatak

Propozicija

U svakom grafu G vrijedi

$$\omega(G) + \varepsilon(G) \geqslant \nu(G).$$

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U svakom grafu G vrijedi

$$\omega(G) + \varepsilon(G) \geqslant \nu(G).$$

- Ako je G povezani graf, tada je $\omega(G) = 1$ pa je

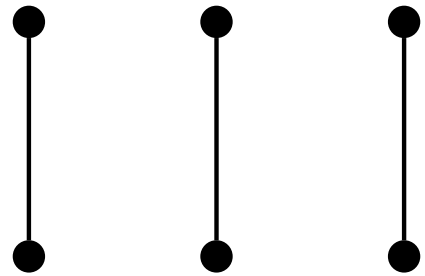
$$\varepsilon \geqslant \nu - 1.$$

Zadatak 4

Postoji li graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$*?*
Postoji li povezani graf s navedenim nizom stupnjeva vrhova?
Ukoliko u nekom slučaju takav graf postoji, navedite jedan primjer takvog grafa. U protivnom, objasnite zašto takav graf ne postoji.

Rješenje

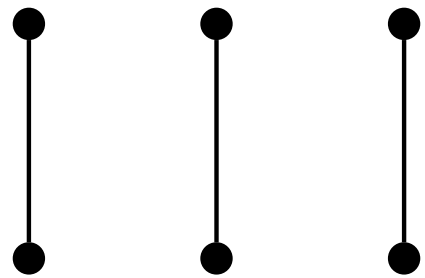
Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.



$$\omega(G) + \varepsilon(G) \geqslant \nu(G)$$

Rješenje

Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.



Povezani graf sa 6 vrhova mora imati barem 5 bridova.

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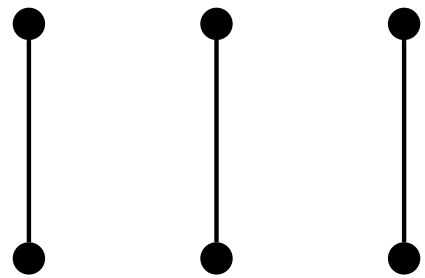
Rješenje

Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.

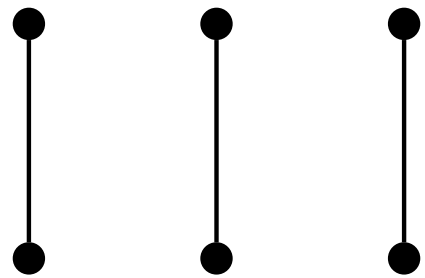


Povezani graf sa 6 vrhova mora imati barem 5 bridova.

$$\sum_{v \in V(G)} d(v) = 2\varepsilon$$

Rješenje

Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.

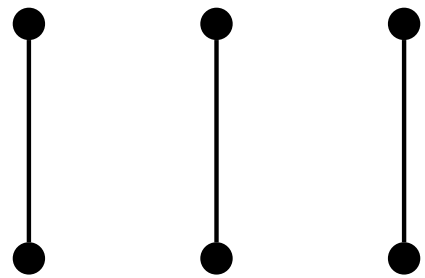


Povezani graf sa 6 vrhova mora imati barem 5 bridova.

$$\sum_{v \in V(G)} d(v) = 2\varepsilon \;\Rightarrow\; 6 = 2\varepsilon$$

Rješenje

Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.

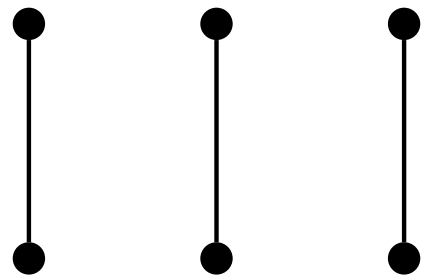


Povezani graf sa 6 vrhova mora imati barem 5 bridova.

$$\sum_{v \in V(G)} d(v) = 2\varepsilon \;\Rightarrow\; 6 = 2\varepsilon \;\Rightarrow\; \varepsilon = 3$$

Rješenje

Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.



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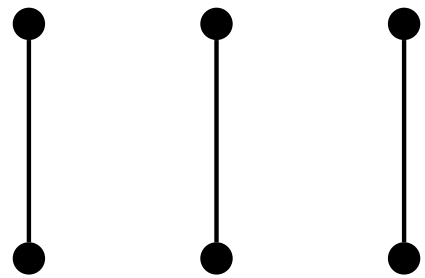

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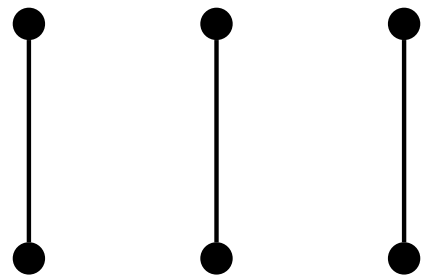
Rješenje

Postoji graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.



Povezani graf sa 6 vrhova mora imati barem 5 bridova.

$$\sum_{v \in V(G)} d(v) = 2\varepsilon \;\Rightarrow\; 6 = 2\varepsilon \;\Rightarrow\; \varepsilon = 3$$

Ne postoji povezani graf s nizom stupnjeva vrhova $1, 1, 1, 1, 1, 1$.

peti zadatak

Teorem (karakterizacija bipartitnih grafova)

Graf G je bipartitni ako i samo ako ne sadrži cikluse neparne duljine.

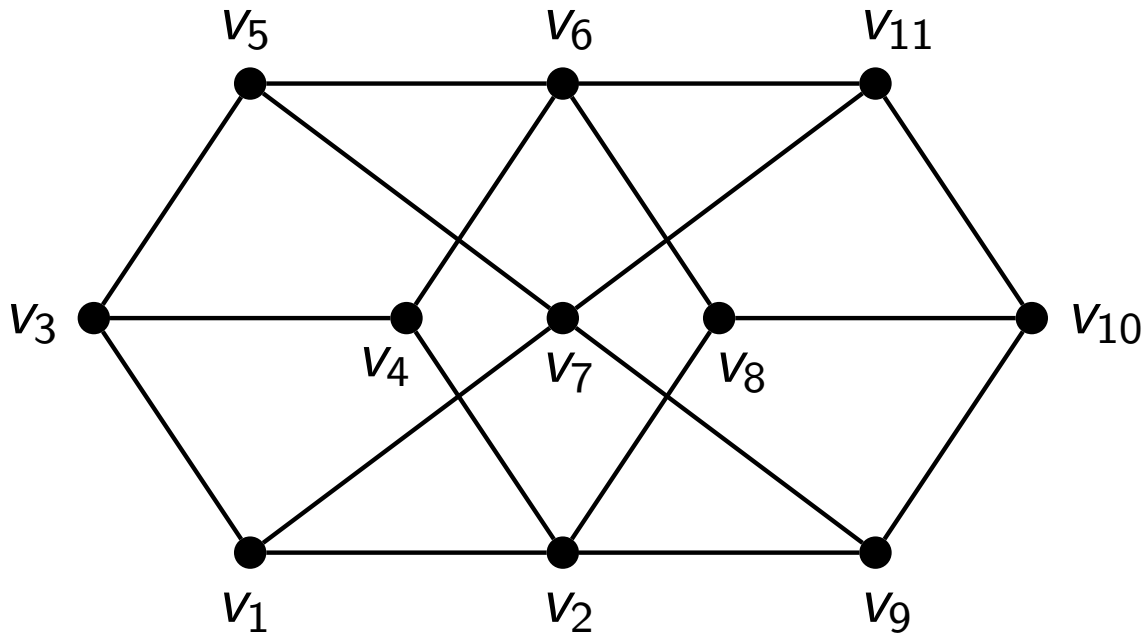
Teorem (karakterizacija bipartitnih grafova)

Graf G je bipartitni ako i samo ako ne sadrži cikluse neparne duljine.

- Dokaz teorema je konstruktivan i daje algoritam za testiranje bipartitnosti grafa te pronalaženje pripadne biparticije vrhova.

Zadatak 5

Zadan je graf G.



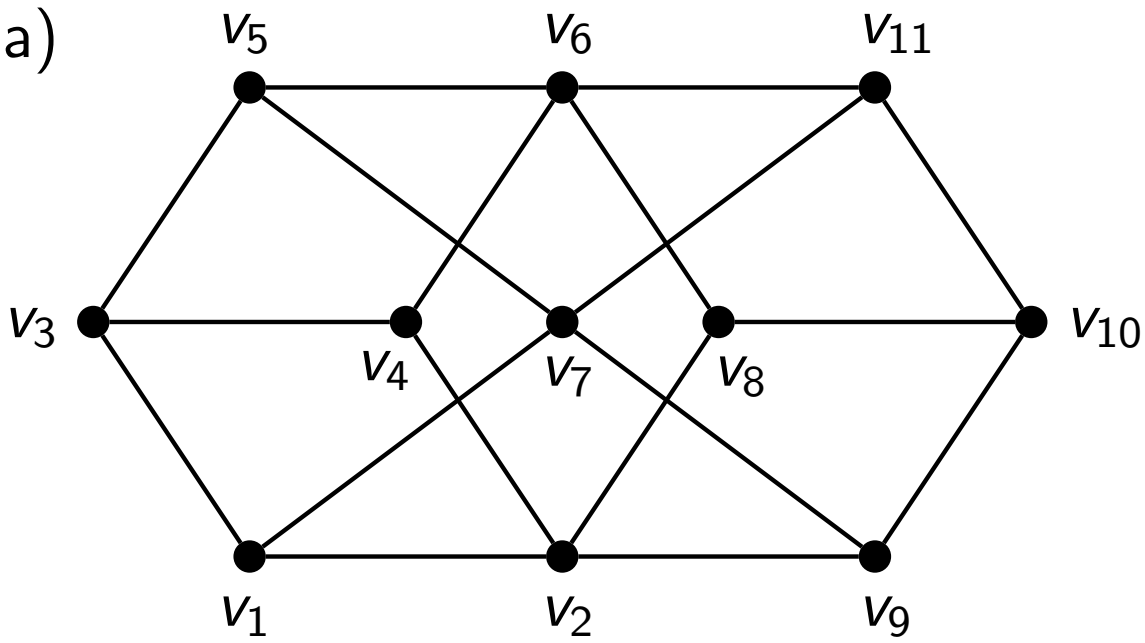
a) *Dokažite da je G bipartitni graf i nacrtajte graf G tako da se na slici jasno vidi njegova biparticija vrhova.*

b) *Odredite struk grafa G.*

c) *Odredite sve rezne bridove i rezne vrhove u grafu $G - \{v_2, v_6\}$.*

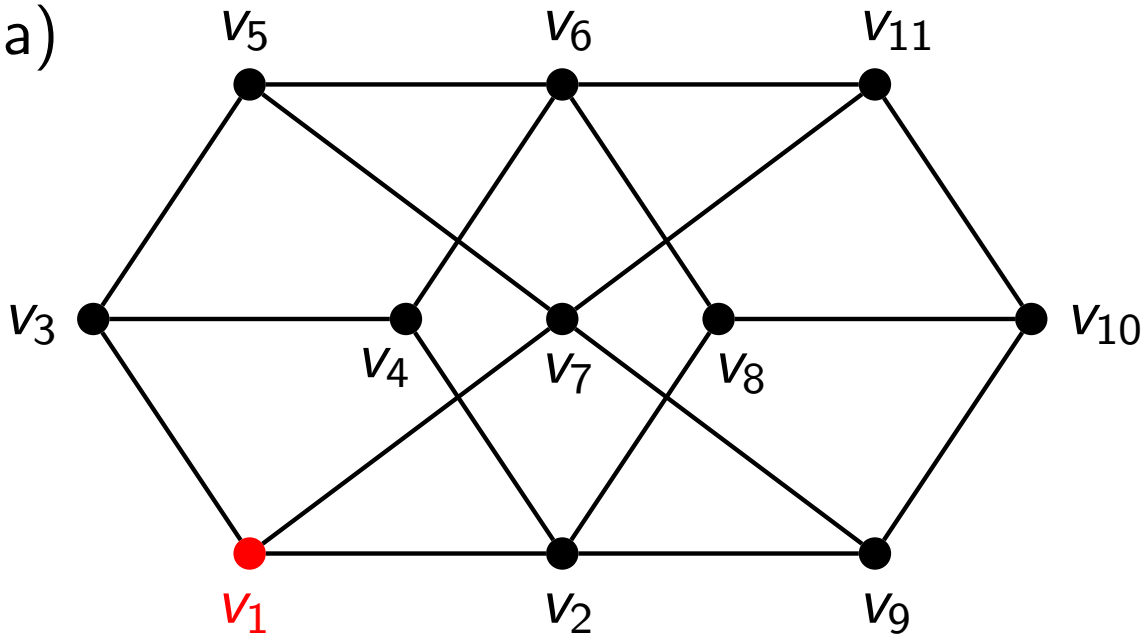
Rješenje

a)



Rješenje

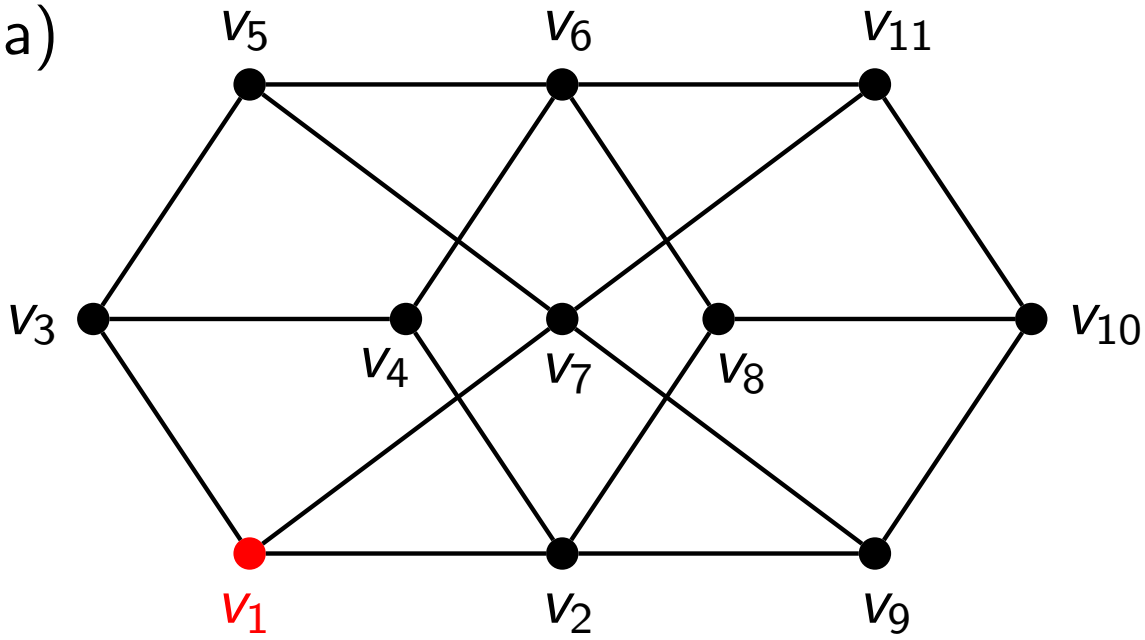
a)



- Odaberemo neki vrh, npr. vrh v_1.

Rješenje

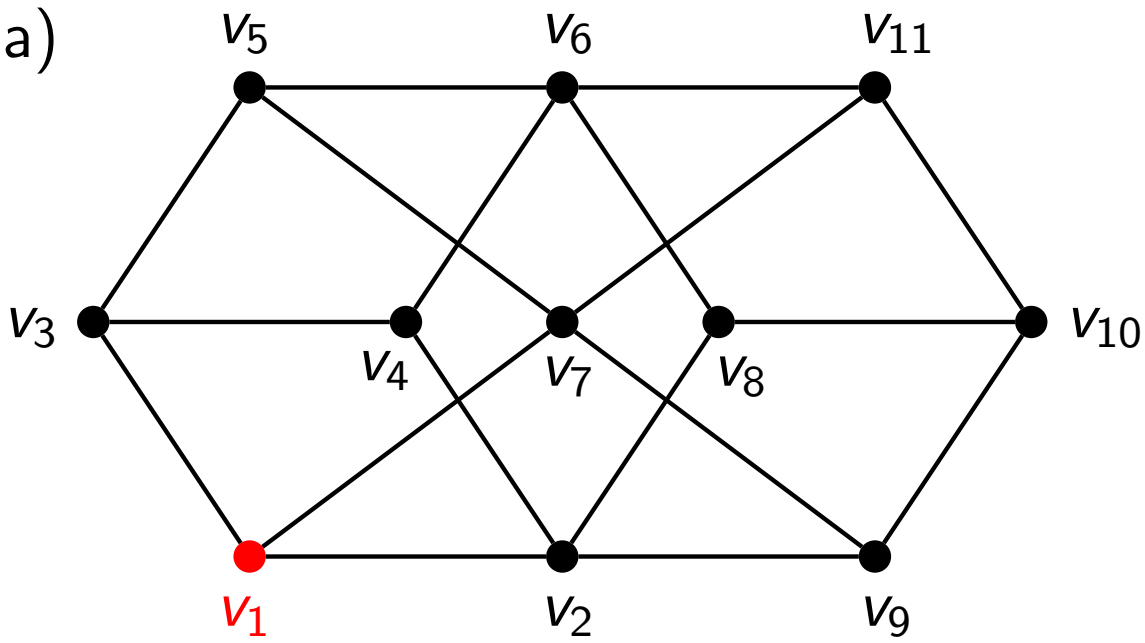
a)



- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1

Rješenje

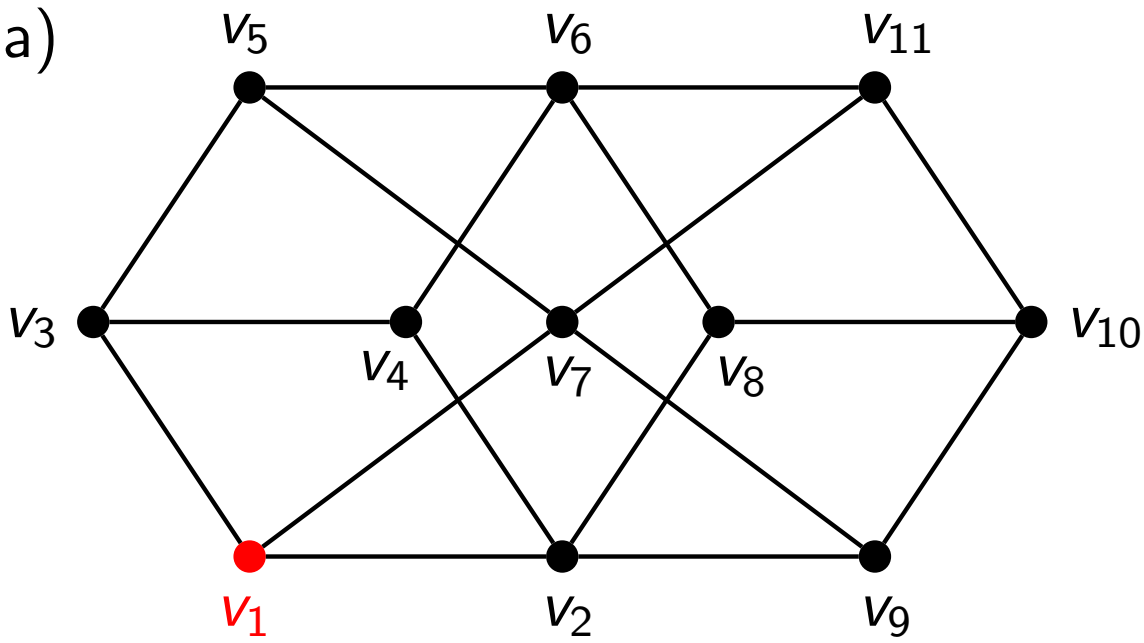
a)



- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

Rješenje

a)



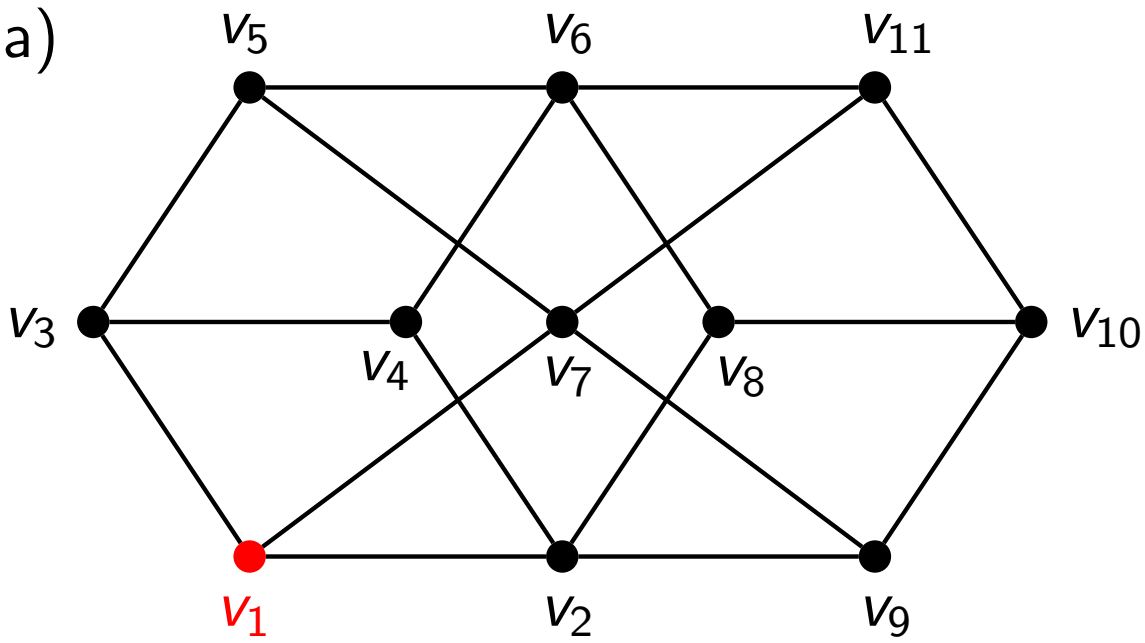
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{$

$Y = \{$

Rješenje

a)



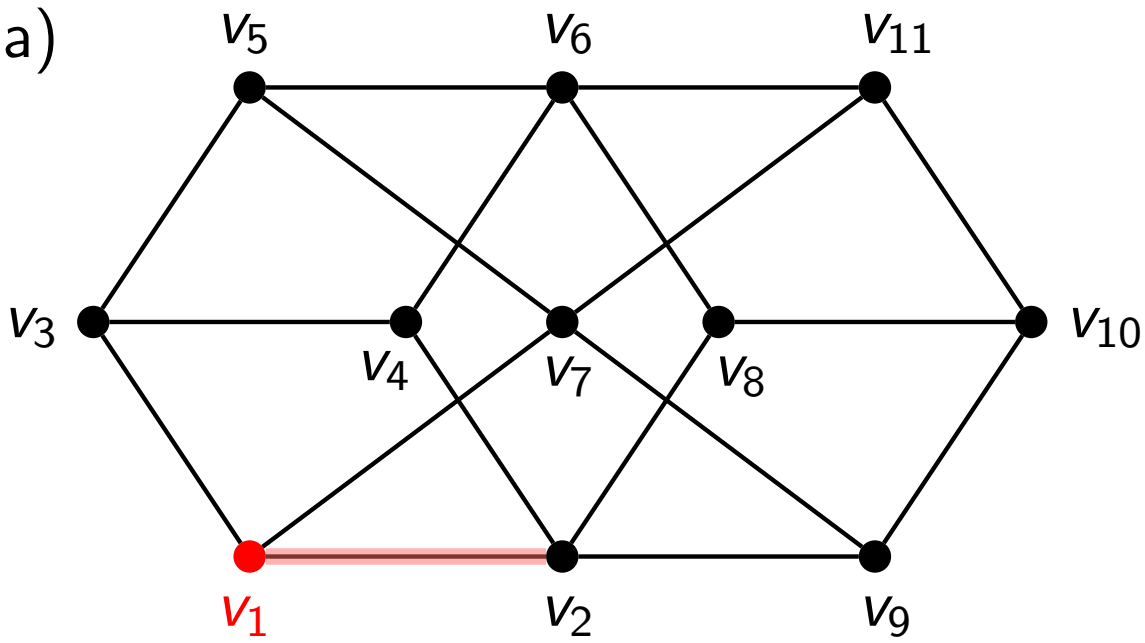
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1$

$Y = \{$

Rješenje

a)



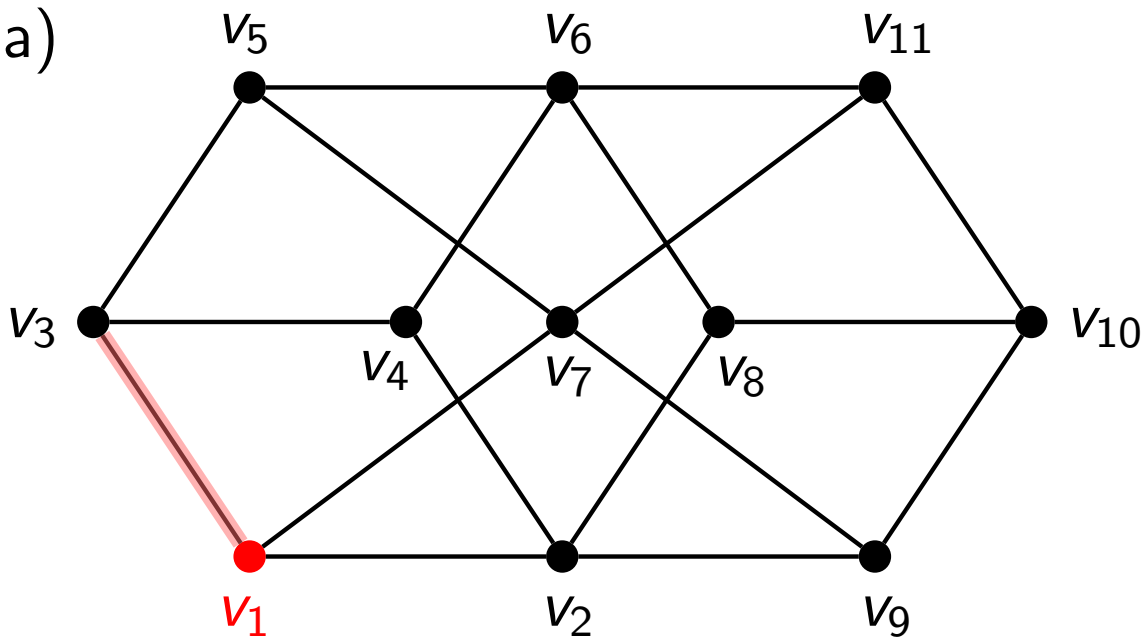
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1$

$Y = \{v_2$

Rješenje

a)



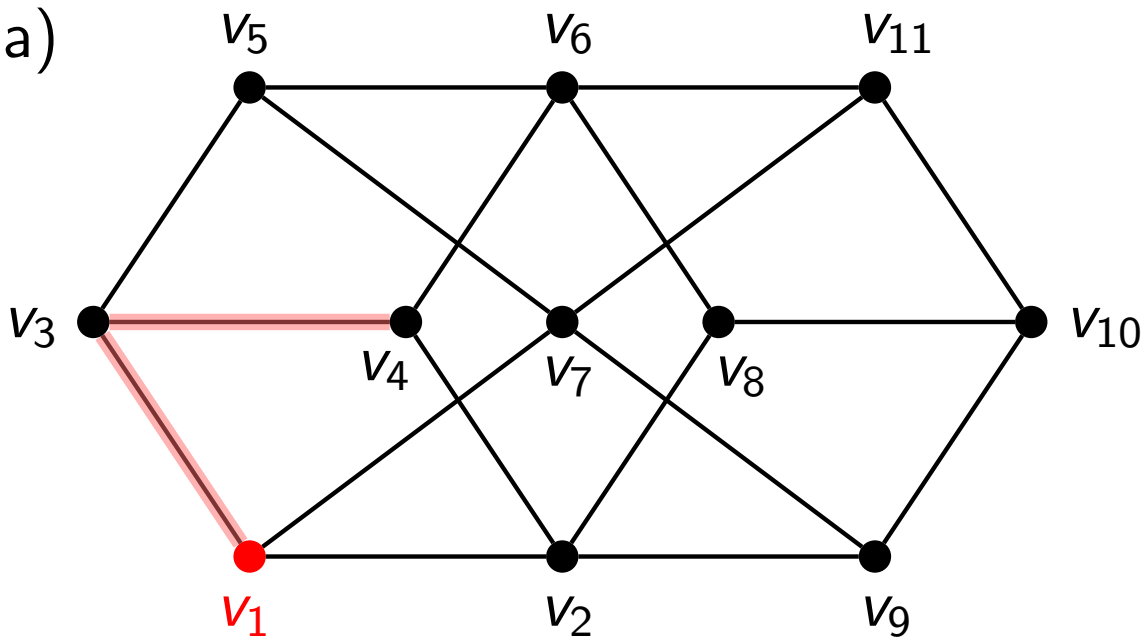
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1$

$Y = \{v_2, v_3$

Rješenje

a)



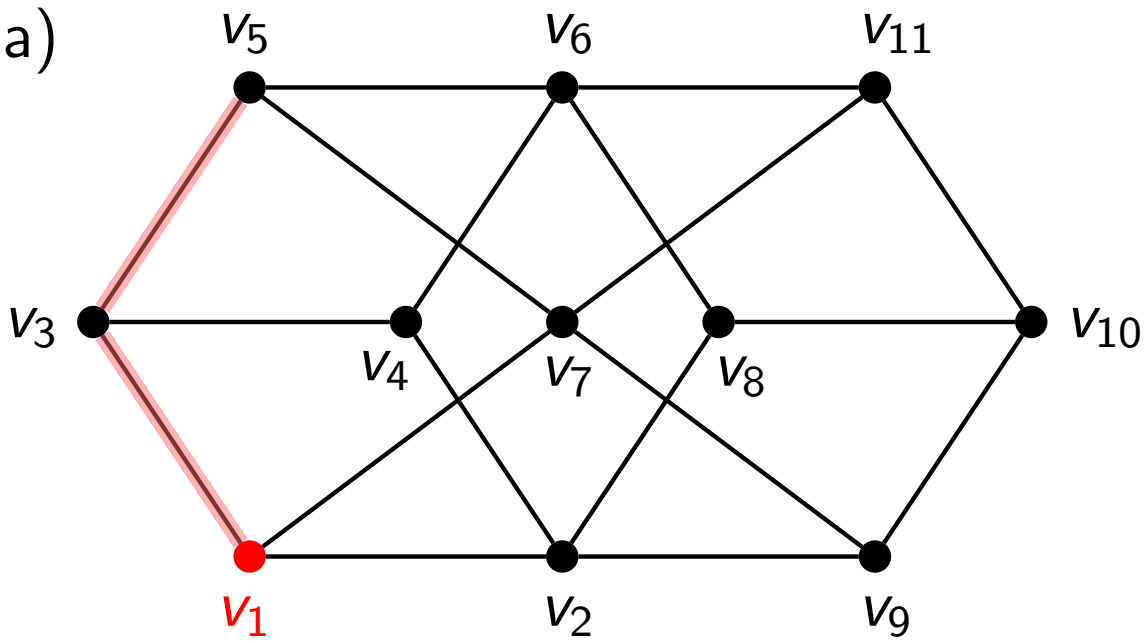
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4$

$Y = \{v_2, v_3$

Rješenje

a)



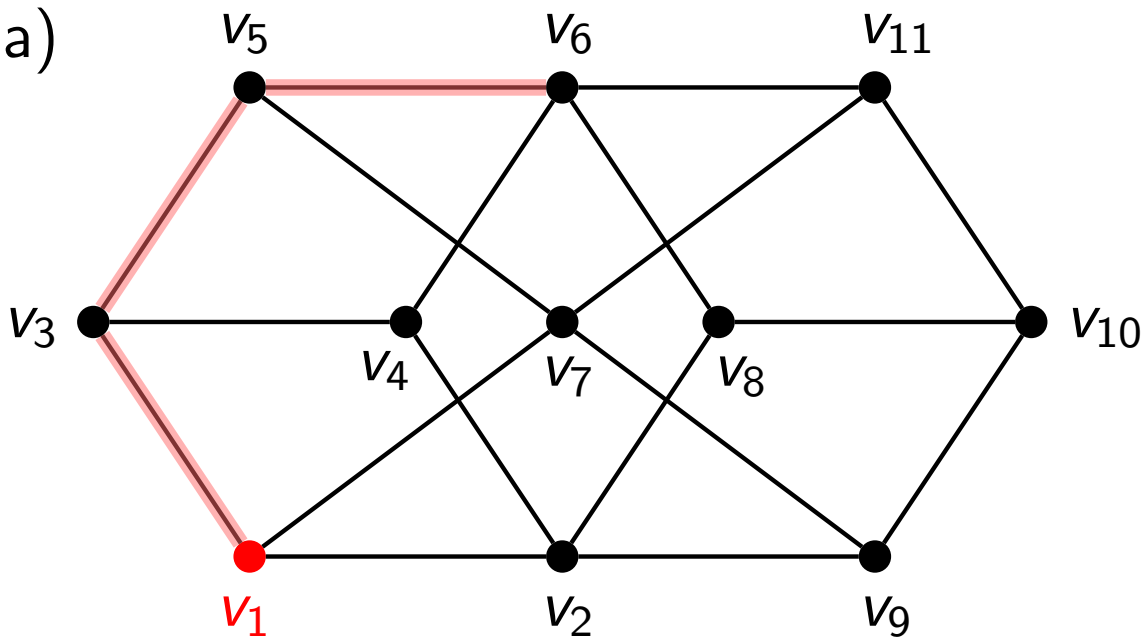
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5$

$Y = \{v_2, v_3$

Rješenje

a)



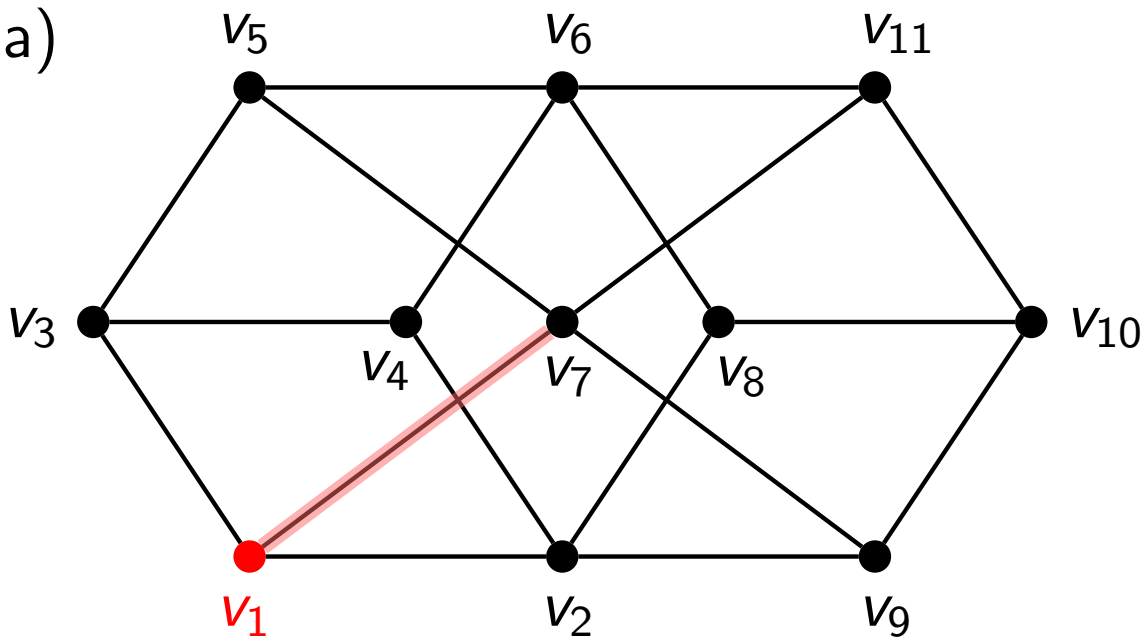
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5$

$Y = \{v_2, v_3, v_6$

Rješenje

a)



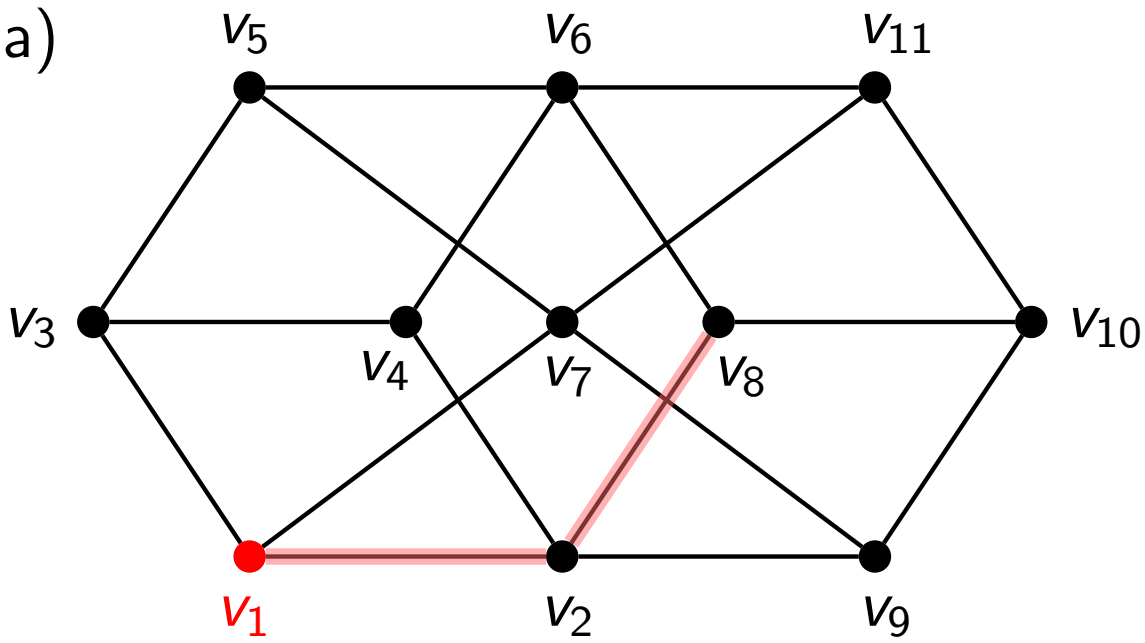
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5$

$Y = \{v_2, v_3, v_6, v_7$

Rješenje

a)



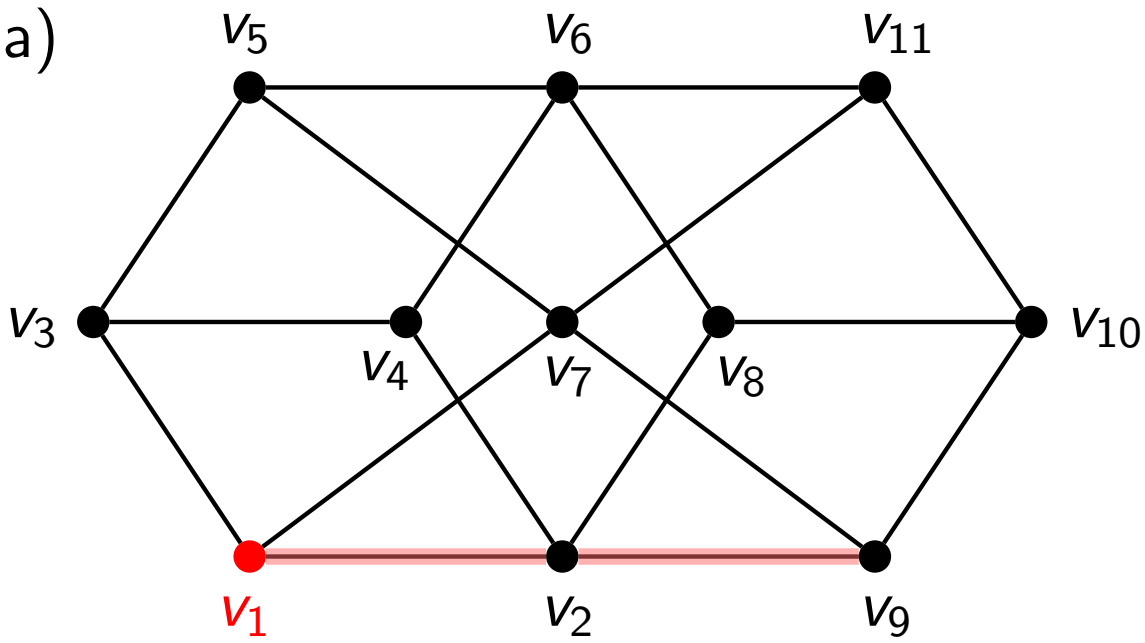
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5, v_8$

$Y = \{v_2, v_3, v_6, v_7$

Rješenje

a)



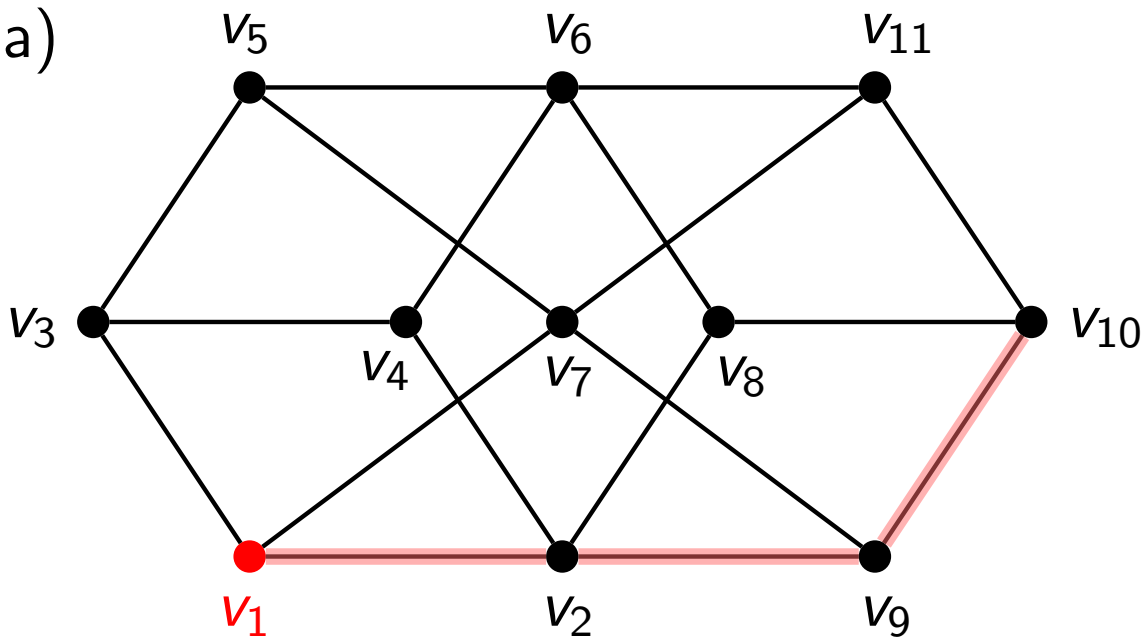
- Odaberemo neki vrh, npr. vrh v_1.
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5, v_8, v_9$

$Y = \{v_2, v_3, v_6, v_7$

Rješenje

a)



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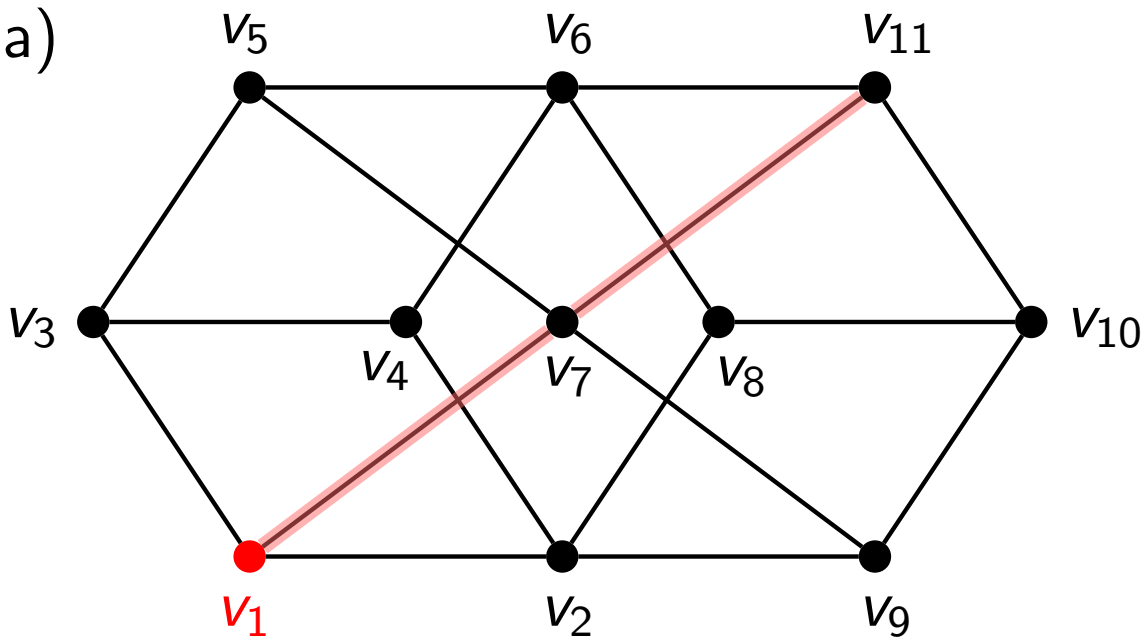
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5, v_8, v_9$

$Y = \{v_2, v_3, v_6, v_7, v_{10}$

Rješenje

a)



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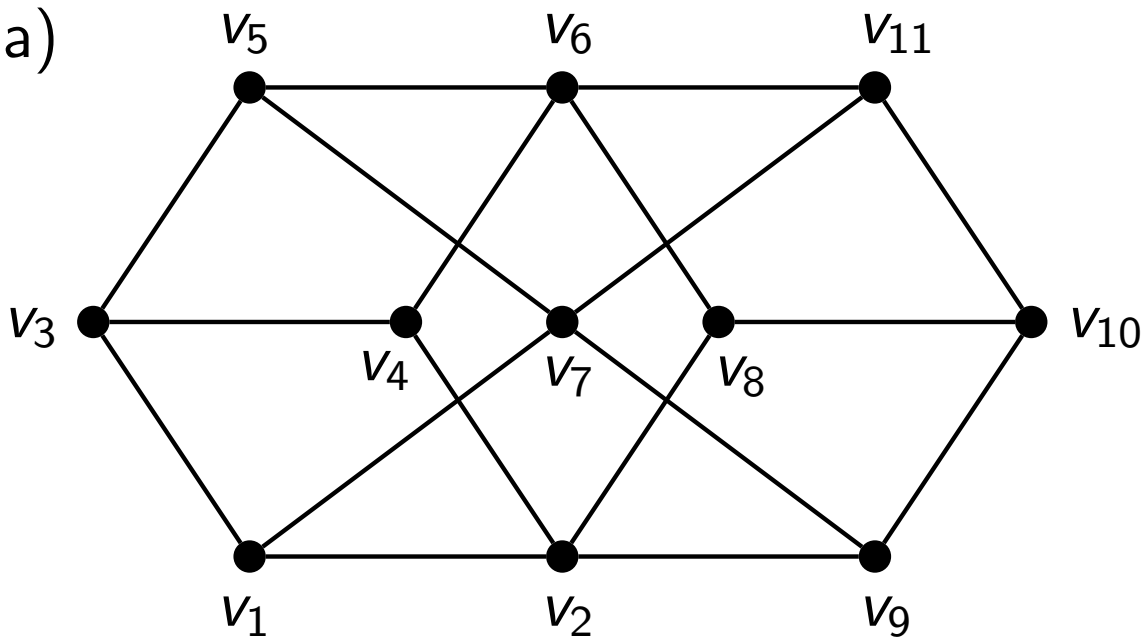
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

$X = \{v_1, v_4, v_5, v_8, v_9, v_{11}$

$Y = \{v_2, v_3, v_6, v_7, v_{10}$

Rješenje

a)



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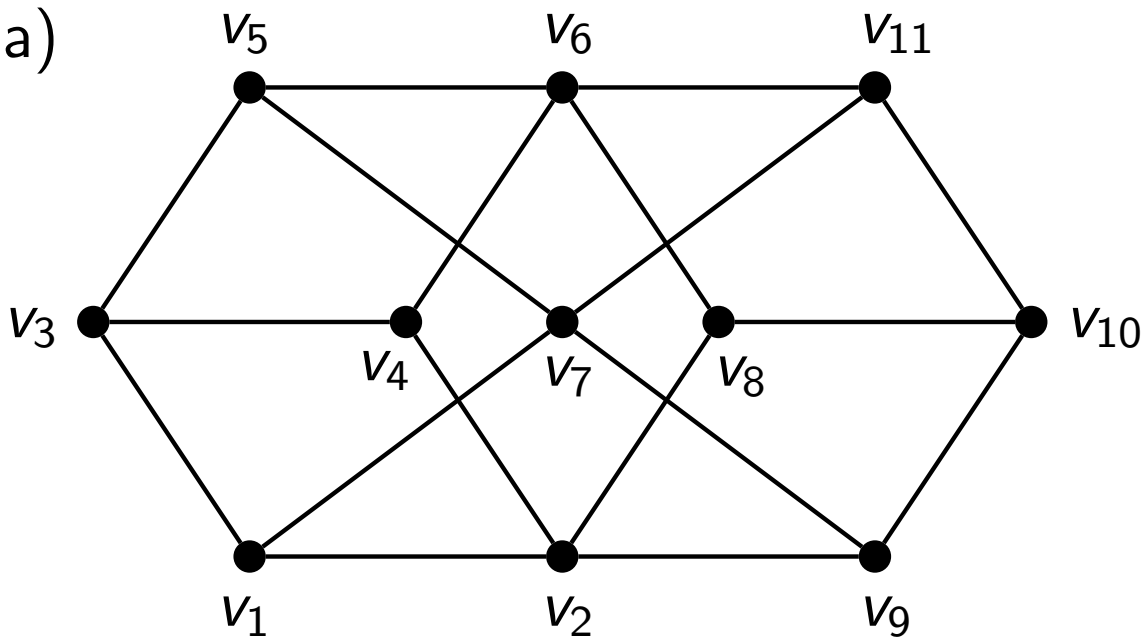
$X = \{v_1, v_4, v_5, v_8, v_9, v_{11}\}$

$Y = \{v_2, v_3, v_6, v_7, v_{10}\}$

Rješenje

a)



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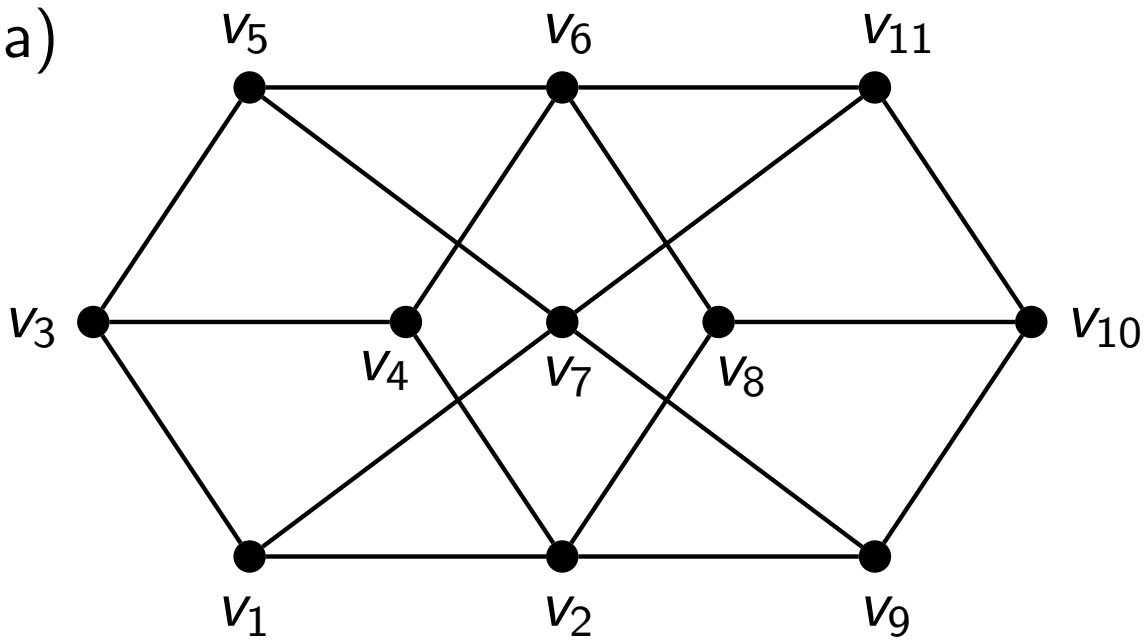
$X = \{v_1, v_4, v_5, v_8, v_9, v_{11}\} \longleftarrow$ svi vrhovi u X su međusobno nesusjedni

$Y = \{v_2, v_3, v_6, v_7, v_{10}\}$

Rješenje

a)



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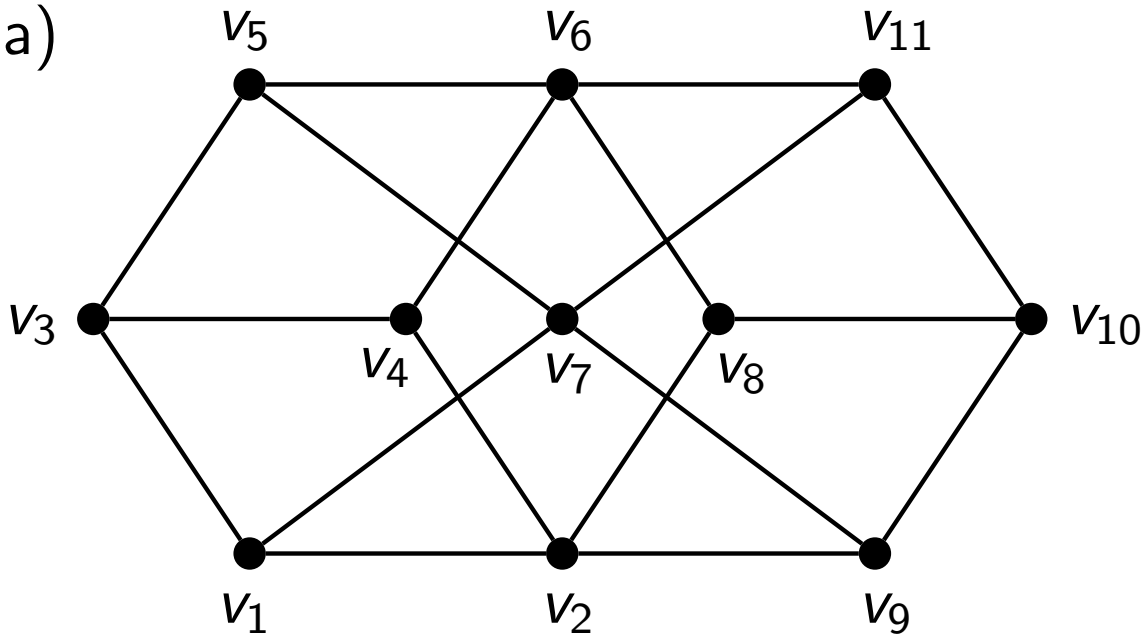
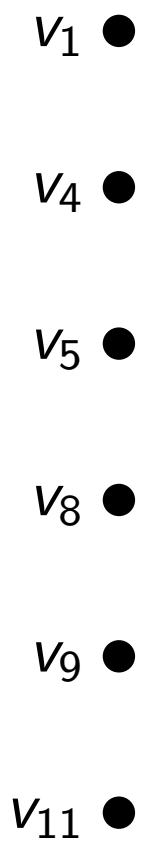
$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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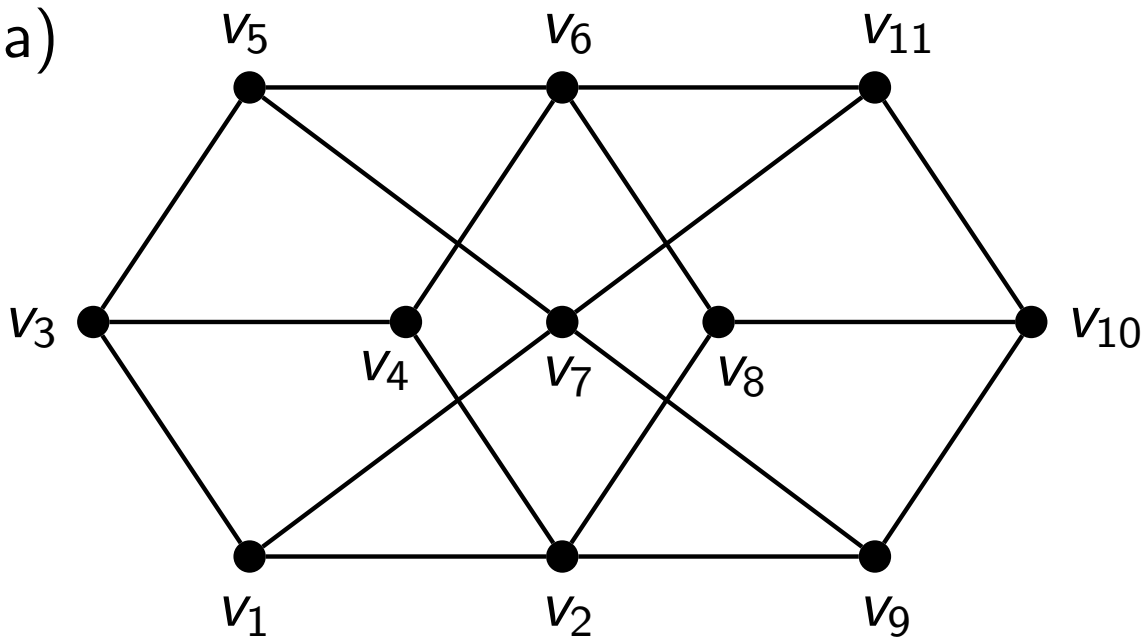
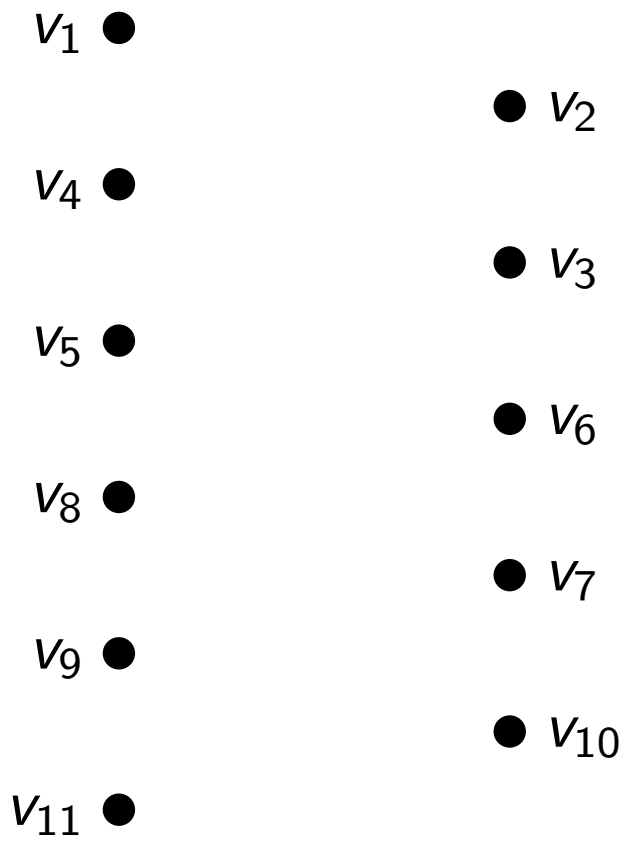
$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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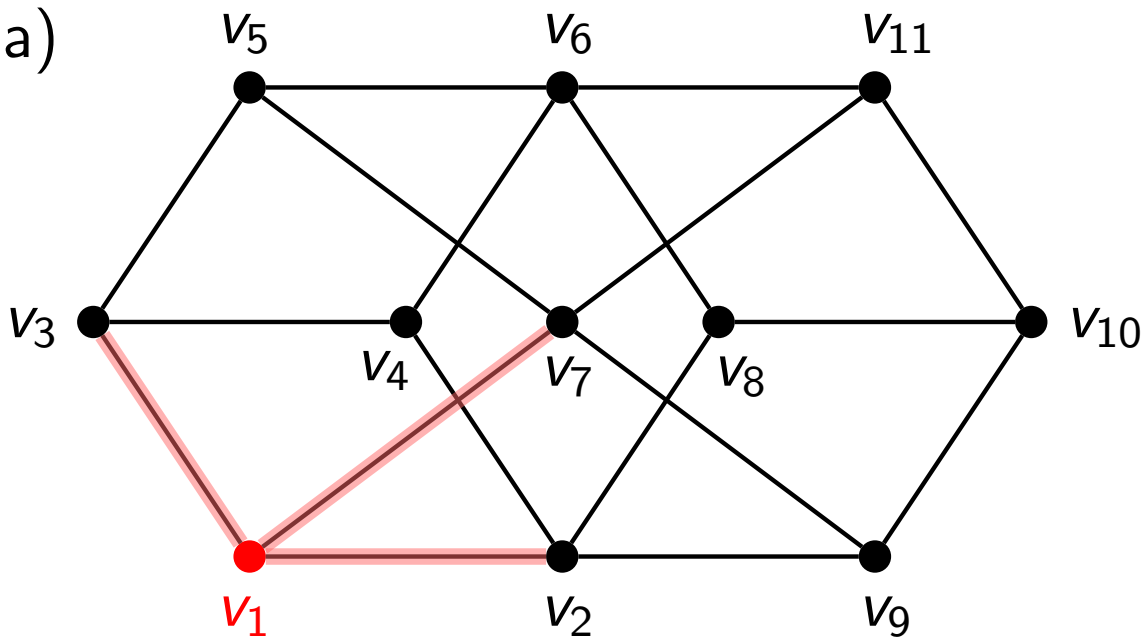
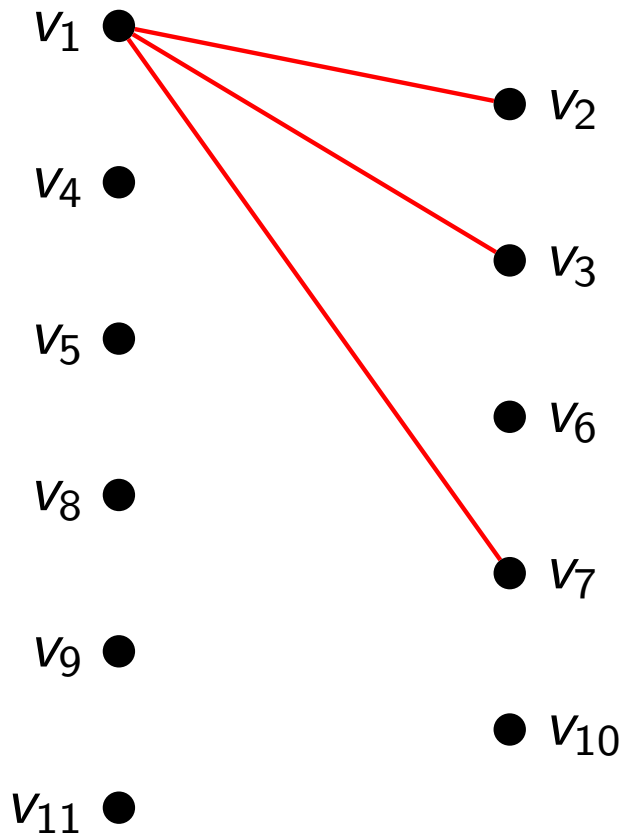
$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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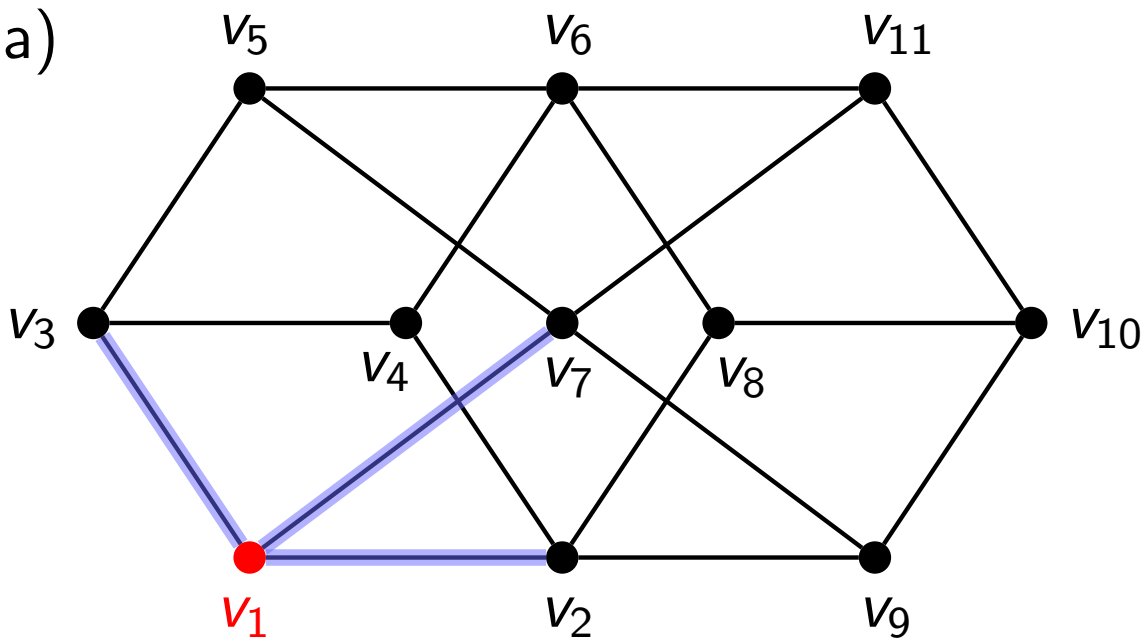
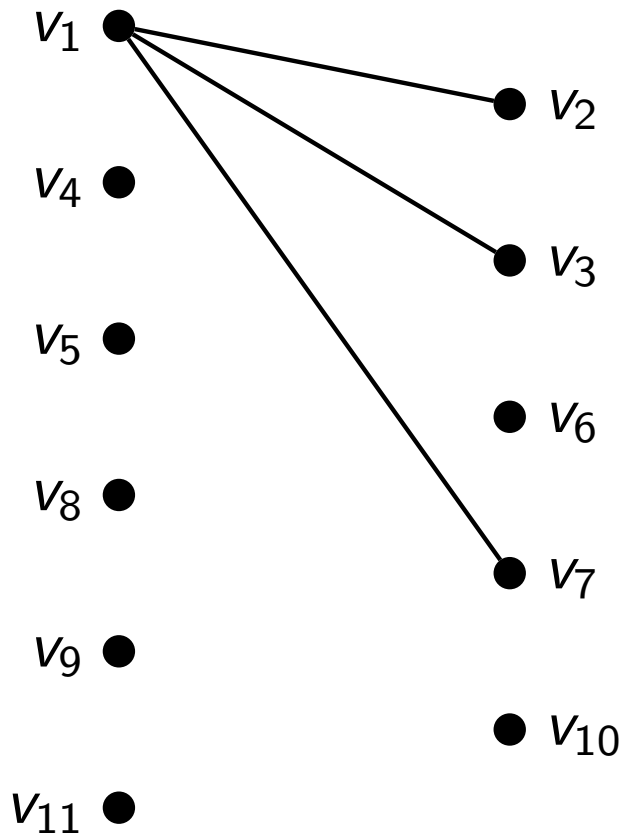
$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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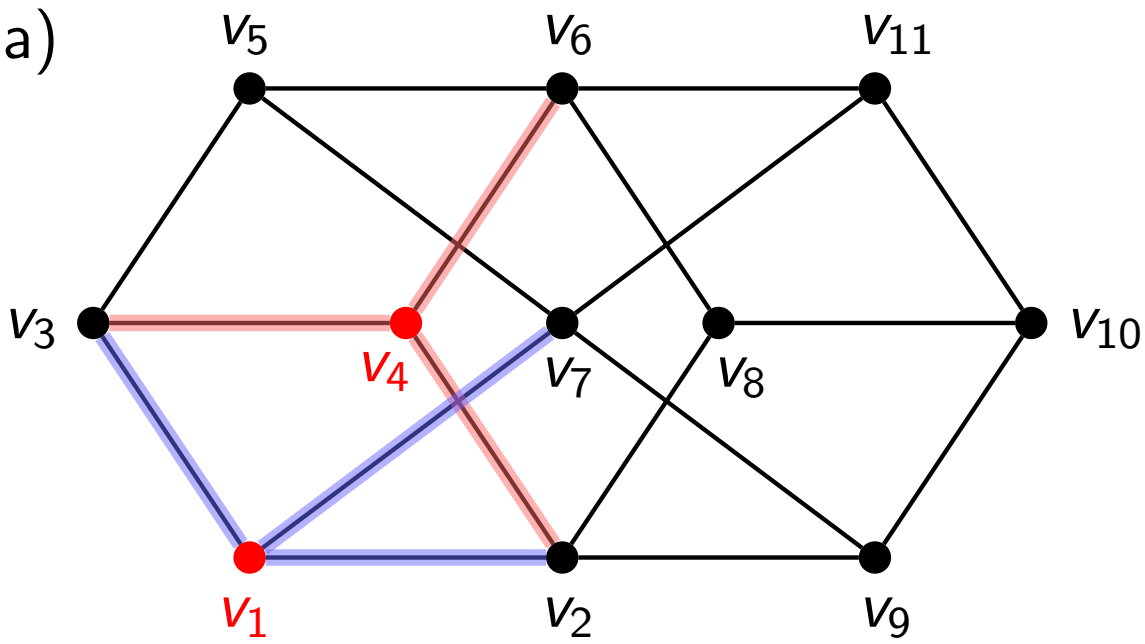
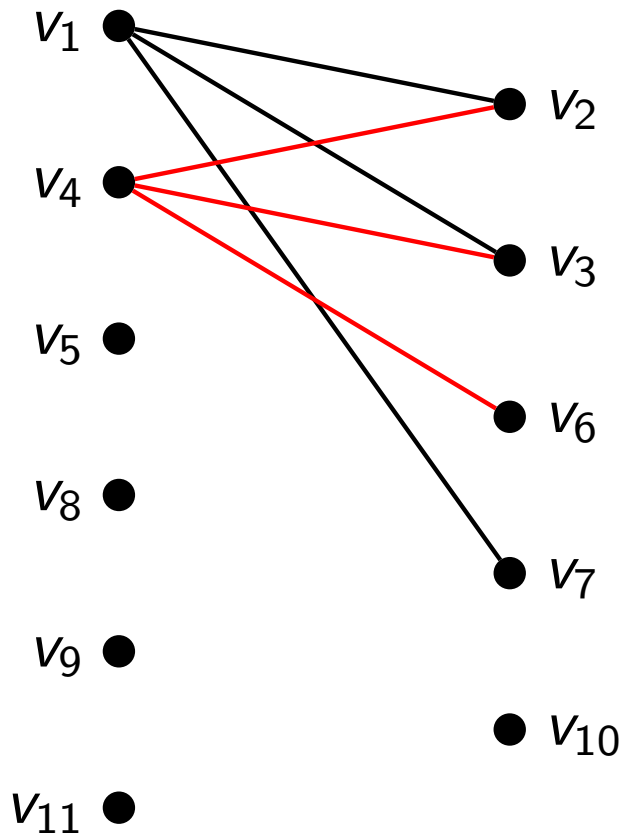
$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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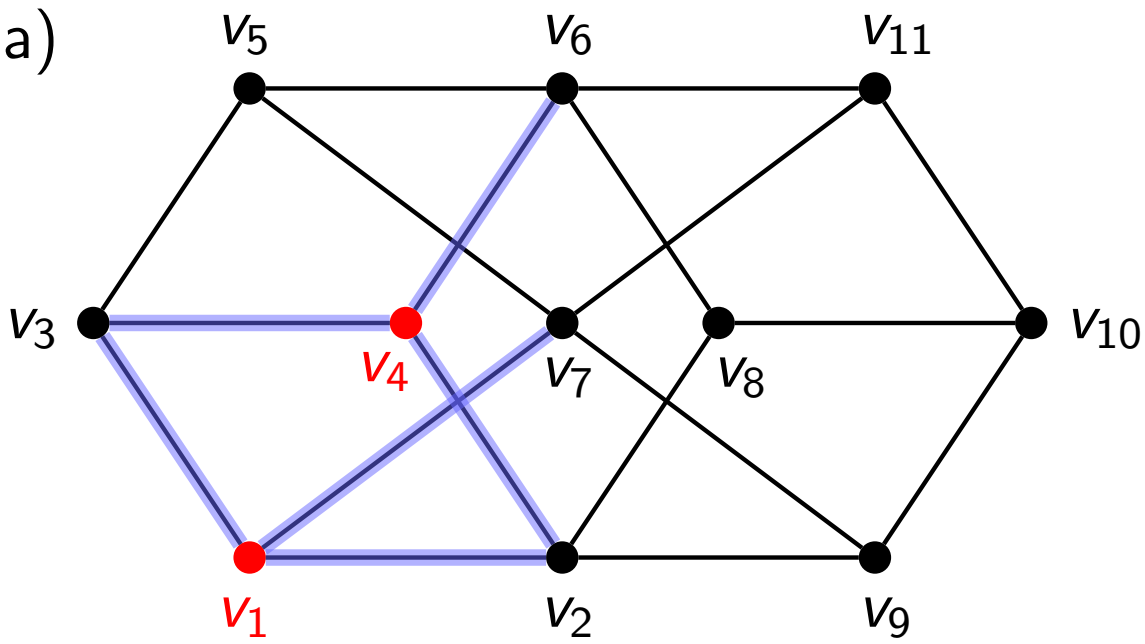
- $X \longleftarrow$ skup svih vrhova na parnoj udaljenosti od vrha v_1
- $Y \longleftarrow$ skup svih vrhova na neparnoj udaljenosti od vrha v_1

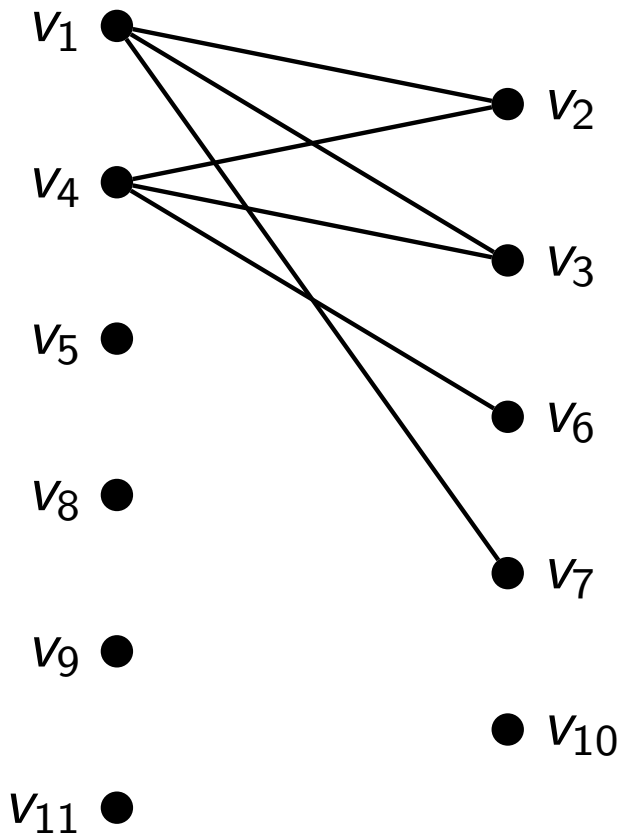
$X = \{v_1, v_4, v_5, v_8, v_9, v_{11}\} \longleftarrow$ svi vrhovi u X su međusobno nesusjedni

$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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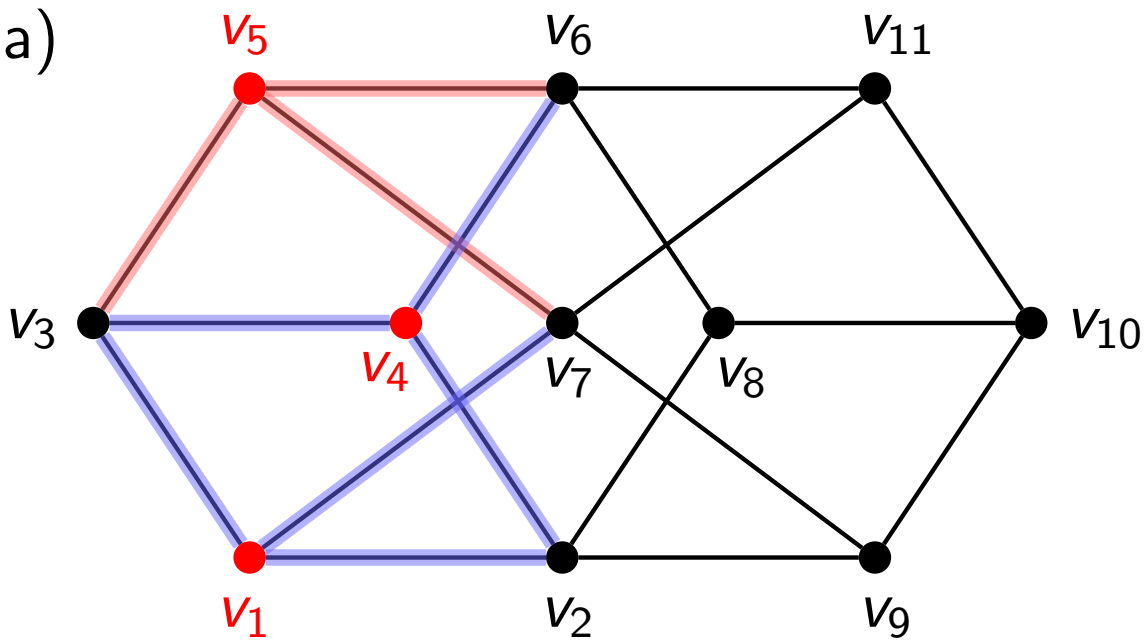
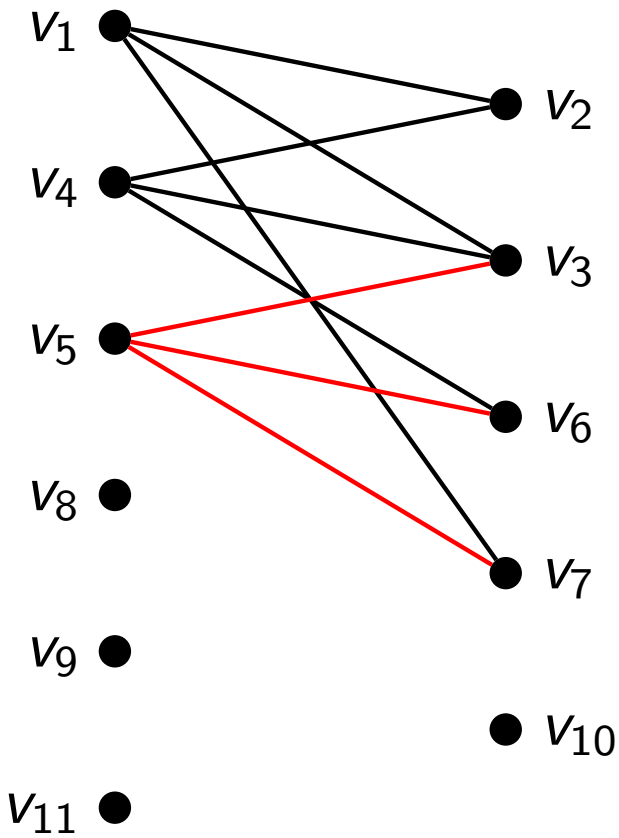
$X = \{v_1, v_4, v_5, v_8, v_9, v_{11}\} \longleftarrow$ svi vrhovi u X su međusobno nesusjedni

$Y = \{v_2, v_3, v_6, v_7, v_{10}\} \longleftarrow$ svi vrhovi u Y su međusobno nesusjedni

Rješenje

a)





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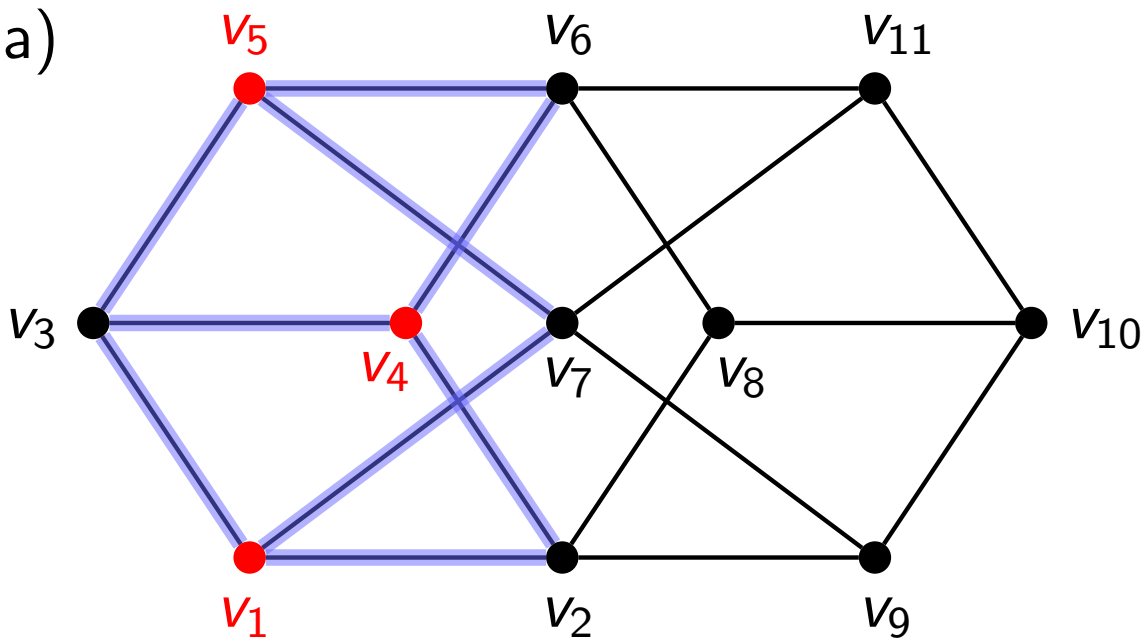
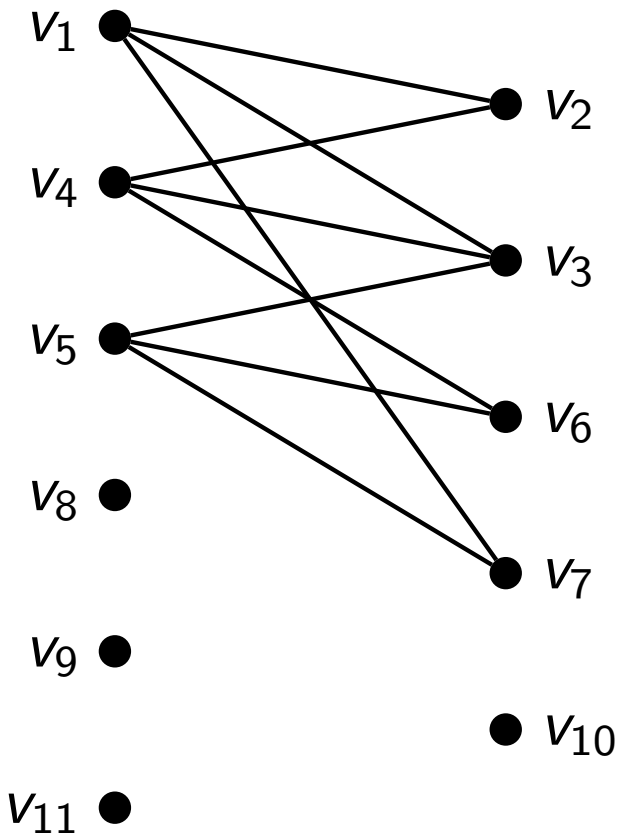
Rješenje

a)





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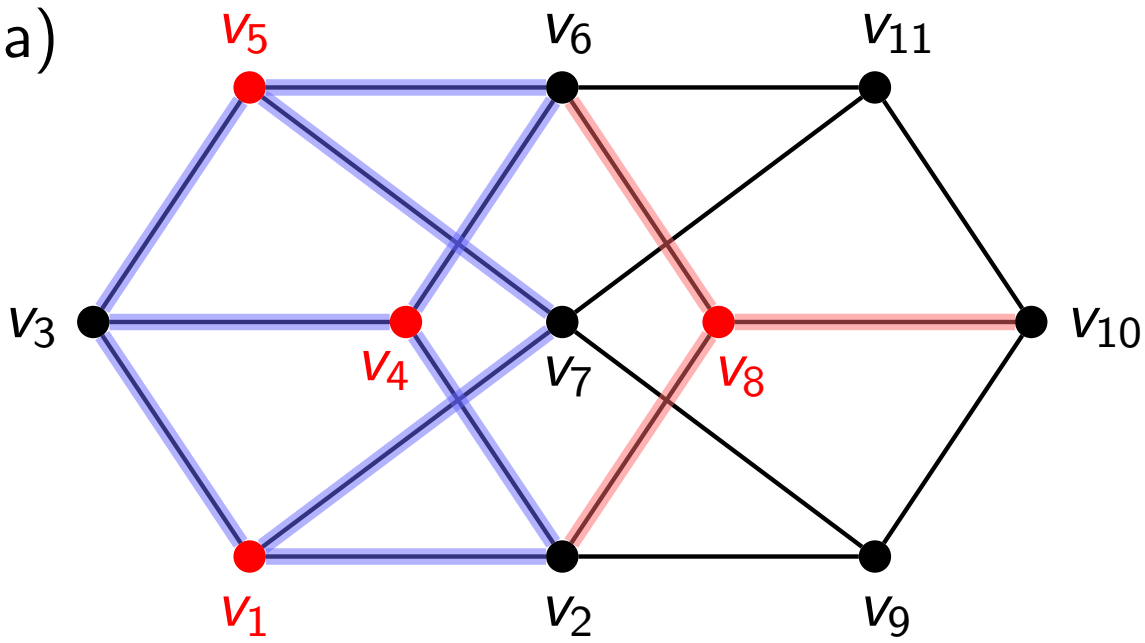
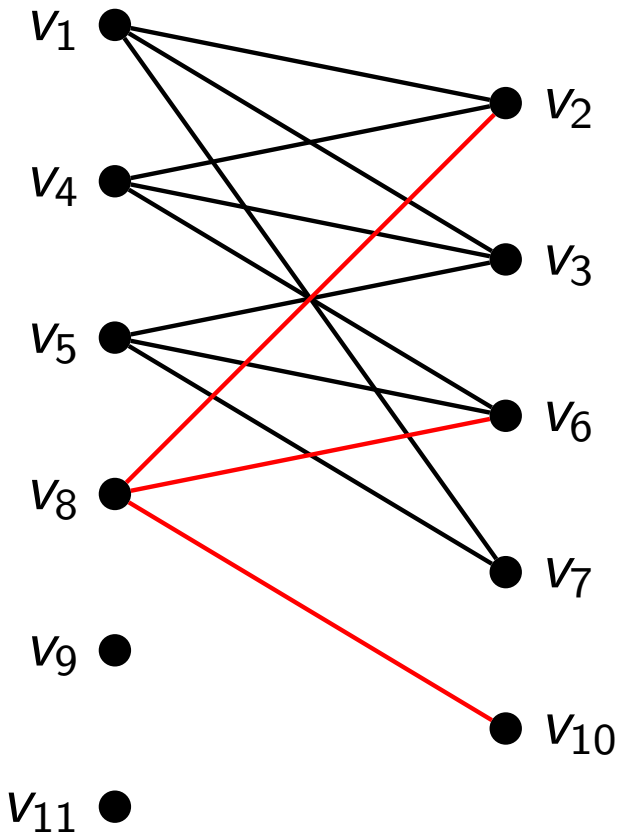
Rješenje

a)





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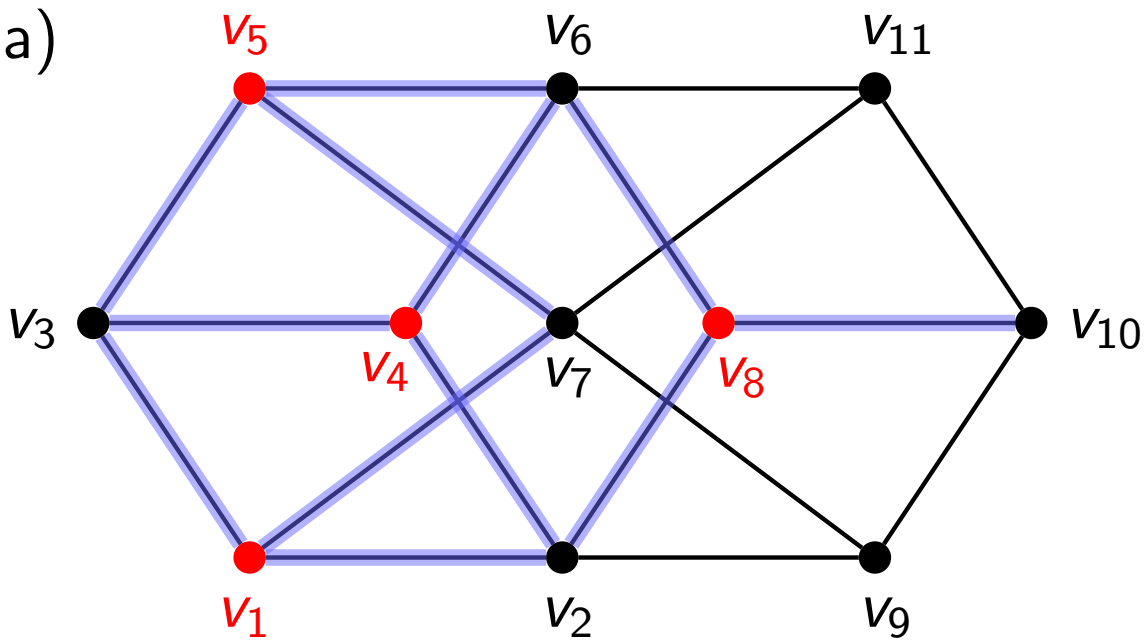
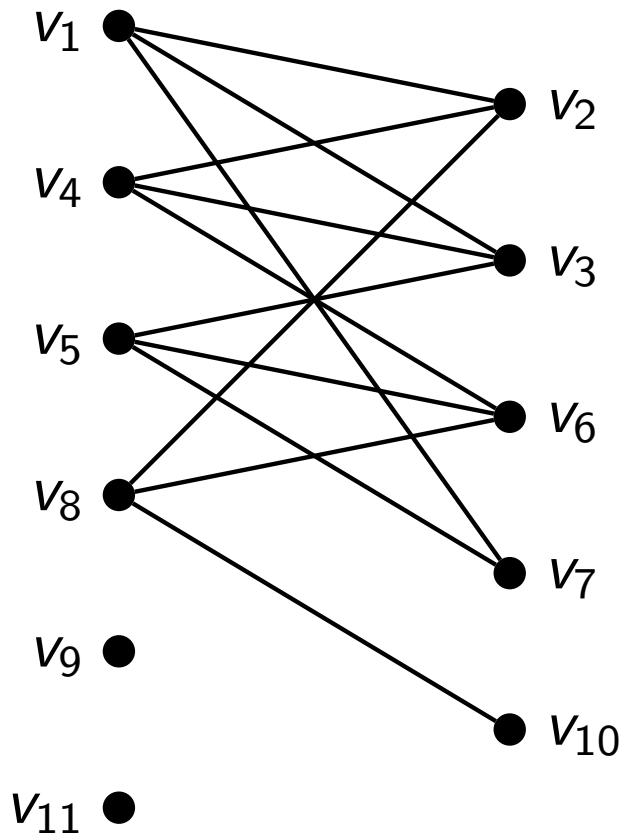
Rješenje

a)





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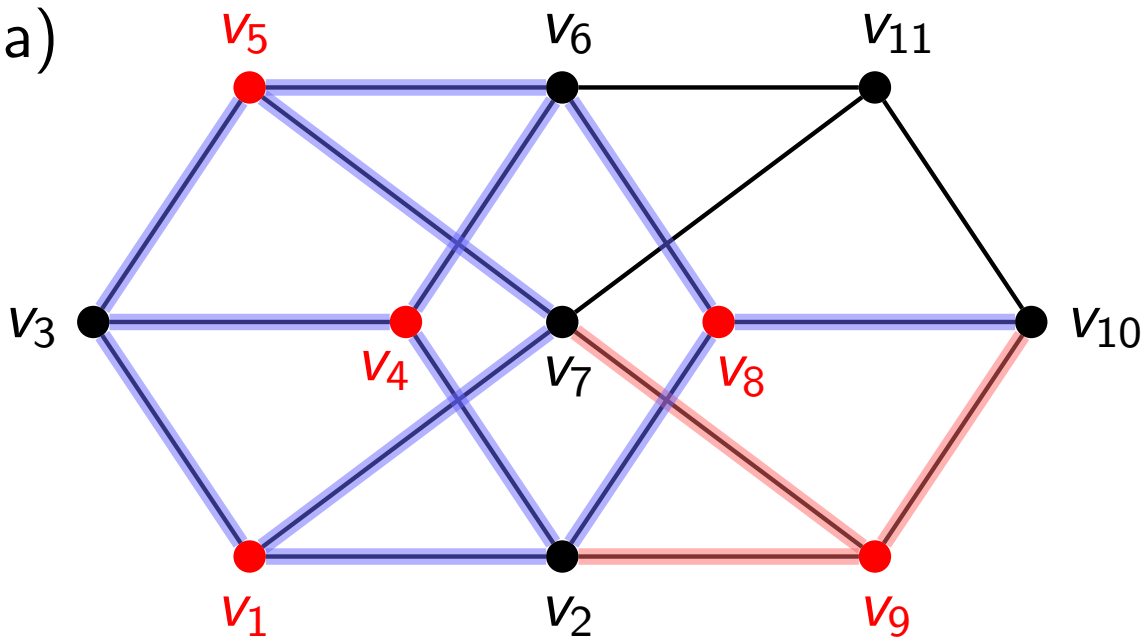
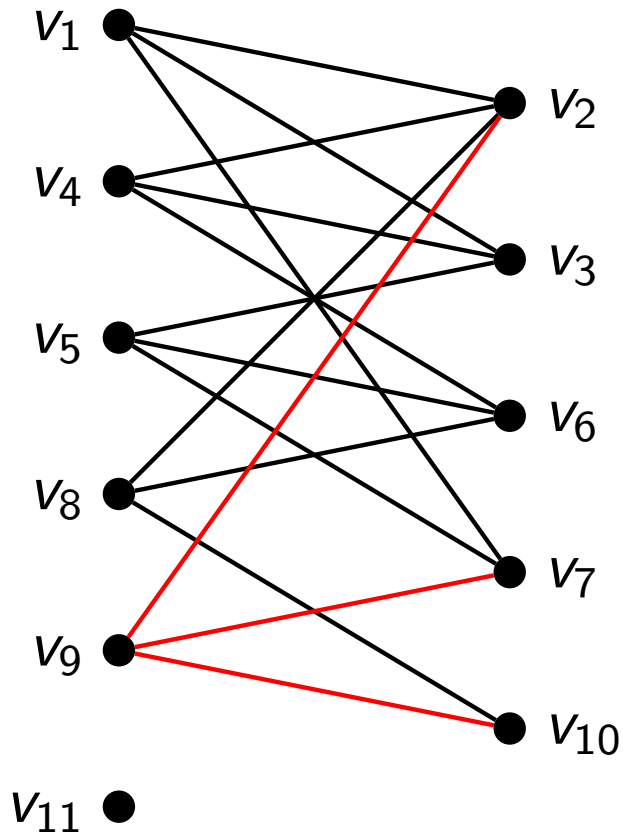
Rješenje

a)





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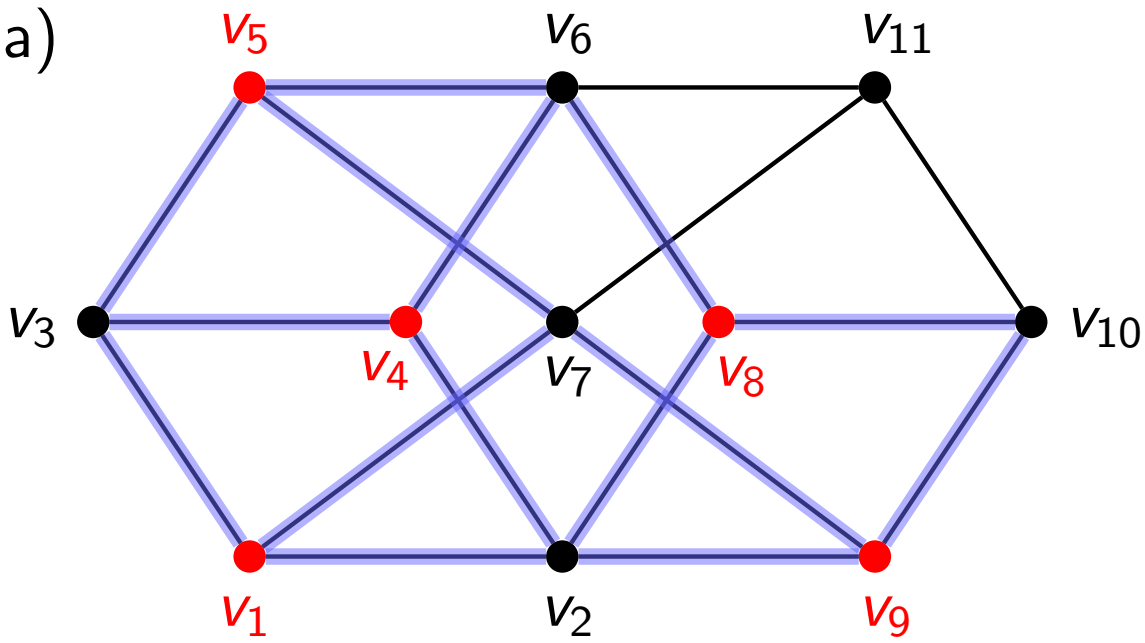
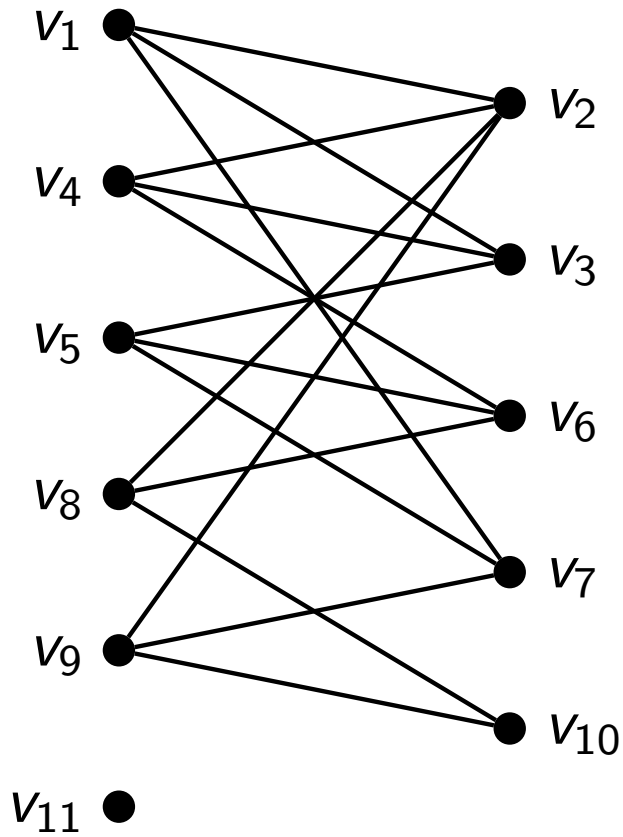
Rješenje

a)





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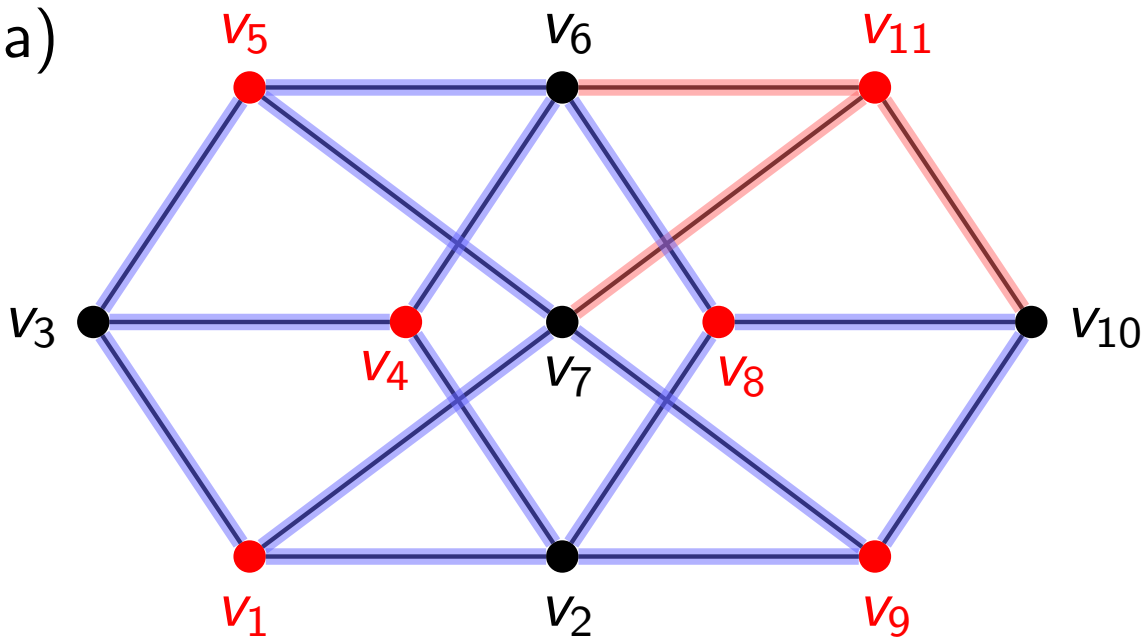
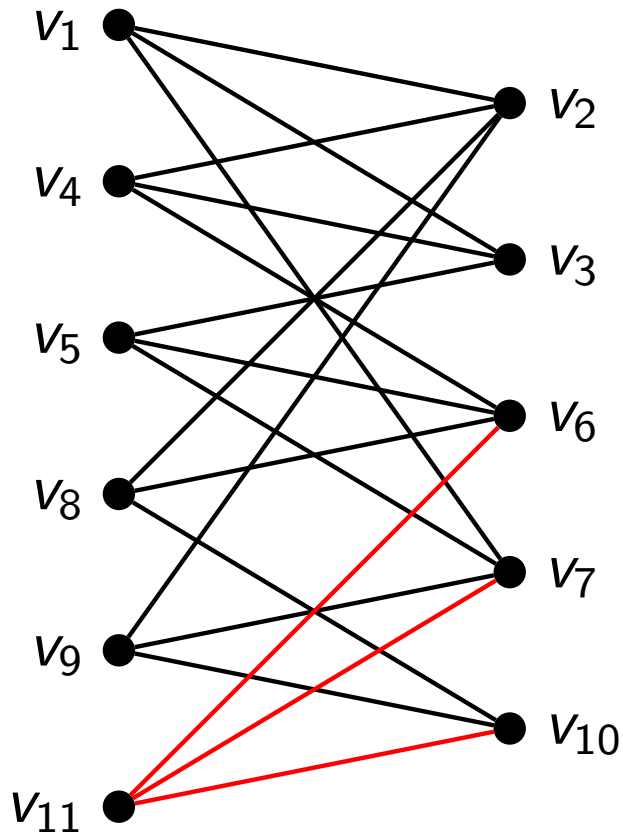
Rješenje

a)





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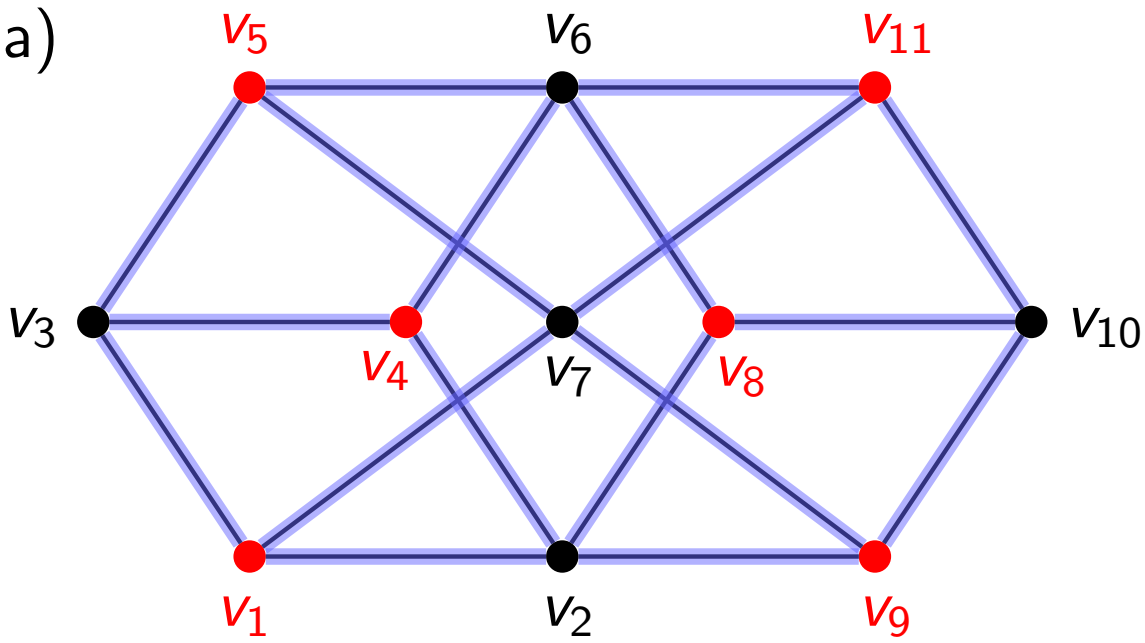
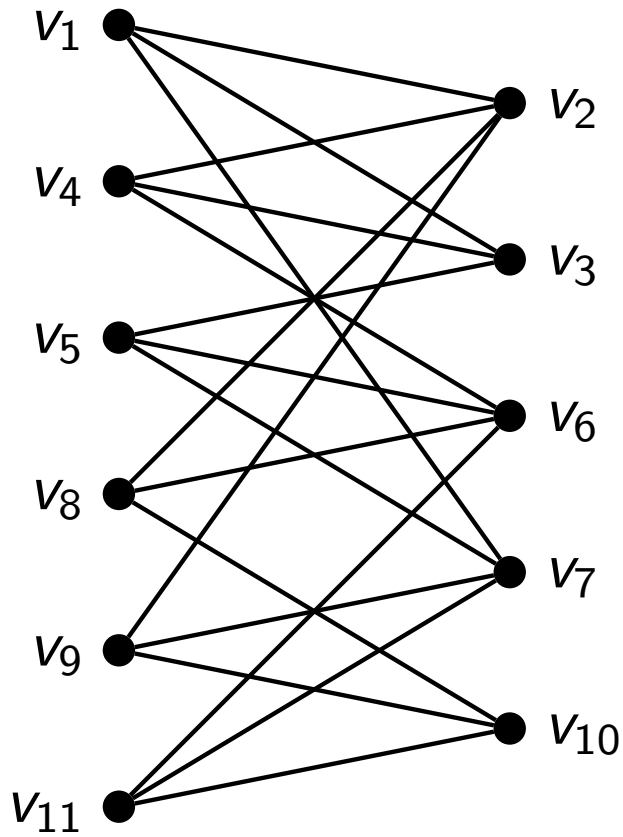
Rješenje

a)





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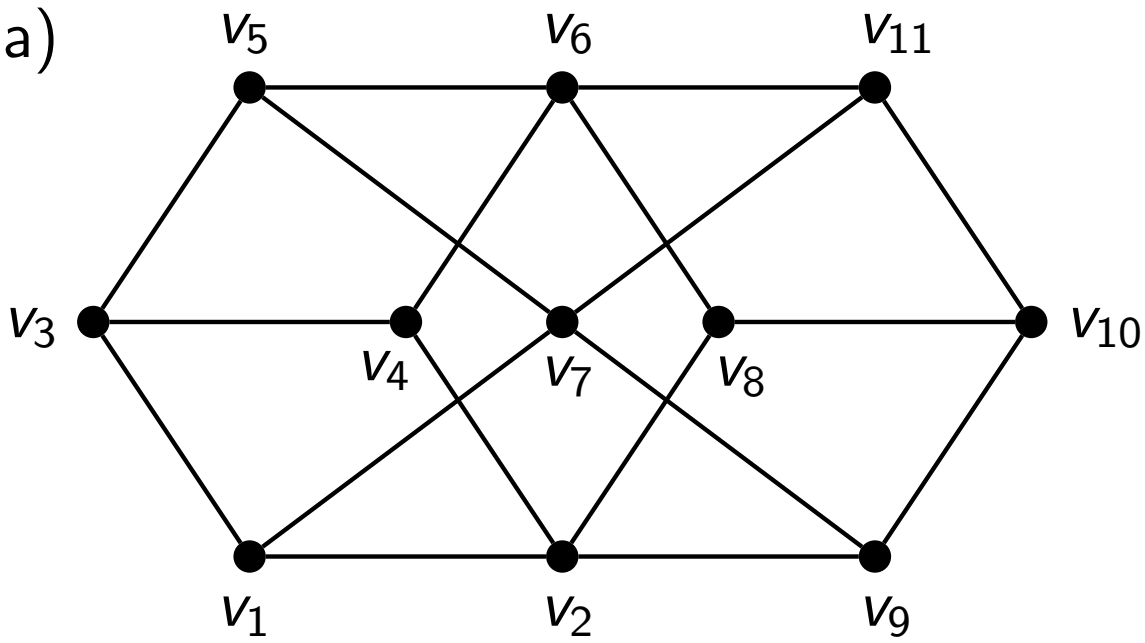
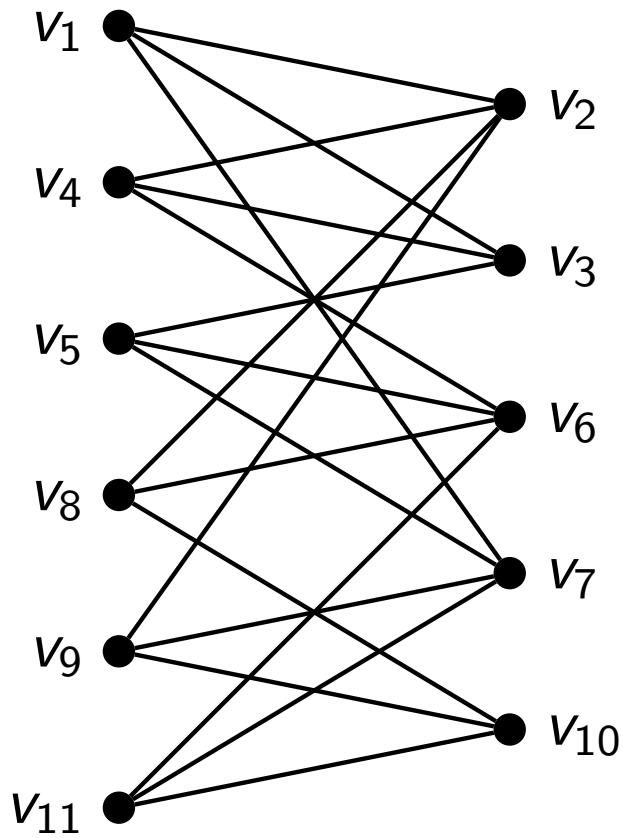
Rješenje

a)





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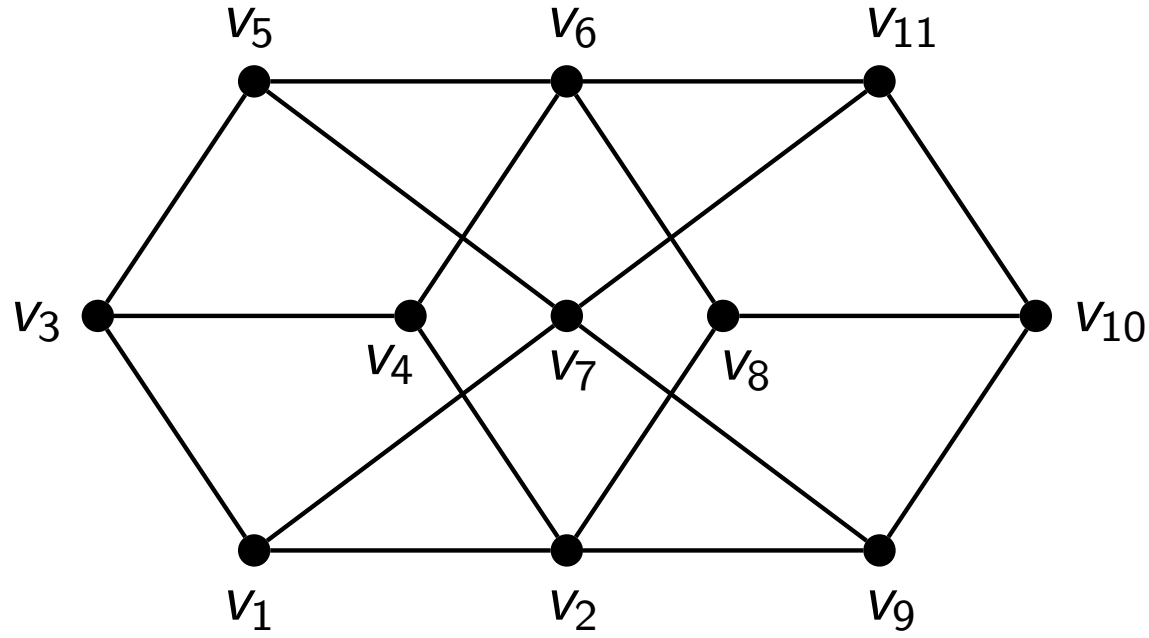
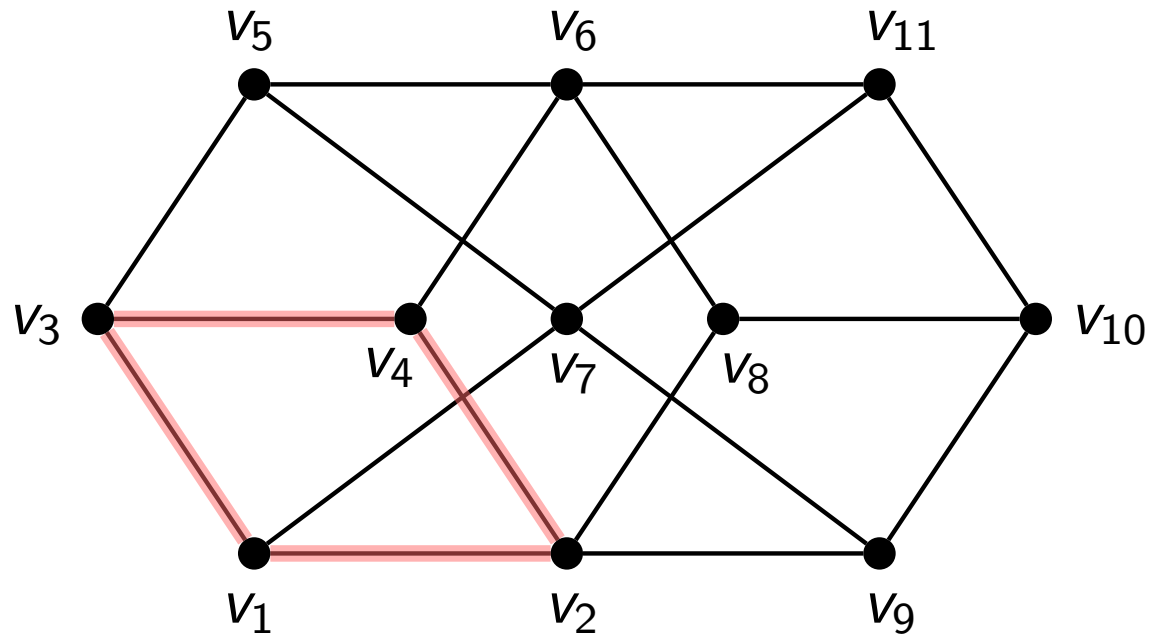
b)

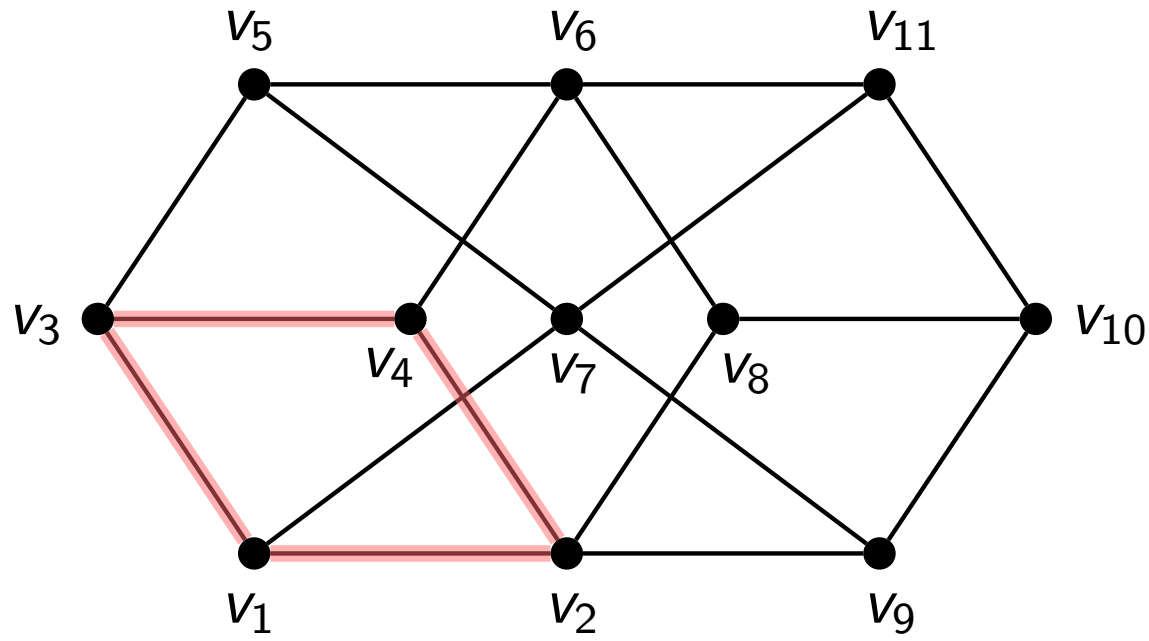


b)



Struk grafa G jednak je 4 jer je npr. $v_1v_2v_4v_3v_1$ jedan ciklus najmanje duljine u grafu G.

b)

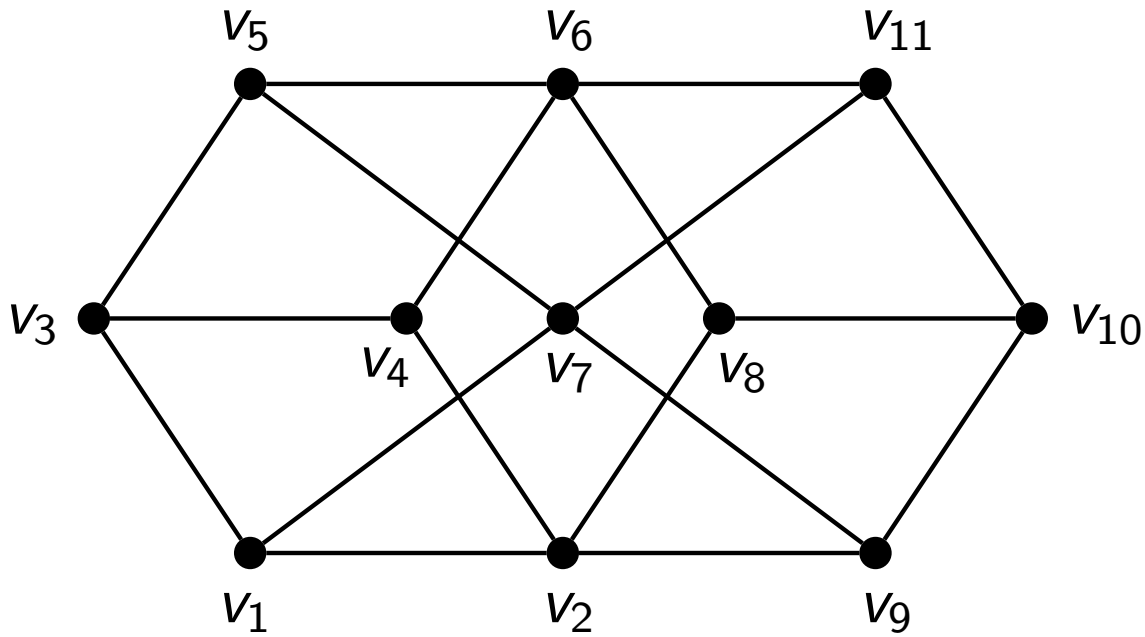


Struk grafa G jednak je 4 jer je npr. $v_1 v_2 v_4 v_3 v_1$ jedan ciklus najmanje duljine u grafu G.

Može li struk bipartitnog grafa biti neparni broj?

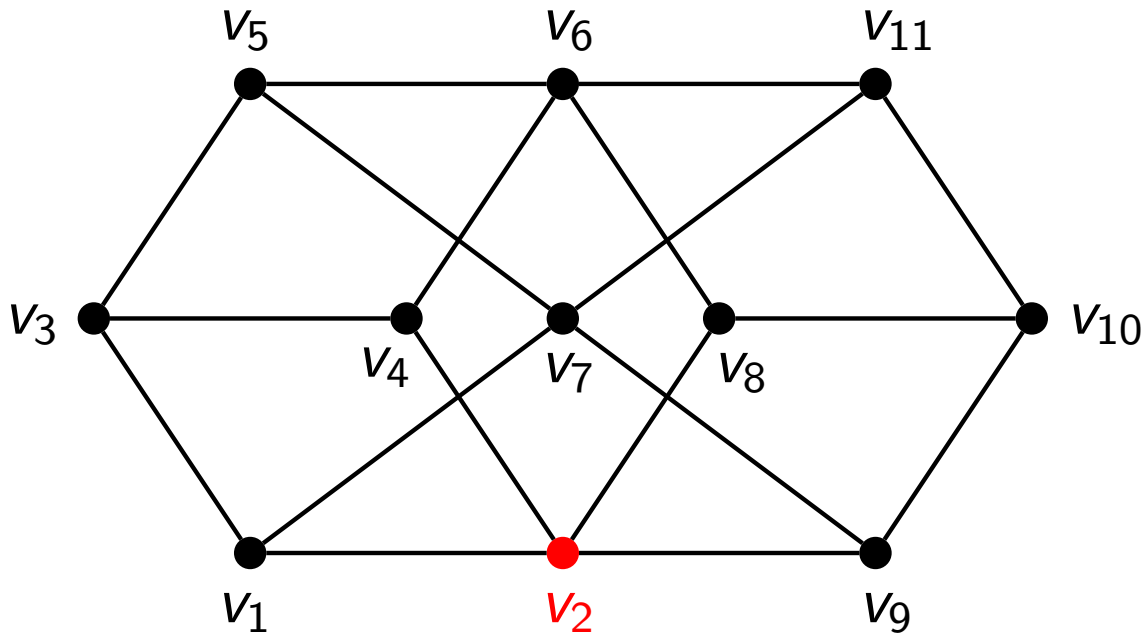
c)

G



c)

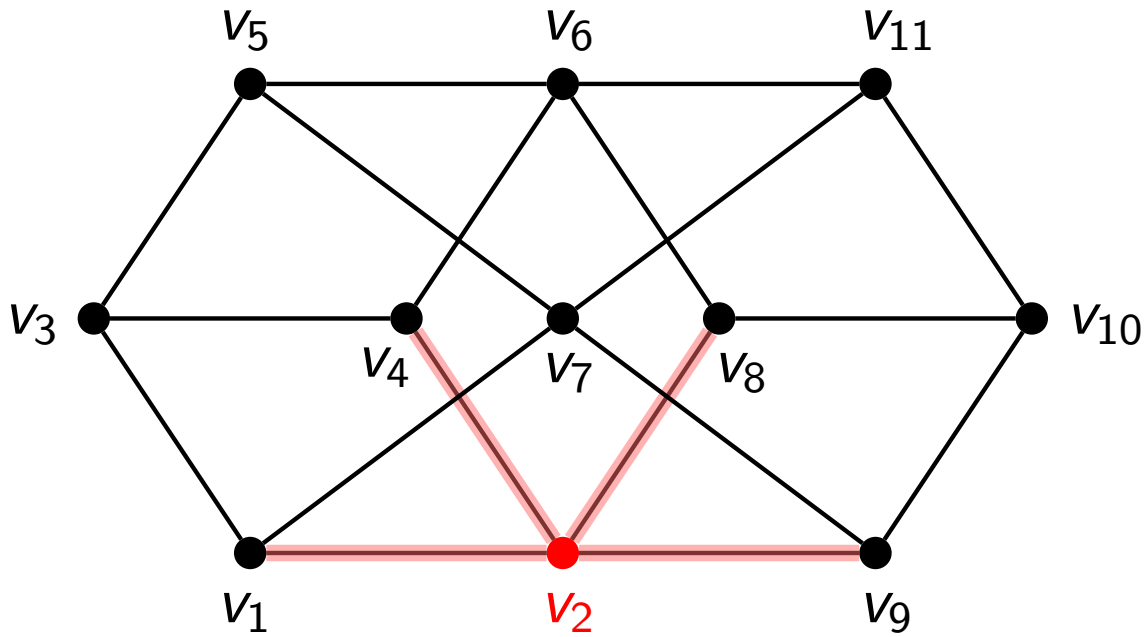
G



Dobivanje grafa $G - \{v_2, v_6\}$

c)

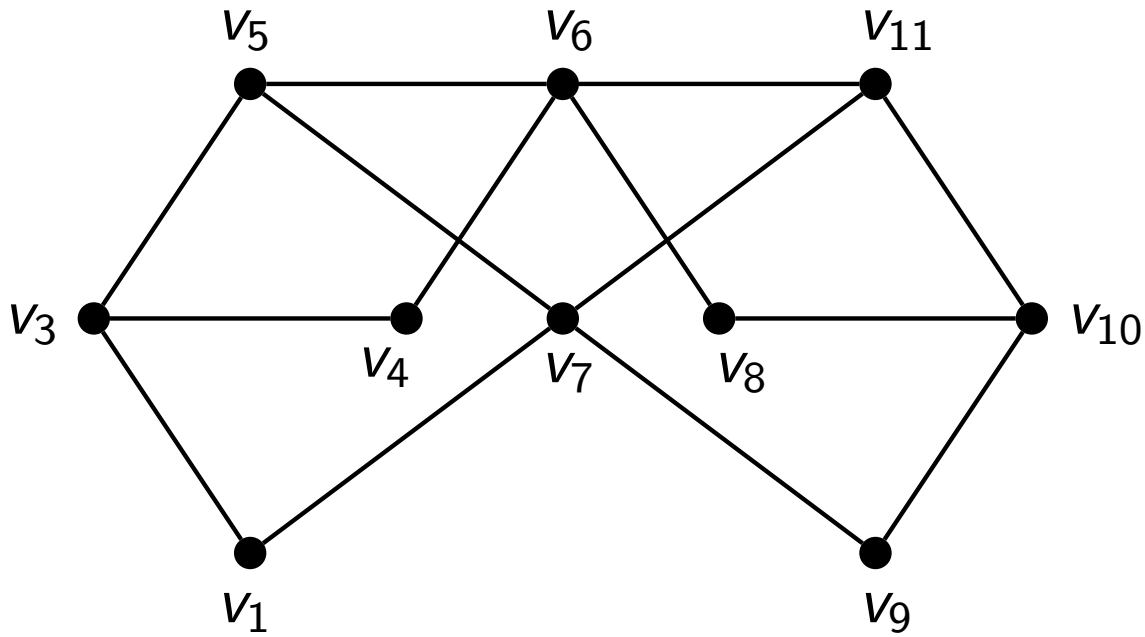
G



Dobivanje grafa $G - \{v_2, v_6\}$

c)

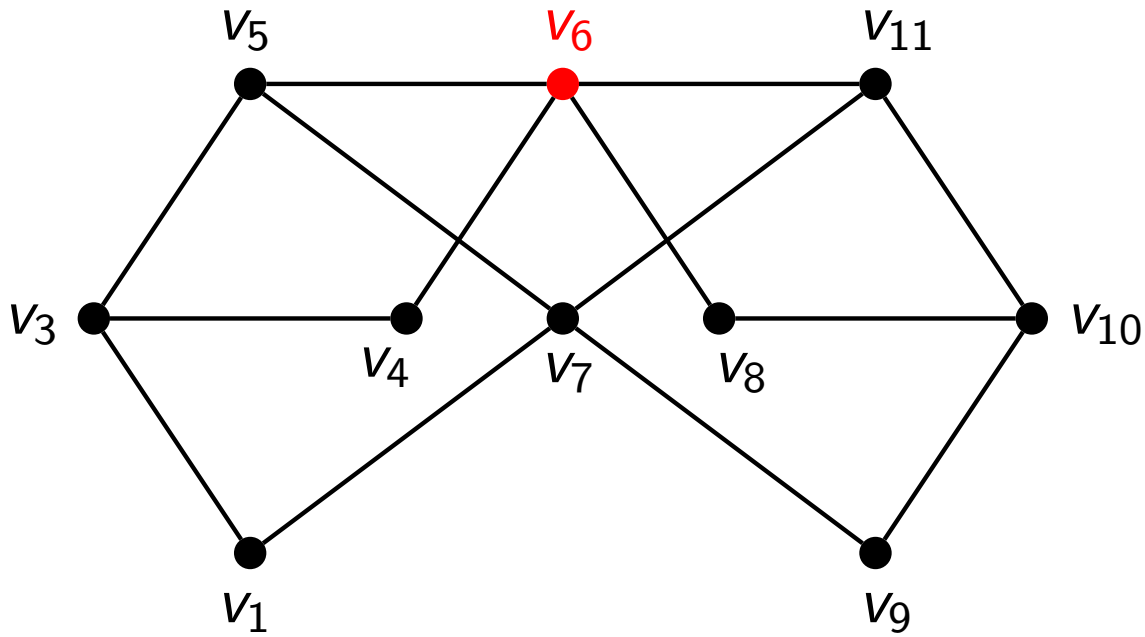
$G - v_2$



Dobivanje grafa $G - \{v_2, v_6\}$

c)

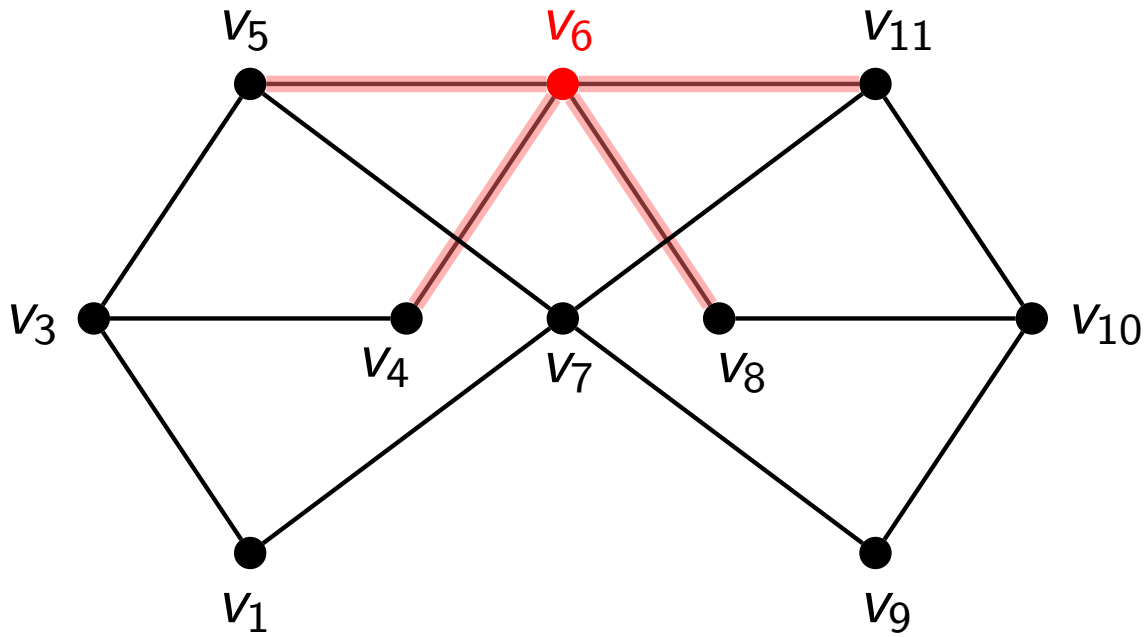
$G - v_2$



Dobivanje grafa $G - \{v_2, v_6\}$

c)

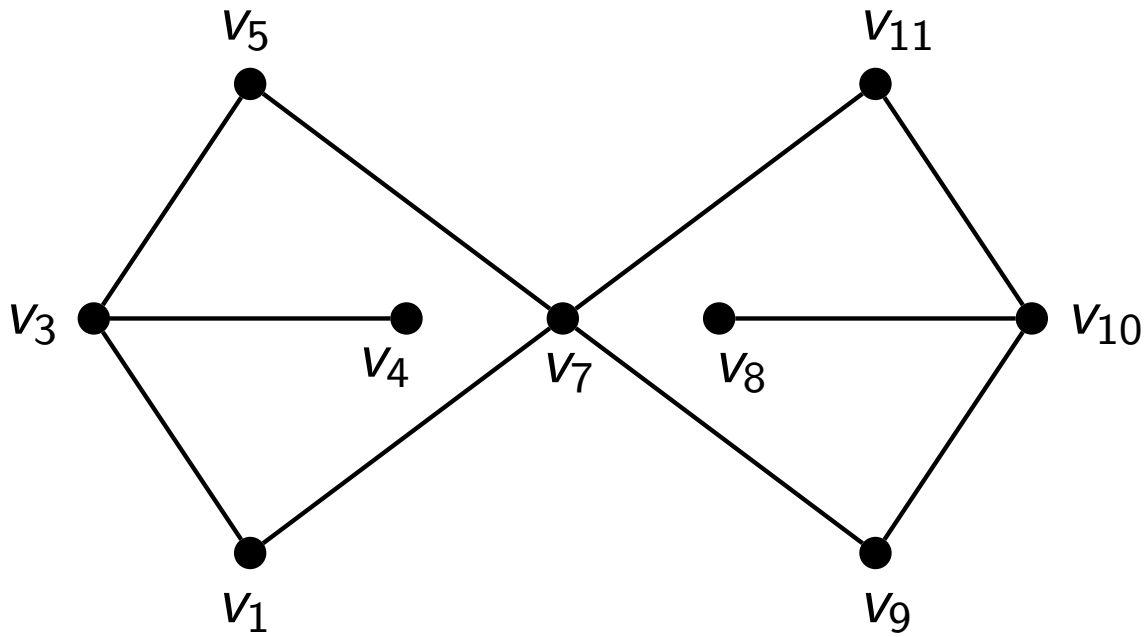
$G - v_2$



Dobivanje grafa $G - \{v_2, v_6\}$

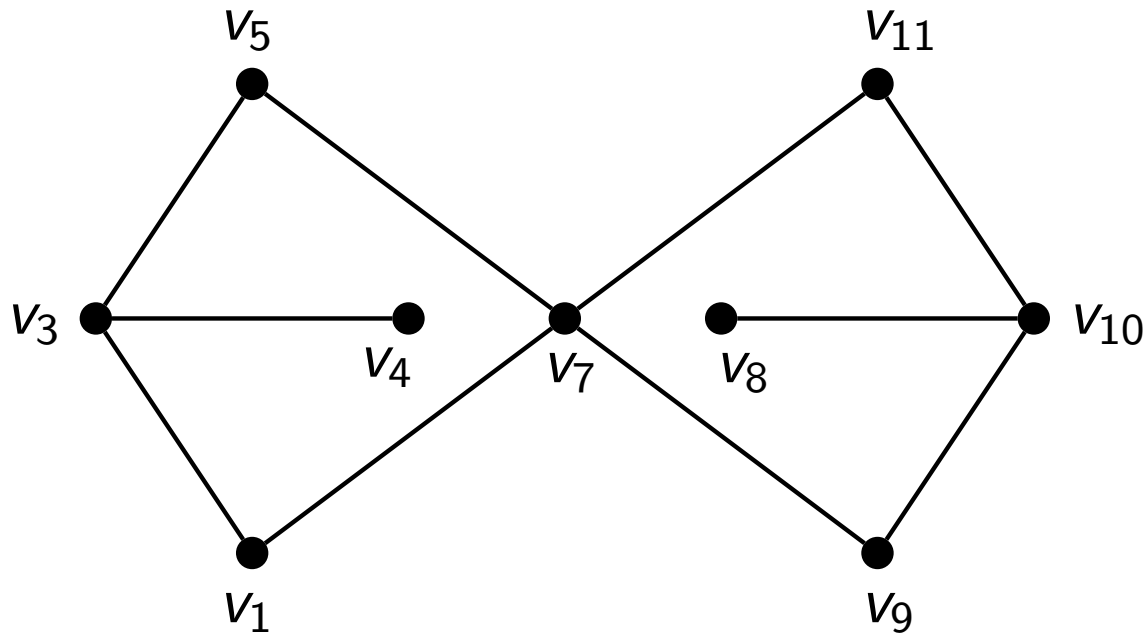
c)

$G - \{v_2, v_6\}$



c)

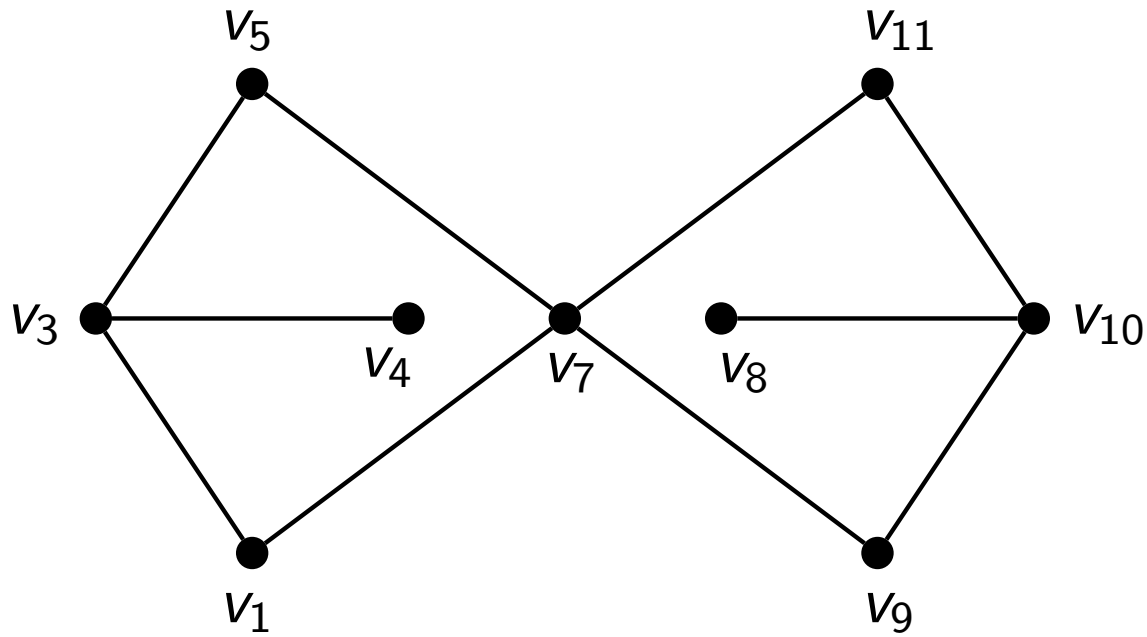
$G - \{v_2, v_6\}$



- Rezni vrhovi u grafu $G - \{v_2, v_6\}$

c)

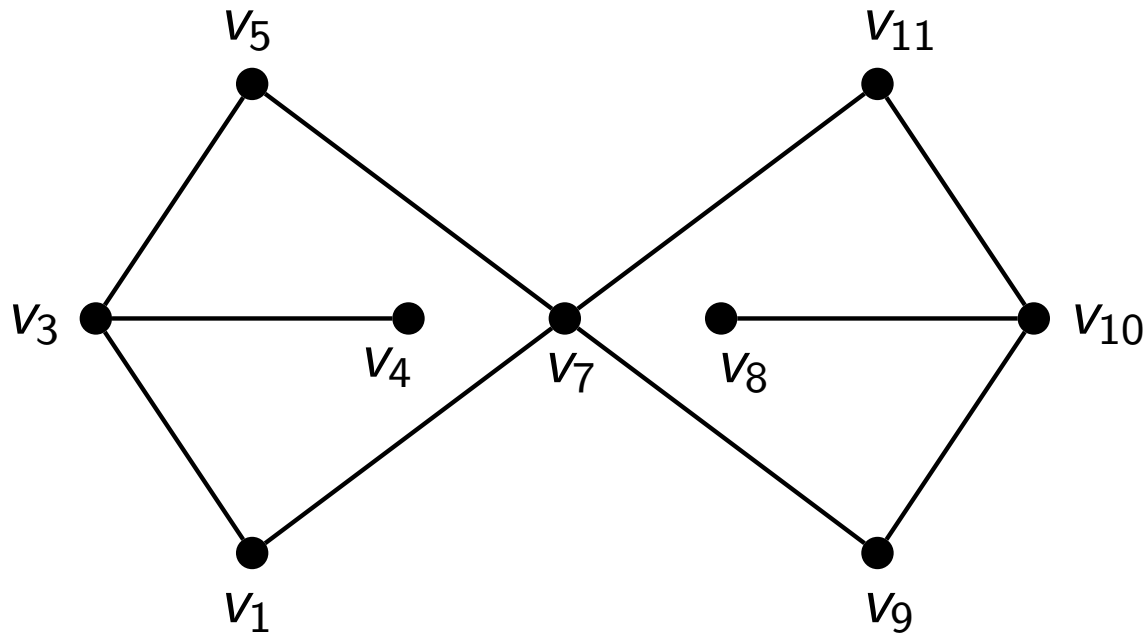
$G - \{v_2, v_6\}$



- Rezni vrhovi u grafu $G - \{v_2, v_6\}$ v_3, v_7, v_{10}

c)

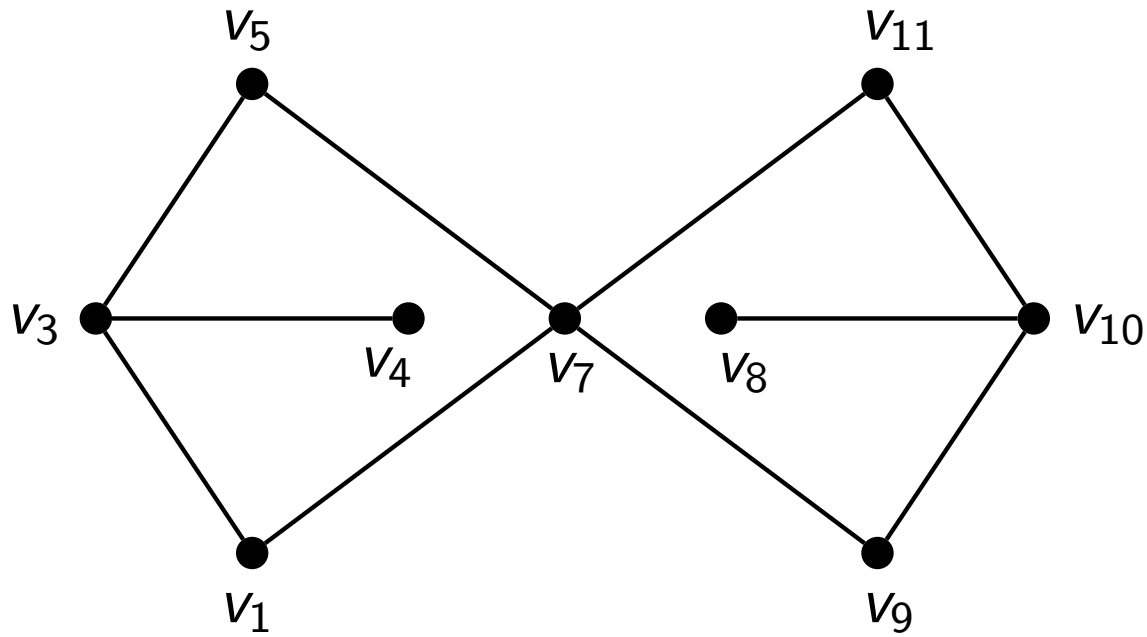
$G - \{v_2, v_6\}$



- Rezni vrhovi u grafu $G - \{v_2, v_6\}$ v_3, v_7, v_{10}
- Rezni bridovi u grafu $G - \{v_2, v_6\}$

c)

$G - \{v_2, v_6\}$



- Rezni vrhovi u grafu $G - \{v_2, v_6\}$ v_3, v_7, v_{10}
- Rezni bridovi u grafu $G - \{v_2, v_6\}$ $\{v_3, v_4\}, \{v_8, v_{10}\}$

šesti zadatak

Teorem (karakterizacija Eulerovih grafova)

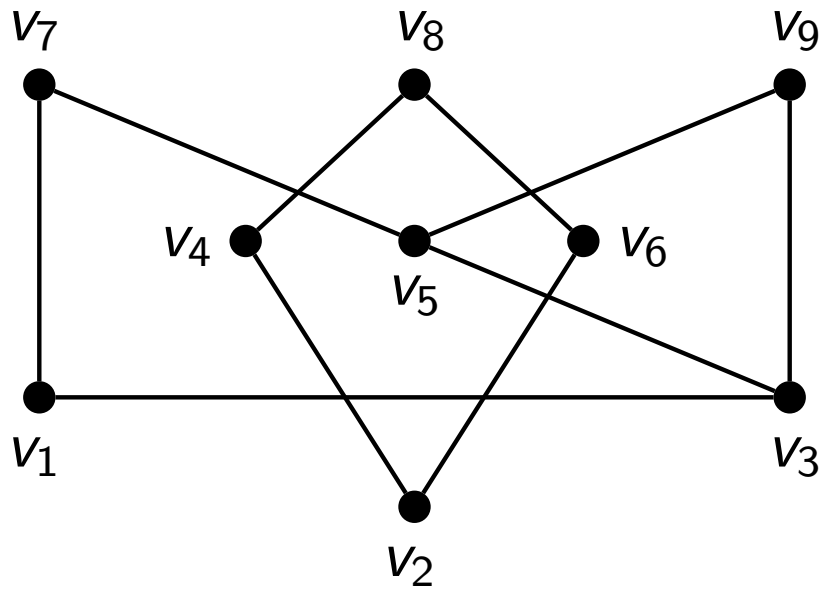
Neprazni povezani graf G je Eulerov graf ako i samo ako su svi vrhovi u grafu G parnog stupnja.

Korolar

Povezani graf G ima Eulerovu stazu ako i samo ako G ima najviše dva vrha neparnog stupnja.

Zadatak 6

Zadan je graf G.



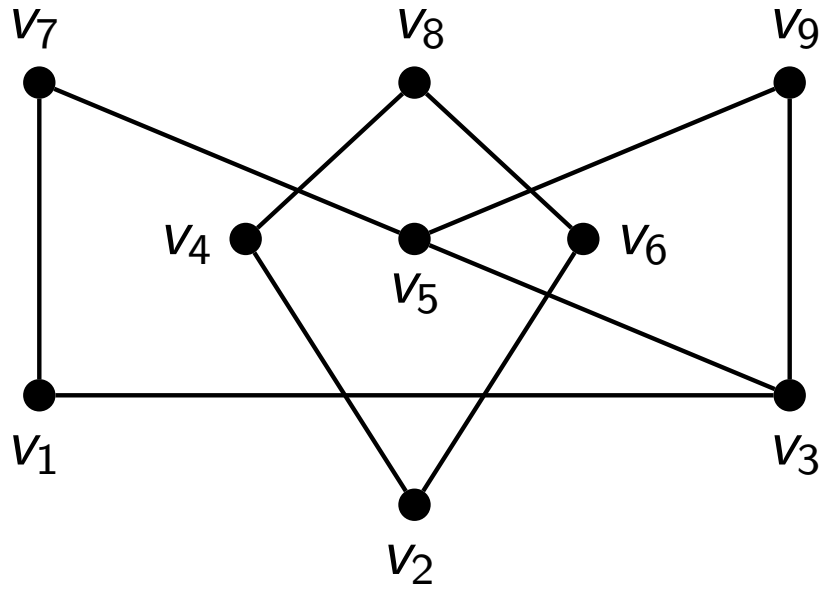
a) *Je li G povezani graf? Obrazložite svoj odgovor.*

b) *Je li G bipartitni graf? Obrazložite svoj odgovor.*

c) *Postoji li u grafu G Eulerova staza? Obrazložite svoj odgovor.*

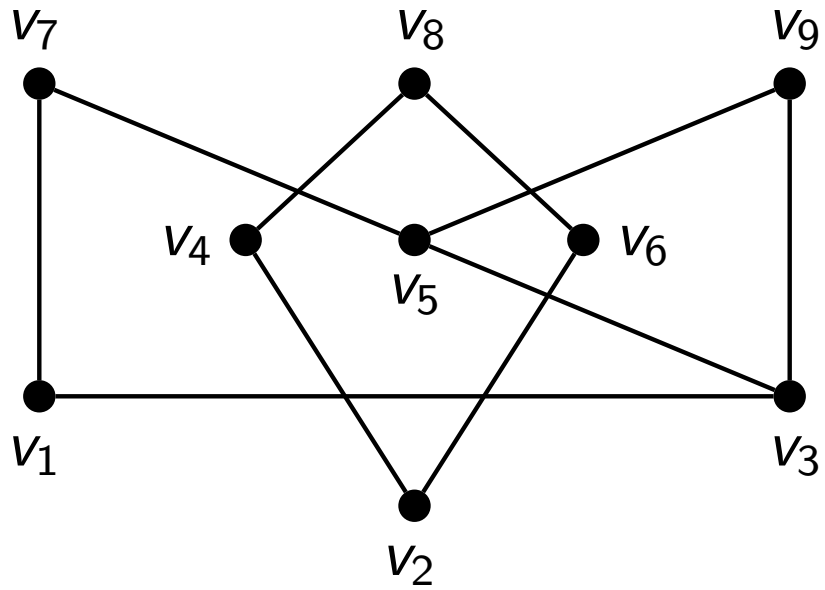
d) *Je li moguće dodavanjem samo jednog brida u graf G dobiti graf koji će imati Eulerovu turu ili Eulerovu stazu? Obrazložite svoj odgovor.*

Rješenje



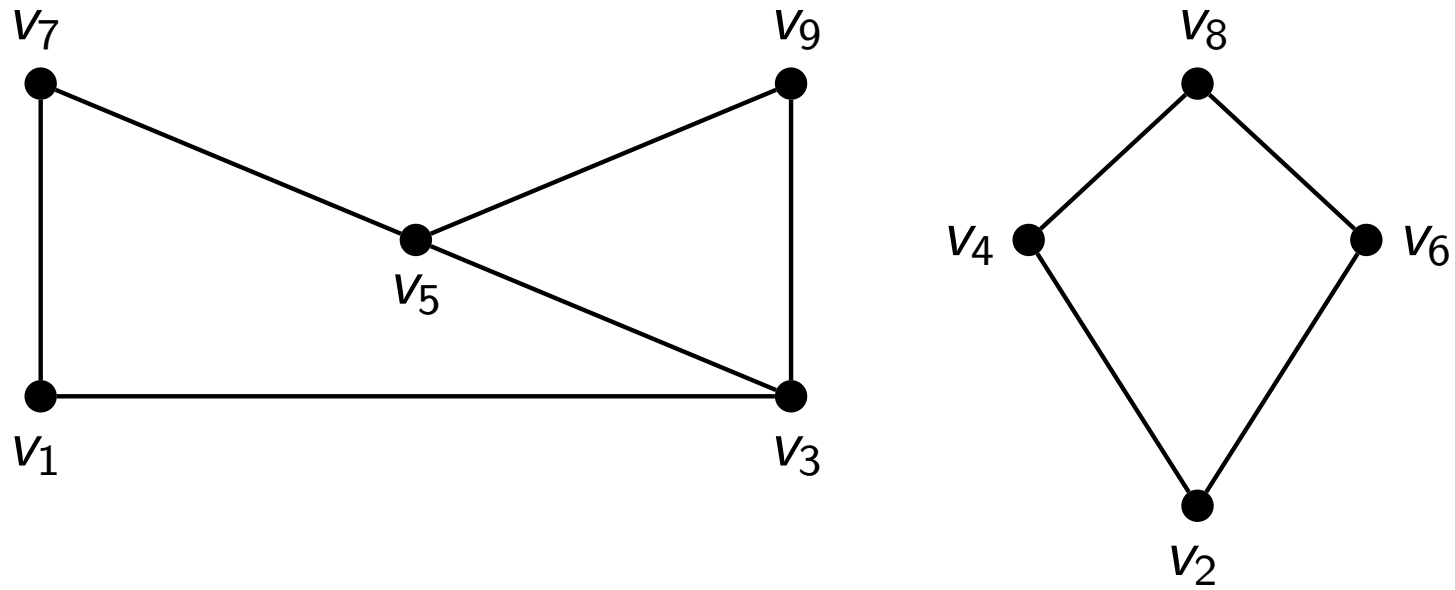
a)

Rješenje



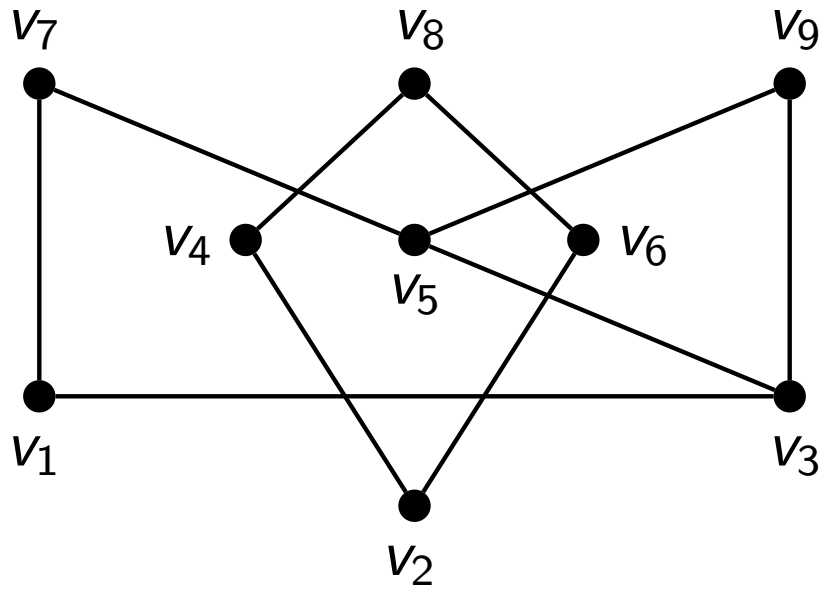
a) G nije povezani graf jer ima dvije komponente povezanosti

Rješenje



a) G nije povezani graf jer ima dvije komponente povezanosti $G[\{v_2, v_4, v_6, v_8\}]$ i $G[\{v_1, v_3, v_5, v_7, v_9\}]$.

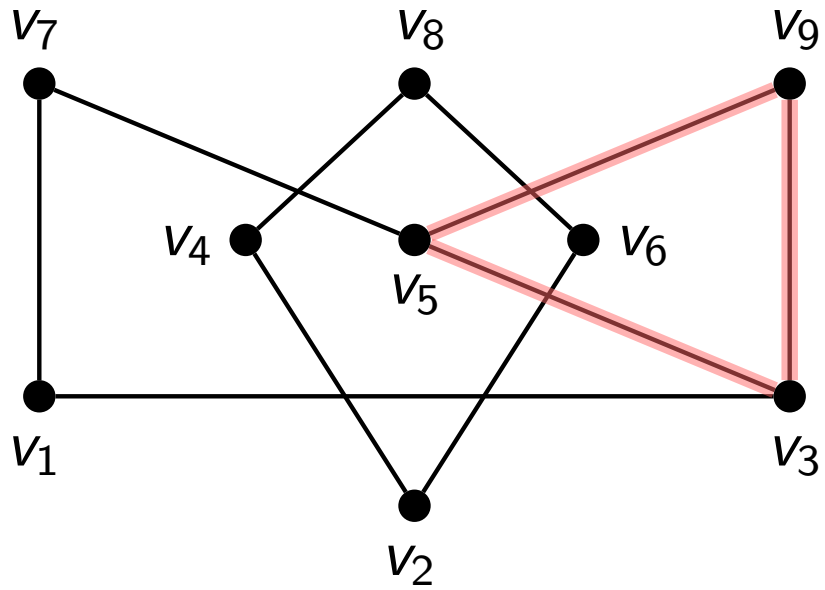
Rješenje



a) G nije povezani graf jer ima dvije komponente povezanosti $G[\{v_2, v_4, v_6, v_8\}]$ i $G[\{v_1, v_3, v_5, v_7, v_9\}]$.

b)

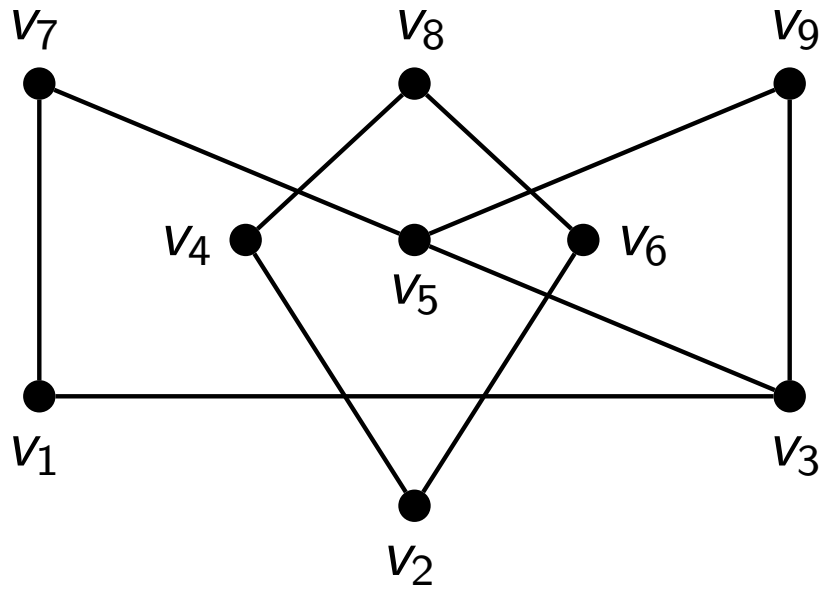
Rješenje



a) G nije povezani graf jer ima dvije komponente povezanosti $G[\{v_2, v_4, v_6, v_8\}]$ i $G[\{v_1, v_3, v_5, v_7, v_9\}]$.

b) G nije bipartitni graf jer sadrži cikluse neparnih duljina, npr. ciklus $v_3 v_5 v_9 v_3$.

Rješenje

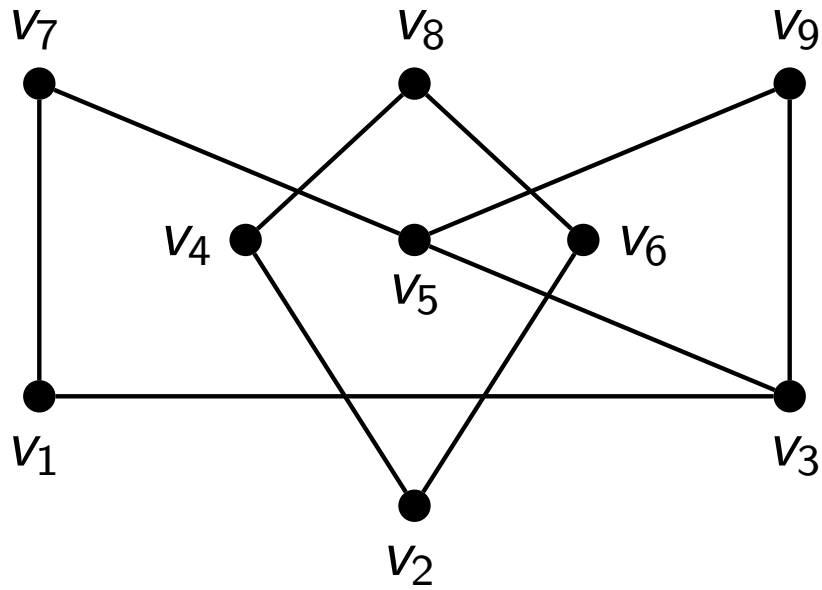


a) G nije povezani graf jer ima dvije komponente povezanosti $G[\{v_2, v_4, v_6, v_8\}]$ i $G[\{v_1, v_3, v_5, v_7, v_9\}]$.

b) G nije bipartitni graf jer sadrži cikluse neparnih duljina, npr. ciklus $v_3 v_5 v_9 v_3$.

c)

Rješenje



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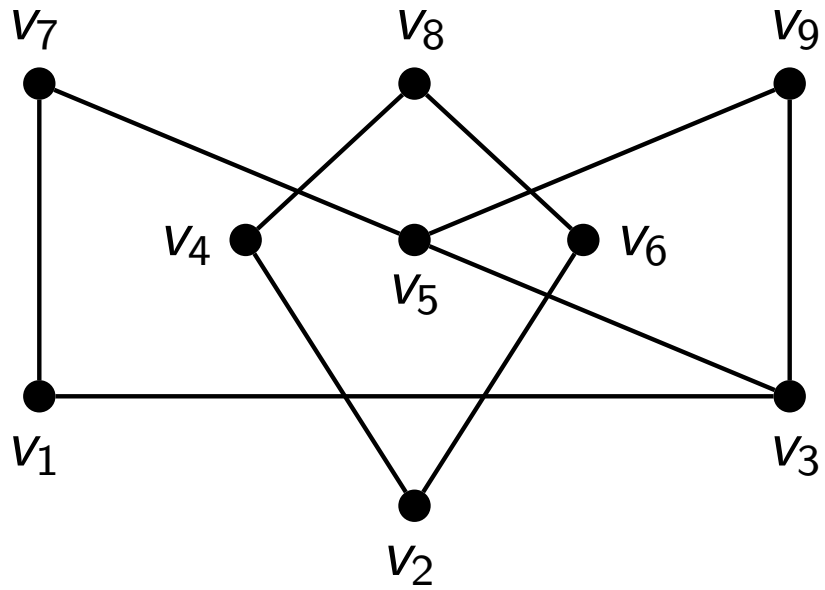
b) G nije bipartitni graf jer sadrži cikluse neparnih duljina, npr. ciklus $v_3 v_5 v_9 v_3$.

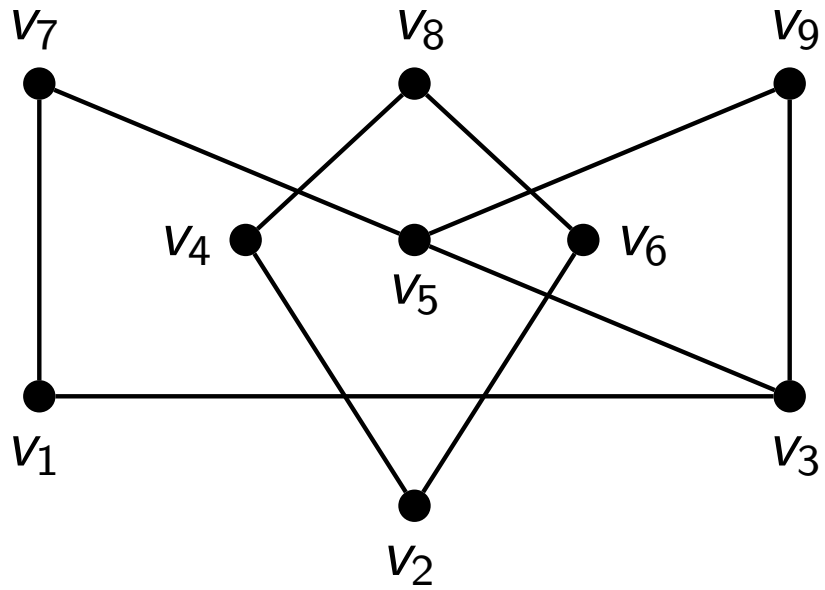
c) Graf G ima točno dva vrha neparnog stupnja v_3 i v_5, ali ipak u grafu G ne postoji Eulerova staza jer G nije povezani graf.

Rješenje



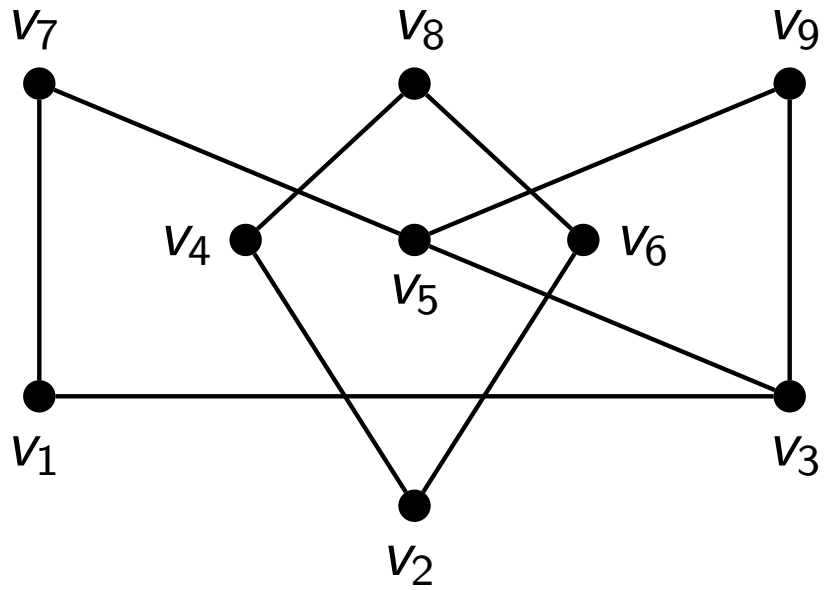
d)

Rješenje



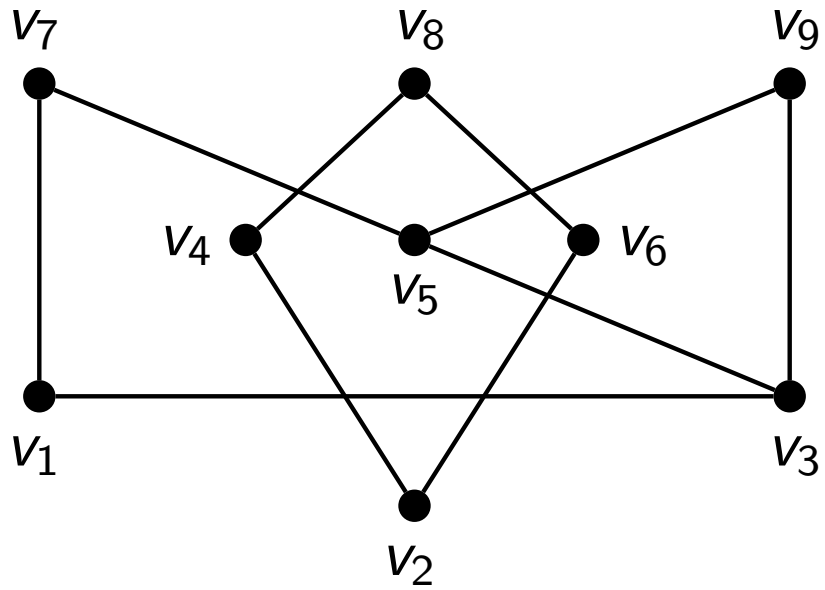
d) Kako graf G ima dvije komponente povezanosti, treba dodati brid koji će spojiti te dvije komponente povezanosti tako da dobijemo povezani graf.

Rješenje



d) Kako graf G ima dvije komponente povezanosti, treba dodati brid koji će spojiti te dvije komponente povezanosti tako da dobijemo povezani graf. Kako su u $G[\{v_2, v_4, v_6, v_8\}]$ svi vrhovi parnog stupnja, dodavanjem spomenutog brida u novom grafu neće svi vrhovi biti parnog stupnja.

Rješenje



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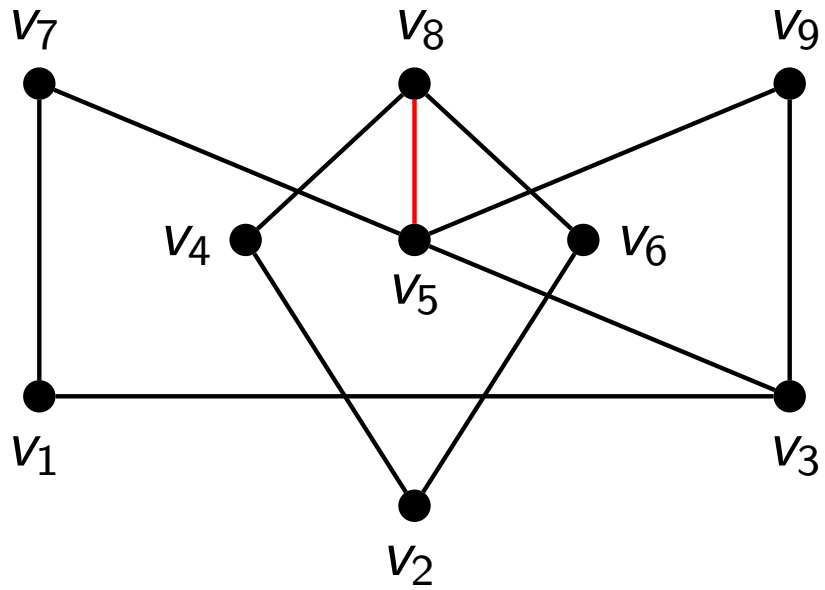
Rješenje



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sedmi zadatak

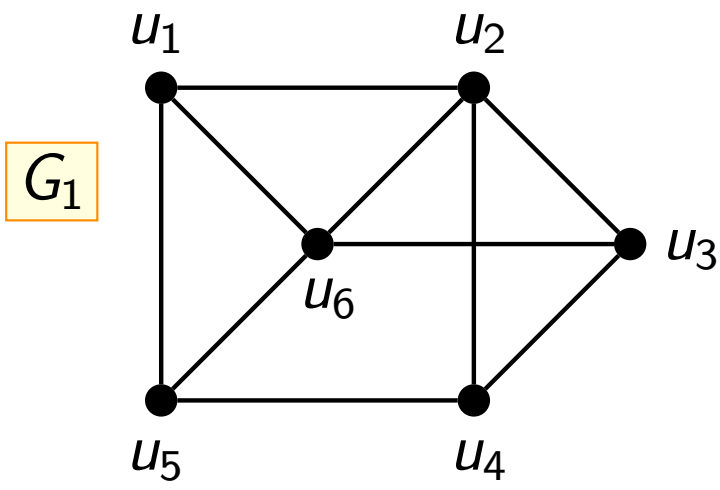
Teorem (Dirac)

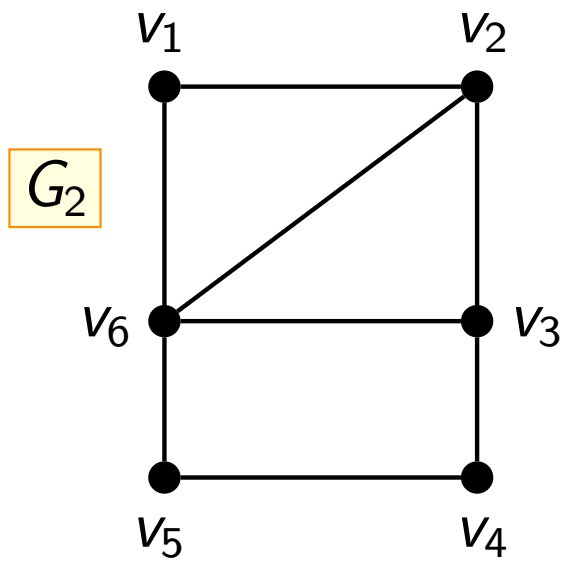
Neka je G jednostavni graf u kojemu je broj vrhova $\nu(G) \geqslant 3$ i $\delta(G) \geqslant \frac{\nu}{2}$. Tada je G Hamiltonov graf.

- Diracov teorem daje dovoljan uvjet na temelju kojeg se može zaključiti da je jednostavni graf Hamiltonov ako zadovoljava taj uvjet.
- Međutim, obrat Diracovog teorema ne vrijedi.
- Drugim riječima, uvjet iz Diracovog teorema nije ujedno i nužan uvjet. Postoje Hamiltonovi grafovi koji ne zadovoljavaju uvjet iz Diracovog teorema.

Zadatak 7

Zadani su grafovi G_1 i G_2.



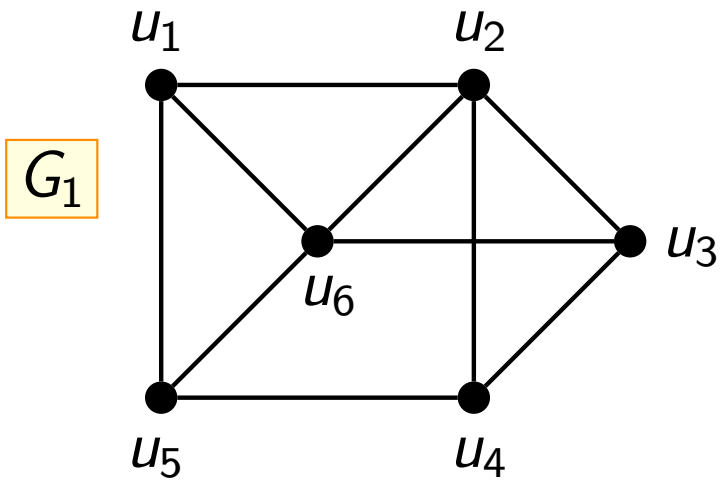


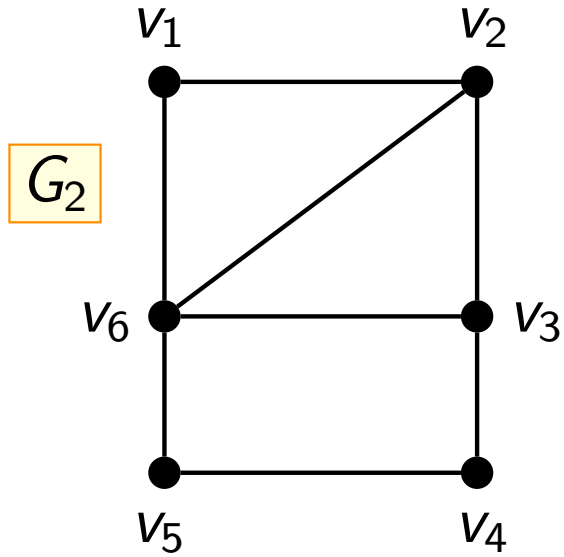
a) *Može li se neki od grafova G_1 i G_2 nacrtati bez podizanja olovke s papira tako da se ne prolazi po već nacrtanim bridovima? Obrazložite svoj odgovor.*

b) *Jesu li G_1 i G_2 Hamiltonovi grafovi? Obrazložite svoj odgovor.*

c) *Možemo li pomoću Diracovog teorema zaključiti je li neki od zadanih grafova Hamiltonov graf? Obrazložite svoj odgovor.*

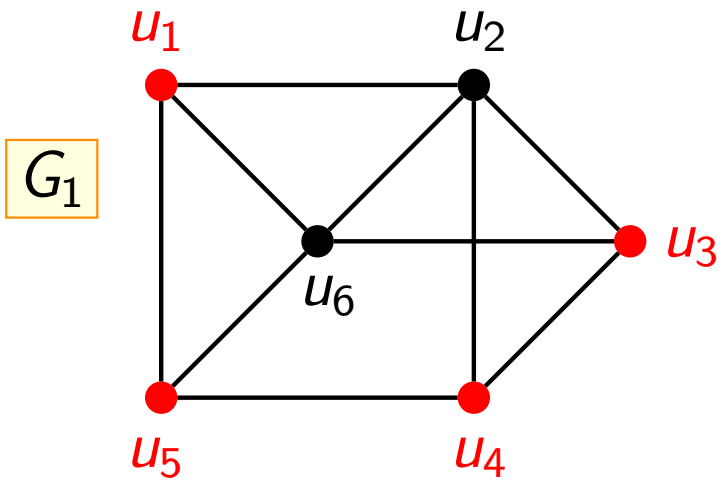
Rješenje

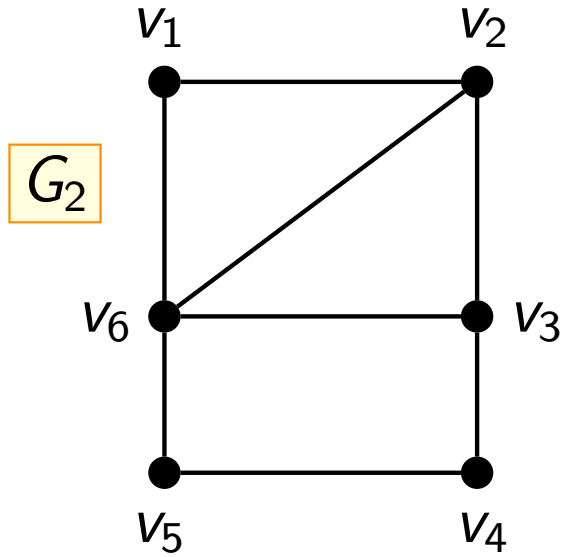




a)

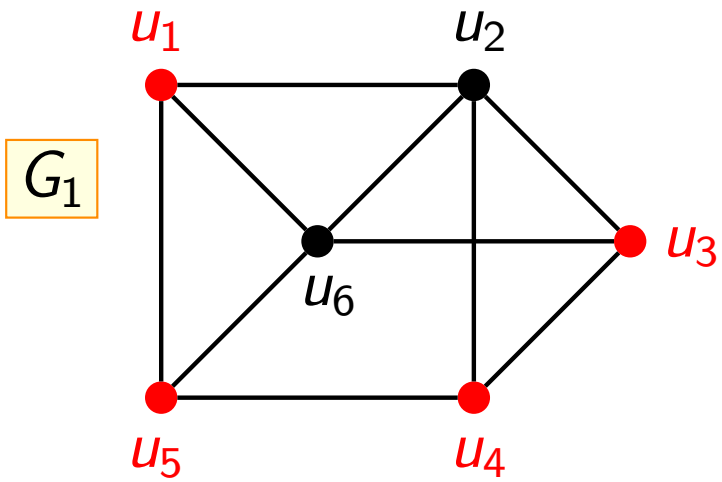
Rješenje

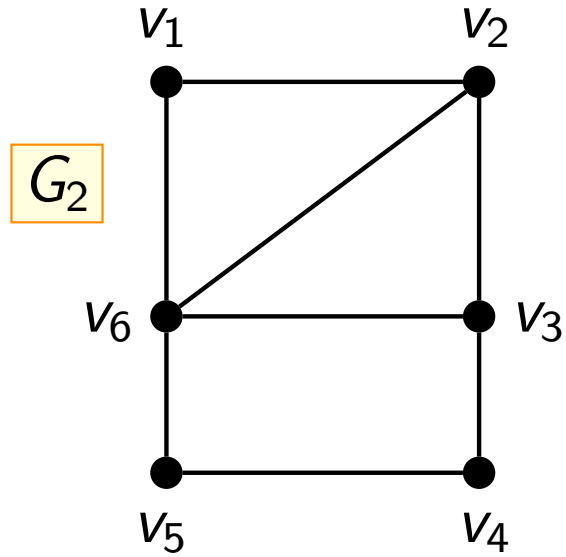




a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu.

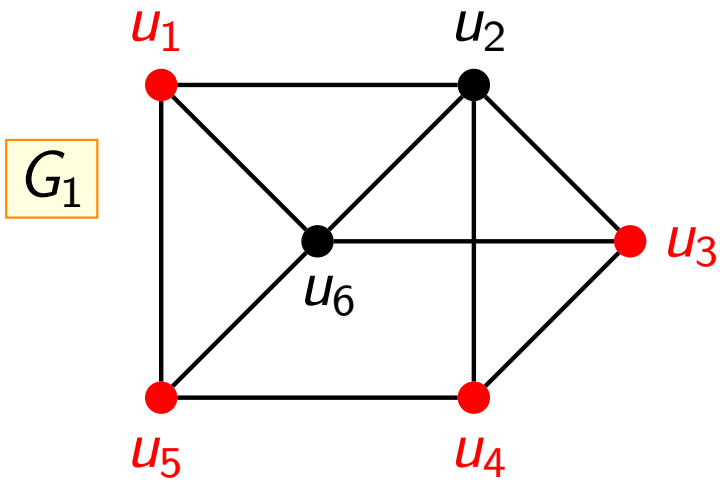
Rješenje

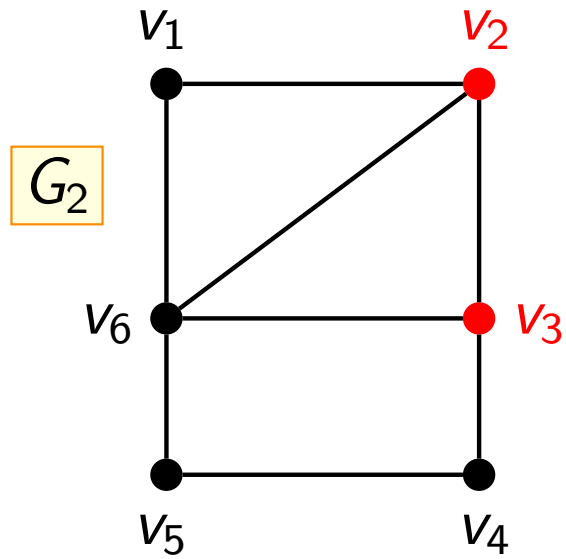




a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Rješenje

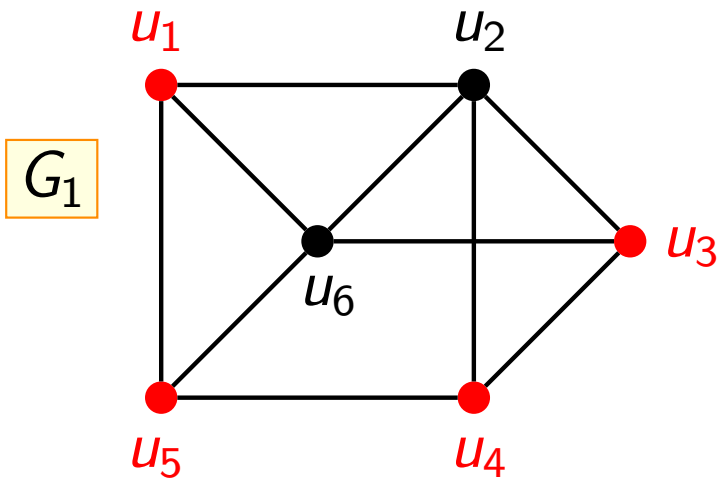


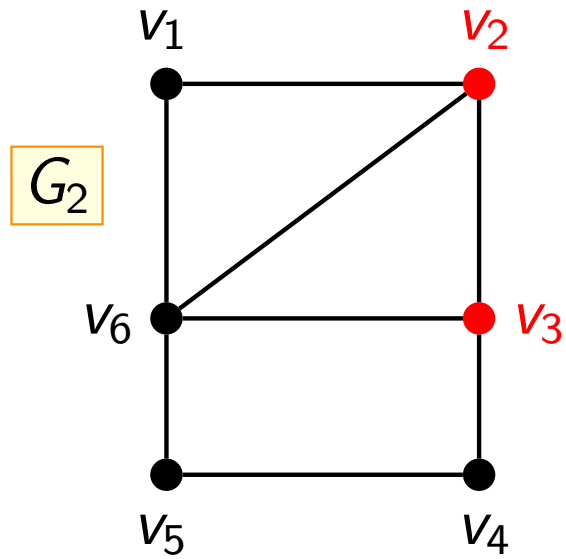


a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu.

Rješenje





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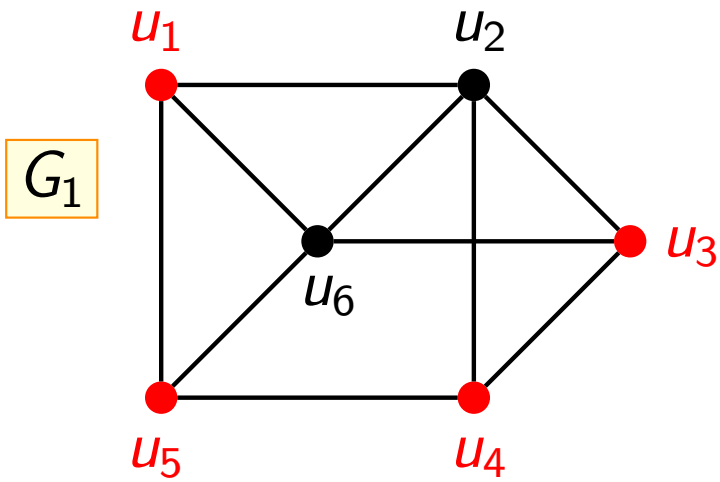
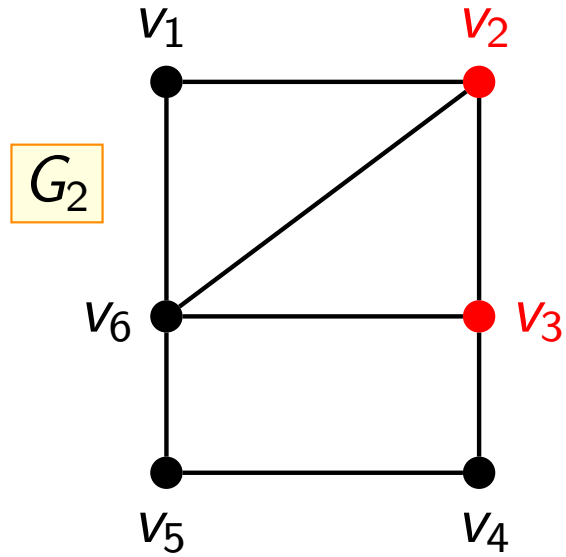
Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

Rješenje





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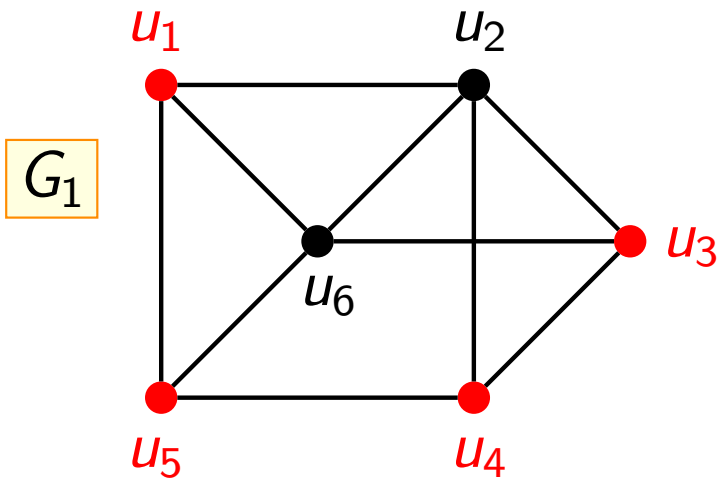
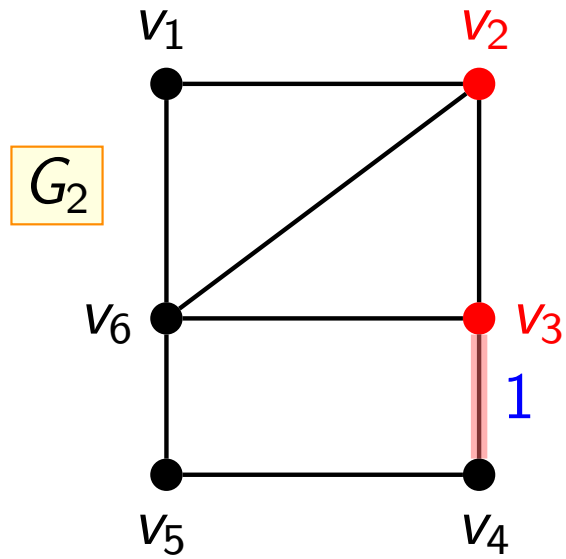
Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

v_3

Rješenje



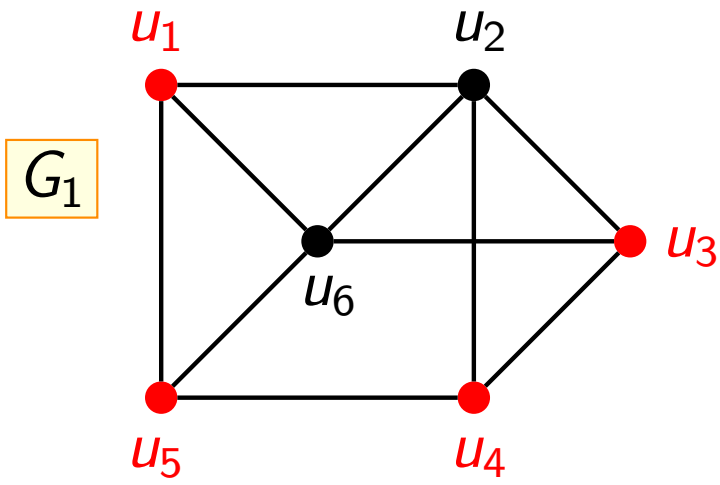


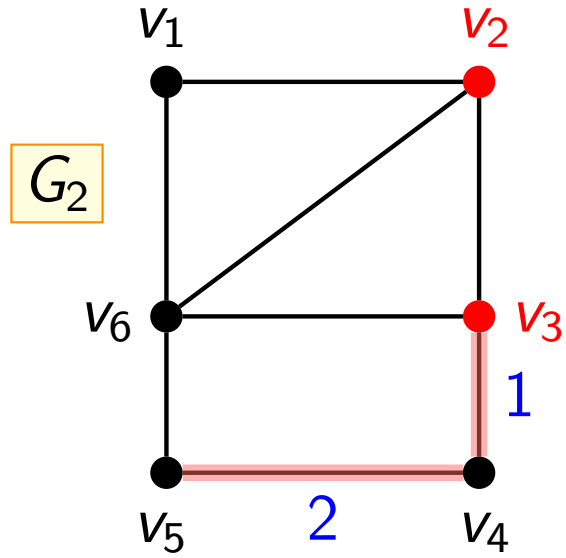
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4$$

Rješenje



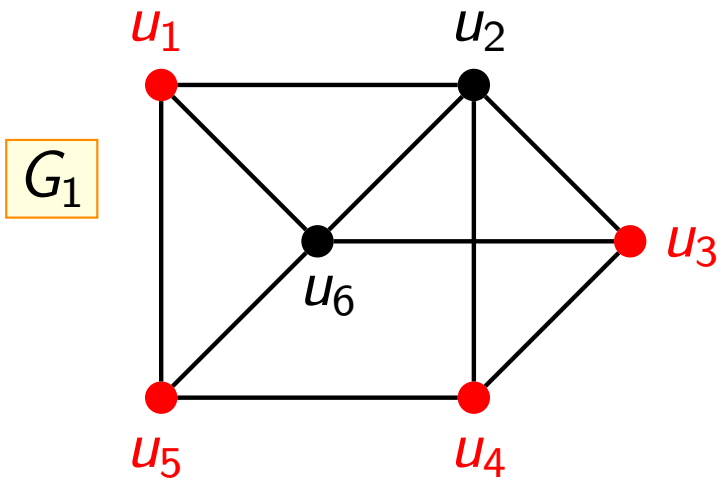


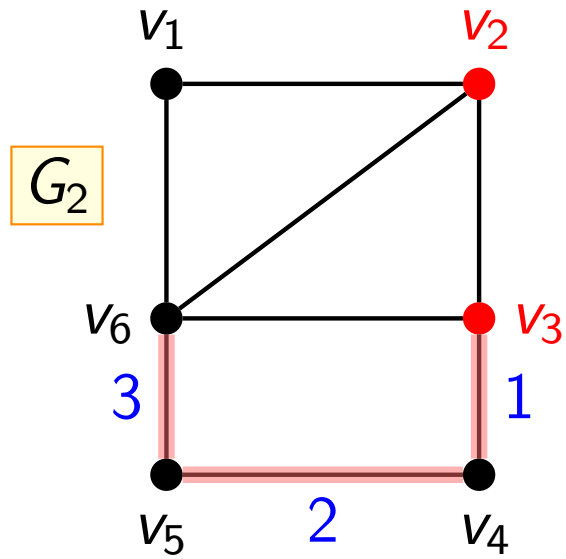
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4\, v_5$$

Rješenje



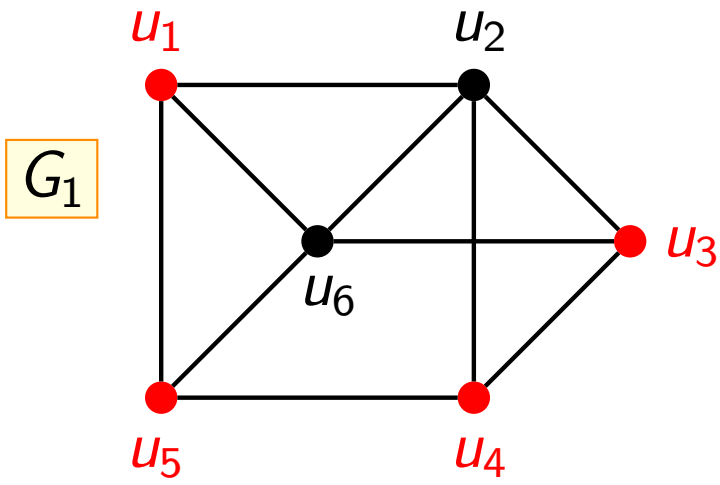


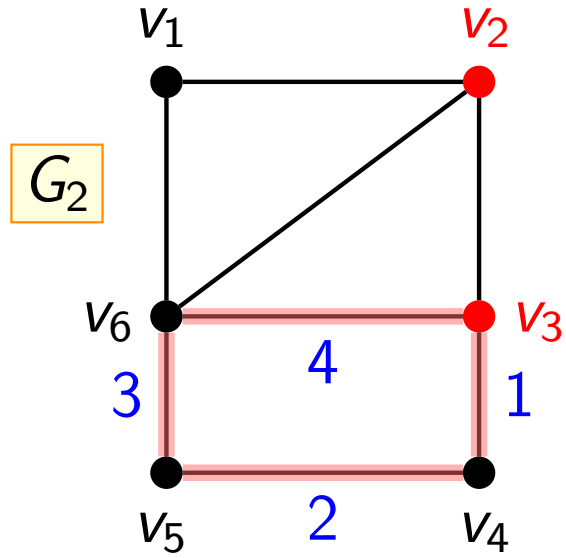
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4\, v_5\, v_6$$

Rješenje



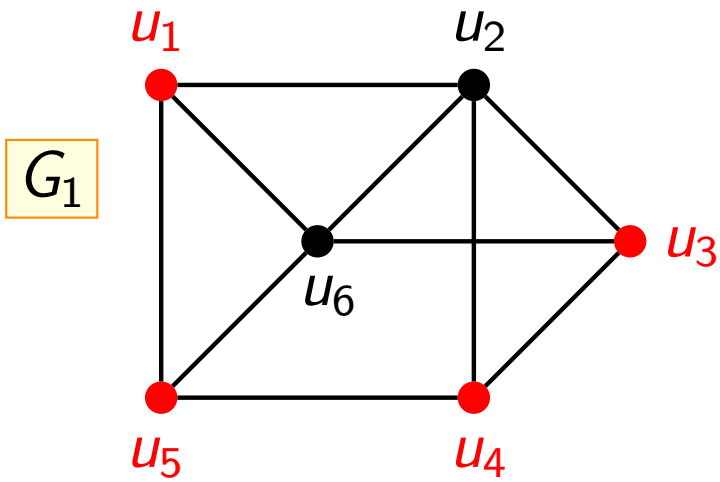


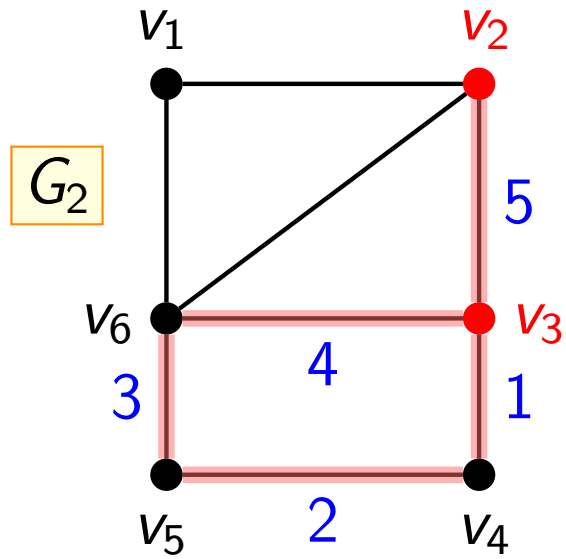
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4\, v_5\, v_6\, v_3$$

Rješenje



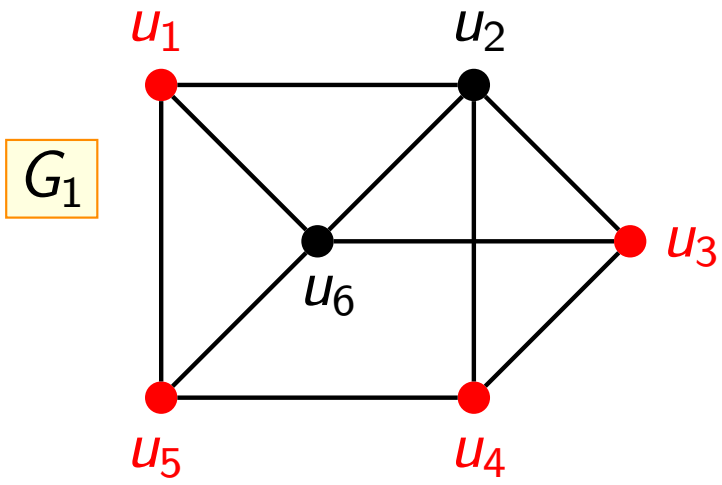


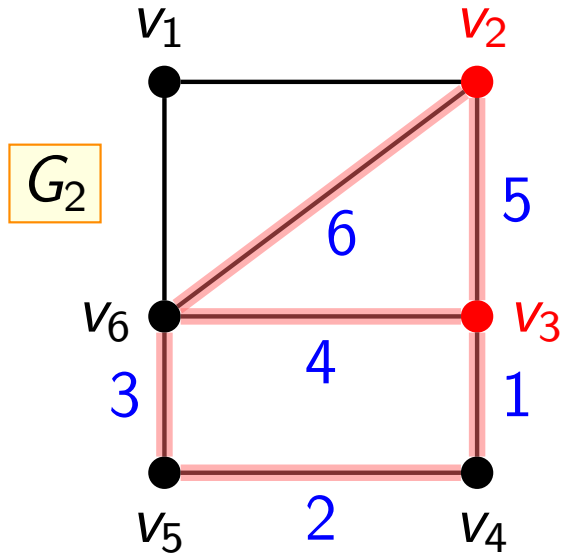
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4\, v_5\, v_6\, v_3\, v_2$$

Rješenje



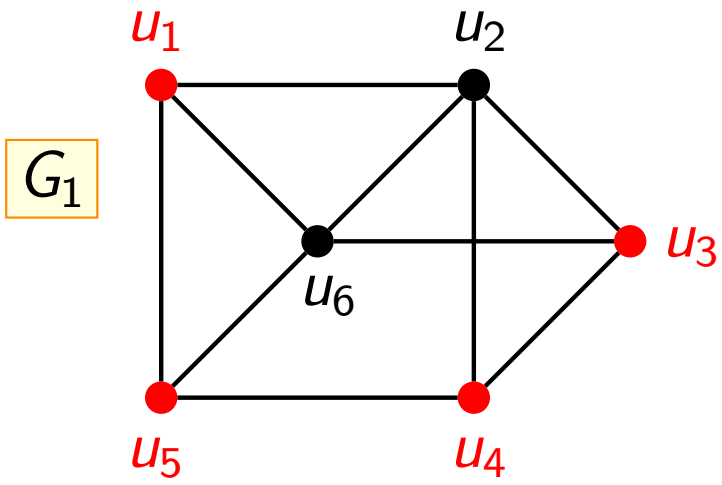


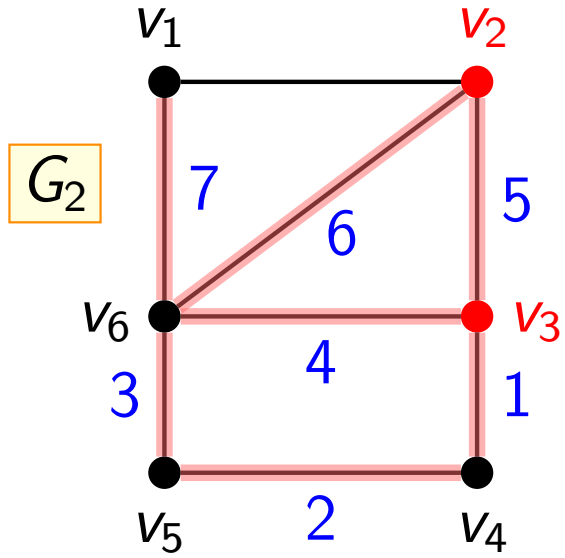
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4\, v_5\, v_6\, v_3\, v_2\, v_6$$

Rješenje



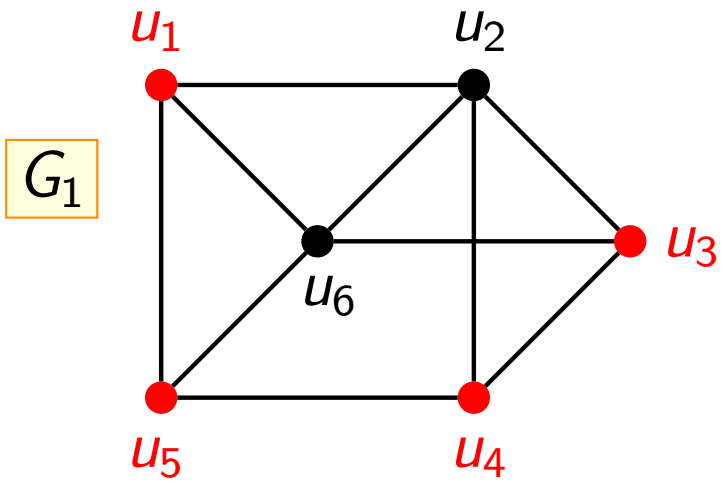


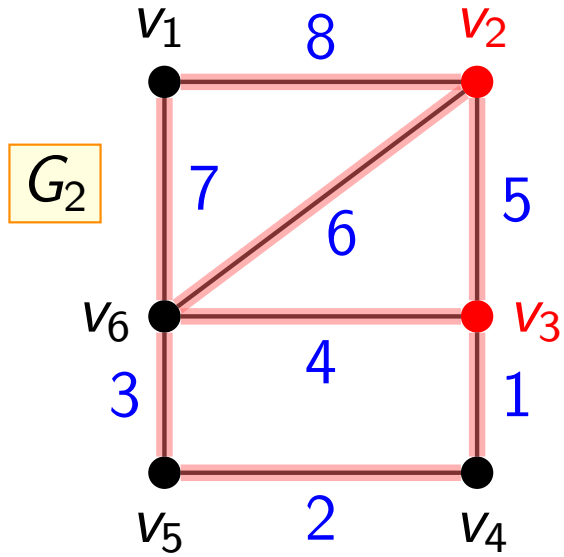
a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

$$v_3\, v_4\, v_5\, v_6\, v_3\, v_2\, v_6\, v_1$$

Rješenje

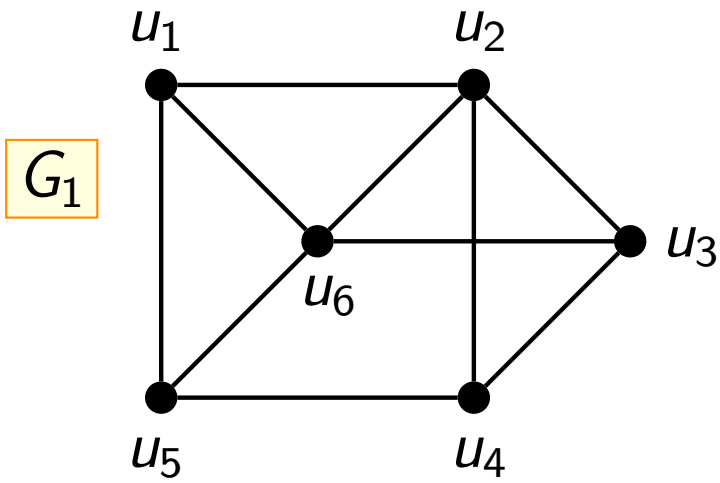


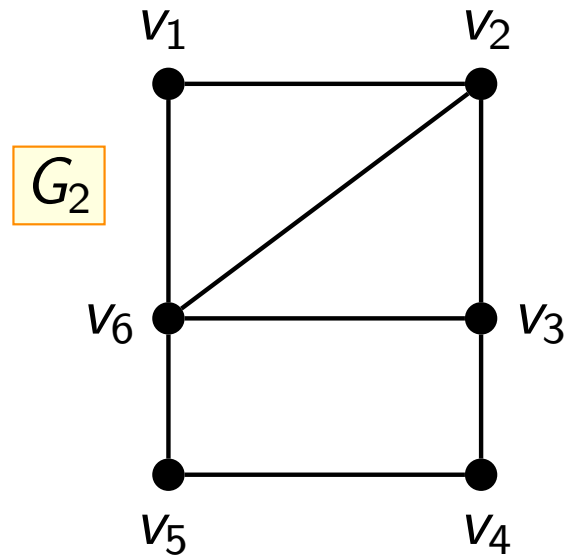


a) Graf G_1 ima 4 vrha neparnog stupnja pa nema Eulerovu stazu. Stoga se graf G_1 ne može nacrtati bez podizanja olovke s papira.

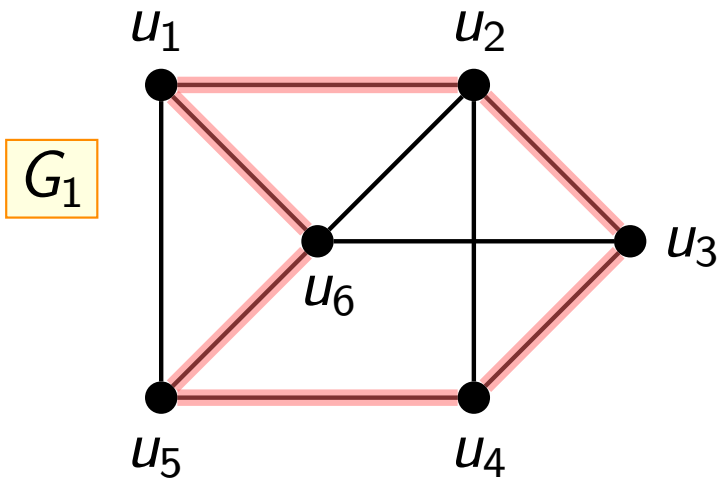
Graf G_2 ima točno dva vrha neparnog stupnja pa ima Eulerovu stazu. Stoga se graf G_2 može nacrtati bez podizanja olovke s papira.

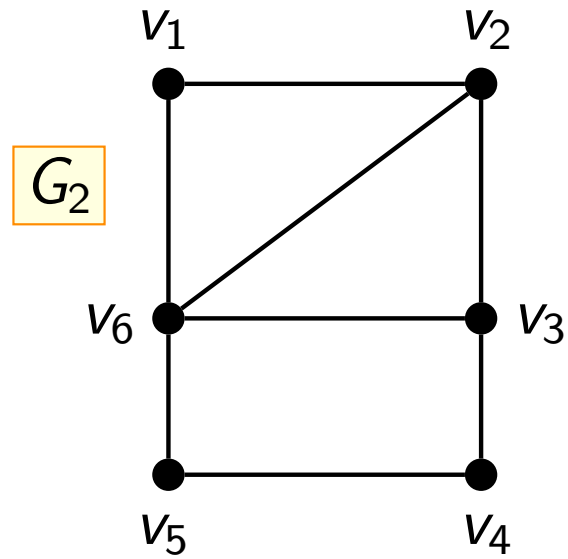
$$v_3\, v_4\, v_5\, v_6\, v_3\, v_2\, v_6\, v_1\, v_2$$



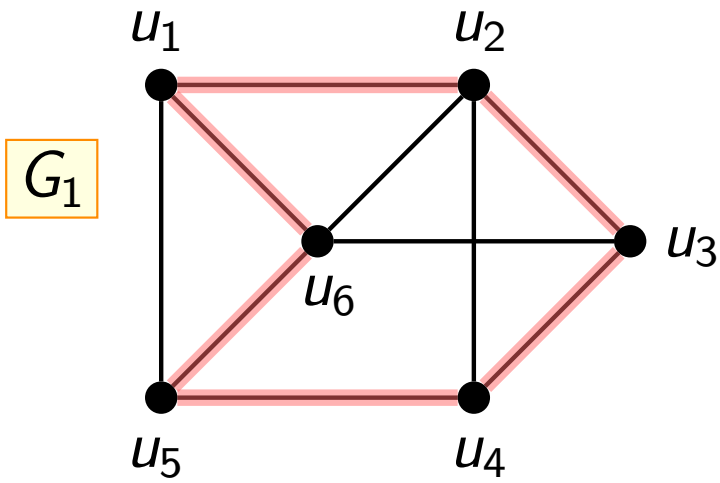


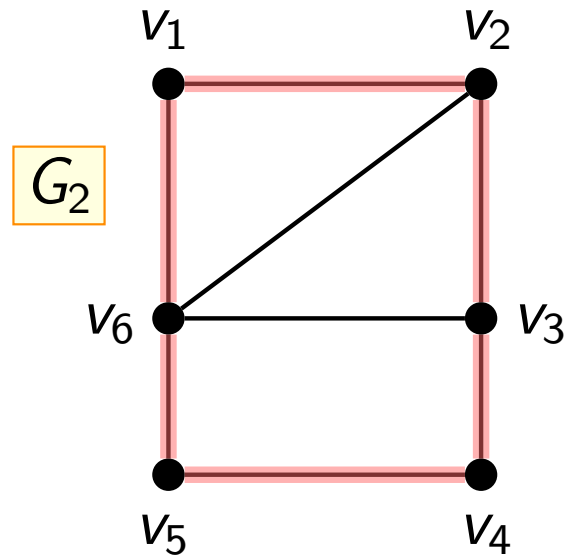
b)





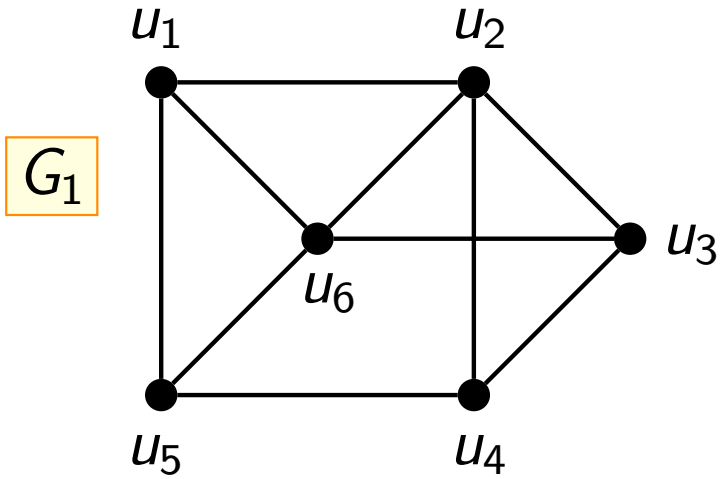
b) Graf G_1 je Hamiltonov graf jer je $u_5u_6u_1u_2u_3u_4u_5$ jedan ciklus u G_1 koji sadrži sve vrhove od G_1.

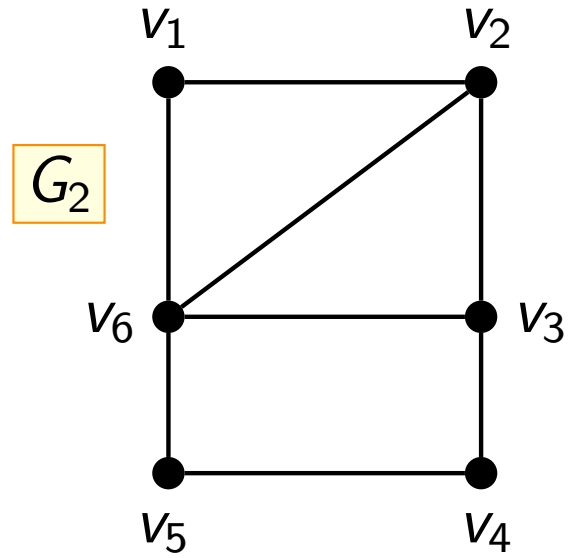




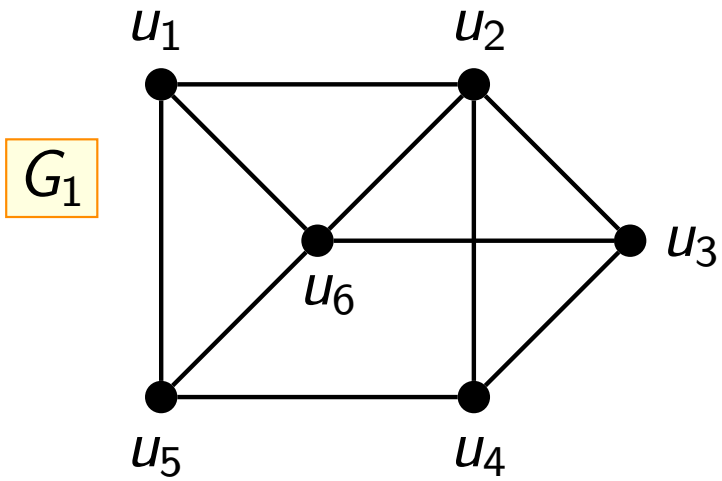
b) Graf G_1 je Hamiltonov graf jer je $u_5u_6u_1u_2u_3u_4u_5$ jedan ciklus u G_1 koji sadrži sve vrhove od G_1.

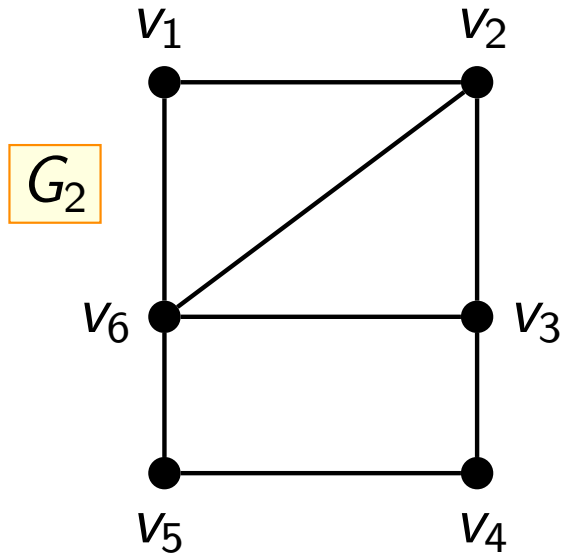
Graf G_2 je Hamiltonov graf jer je $v_6v_1v_2v_3v_4v_5v_6$ jedan ciklus u G_2 koji sadrži sve vrhove od G_2.



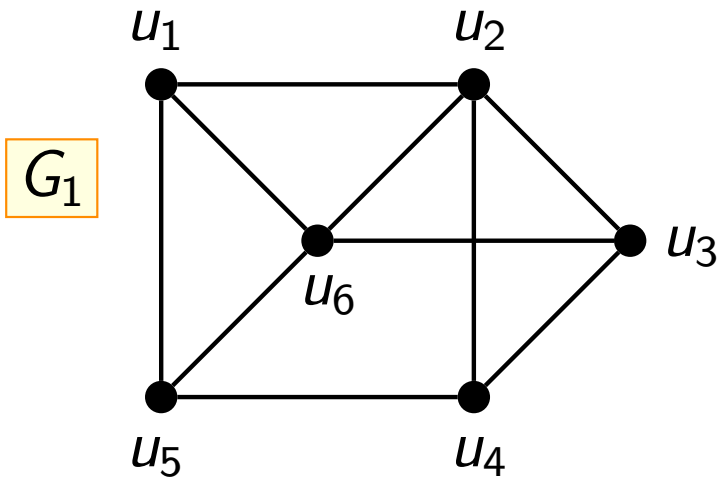


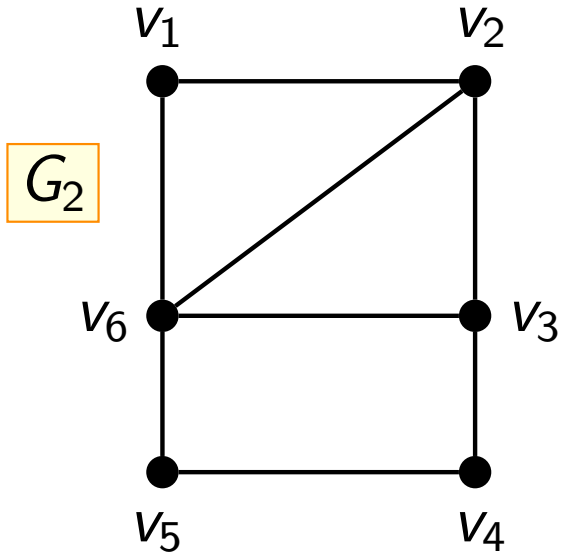
c)





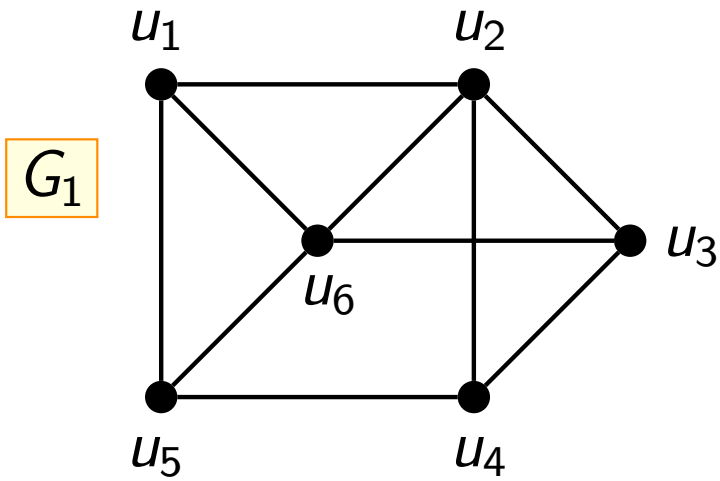
c) $\nu(G_1) = 6,\ \delta(G_1) = 3$

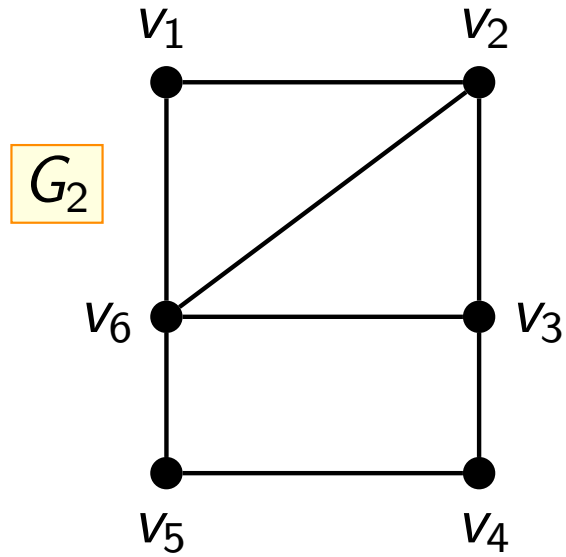




c) $\nu(G_1) = 6,\ \delta(G_1) = 3$

G_1 je jednostavni graf s barem tri vrha i vrijedi $\delta(G_1) \geqslant \dfrac{\nu(G_1)}{2}$.





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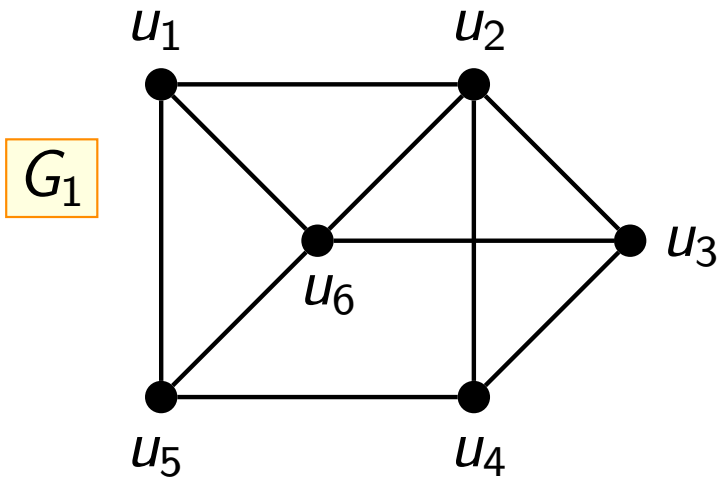
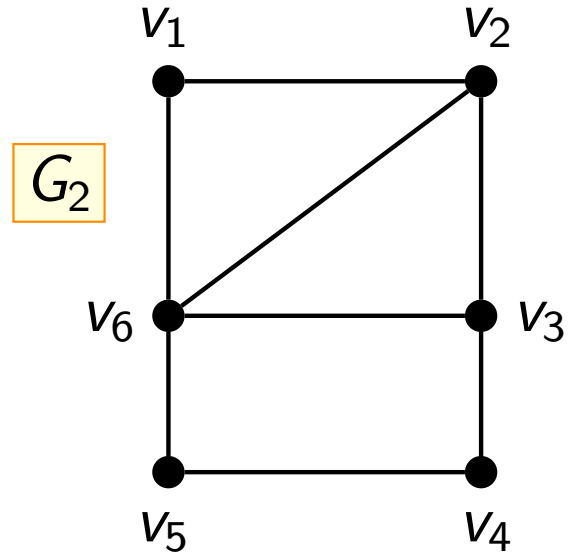
Stoga na temelju Diracovog teorema možemo zaključiti da je G_1 Hamiltonov graf.



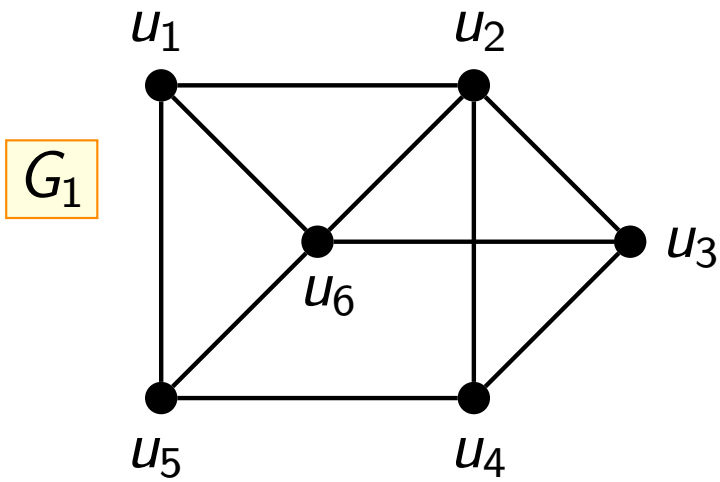


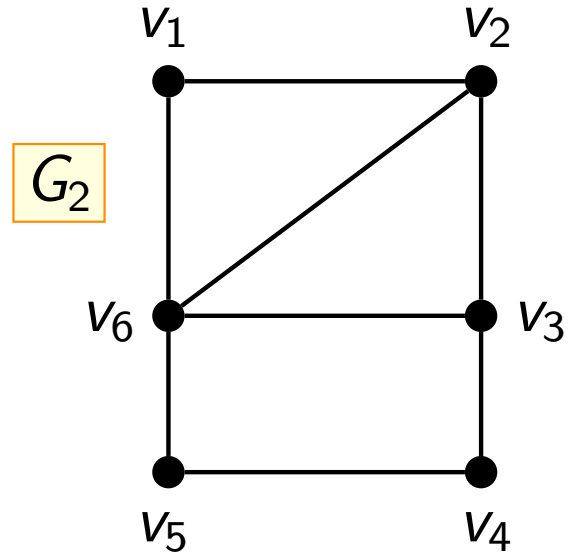
c) $\nu(G_1) = 6,\ \delta(G_1) = 3$

G_1 je jednostavni graf s barem tri vrha i vrijedi $\delta(G_1) \geqslant \frac{\nu(G_1)}{2}$.

Stoga na temelju Diracovog teorema možemo zaključiti da je G_1 Hamiltonov graf.

$\nu(G_2) = 6,\ \delta(G_2) = 2$





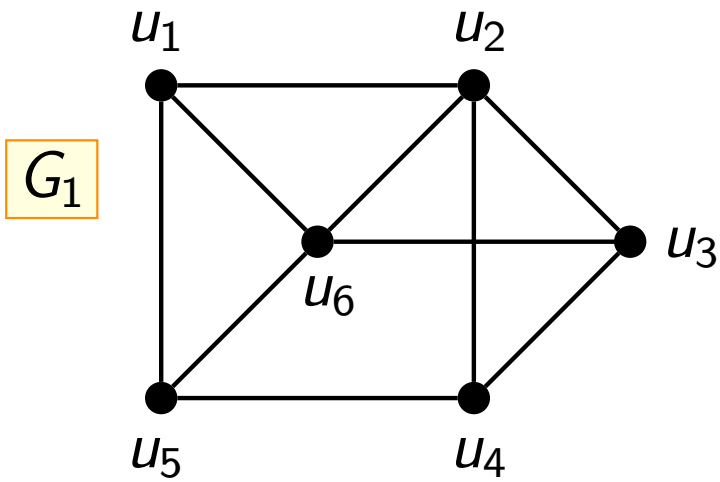
c) $\nu(G_1) = 6,\ \delta(G_1) = 3$

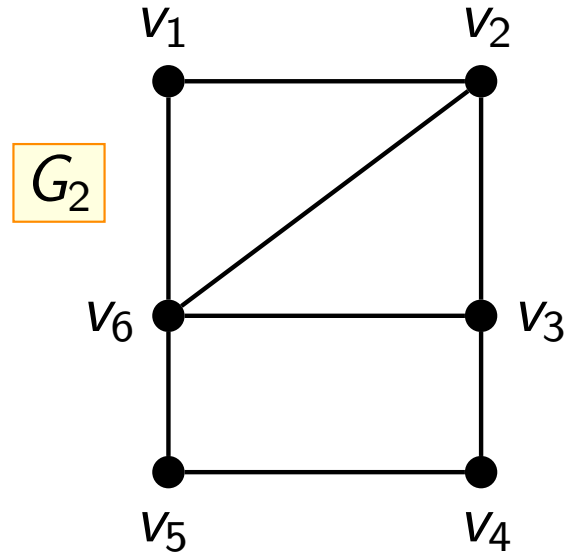
G_1 je jednostavni graf s barem tri vrha i vrijedi $\delta(G_1) \geqslant \frac{\nu(G_1)}{2}$.

Stoga na temelju Diracovog teorema možemo zaključiti da je G_1 Hamiltonov graf.

$\nu(G_2) = 6,\ \delta(G_2) = 2$

G_2 je jednostavni graf s barem tri vrha, no uvjet $\delta(G_2) \geqslant \frac{\nu(G_2)}{2}$ nije zadovoljen.





c) $\nu(G_1) = 6,\ \delta(G_1) = 3$

G_1 je jednostavni graf s barem tri vrha i vrijedi $\delta(G_1) \geqslant \frac{\nu(G_1)}{2}$.

Stoga na temelju Diracovog teorema možemo zaključiti da je G_1 Hamiltonov graf.

$\nu(G_2) = 6,\ \delta(G_2) = 2$

G_2 je jednostavni graf s barem tri vrha, no uvjet $\delta(G_2) \geqslant \frac{\nu(G_2)}{2}$ nije zadovoljen. Stoga na temelju Diracovog teorema ne možemo zaključiti je li G_2 Hamiltonov graf.

osmi zadatak

Teorem (nužan uvjet za Hamiltonov graf)

Ako je G Hamiltonov graf, tada za svaki $\emptyset \neq S \subset V(G)$ vrijedi $\omega(G - S) \leqslant k(S)$.

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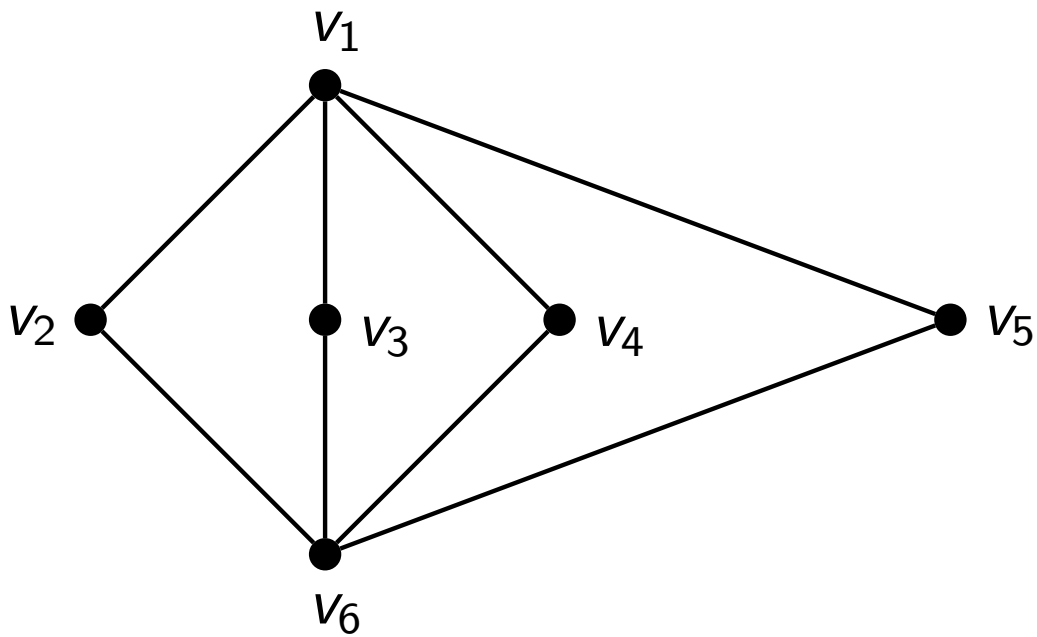
Ako je G Hamiltonov graf, tada za svaki $\emptyset \neq S \subset V(G)$ vrijedi $\omega(G - S) \leqslant k(S)$.

Kontrapozicija

Ako postoji $\emptyset \neq S \subset V(G)$ takav da vrijedi $\omega(G - S) > k(S)$, tada G nije Hamiltonov graf.

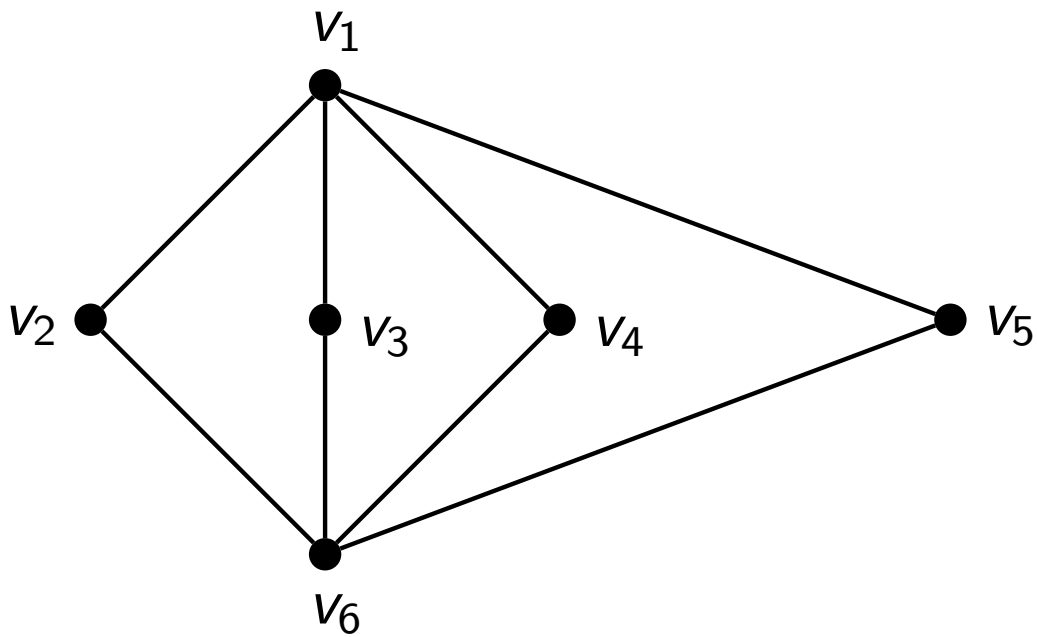
Zadatak 8

Dokažite da graf G nije Hamiltonov graf.



Zadatak 8

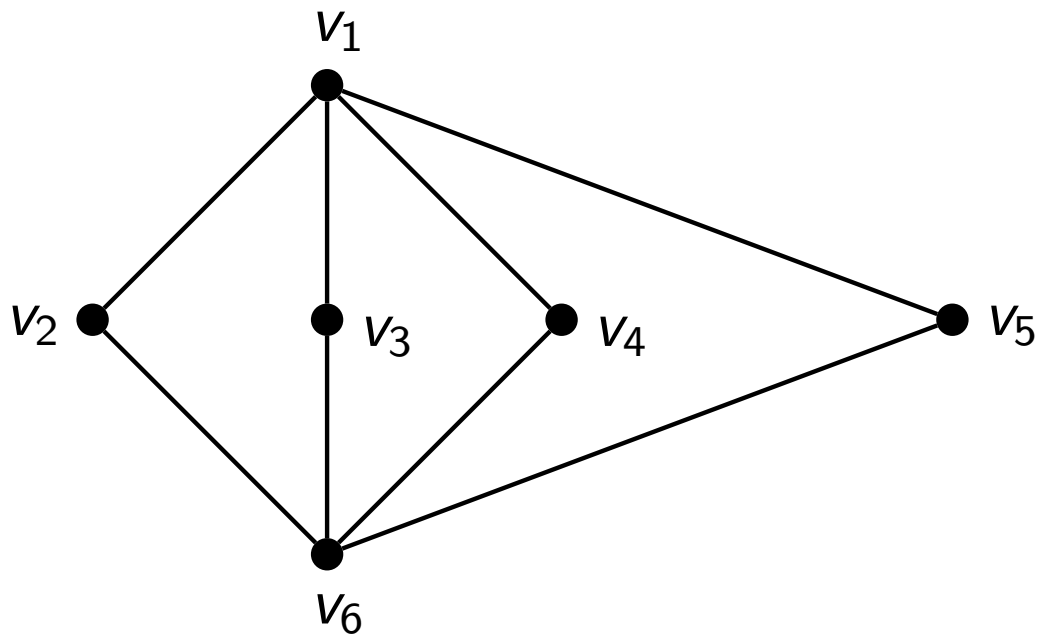
Dokažite da graf G nije Hamiltonov graf.



Rješenje

Zadatak 8

Dokažite da graf G nije Hamiltonov graf.

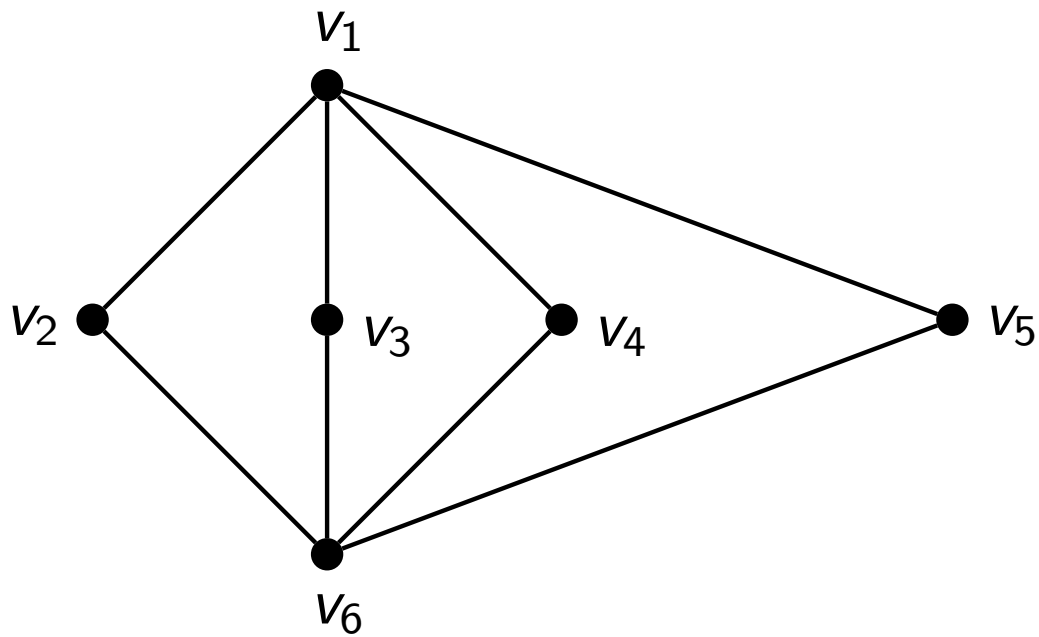


Rješenje

- $S = \{v_1, v_6\}$

Zadatak 8

Dokažite da graf G nije Hamiltonov graf.

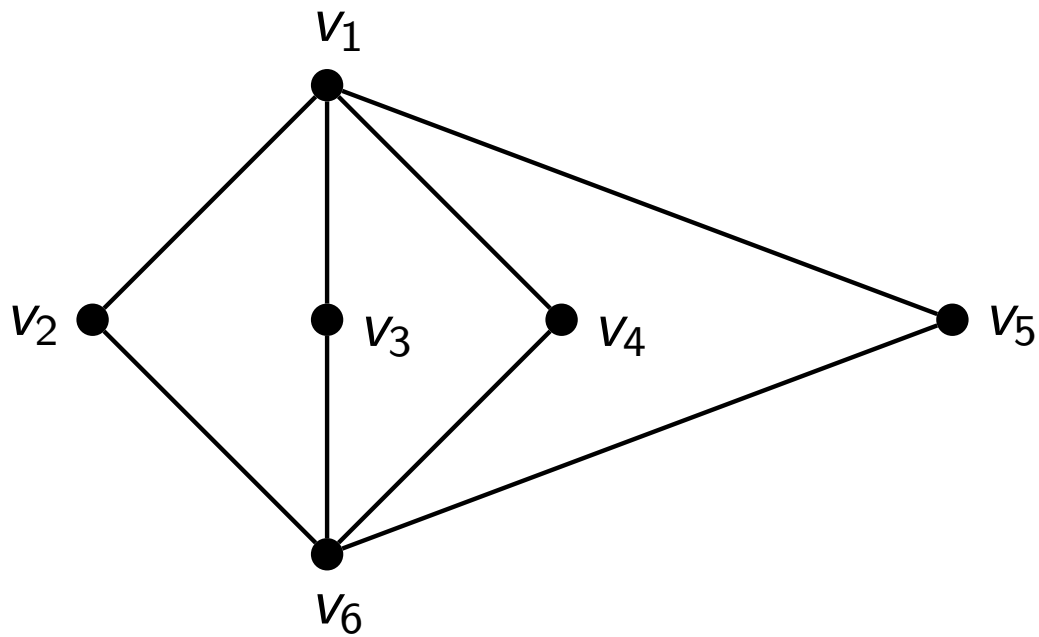


Rješenje

- $S = \{v_1, v_6\}, \quad k(S) = 2$

Zadatak 8

Dokažite da graf G nije Hamiltonov graf.



Rješenje

- $S = \{v_1, v_6\}, \quad k(S) = 2, \quad \omega(G - S) =$

Zadatak 8

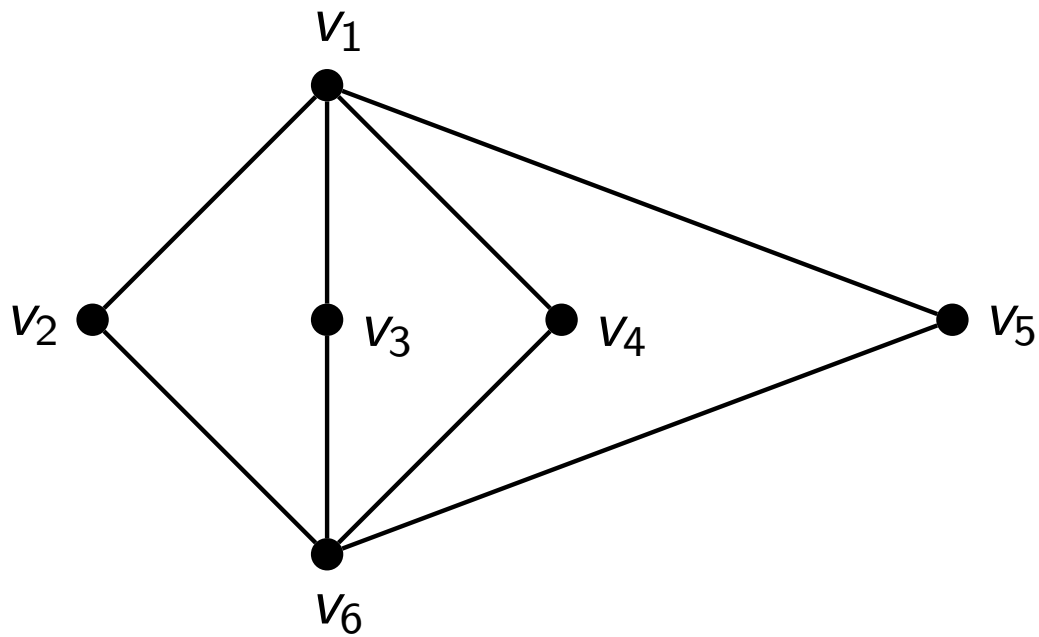
Dokažite da graf G nije Hamiltonov graf.

Rješenje

- $S = \{v_1, v_6\}, \quad k(S) = 2, \quad \omega(G - S) = 4$

Zadatak 8

Dokažite da graf G nije Hamiltonov graf.

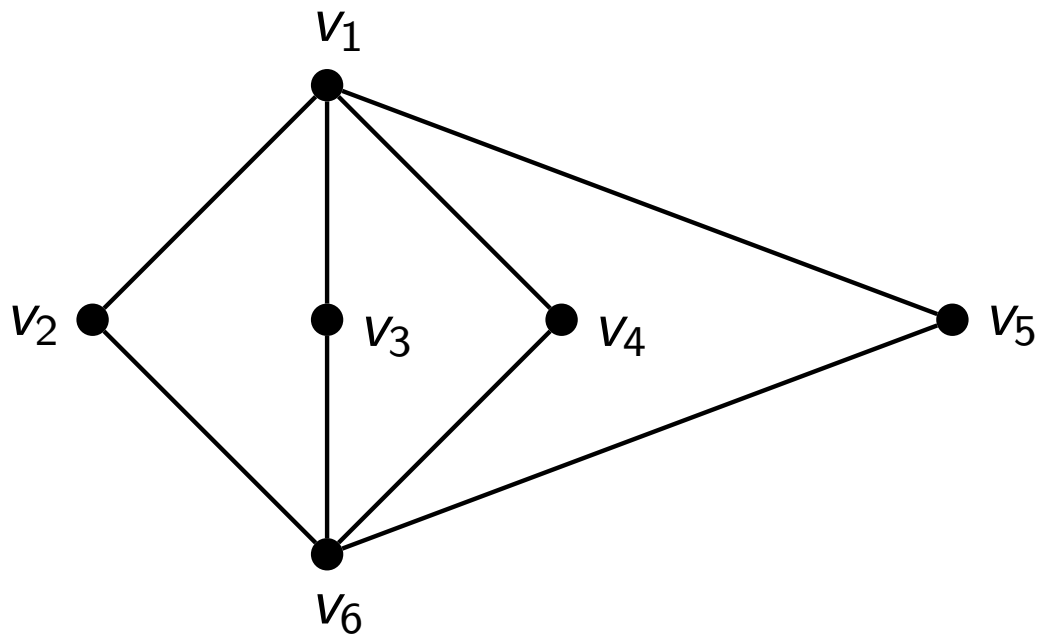


Rješenje

- $S = \{v_1, v_6\}, \quad k(S) = 2, \quad \omega(G - S) = 4$
- $\omega(G - S) > k(S)$

Zadatak 8

Dokažite da graf G nije Hamiltonov graf.



Rješenje

- $S = \{v_1, v_6\}, \quad k(S) = 2, \quad \omega(G - S) = 4$
- $\omega(G - S) > k(S) \longrightarrow G$ nije Hamiltonov graf

Teorem (nužan uvjet za Hamiltonov graf)

Ako je G Hamiltonov graf, tada za svaki $\emptyset \neq S \subset V(G)$ vrijedi $\omega(G - S) \leqslant k(S)$.

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- Obrat gornje tvrdnje ne vrijedi.

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Ako je G Hamiltonov graf, tada za svaki $\emptyset \neq S \subset V(G)$ vrijedi $\omega(G - S) \leqslant k(S)$.

- Obrat gornje tvrdnje ne vrijedi.

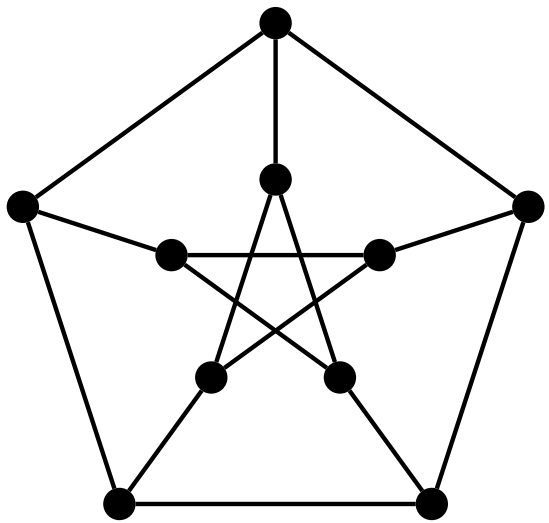
Kontraprimjer

Teorem (nužan uvjet za Hamiltonov graf)

Ako je G Hamiltonov graf, tada za svaki $\emptyset \neq S \subset V(G)$ vrijedi $\omega(G - S) \leqslant k(S)$.

- Obrat gornje tvrdnje ne vrijedi.

Kontraprimjer Petersenov graf



SAGE kod

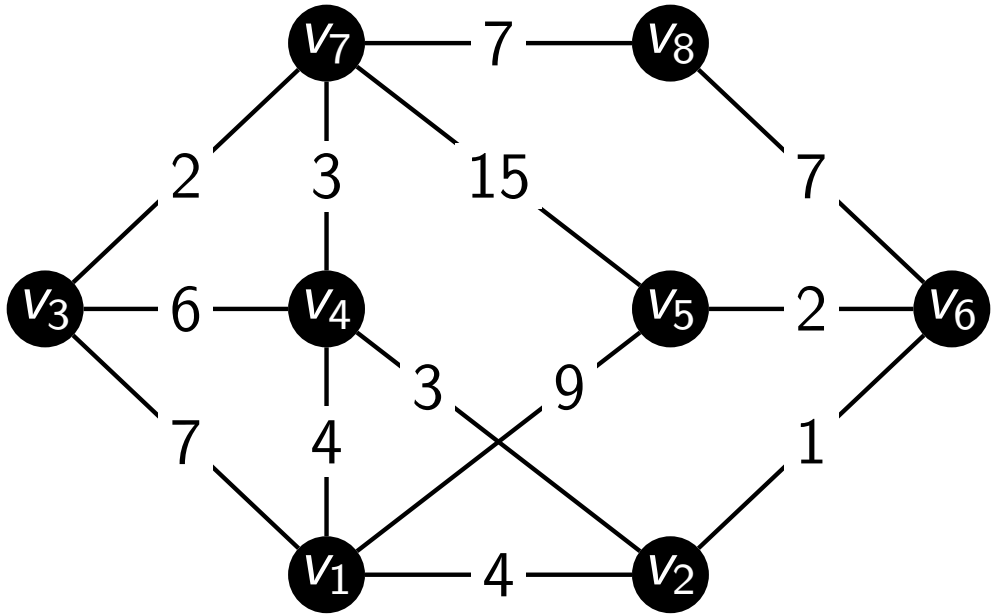
- Donji SAGE kod provjerava da u Petersenovom grafu P zaista za svaki $\emptyset \neq S \subset V(P)$ vrijedi $\omega(P - S) \leqslant k(S)$.
- Međutim, Petersenov graf nije Hamiltonov graf.

```
komb = Combinations(range(10))
for k in komb:
  P = graphs.PetersenGraph()
  P.delete_vertices(k)
  if P.connected_components_number() > len(k): print(k)
print("Gotovo!")
```

deveti zadatak

Zadatak 9

Zadan je težinski graf G.

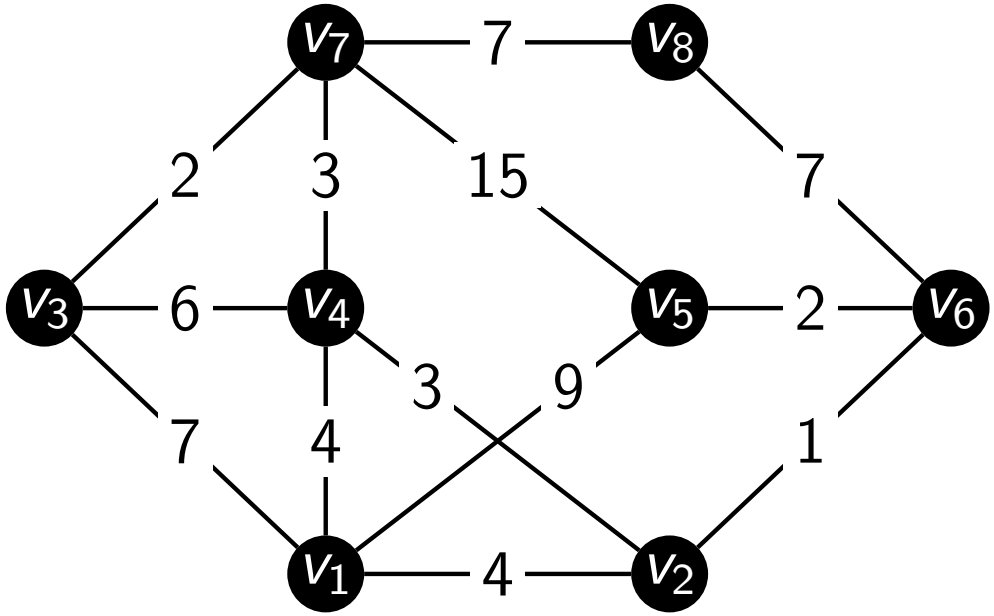


a) *Pomoću Dijkstrinog algoritma odredite najkraće putove od vrha v_1 do svih preostalih vrhova u težinskom grafu G.*

b) *Pomoću poboljšane verzije Dijkstrinog algoritma odredite najkraće putove od vrha v_1 do svih preostalih vrhova u težinskom grafu G.*

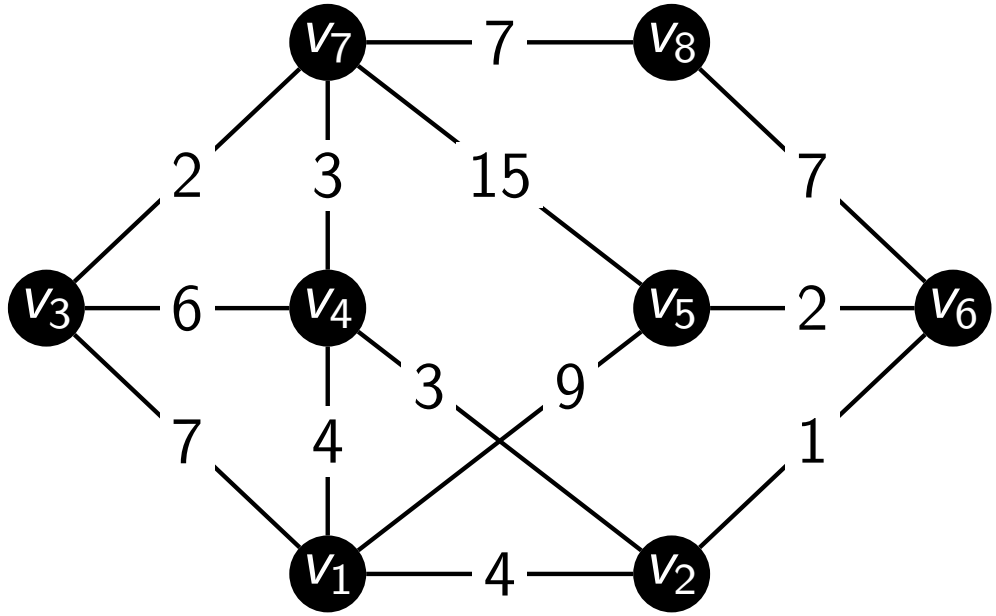
Rješenje

a)



Rješenje

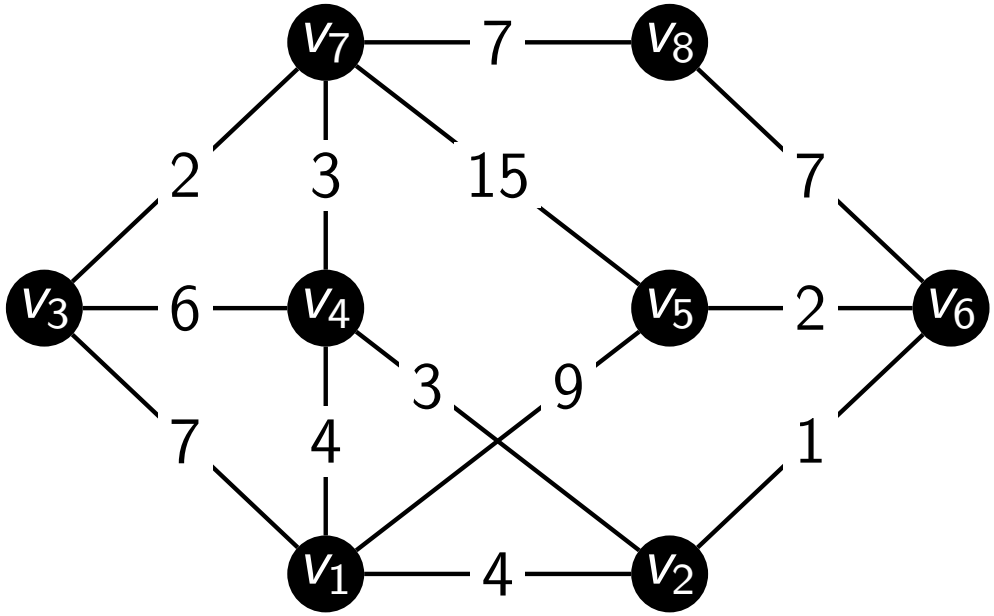
a)



1) $v_1(-, 0)$

Rješenje

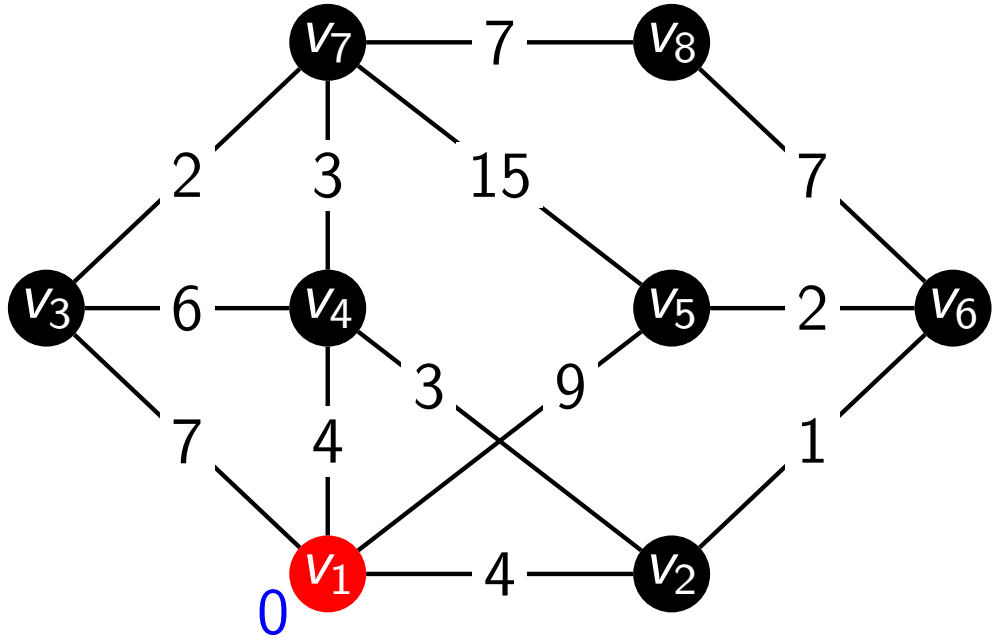
a)



1) $\boxed{v_1(-, 0)}$

Rješenje

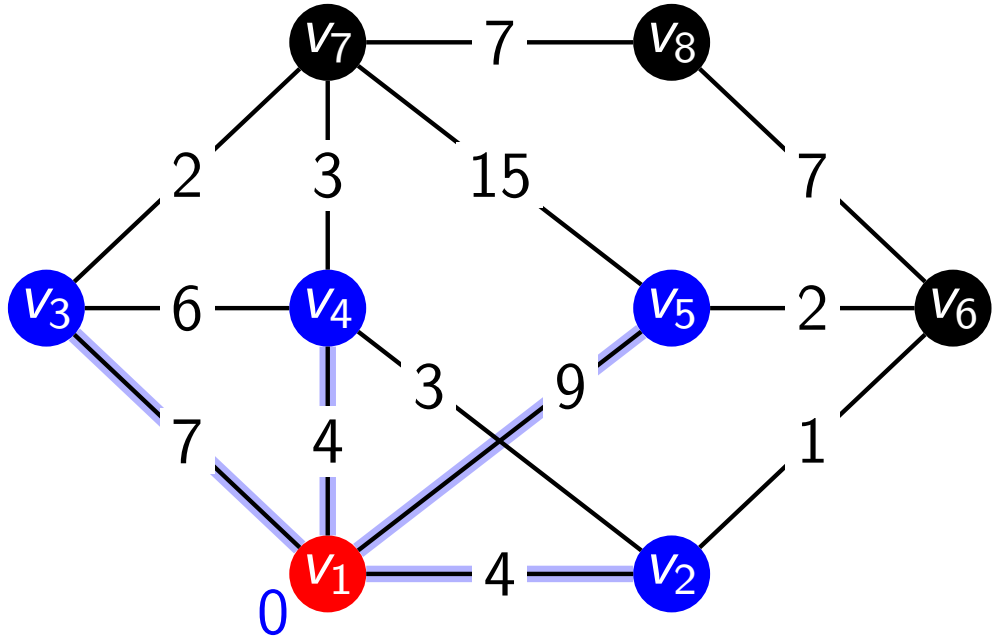
a)



1) $\boxed{v_1(-, 0)}$

Rješenje

a)

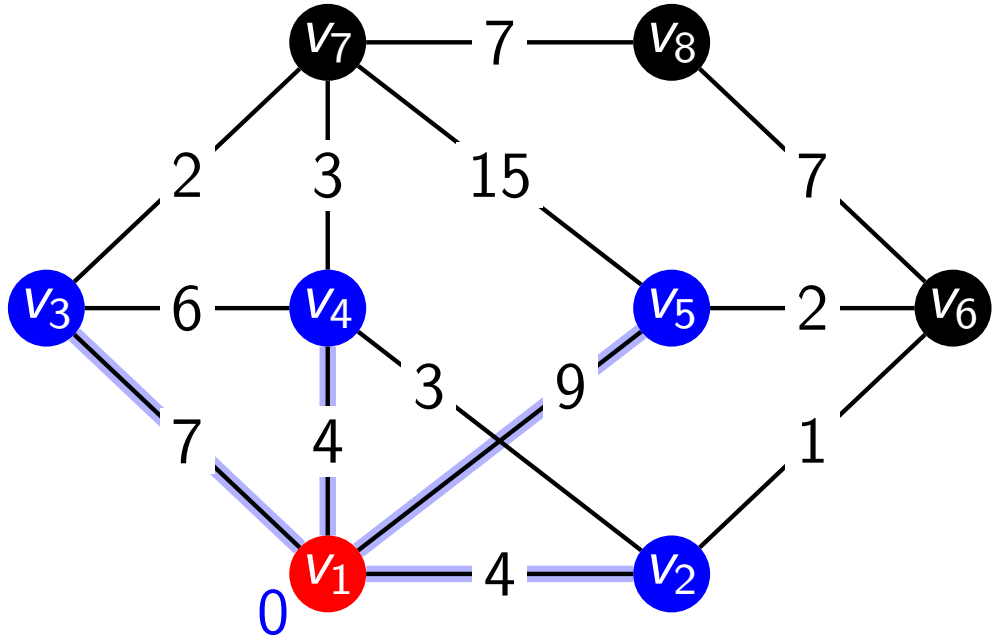


1) $\boxed{v_1(-, 0)}$

2)

Rješenje

a)

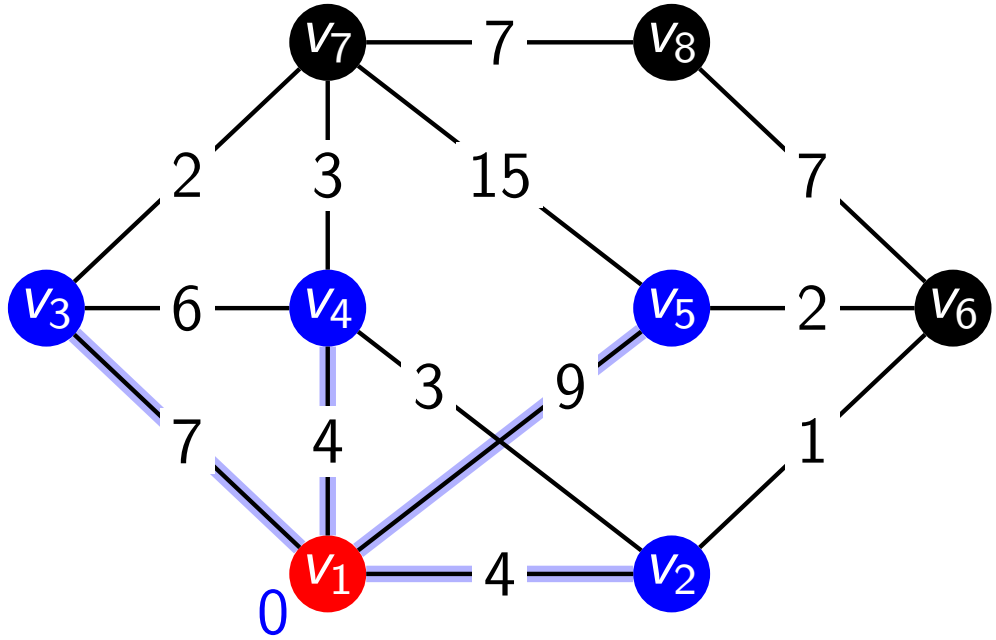


1) $\boxed{v_1(-, 0)}$

2) $v_2(v_1, 4)$

Rješenje

a)

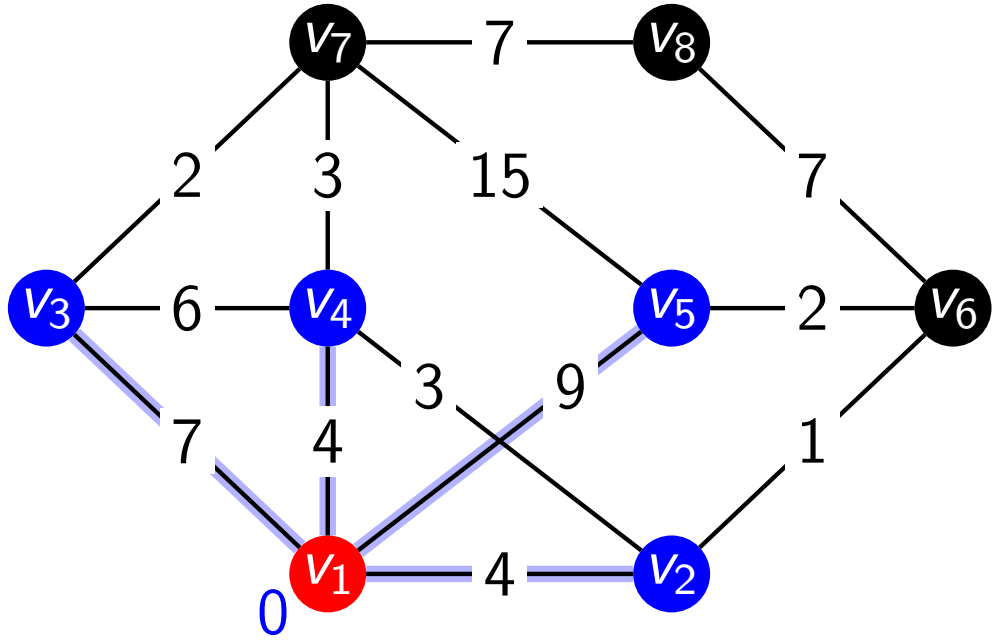


1) $\boxed{v_1(-, 0)}$

2) $v_2(v_1, 4)$, $v_3(v_1, 7)$

Rješenje

a)

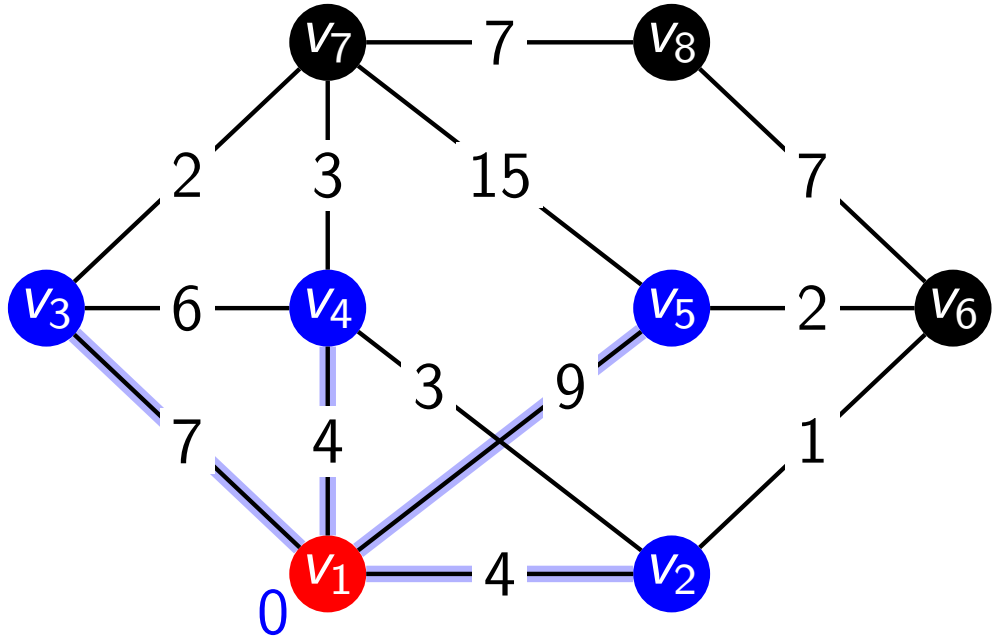


1) $\boxed{v_1(-, 0)}$

2) $v_2(v_1, 4),\ v_3(v_1, 7),\ v_4(v_1, 4)$

Rješenje

a)

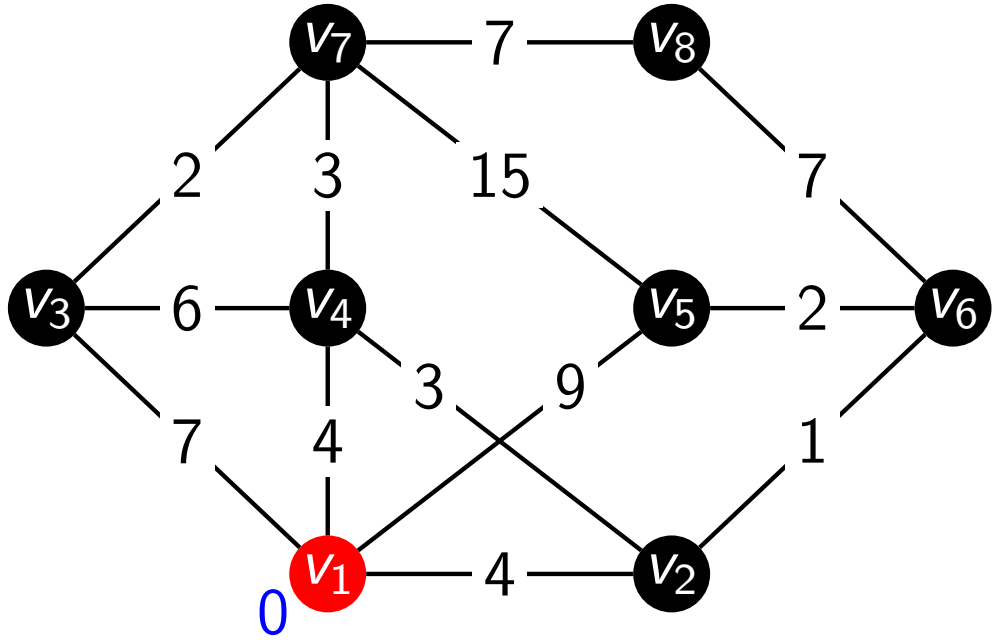


1) $\boxed{v_1(-, 0)}$

2) $v_2(v_1, 4),\ v_3(v_1, 7),\ v_4(v_1, 4),\ v_5(v_1, 9)$

Rješenje

a)

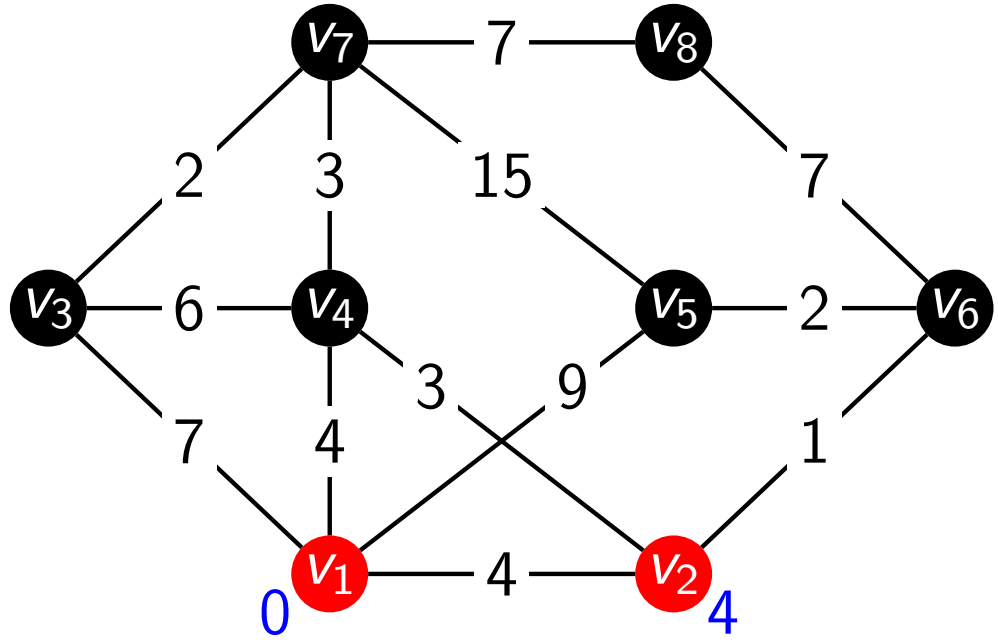


1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

Rješenje

a)

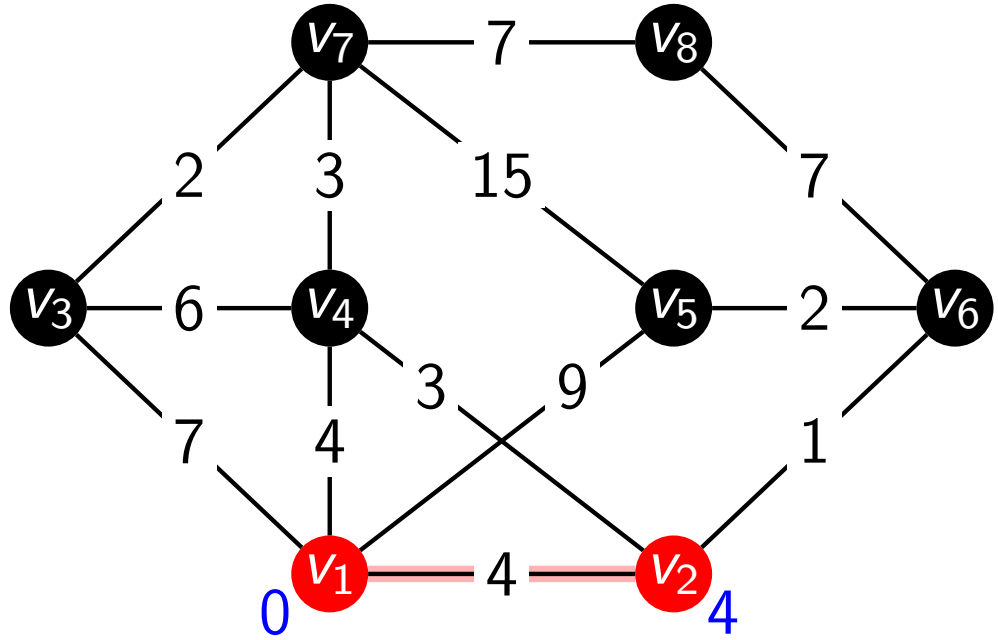


1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

Rješenje

a)

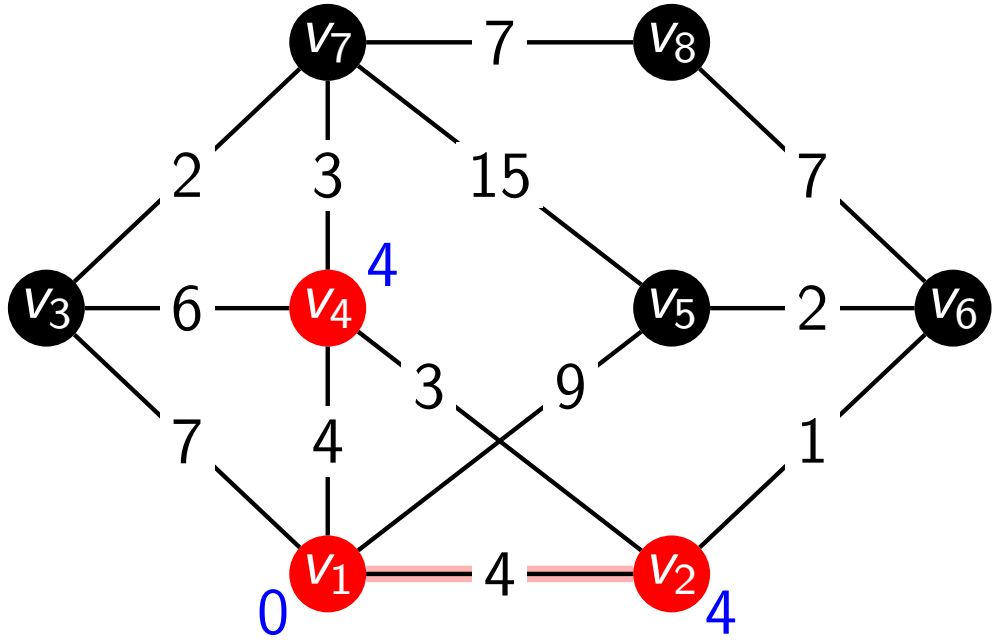


1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

Rješenje

a)

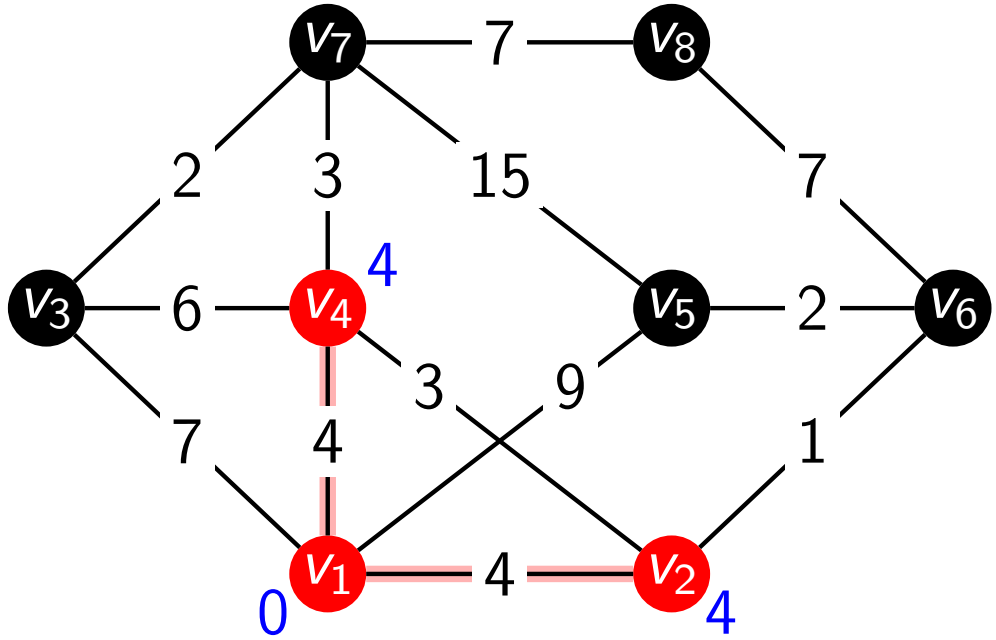


1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

Rješenje

a)

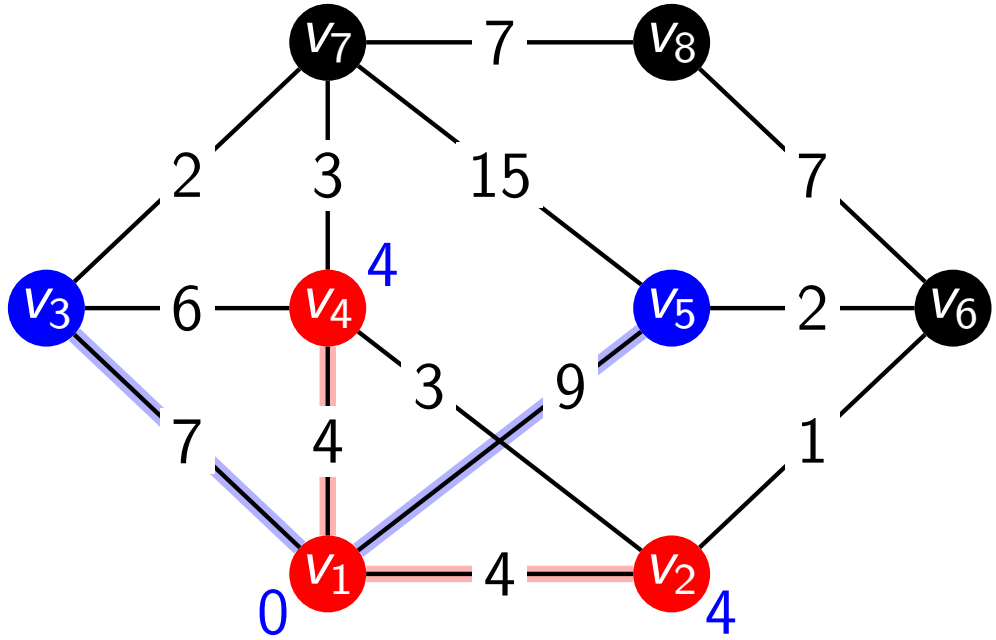


1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

Rješenje

a)



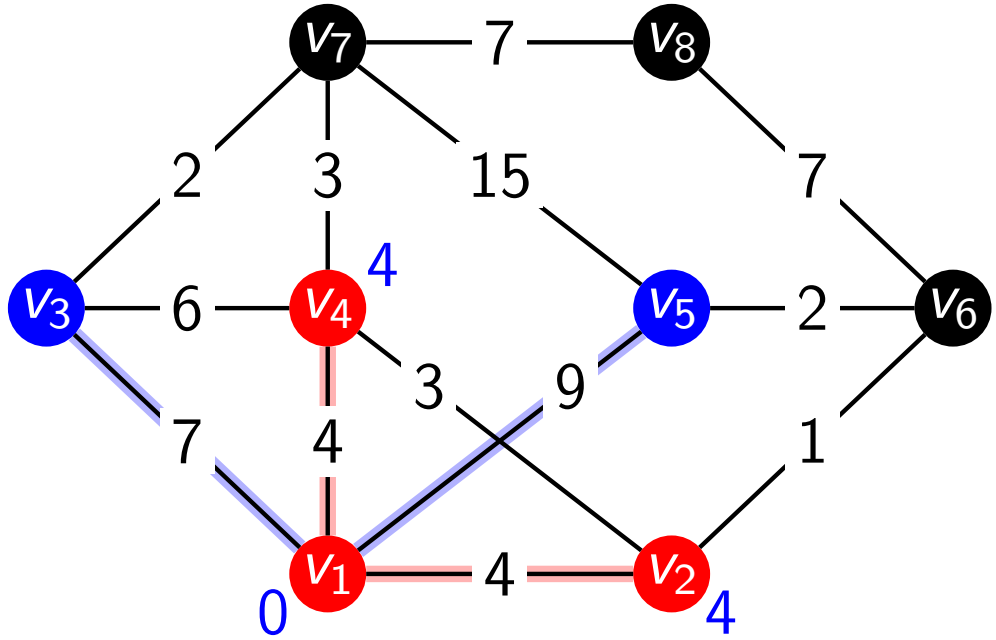
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3)

Rješenje

a)



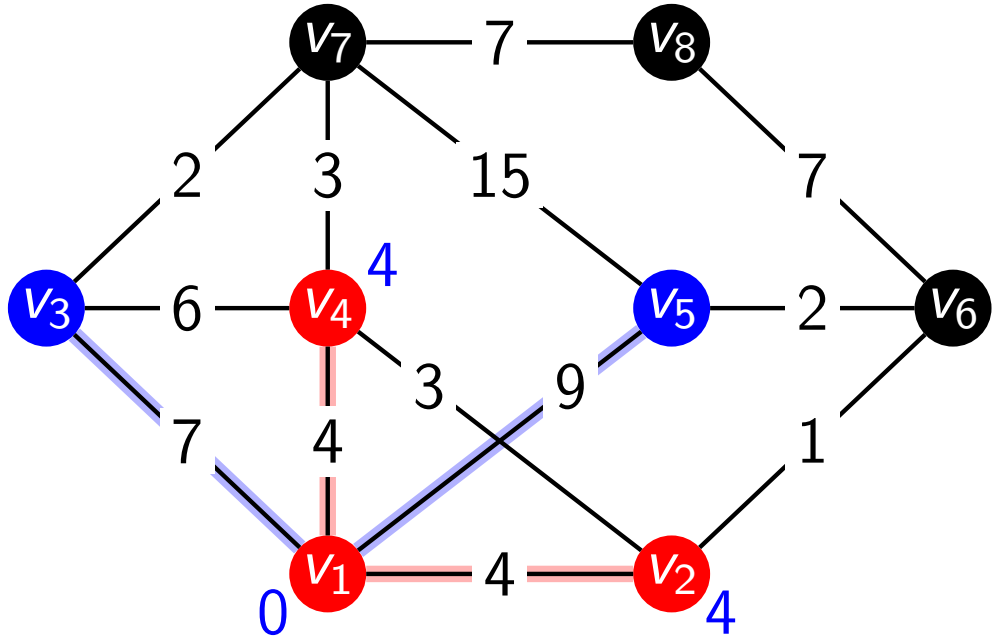
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$

Rješenje

a)



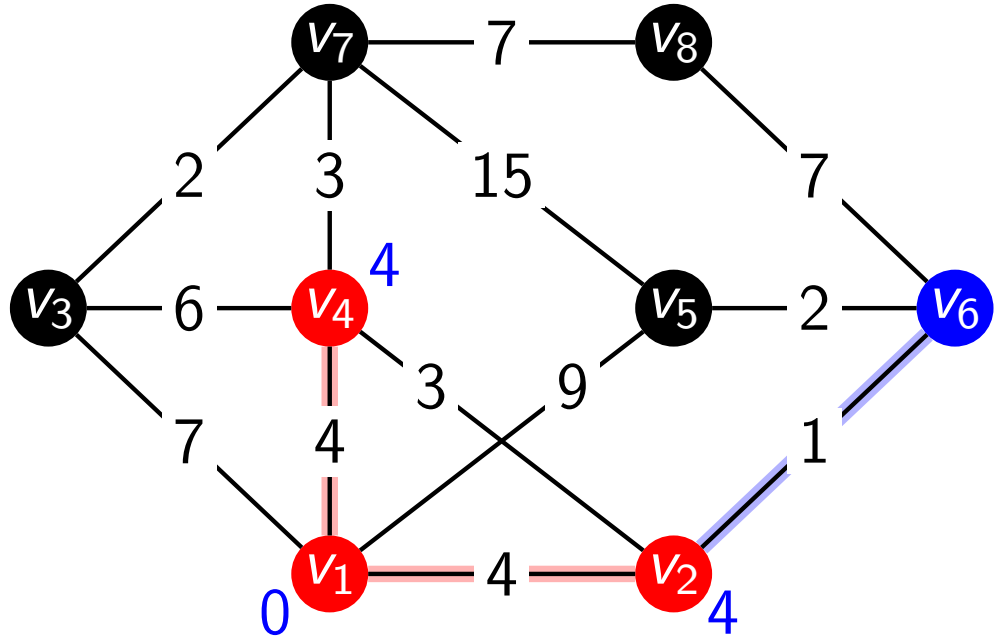
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$

Rješenje

a)



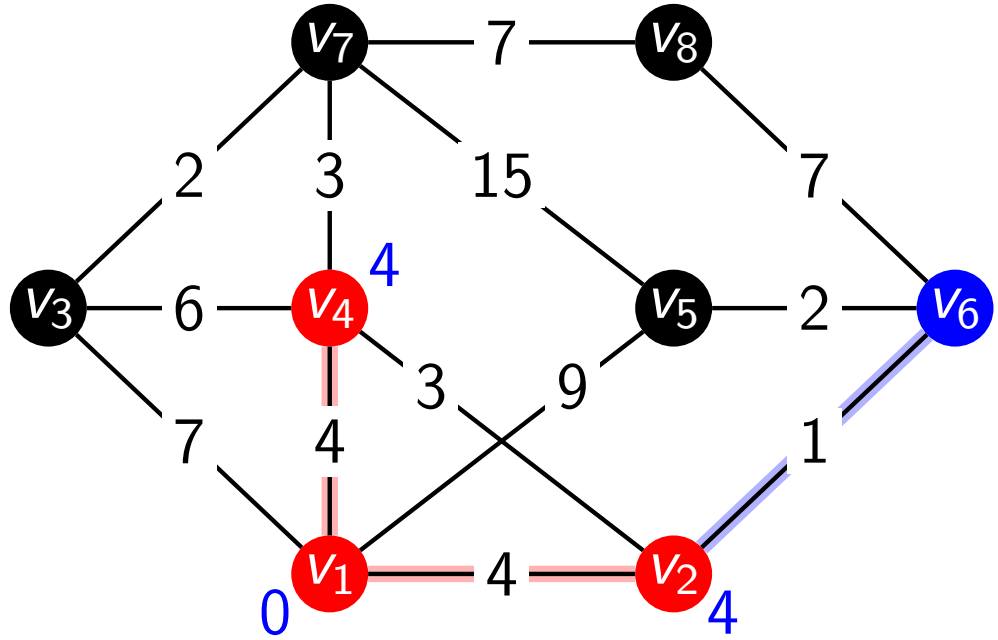
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$

Rješenje

a)



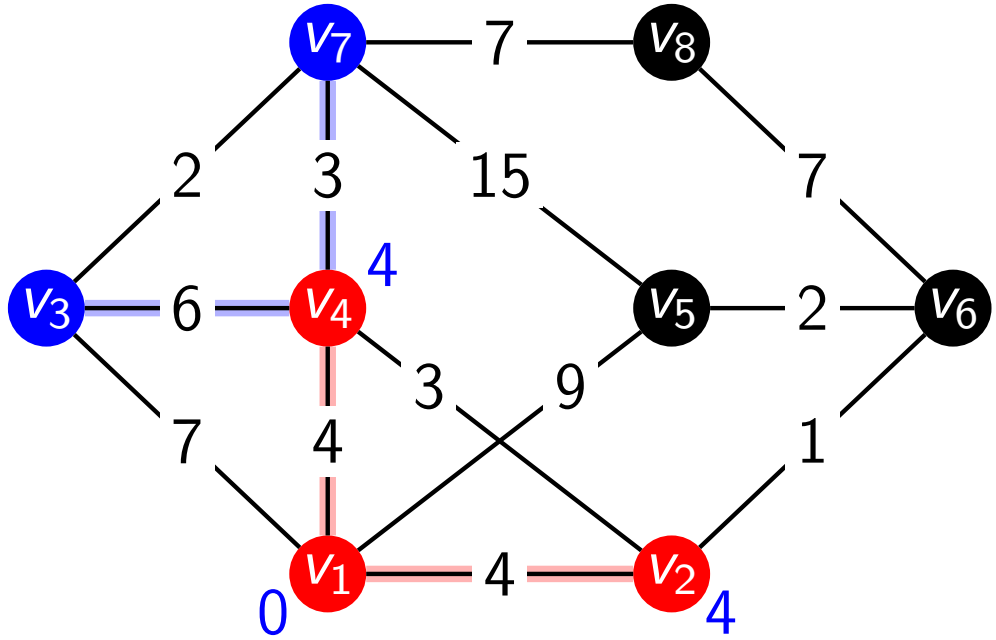
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$, $v_6(v_2, 5)$

Rješenje

a)



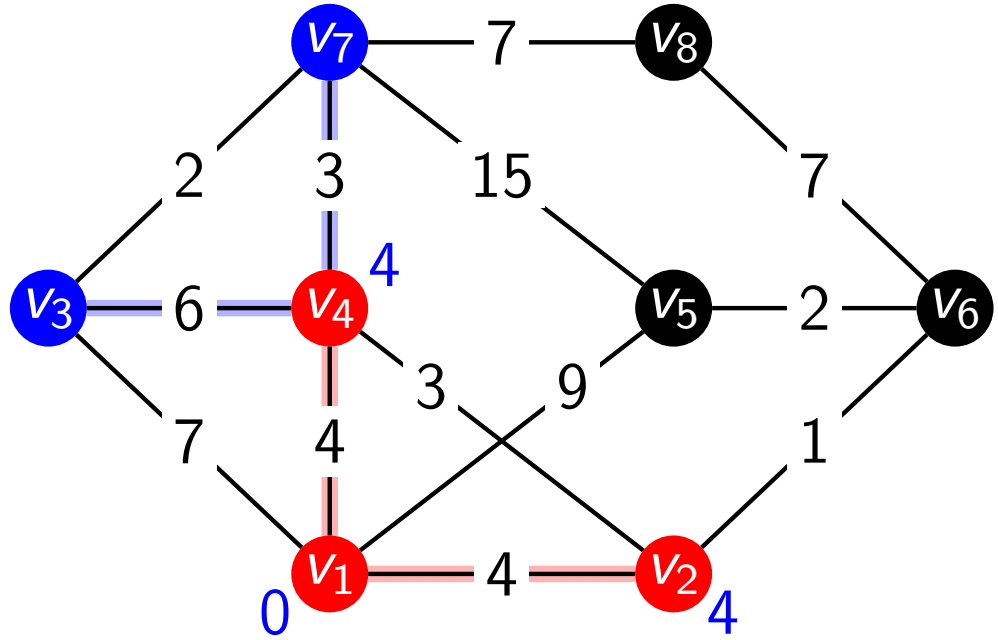
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$, $v_6(v_2, 5)$

Rješenje

a)



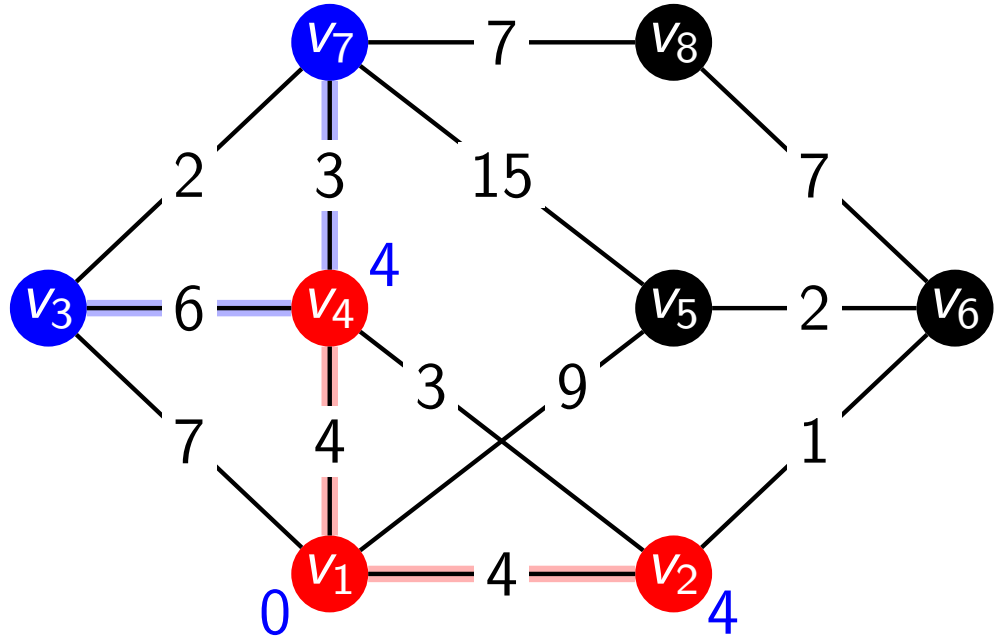
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$, $v_6(v_2, 5)$, $v_3(v_4, 10)$

Rješenje

a)



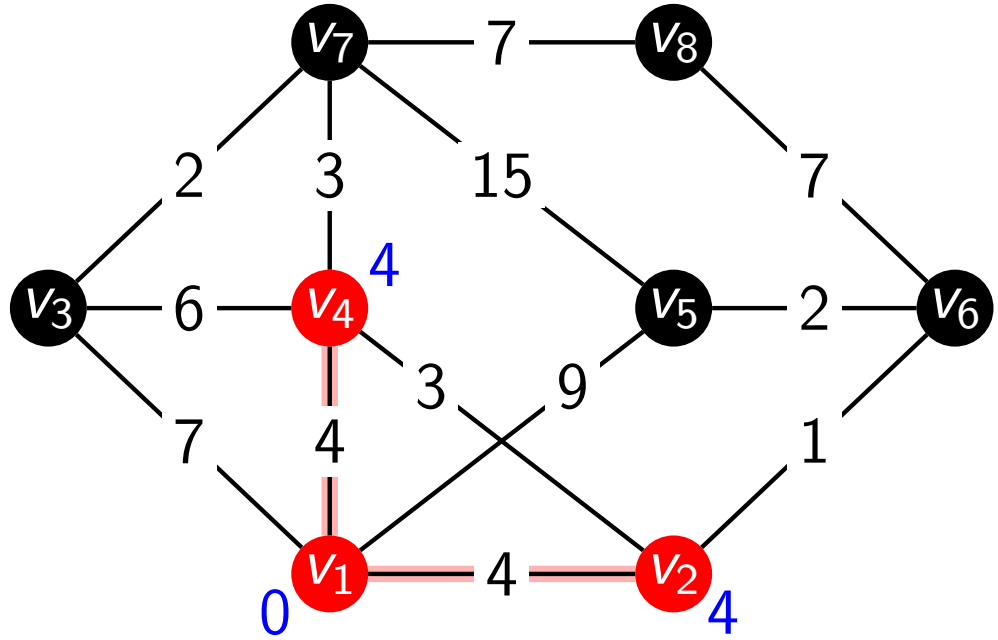
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$, $v_6(v_2, 5)$, $v_3(v_4, 10)$, $v_7(v_4, 7)$

Rješenje

a)



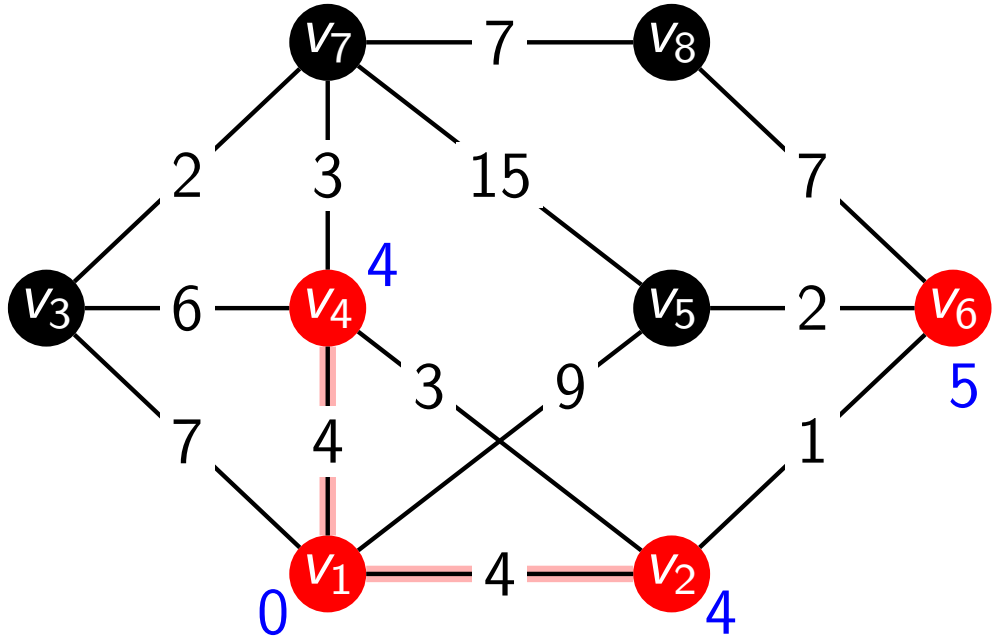
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$, $\boxed{v_6(v_2, 5)}$, $v_3(v_4, 10)$, $v_7(v_4, 7)$

Rješenje

a)



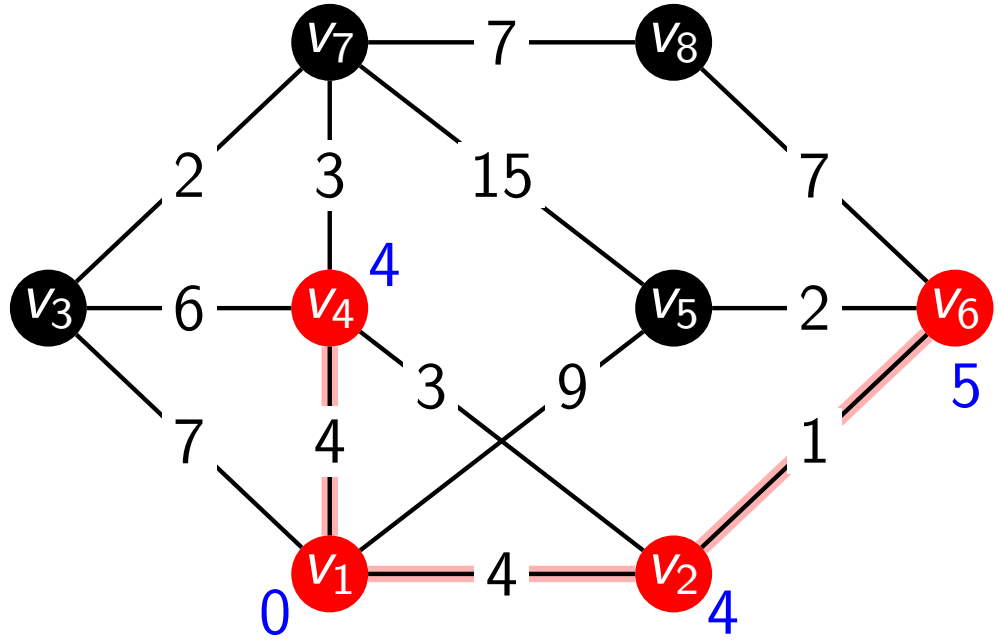
1) $\boxed{v_1(-,0)}$

2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

Rješenje

a)



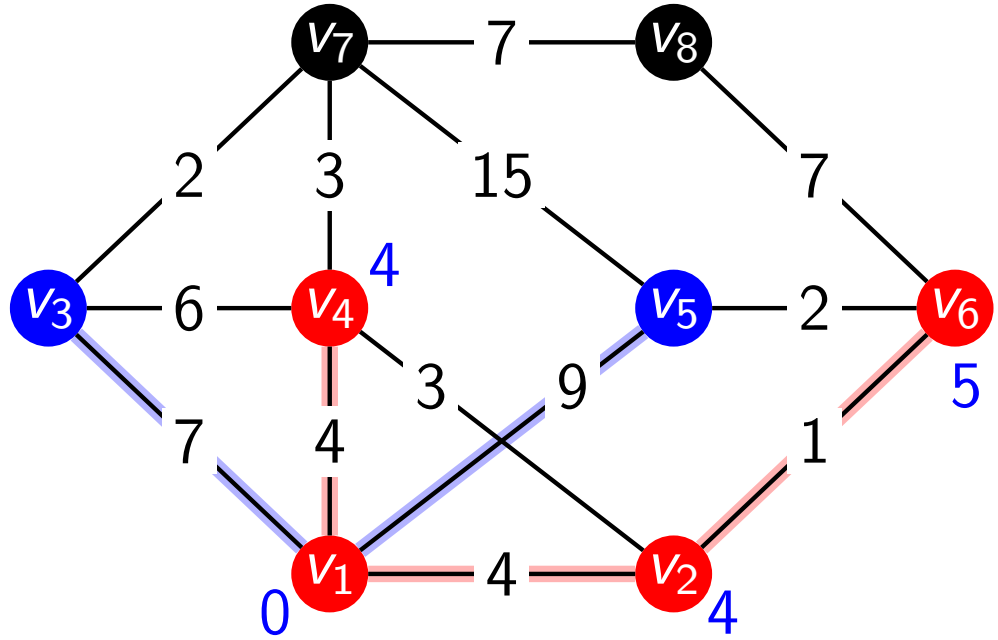
1) $\boxed{v_1(-, 0)}$

2) $\boxed{v_2(v_1, 4)}$, $v_3(v_1, 7)$, $\boxed{v_4(v_1, 4)}$, $v_5(v_1, 9)$

3) $v_3(v_1, 7)$, $v_5(v_1, 9)$, $\boxed{v_6(v_2, 5)}$, $v_3(v_4, 10)$, $v_7(v_4, 7)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

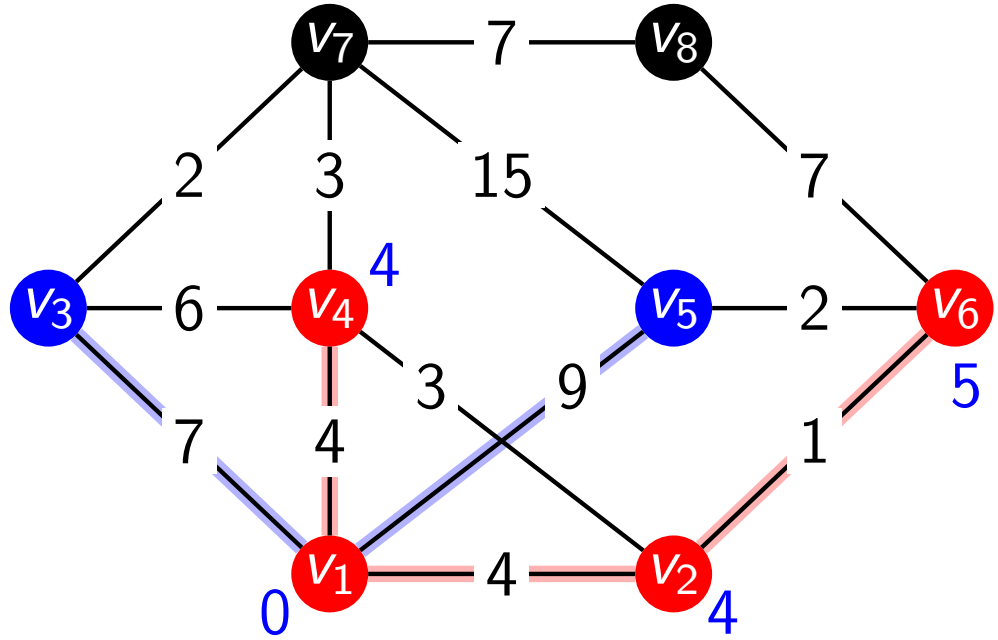
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4)

Rješenje

a)



1) $\boxed{v_1(-,0)}$

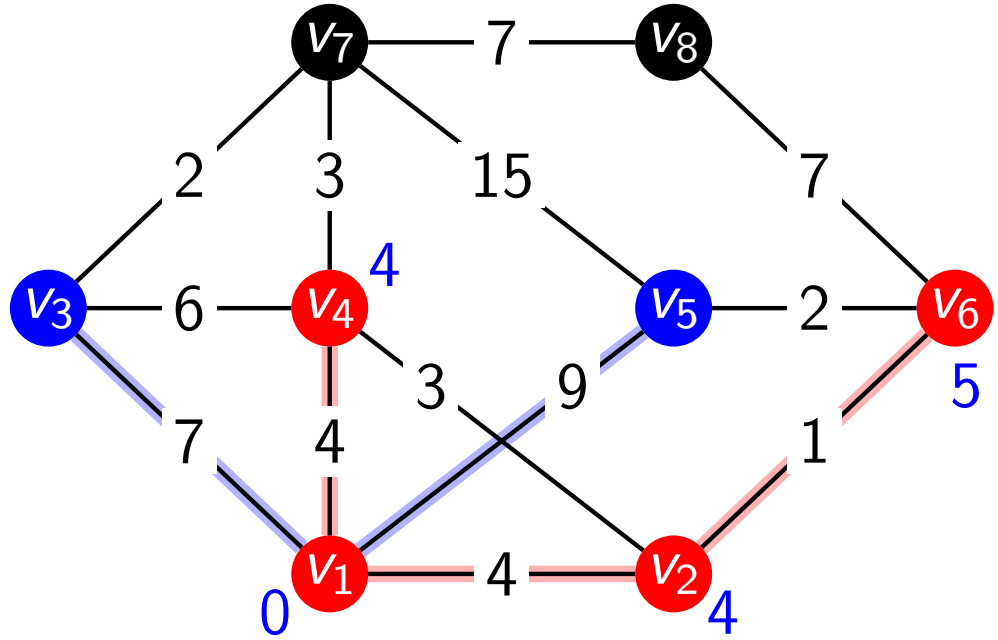
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

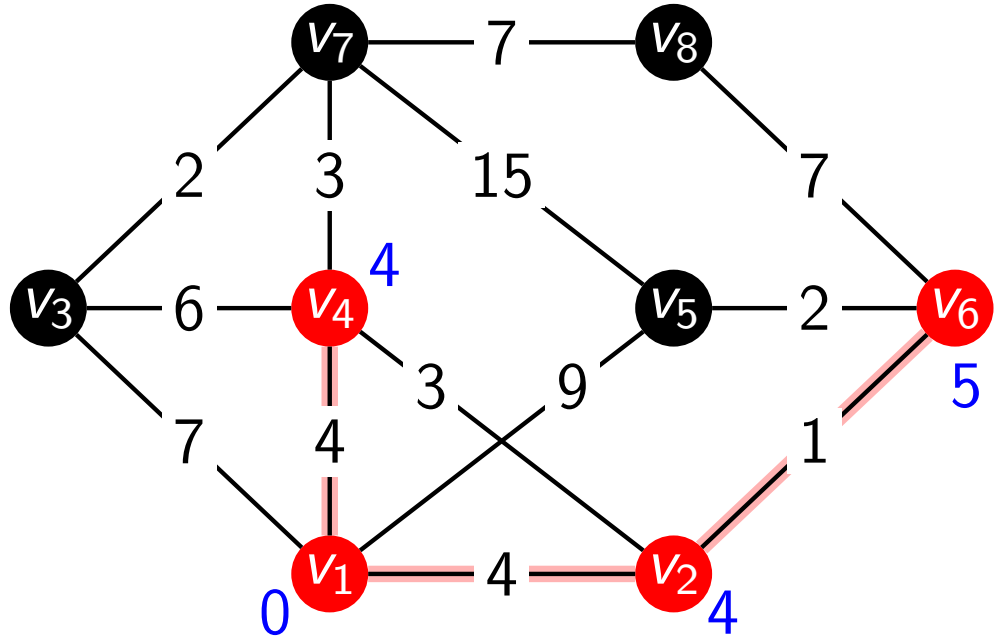
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

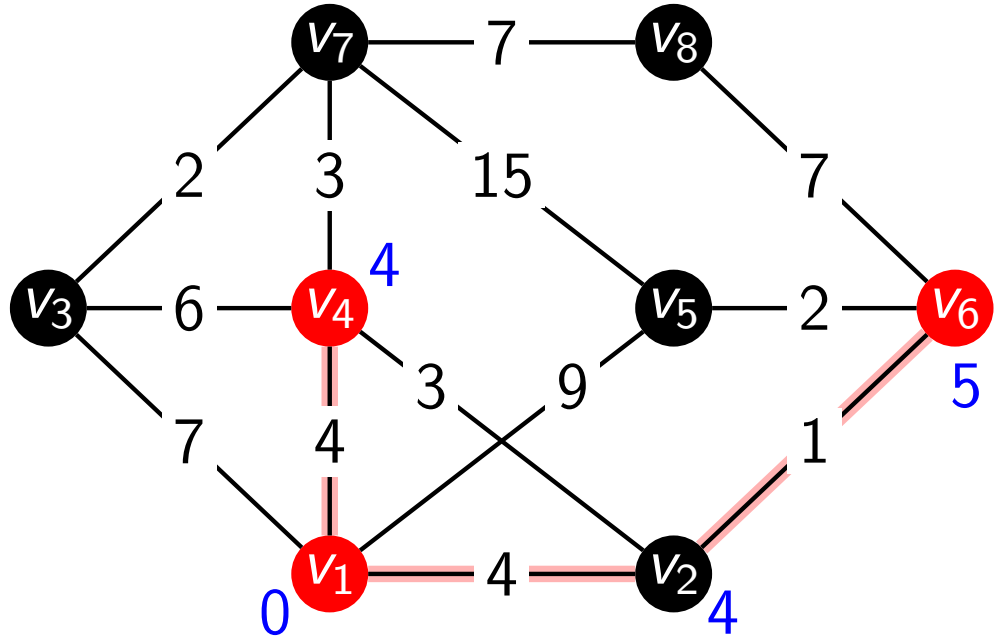
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

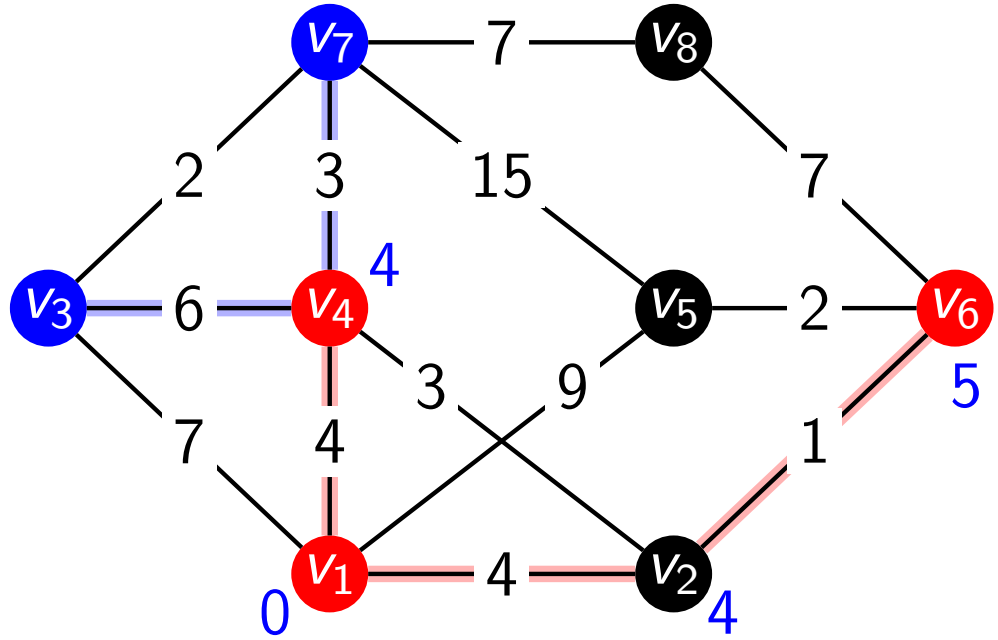
2) $\boxed{v_2(v_1,4)}$, $+$ $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

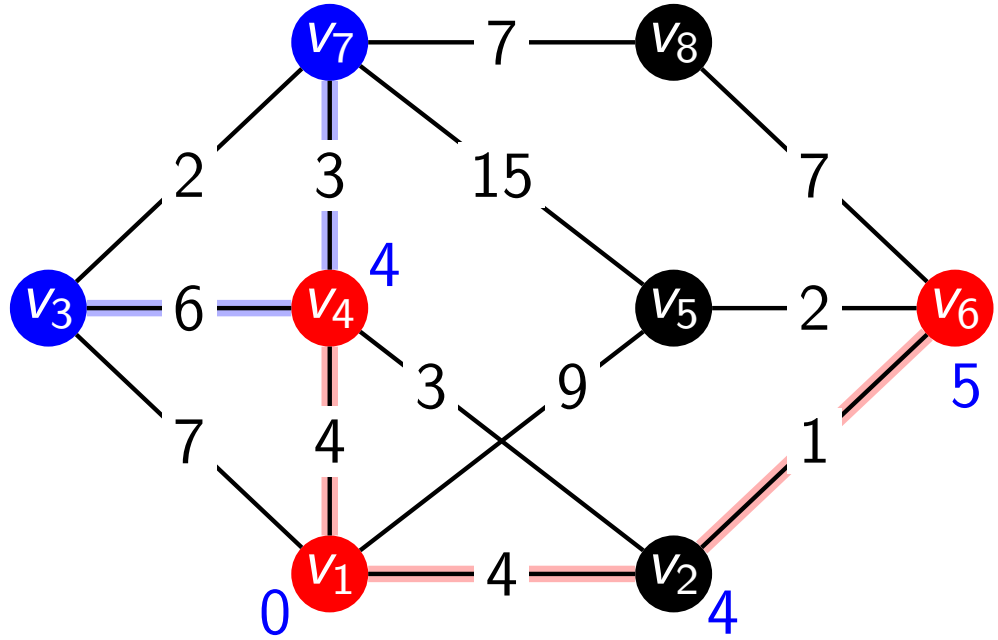
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

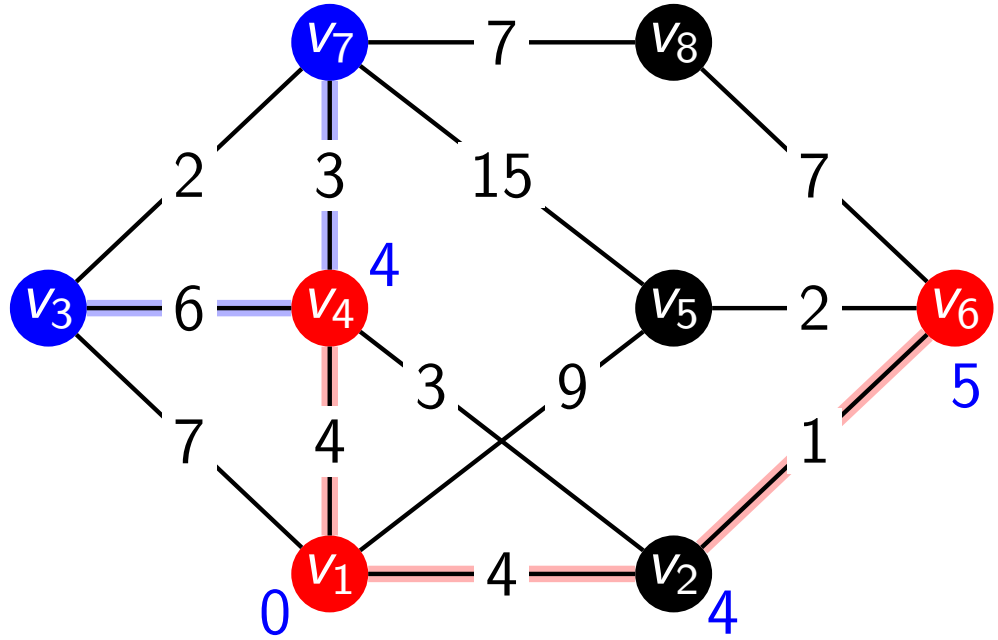
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$, $v_3(v_4,10)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

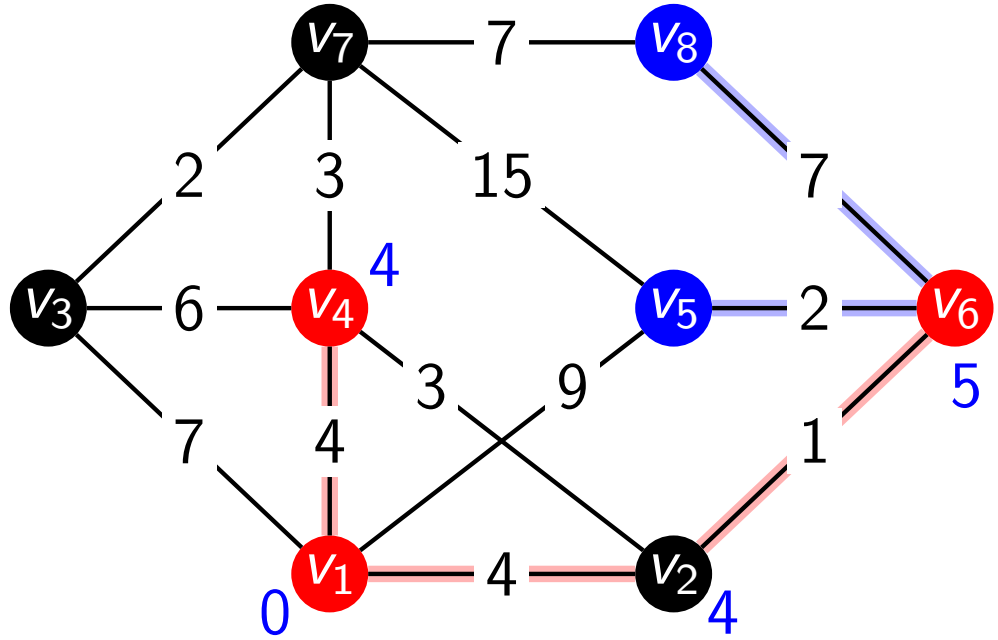
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$, $v_3(v_4,10)$, $v_7(v_4,7)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

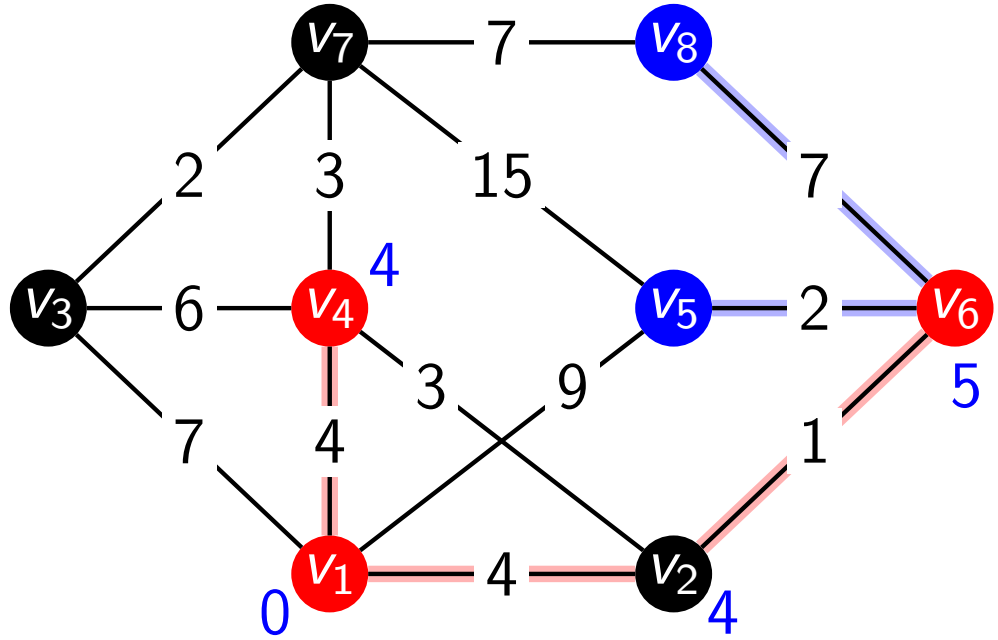
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$, $v_3(v_4,10)$, $v_7(v_4,7)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

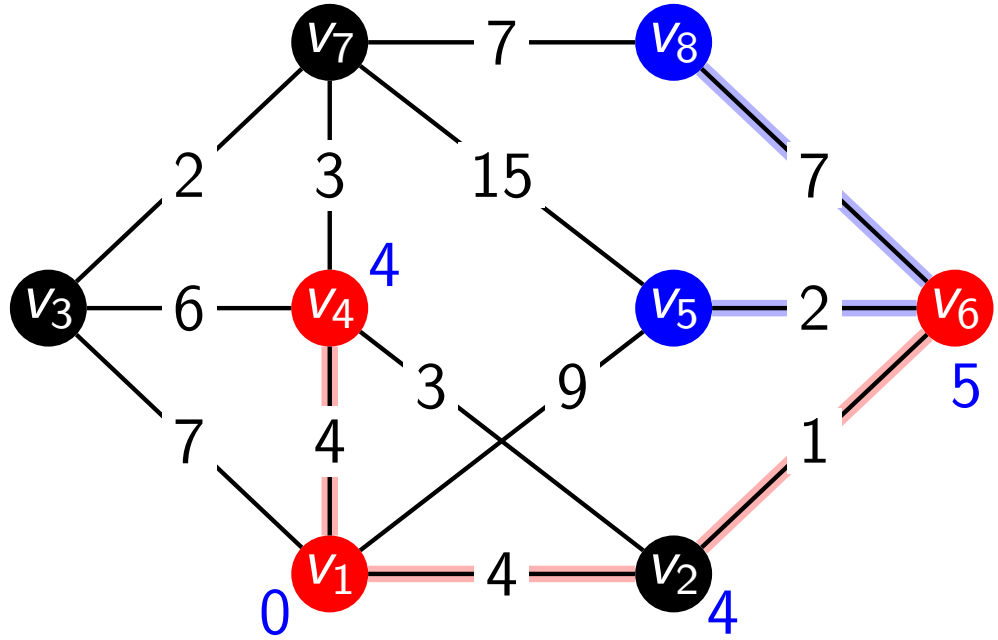
2) $\boxed{v_2(v_1,4)}$, $+$ $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$, $v_3(v_4,10)$, $v_7(v_4,7)$, $v_5(v_6,7)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

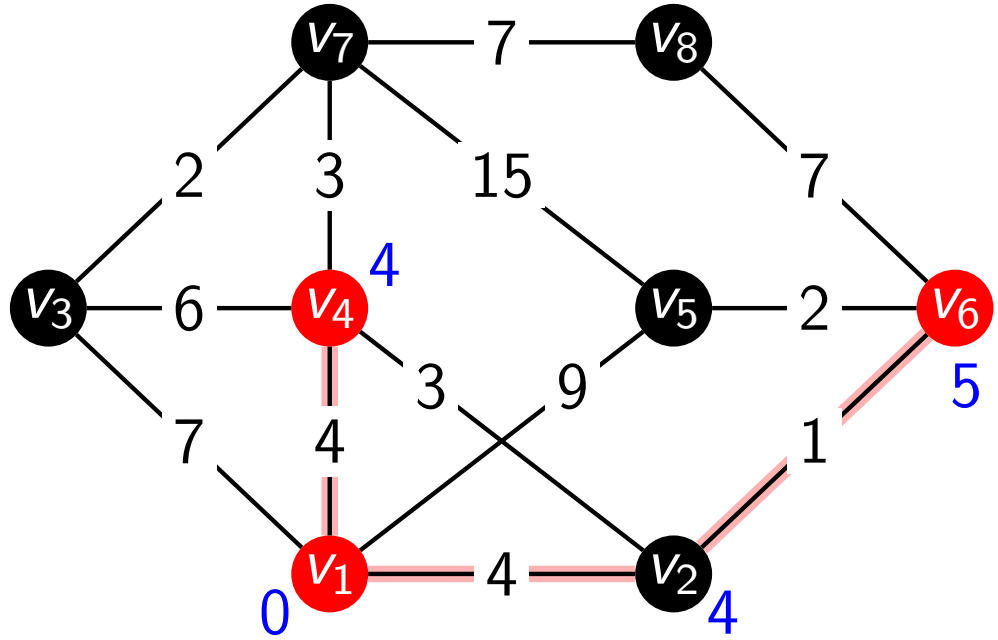
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $v_3(v_1,7)$, $v_5(v_1,9)$, $v_3(v_4,10)$, $v_7(v_4,7)$, $v_5(v_6,7)$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

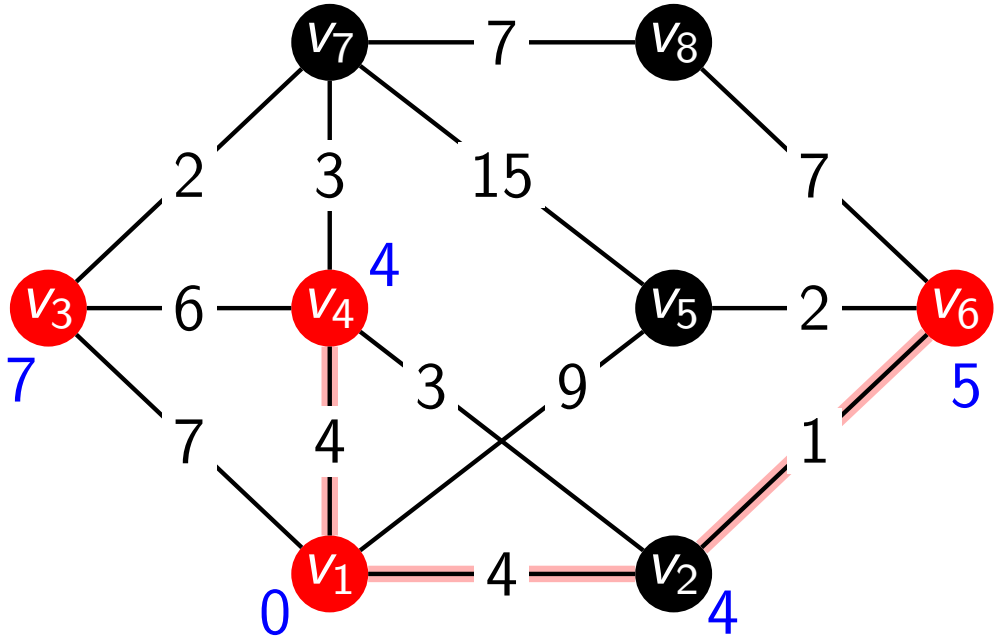
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

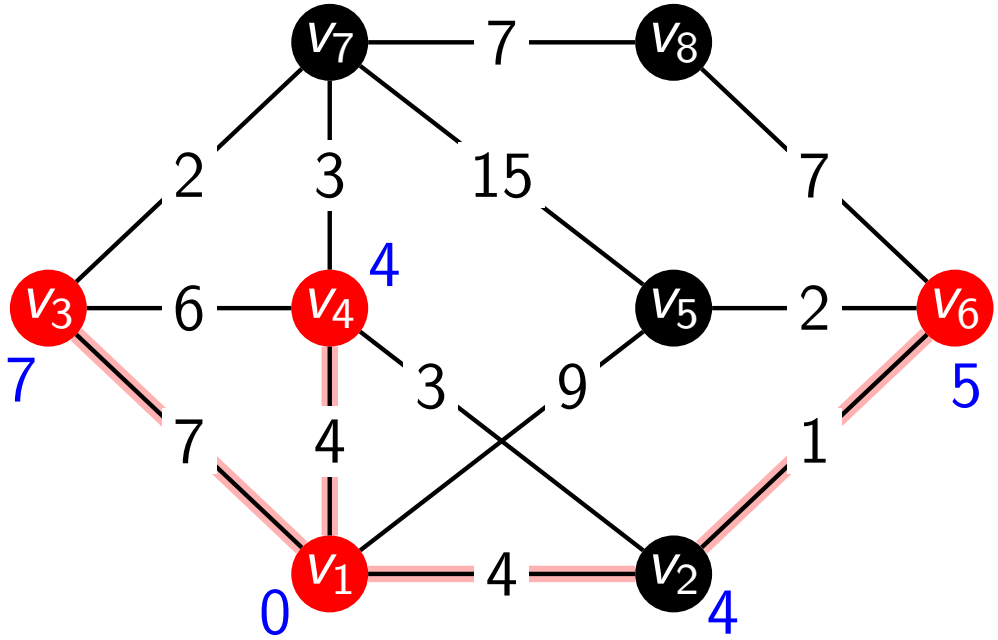
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

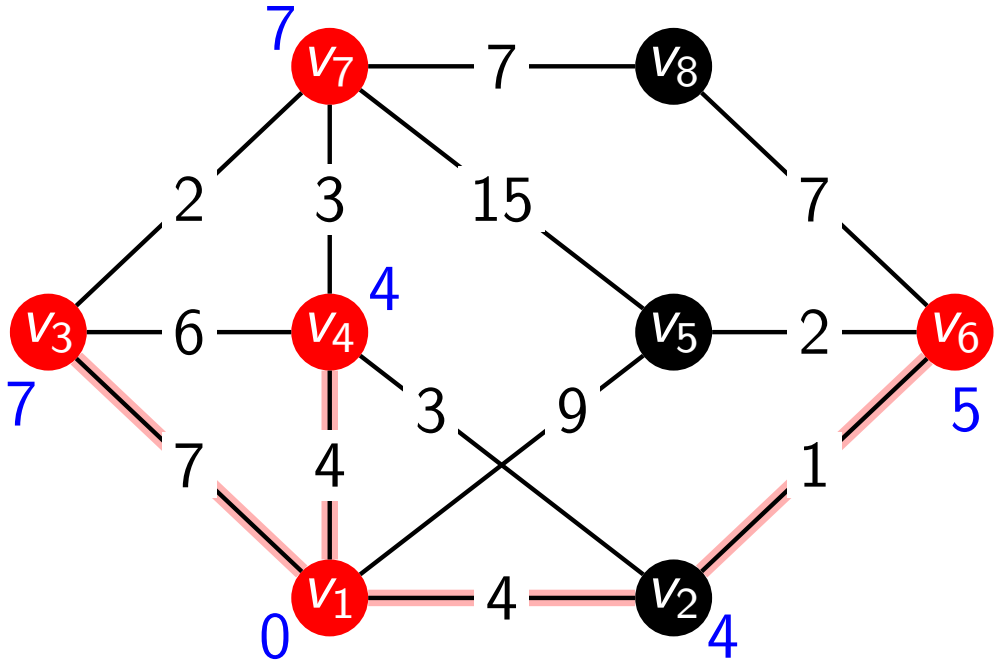
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

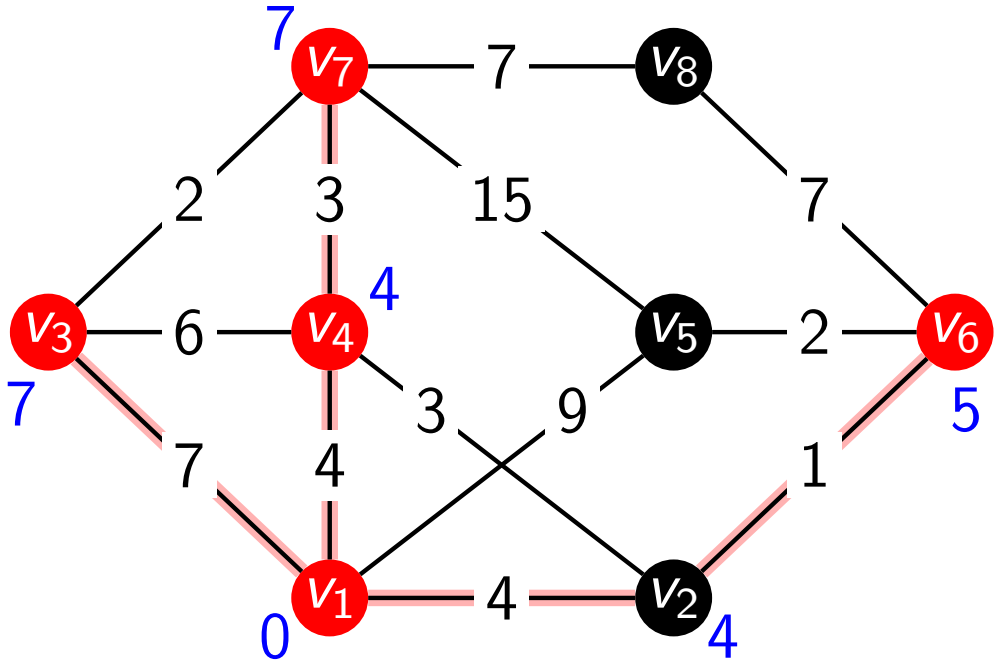
2) $\boxed{v_2(v_1,4)}$, $\boxed{v_3(v_1,7)}$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

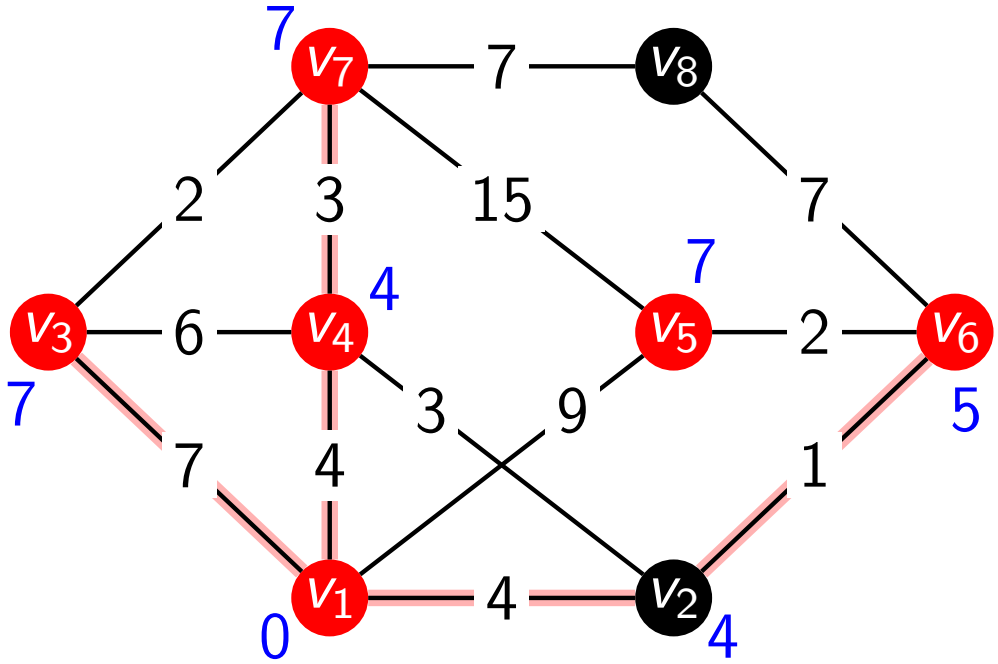
2) $\boxed{v_2(v_1,4)}$, $\boxed{v_3(v_1,7)}$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

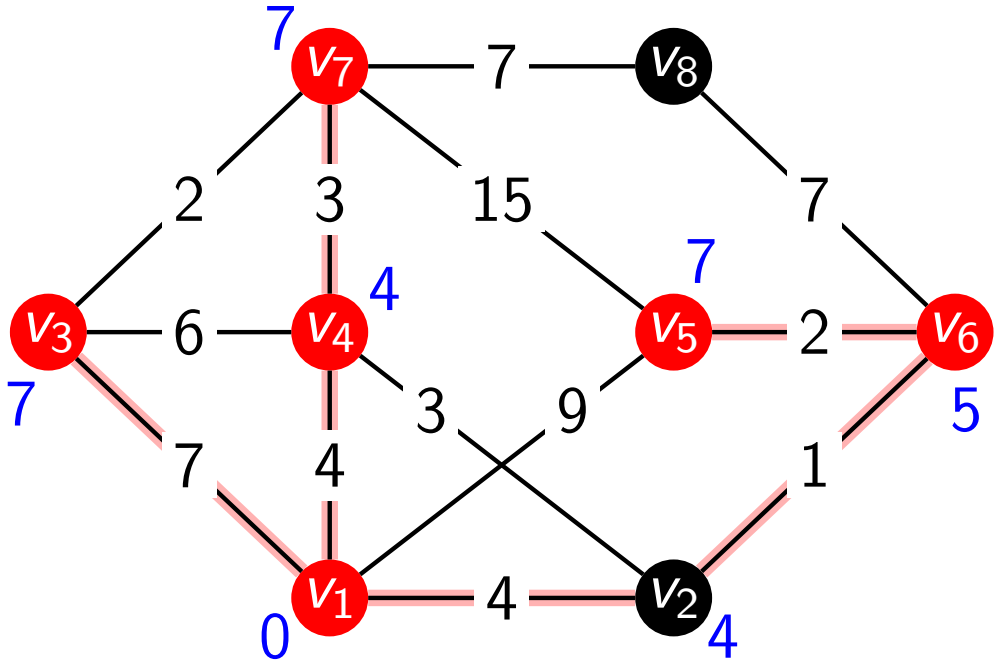
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}$

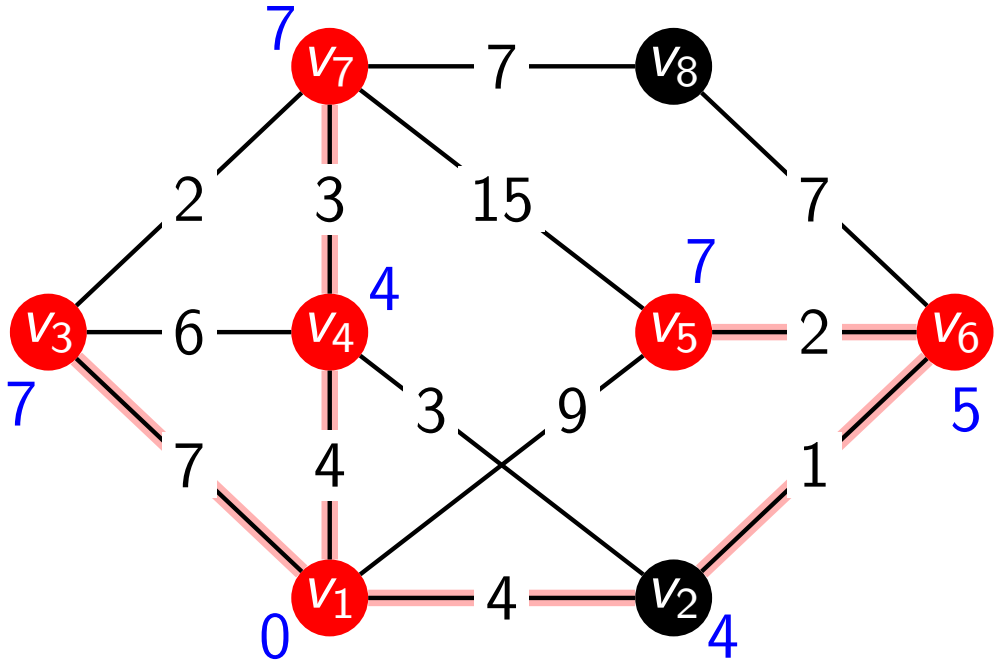
2) $\boxed{v_2(v_1,4)}$, $v_3(v_1,7)$, $\boxed{v_4(v_1,4)}$, $v_5(v_1,9)$

3) $v_3(v_1,7)$, $v_5(v_1,9)$, $\boxed{v_6(v_2,5)}$, $v_3(v_4,10)$, $v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}$, $v_5(v_1,9)$, $v_3(v_4,10)$, $\boxed{v_7(v_4,7)}$, $\boxed{v_5(v_6,7)}$, $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}, v_5(v_1,9)$

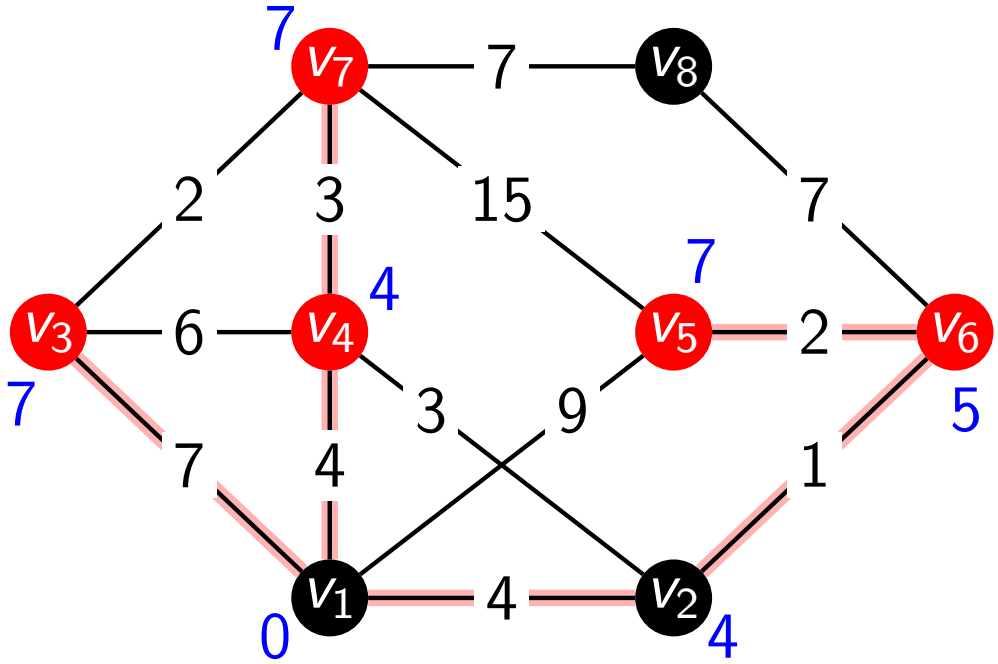
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5)

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}, v_5(v_1,9)$

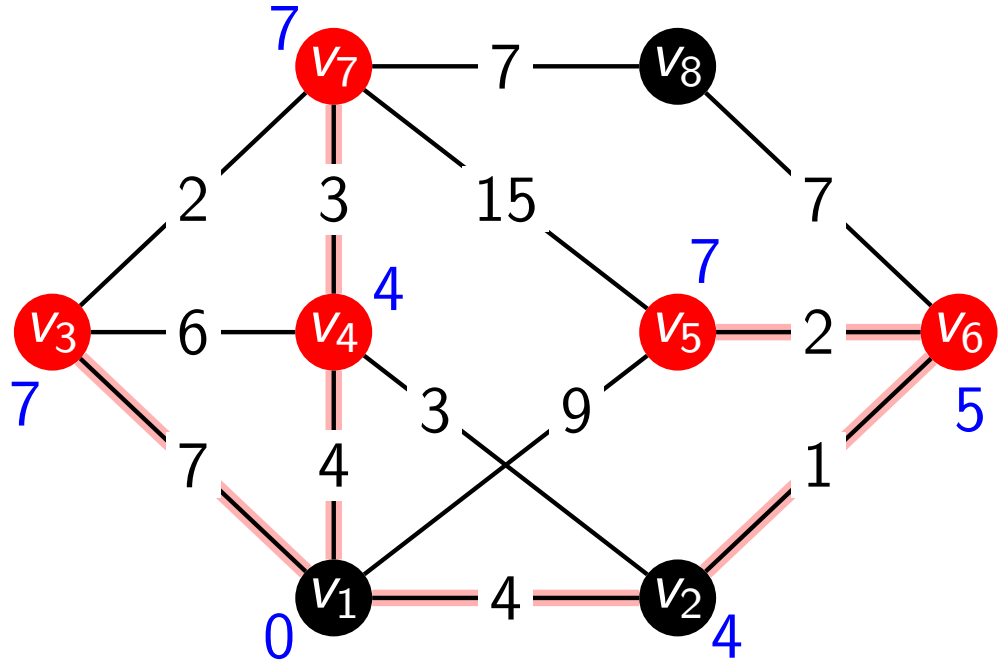
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5)

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

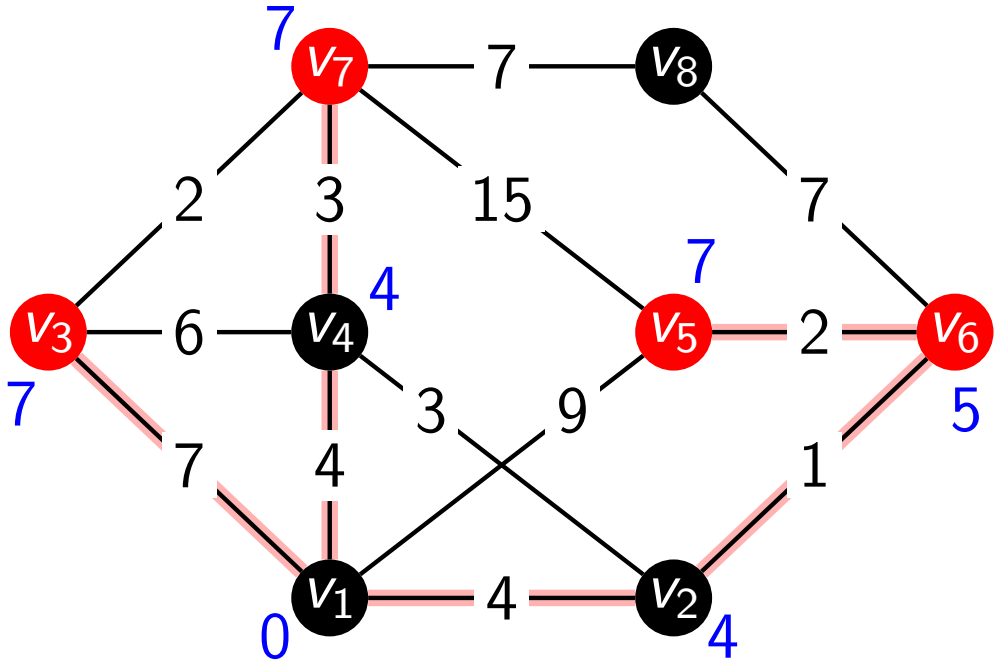
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5)

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

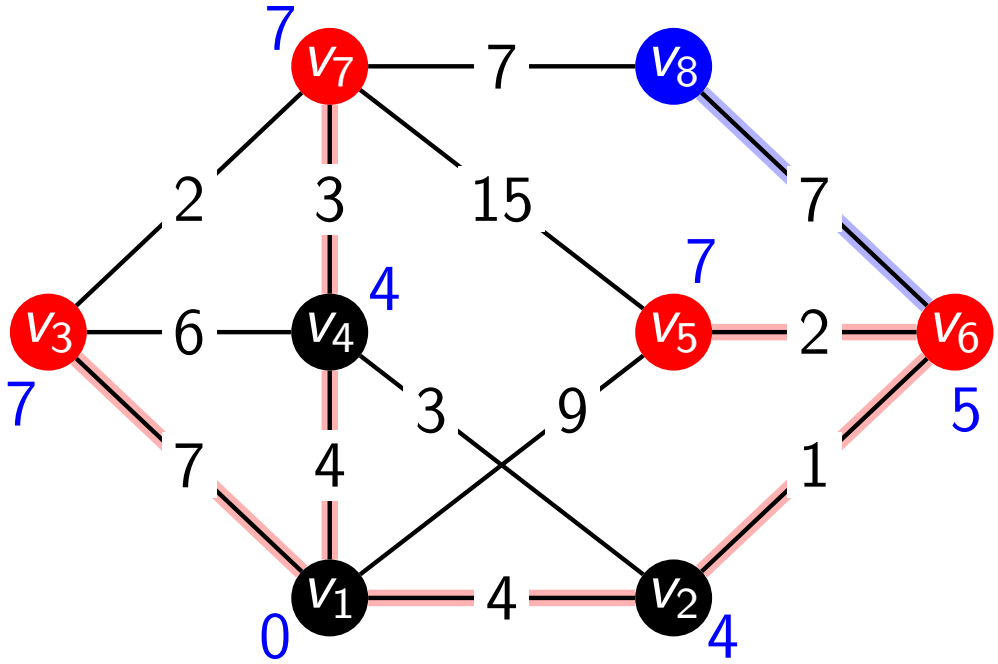
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5)

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

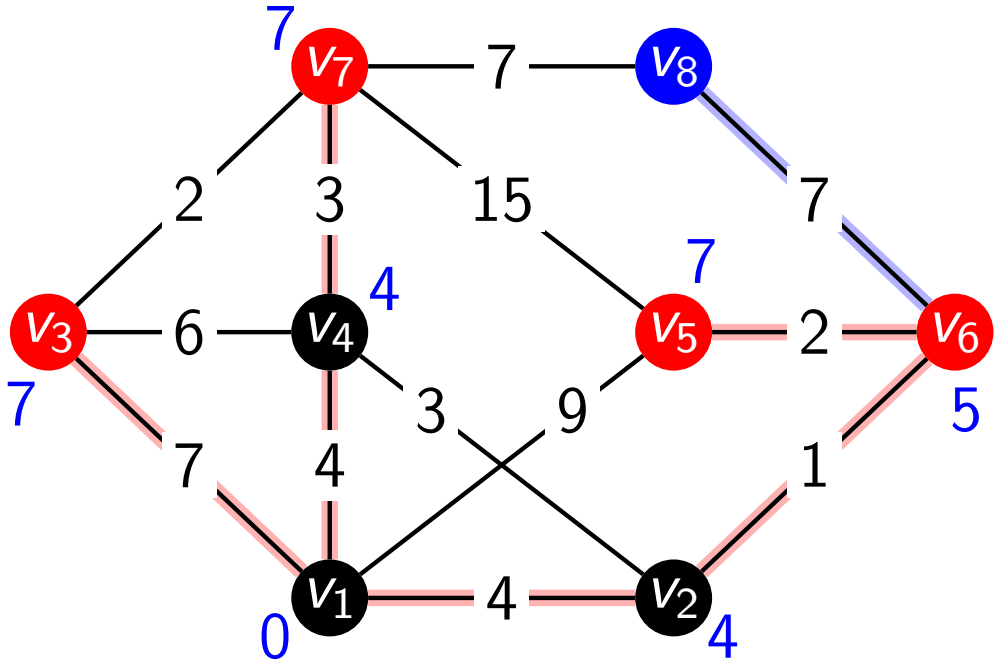
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5)

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

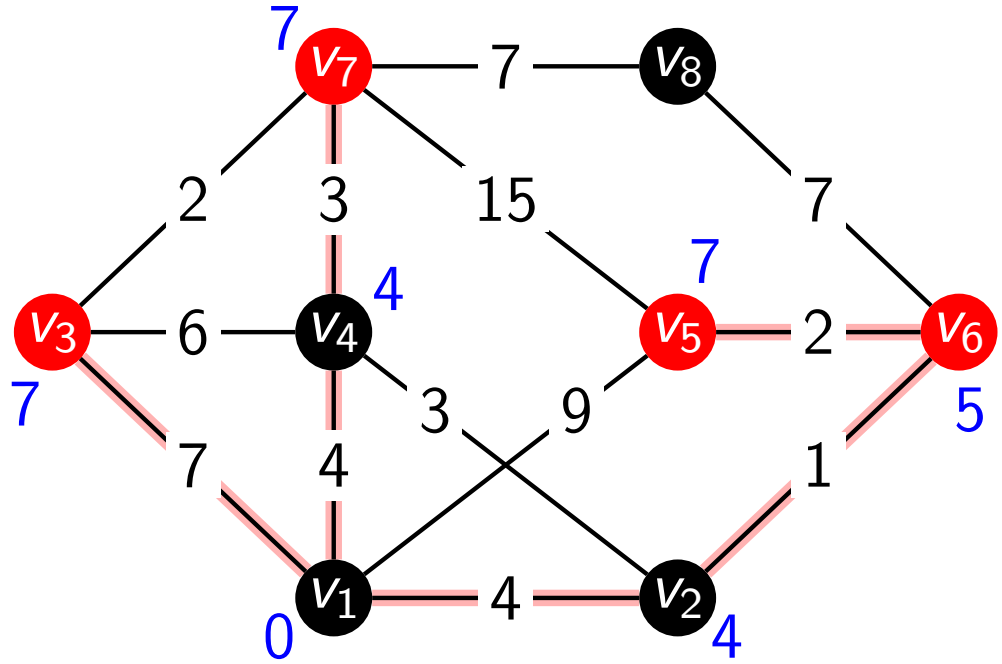
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5) $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

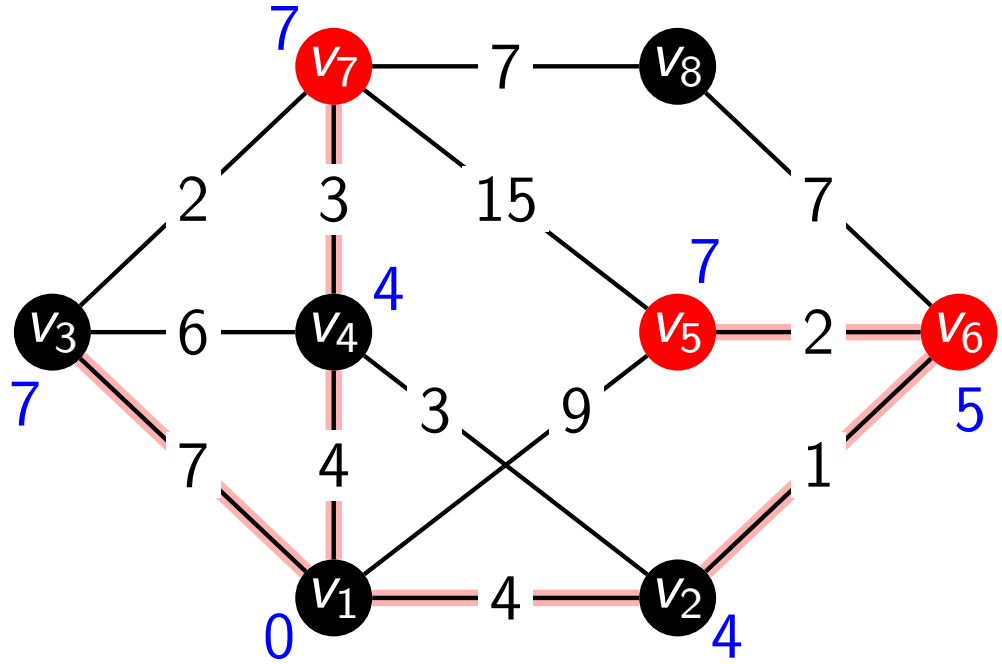
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5) $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

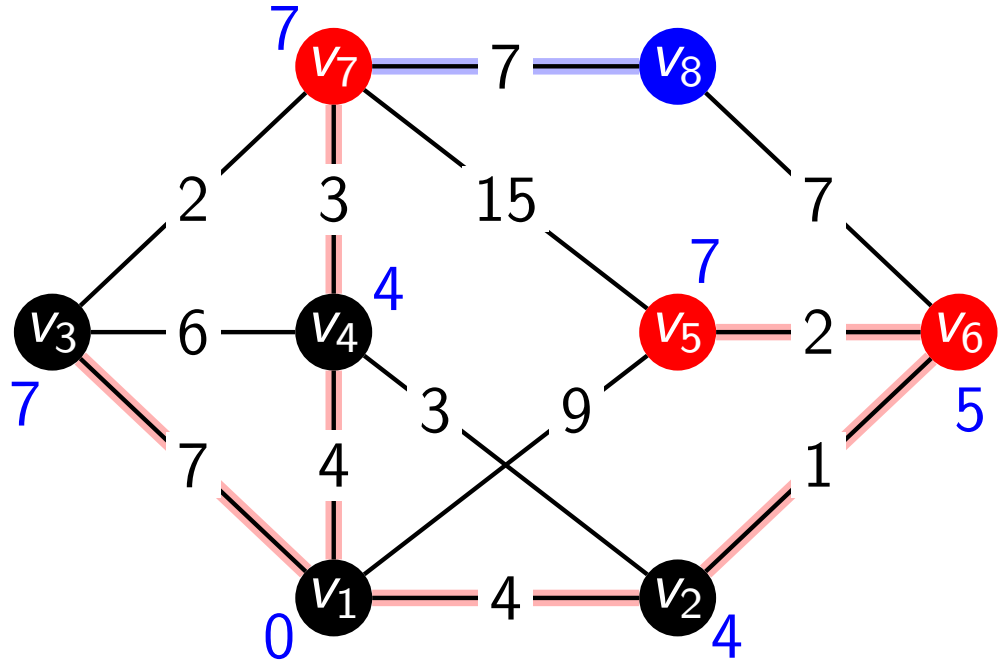
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5) $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

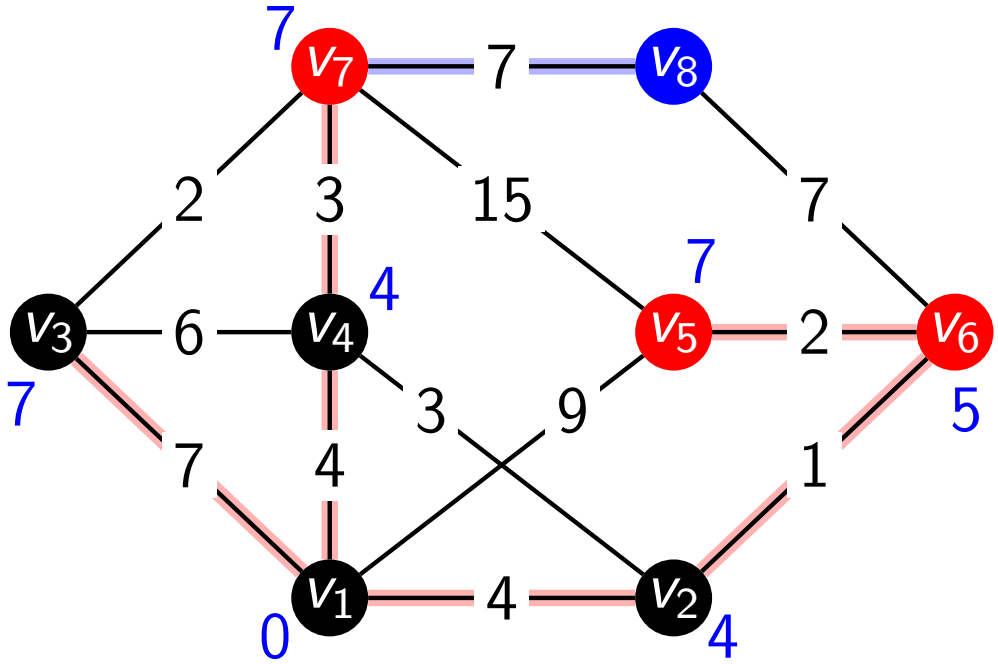
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5) $v_8(v_6,12)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

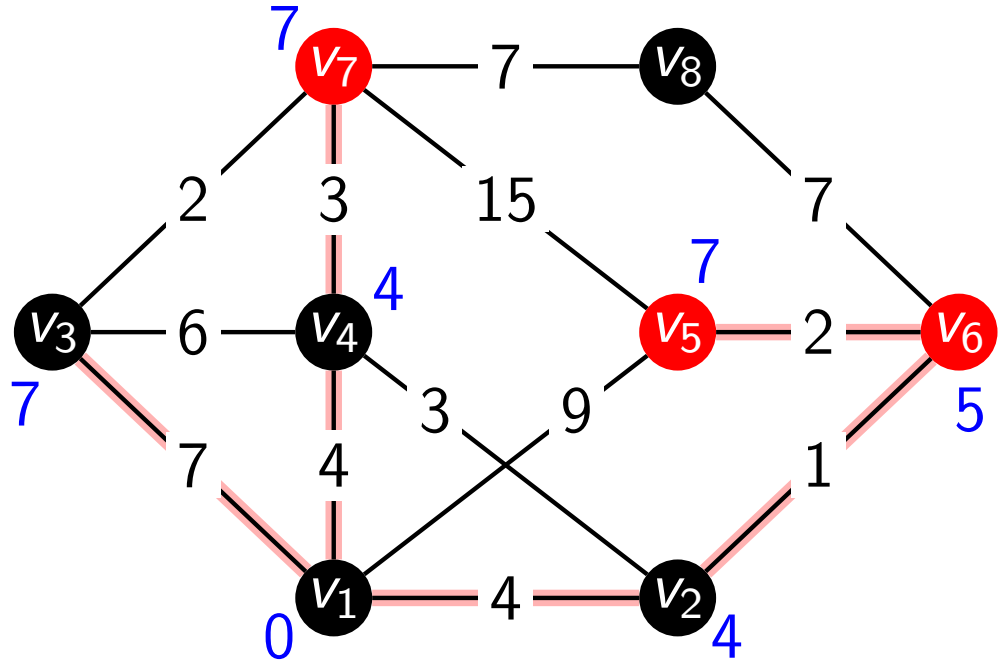
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}, v_8(v_6,12)$

5) $v_8(v_6,12), v_8(v_7,14)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

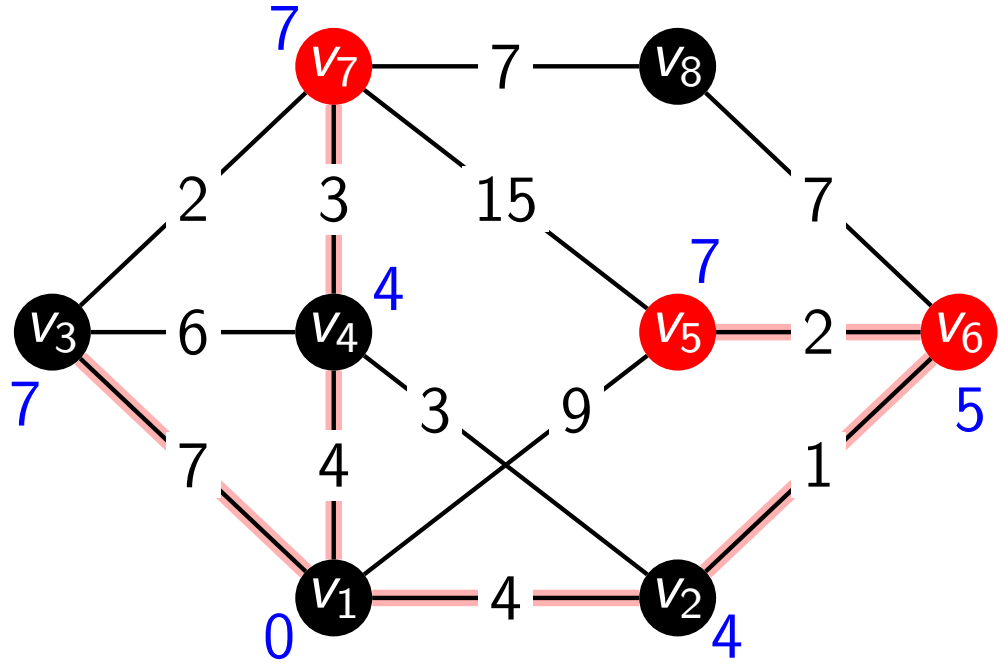
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}^{+}, v_8(v_6,12)$

5) $v_8(v_6,12), v_8(v_7,14)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

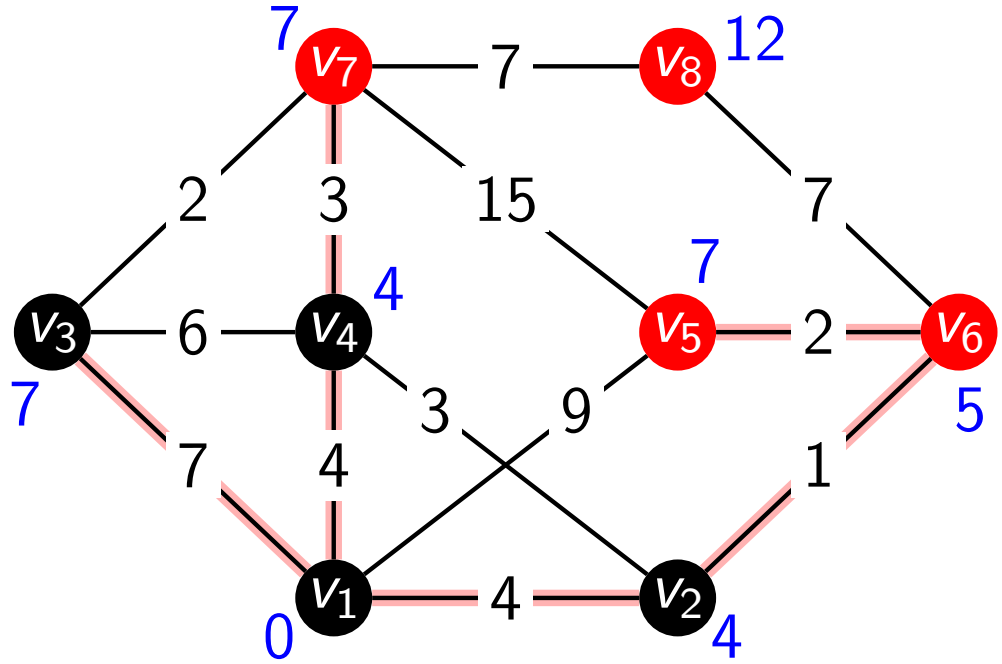
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}^{+}, v_8(v_6,12)$

5) $\boxed{v_8(v_6,12)}, v_8(v_7,14)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

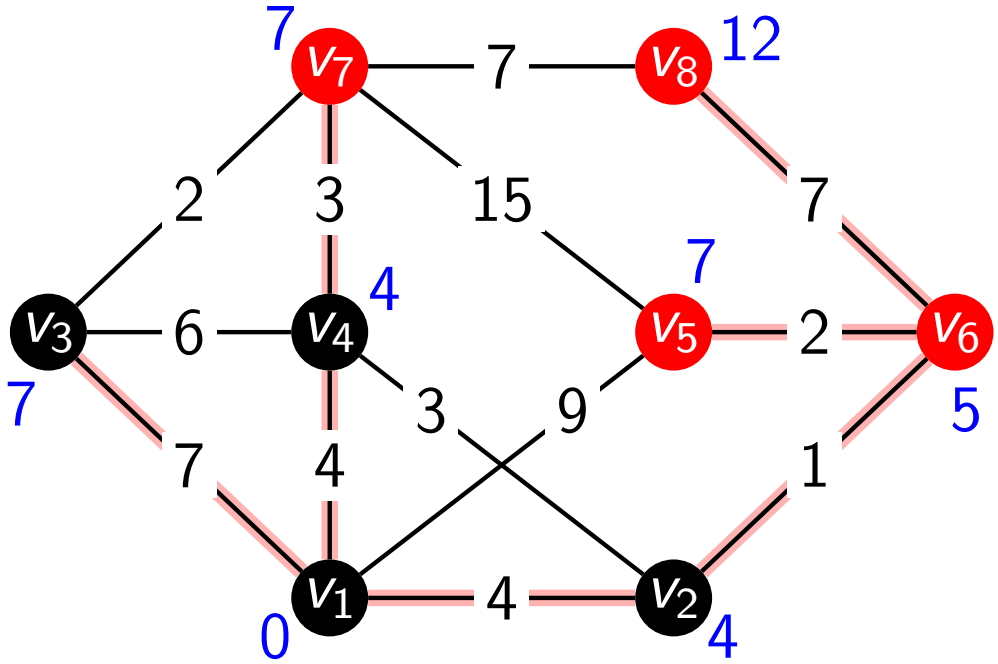
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}^{+}, v_8(v_6,12)$

5) $\boxed{v_8(v_6,12)}, v_8(v_7,14)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

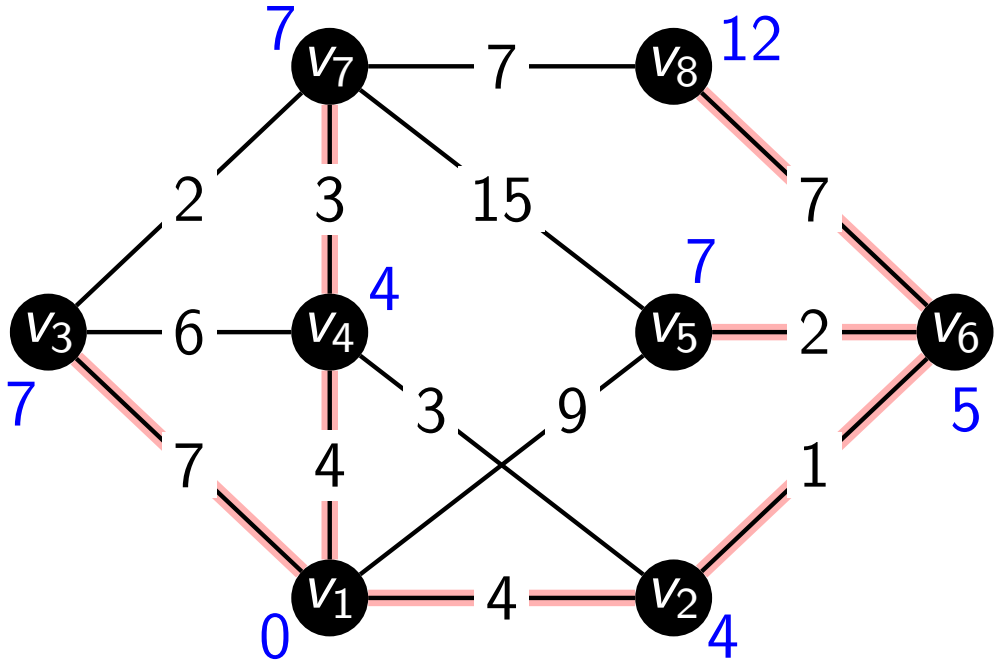
3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}^{+}, v_8(v_6,12)$

5) $\boxed{v_8(v_6,12)}, v_8(v_7,14)$

Rješenje

a)



1) $\boxed{v_1(-,0)}^{+}$

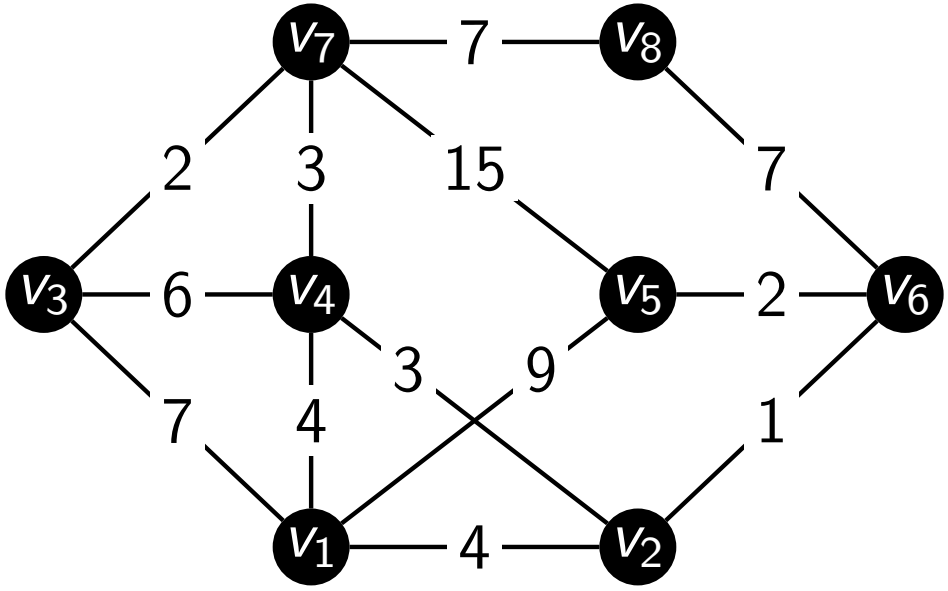
2) $\boxed{v_2(v_1,4)}^{+}, v_3(v_1,7), \boxed{v_4(v_1,4)}^{+}, v_5(v_1,9)$

3) $v_3(v_1,7), v_5(v_1,9), \boxed{v_6(v_2,5)}, v_3(v_4,10), v_7(v_4,7)$

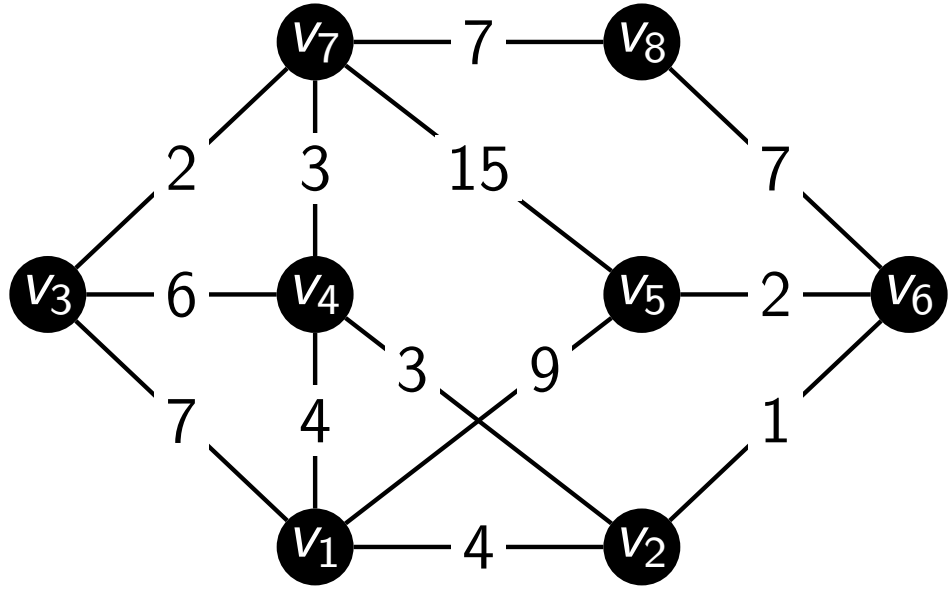
4) $\boxed{v_3(v_1,7)}^{+}, v_5(v_1,9), v_3(v_4,10), \boxed{v_7(v_4,7)}, \boxed{v_5(v_6,7)}^{+}, v_8(v_6,12)$

5) $\boxed{v_8(v_6,12)}, v_8(v_7,14)$

b)

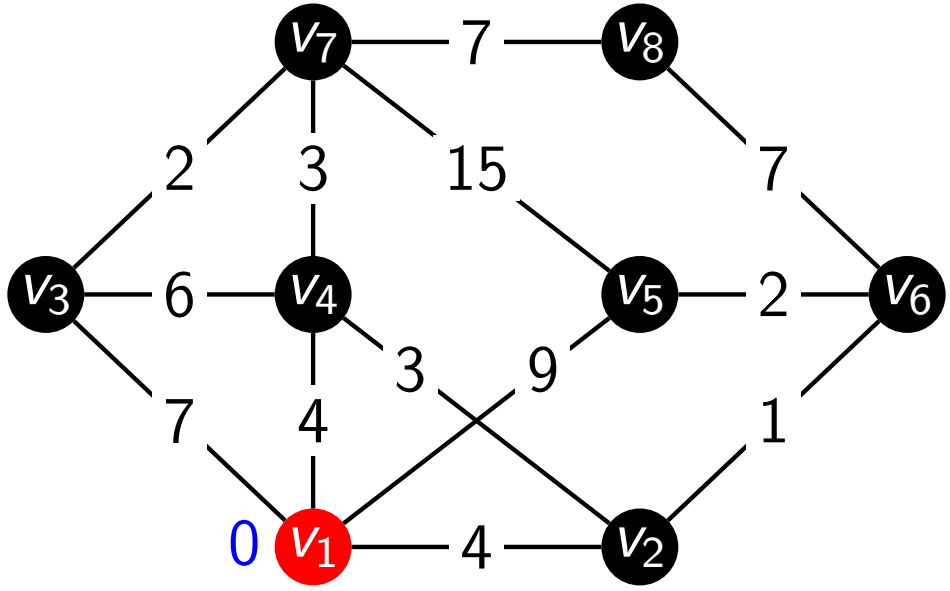


b)



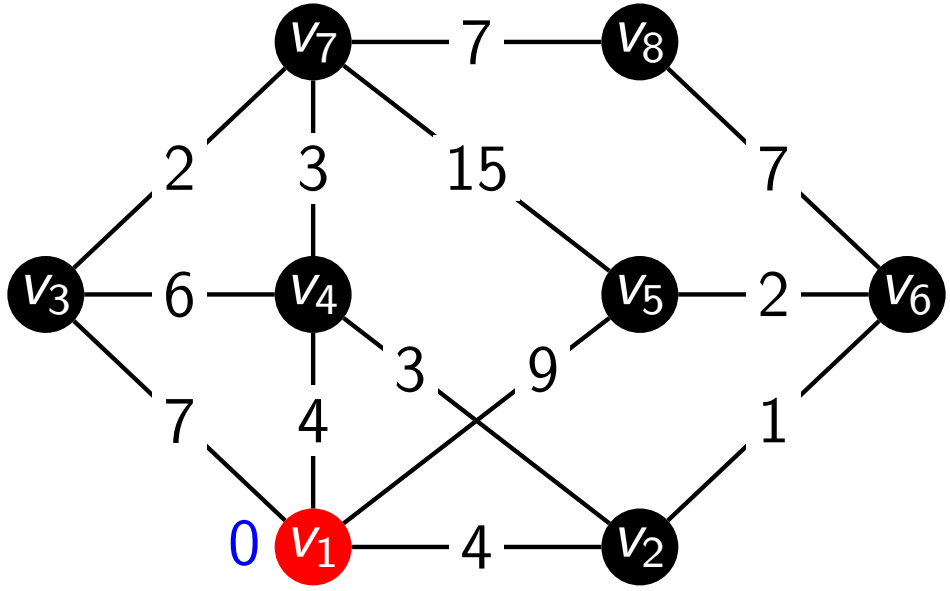
	0							
v_1								
v_2								
v_3								
v_4								
v_5								
v_6								
v_7								
v_8								

b)



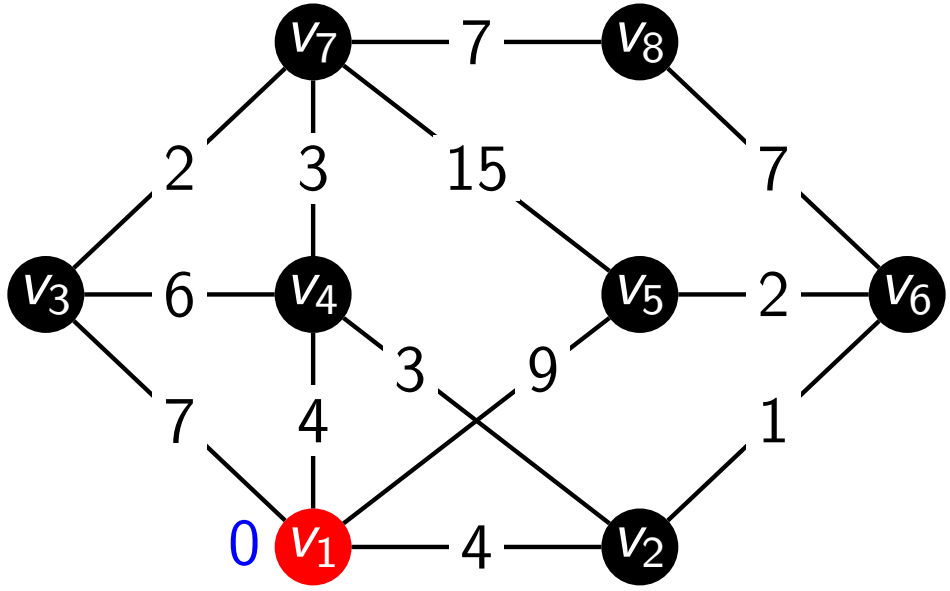
	0							
v_1	$(-, 0)$							
v_2								
v_3								
v_4								
v_5								
v_6								
v_7								
v_8								

b)



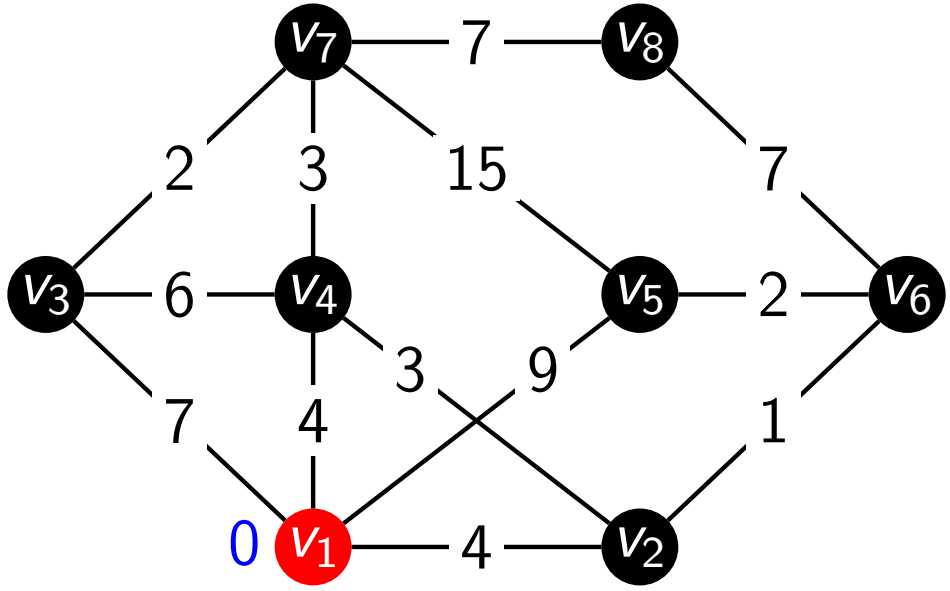
	0							
v_1	$(-, 0)$							
v_2	∞							
v_3	∞							
v_4	∞							
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



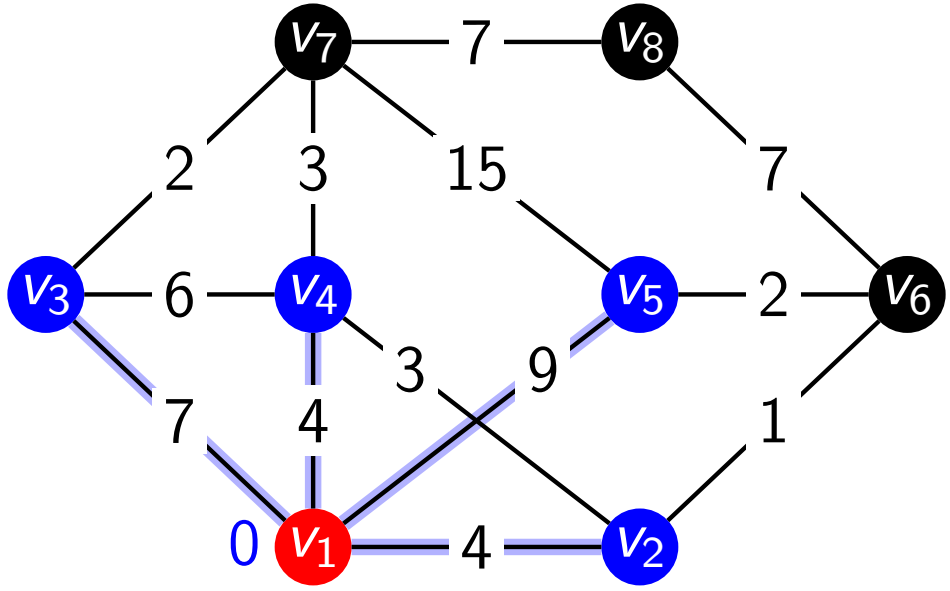
	0	1						
v_1	$(-, 0)$							
v_2	∞							
v_3	∞							
v_4	∞							
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



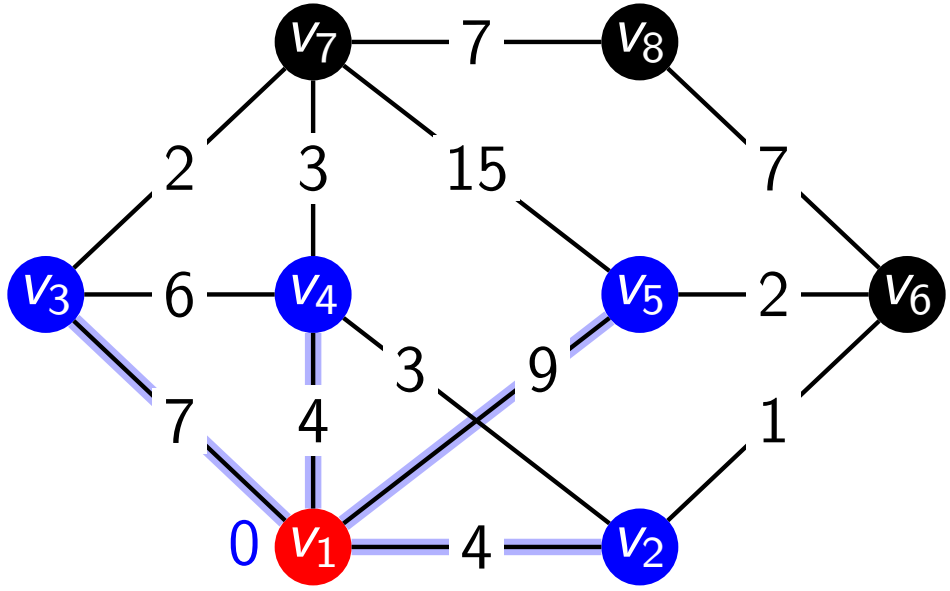
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞							
v_3	∞							
v_4	∞							
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



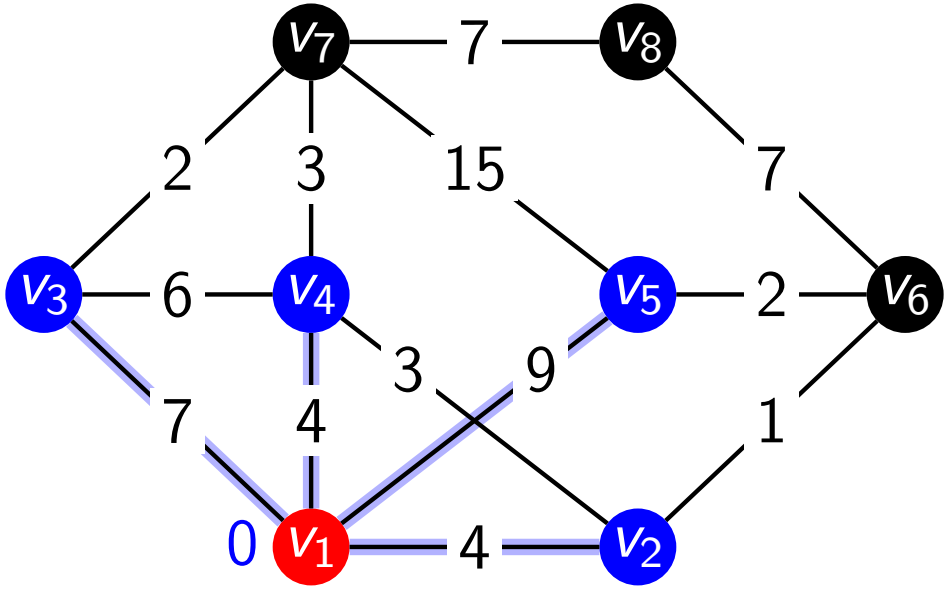
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞							
v_3	∞							
v_4	∞							
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



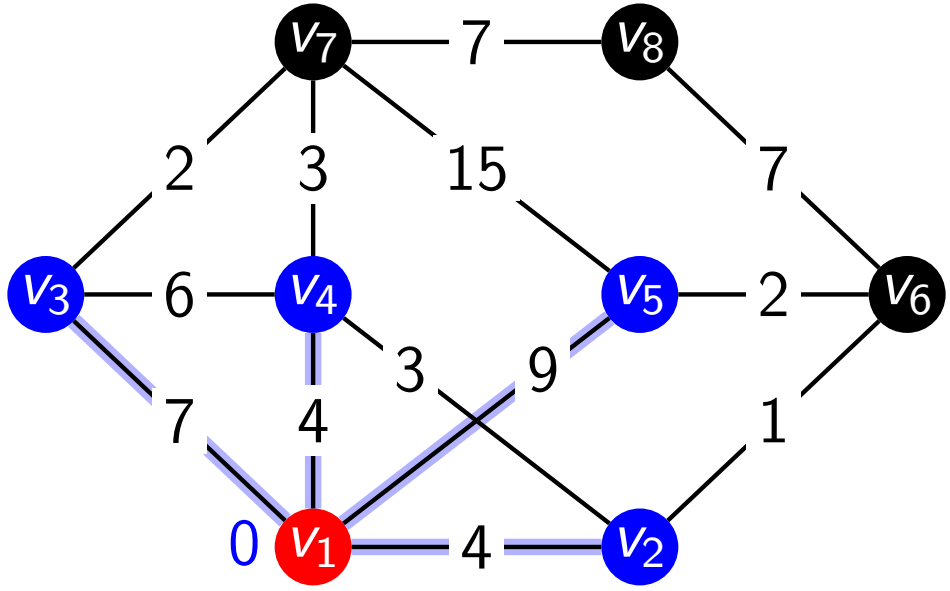
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞							
v_4	∞							
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



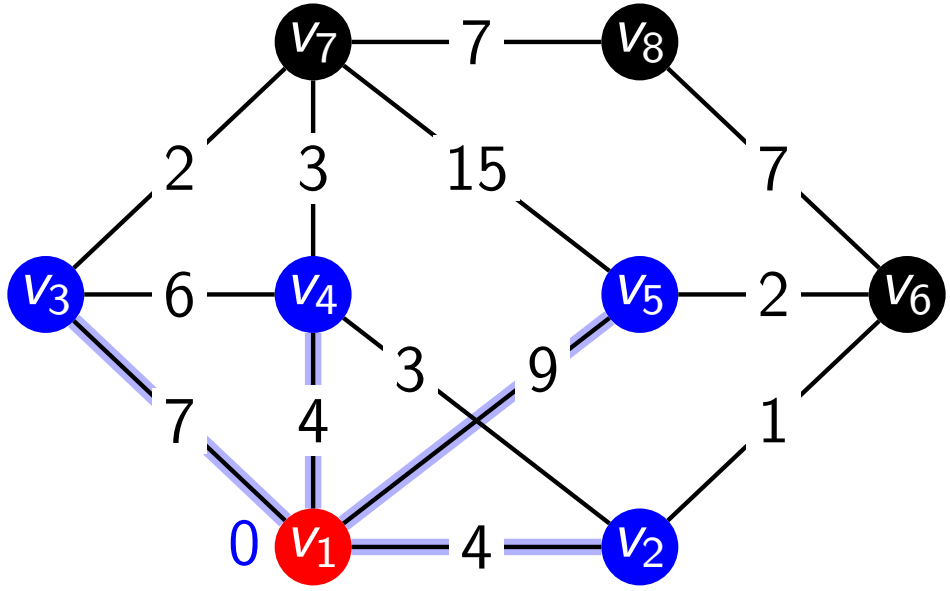
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞							
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



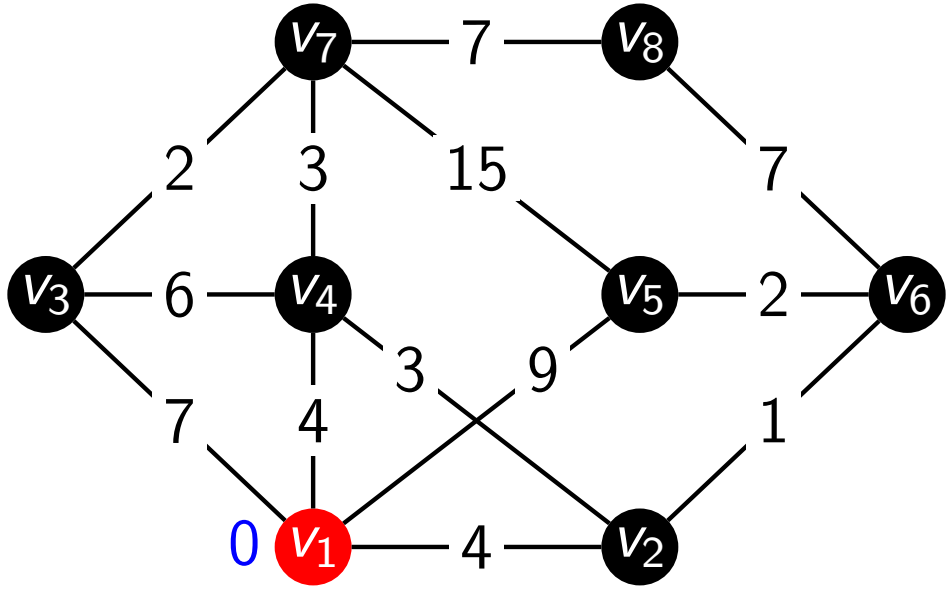
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$v_1, 4$						
v_5	∞							
v_6	∞							
v_7	∞							
v_8	∞							

b)



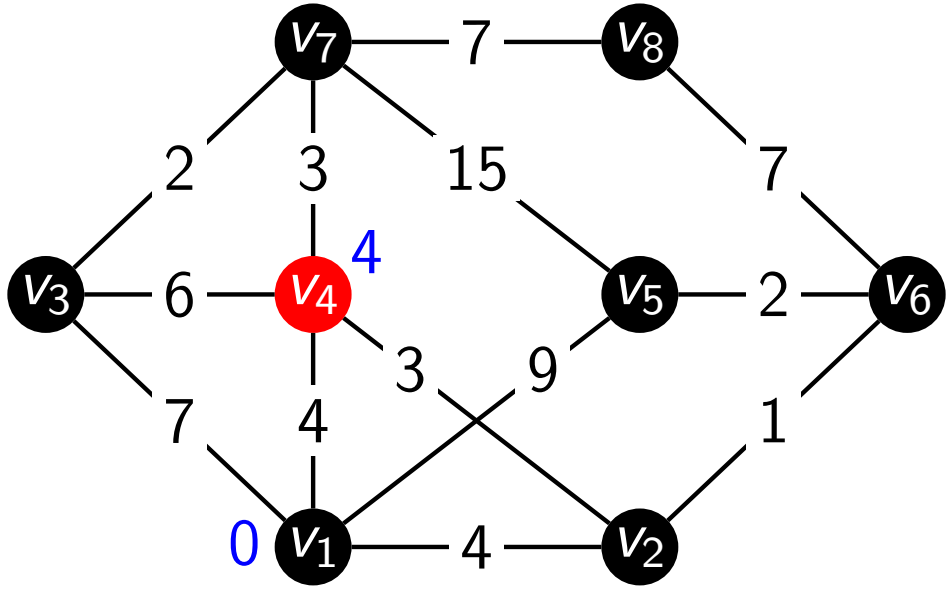
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$v_1, 4$						
v_5	∞	$v_1, 9$						
v_6	∞							
v_7	∞							
v_8	∞							

b)



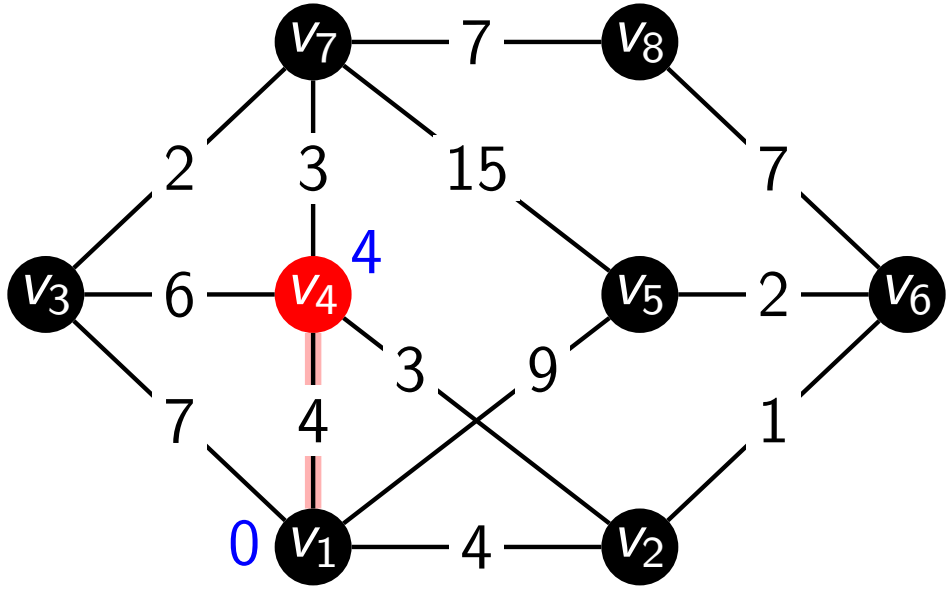
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$v_1, 4$						
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



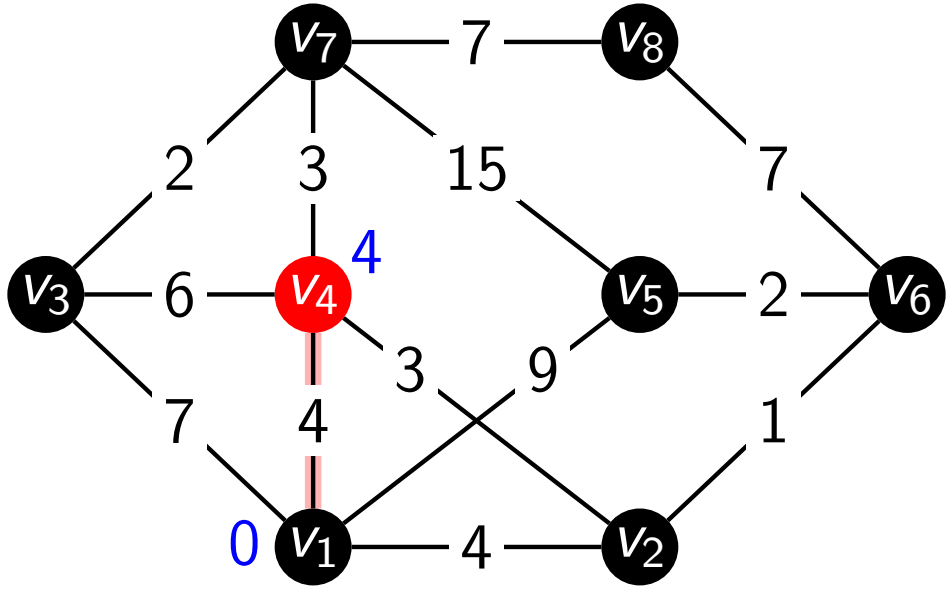
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$(v_1, 4)$						
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



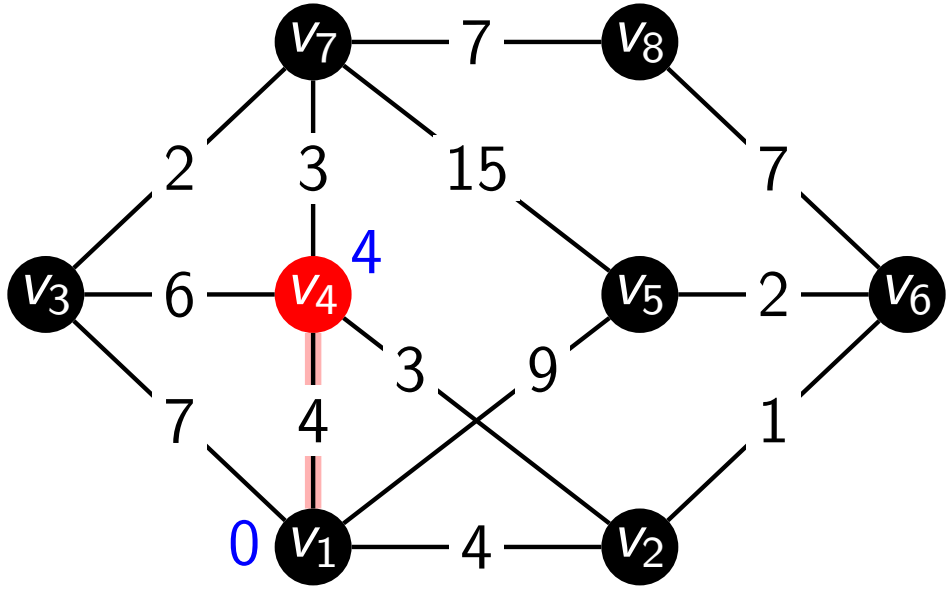
	0	1						
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$(v_1, 4)$						
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



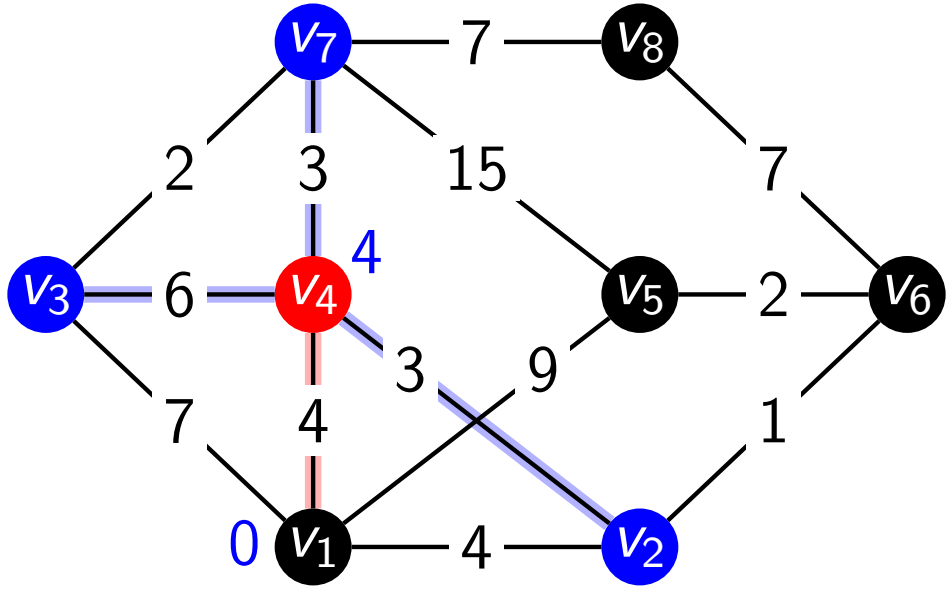
	0	1	2					
v_1	$(-, 0)$	$*$						
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$(v_1, 4)$						
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



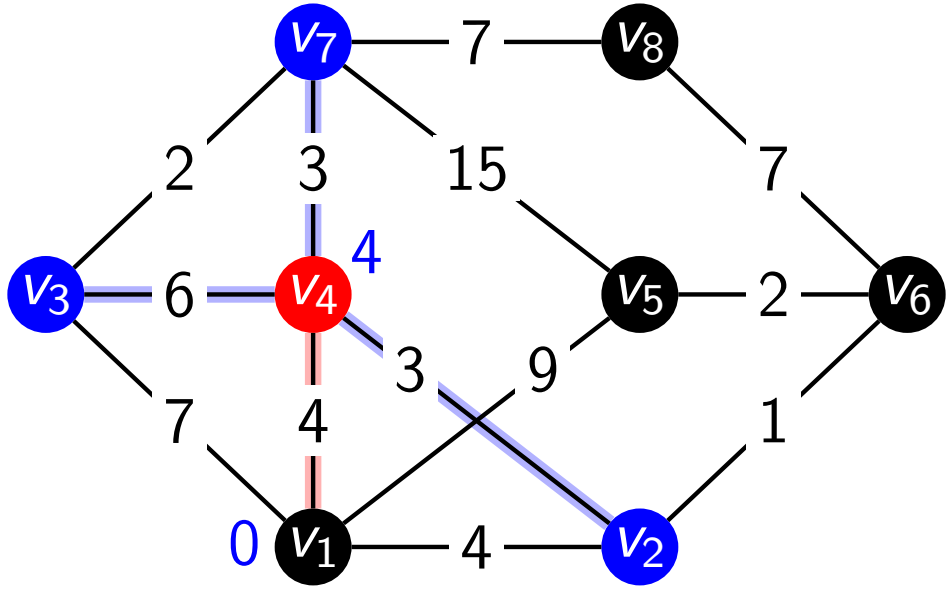
	0	1	2					
v_1	$(-, 0)$	$*$	$*$					
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$(v_1, 4)$	$*$					
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



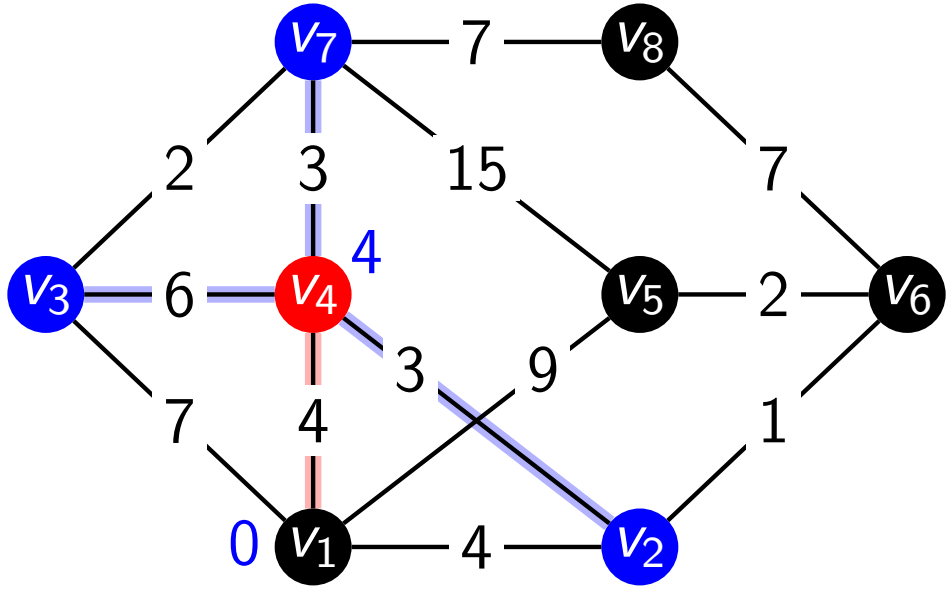
	0	1	2					
v_1	$(-, 0)$	$*$	$*$					
v_2	∞	$v_1, 4$						
v_3	∞	$v_1, 7$						
v_4	∞	$(v_1, 4)$	$*$					
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



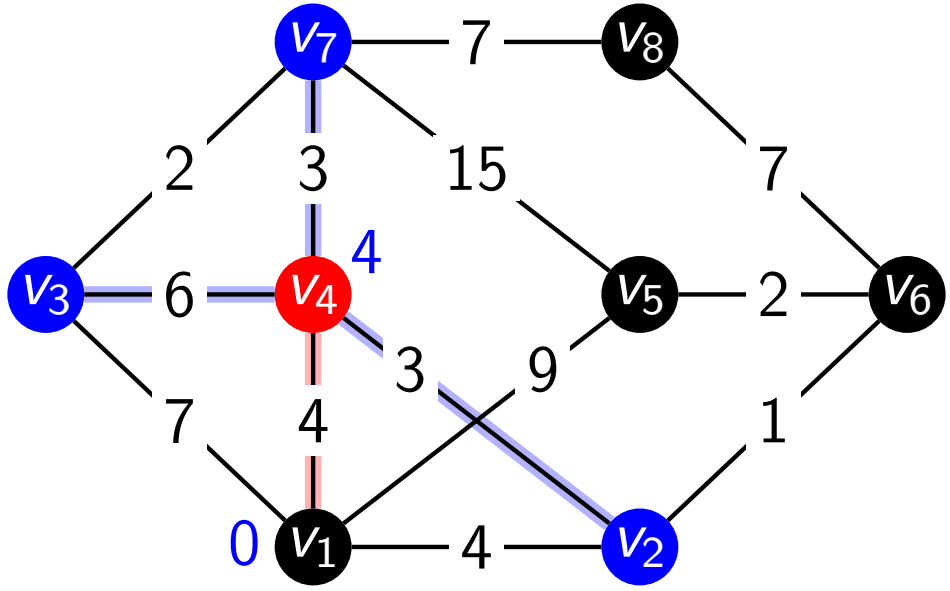
	0	1	2					
v_1	$(-, 0)$	$*$	$*$					
v_2	∞	$v_1, 4$	$v_1, 4$					
v_3	∞	$v_1, 7$						
v_4	∞	$(v_1, 4)$	$*$					
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



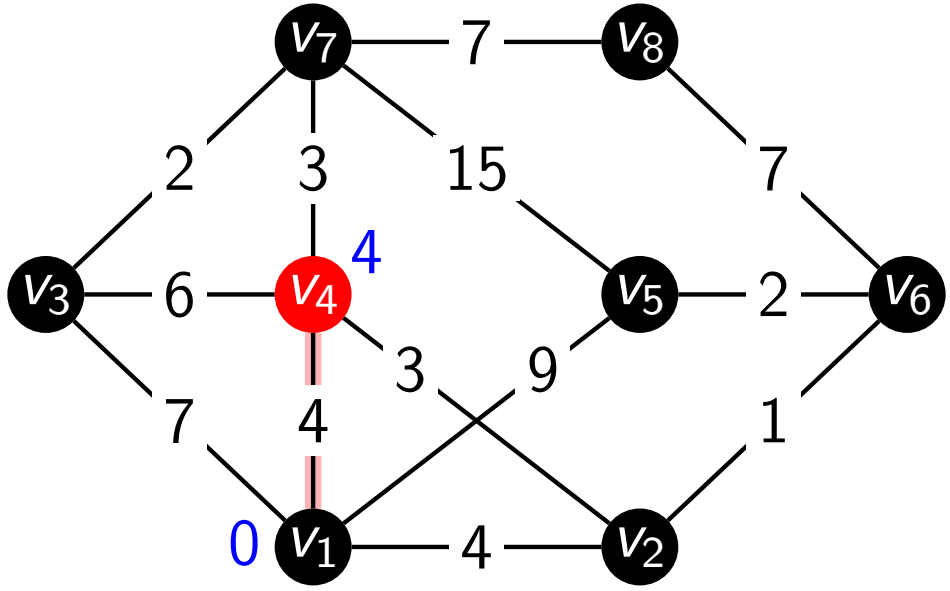
	0	1	2					
v_1	$(-, 0)$	$*$	$*$					
v_2	∞	$v_1, 4$	$v_1, 4$					
v_3	∞	$v_1, 7$	$v_1, 7$					
v_4	∞	$(v_1, 4)$	$*$					
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞						
v_8	∞	∞						

b)



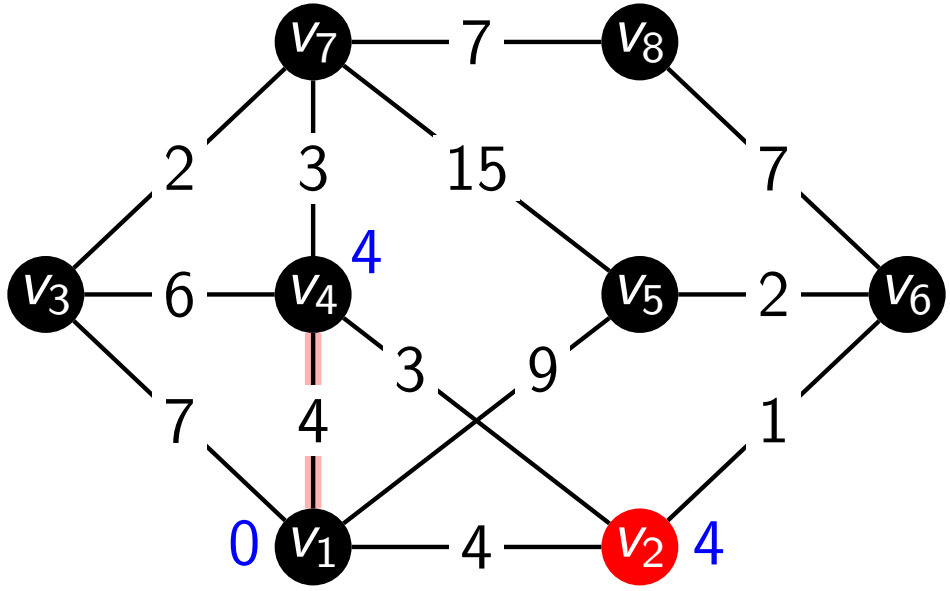
	0	1	2					
v_1	$(-, 0)$	$*$	$*$					
v_2	∞	$v_1, 4$	$v_1, 4$					
v_3	∞	$v_1, 7$	$v_1, 7$					
v_4	∞	$(v_1, 4)$	$*$					
v_5	∞	$v_1, 9$						
v_6	∞	∞						
v_7	∞	∞	$v_4, 7$					
v_8	∞	∞						

b)



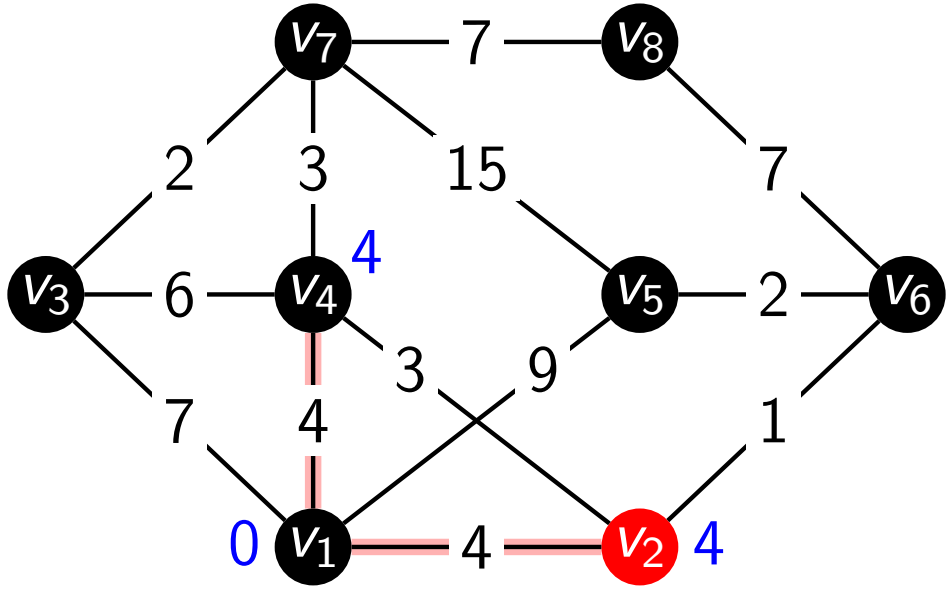
	0	1	2					
v_1	$(-,0)$	$*$	$*$					
v_2	∞	$v_1,4$	$v_1,4$					
v_3	∞	$v_1,7$	$v_1,7$					
v_4	∞	$(v_1,4)$	$*$					
v_5	∞	$v_1,9$	$v_1,9$					
v_6	∞	∞	∞					
v_7	∞	∞	$v_4,7$					
v_8	∞	∞	∞					

b)



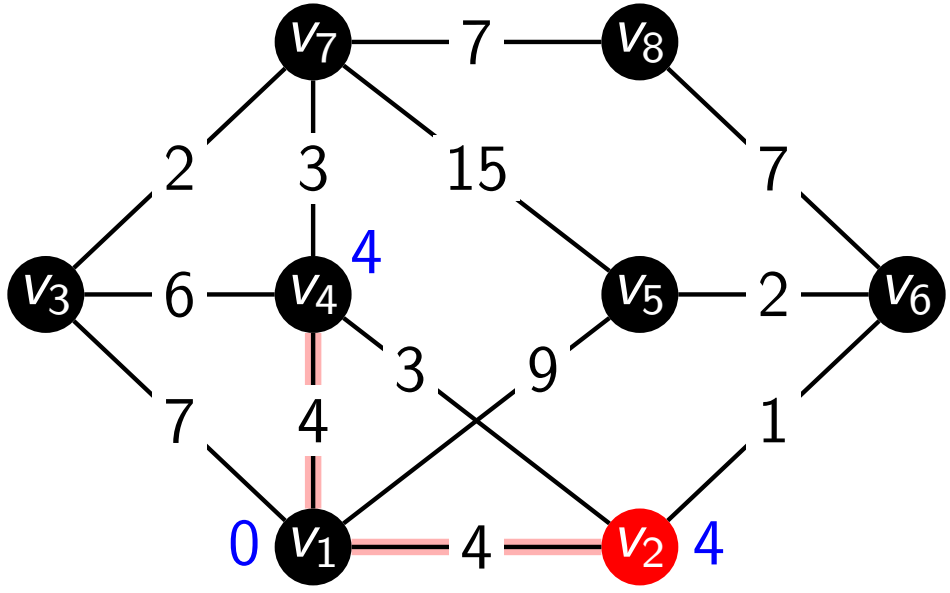
	0	1	2					
v_1	$(-,0)$	$*$	$*$					
v_2	∞	$v_1,4$	$(v_1,4)$					
v_3	∞	$v_1,7$	$v_1,7$					
v_4	∞	$(v_1,4)$	$*$					
v_5	∞	$v_1,9$	$v_1,9$					
v_6	∞	∞	∞					
v_7	∞	∞	$v_4,7$					
v_8	∞	∞	∞					

b)



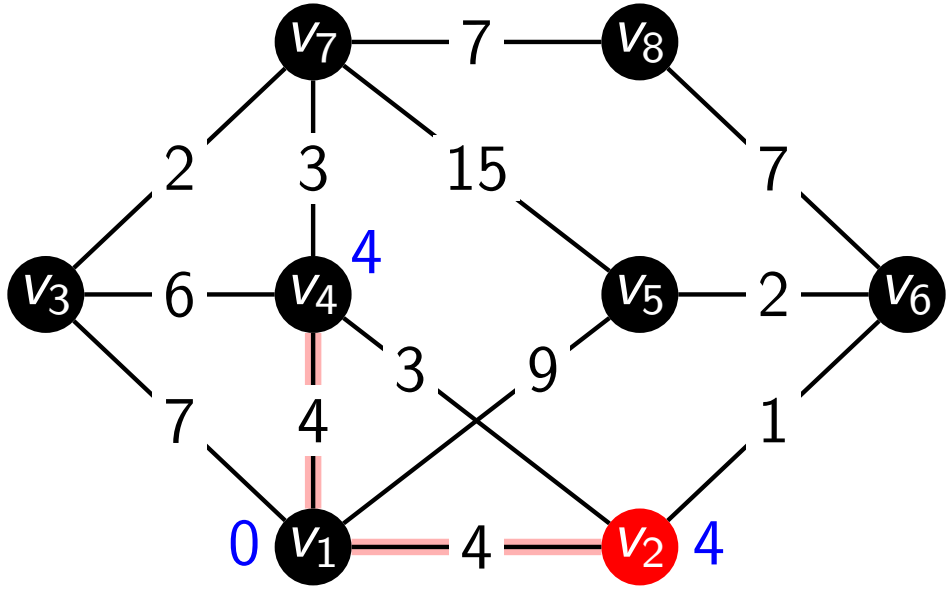
	0	1	2					
v_1	$(-,0)$	$*$	$*$					
v_2	∞	$v_1,4$	$(v_1,4)$					
v_3	∞	$v_1,7$	$v_1,7$					
v_4	∞	$(v_1,4)$	$*$					
v_5	∞	$v_1,9$	$v_1,9$					
v_6	∞	∞	∞					
v_7	∞	∞	$v_4,7$					
v_8	∞	∞	∞					

b)



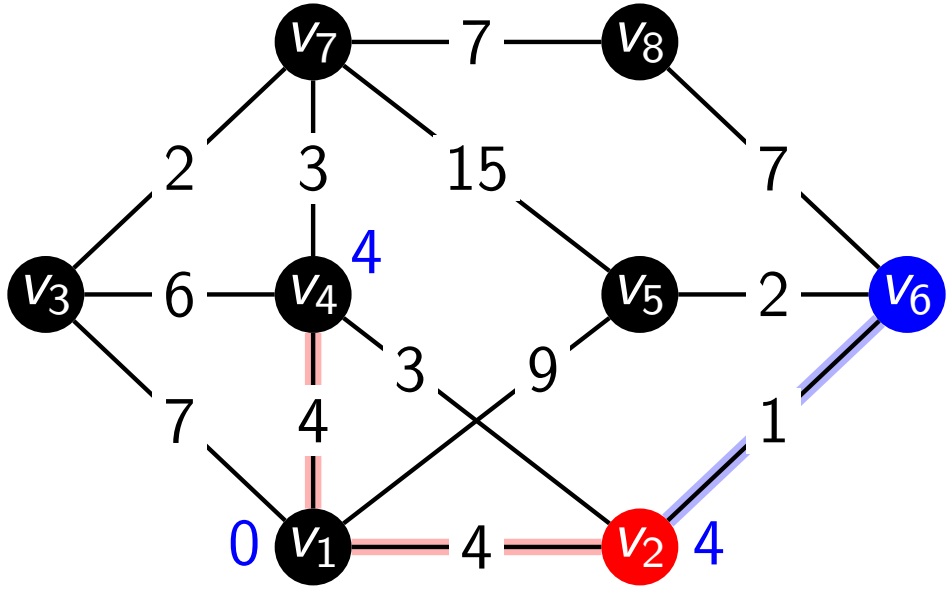
	0	1	2	3				
v_1	$(-, 0)$	$*$	$*$					
v_2	∞	$v_1, 4$	$(v_1, 4)$					
v_3	∞	$v_1, 7$	$v_1, 7$					
v_4	∞	$(v_1, 4)$	$*$					
v_5	∞	$v_1, 9$	$v_1, 9$					
v_6	∞	∞	∞					
v_7	∞	∞	$v_4, 7$					
v_8	∞	∞	∞					

b)



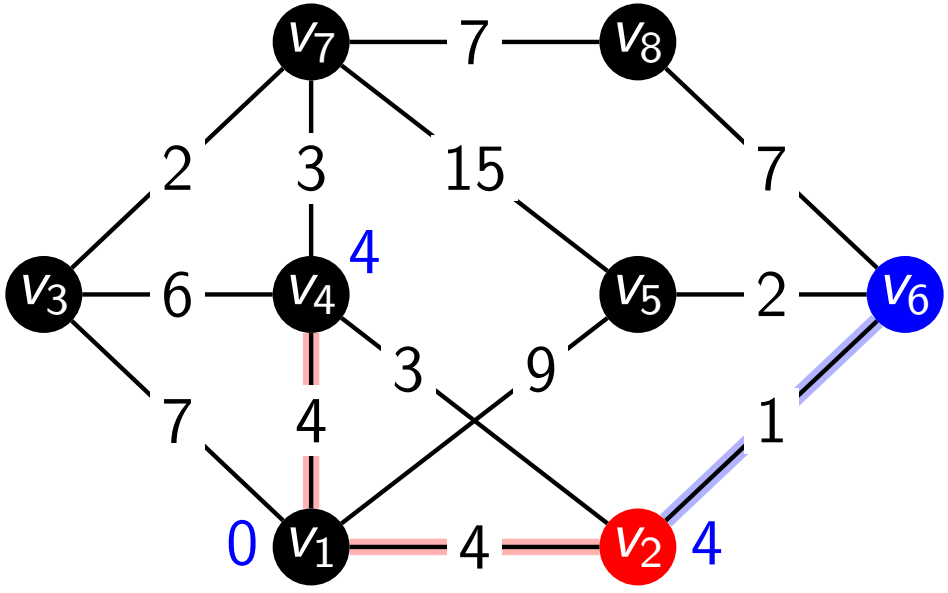
	0	1	2	3				
v_1	$(-,0)$	$*$	$*$	$*$				
v_2	∞	$v_1,4$	$(v_1,4)$	$*$				
v_3	∞	$v_1,7$	$v_1,7$					
v_4	∞	$(v_1,4)$	$*$	$*$				
v_5	∞	$v_1,9$	$v_1,9$					
v_6	∞	∞	∞					
v_7	∞	∞	$v_4,7$					
v_8	∞	∞	∞					

b)



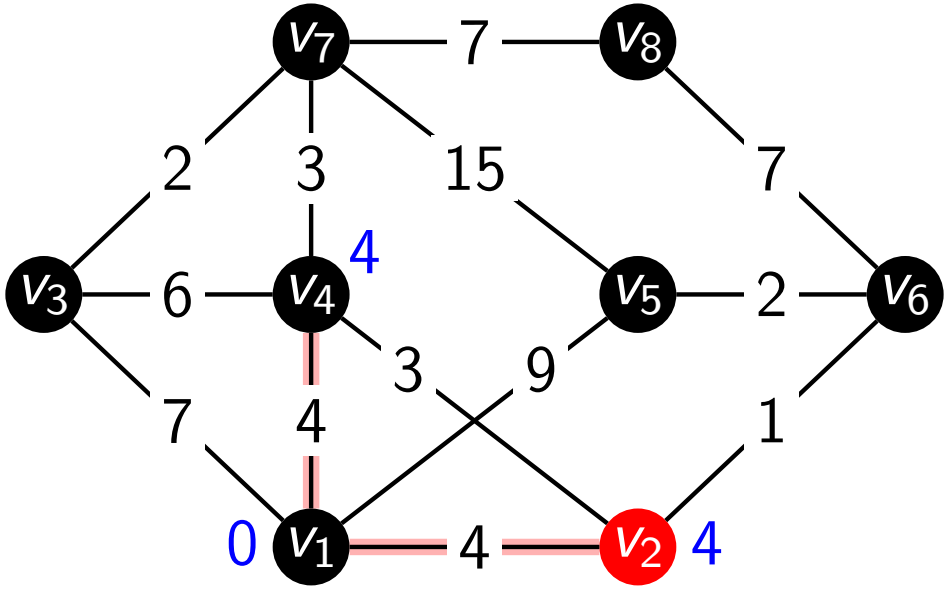
	0	1	2	3				
v_1	$(-,0)$	$*$	$*$	$*$				
v_2	∞	$v_1,4$	$(v_1,4)$	$*$				
v_3	∞	$v_1,7$	$v_1,7$					
v_4	∞	$(v_1,4)$	$*$	$*$				
v_5	∞	$v_1,9$	$v_1,9$					
v_6	∞	∞	∞					
v_7	∞	∞	$v_4,7$					
v_8	∞	∞	∞					

b)



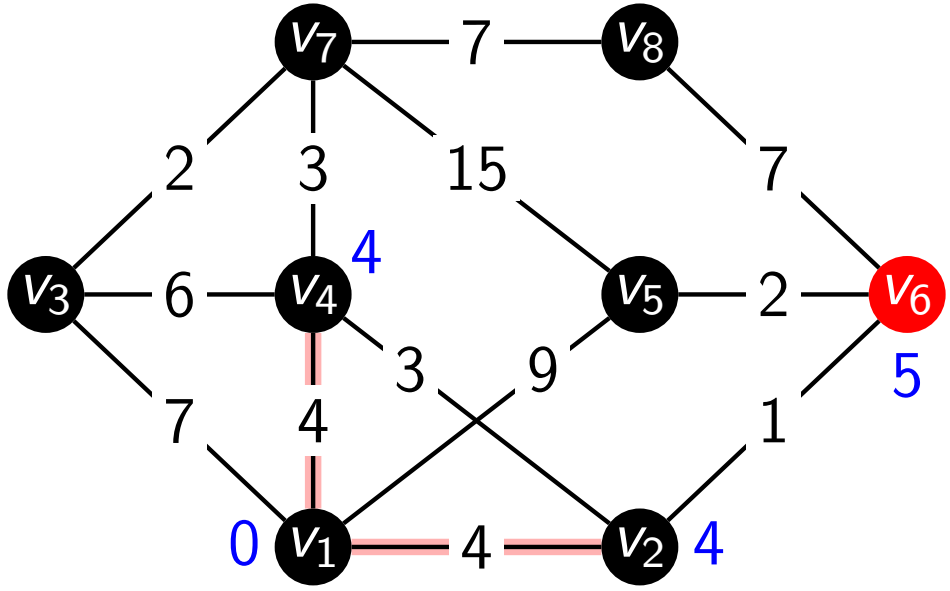
	0	1	2	3				
v_1	$(-,0)$	$*$	$*$	$*$				
v_2	∞	$v_1,4$	$(v_1,4)$	$*$				
v_3	∞	$v_1,7$	$v_1,7$					
v_4	∞	$(v_1,4)$	$*$	$*$				
v_5	∞	$v_1,9$	$v_1,9$					
v_6	∞	∞	∞	$v_2,5$				
v_7	∞	∞	$v_4,7$					
v_8	∞	∞	∞					

b)



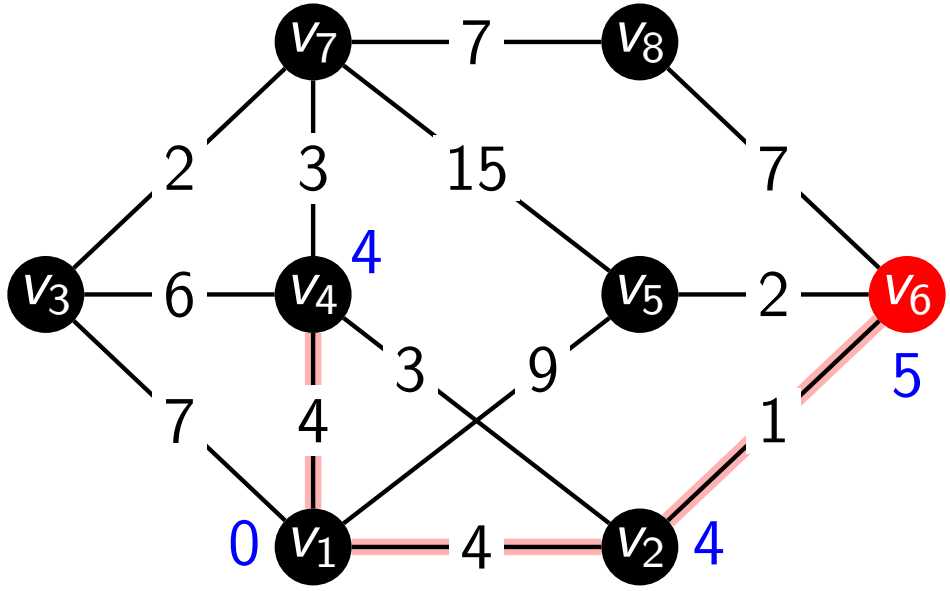
	0	1	2	3				
v_1	$(-,0)$	$*$	$*$	$*$				
v_2	∞	$v_1,4$	$(v_1,4)$	$*$				
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$				
v_4	∞	$(v_1,4)$	$*$	$*$				
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$				
v_6	∞	∞	∞	$v_2,5$				
v_7	∞	∞	$v_4,7$	$v_4,7$				
v_8	∞	∞	∞	∞				

b)



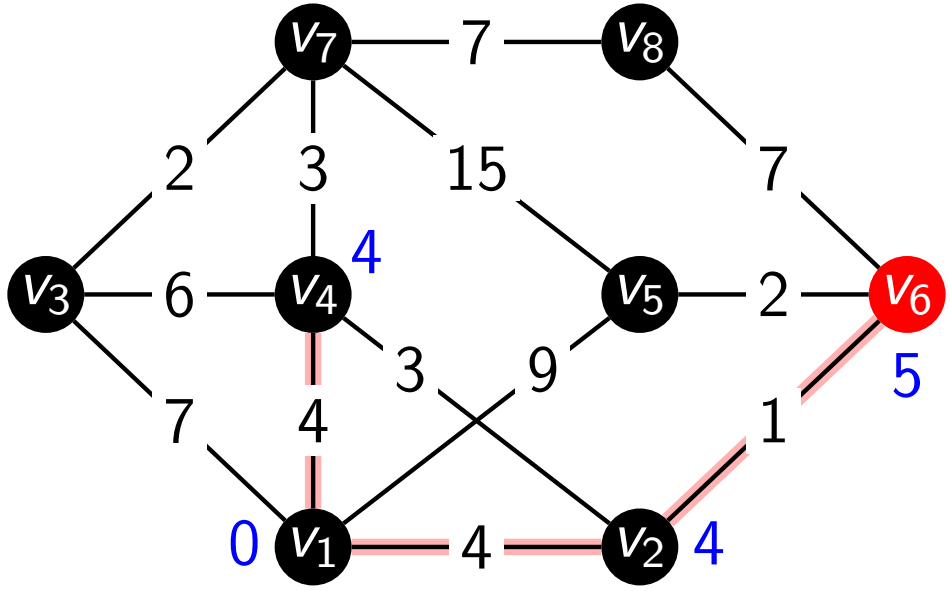
	0	1	2	3				
v_1	$(-, 0)$	$*$	$*$	$*$				
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$				
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$				
v_4	∞	$(v_1, 4)$	$*$	$*$				
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$				
v_6	∞	∞	∞	$(v_2, 5)$				
v_7	∞	∞	$v_4, 7$	$v_4, 7$				
v_8	∞	∞	∞	∞				

b)



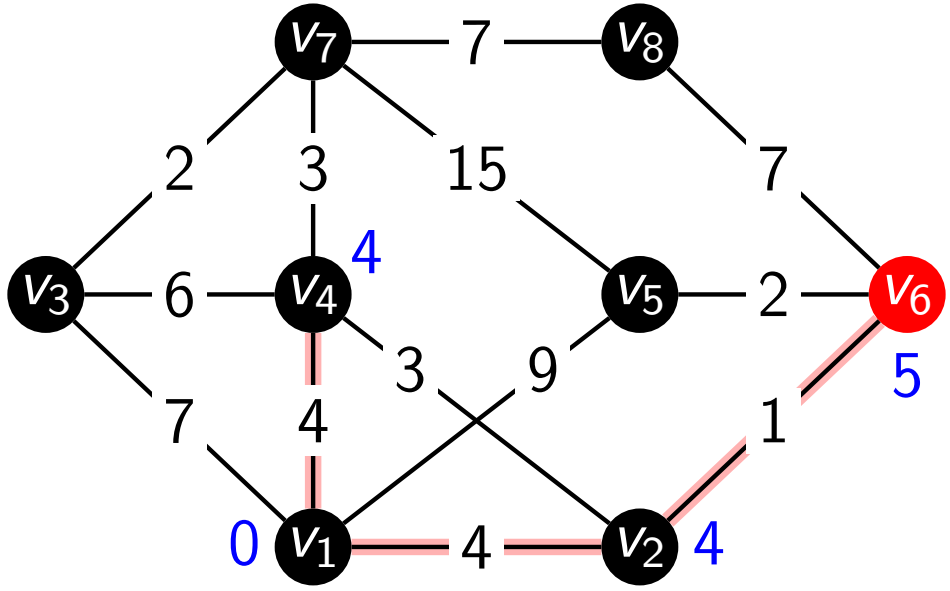
	0	1	2	3				
v_1	$(-, 0)$	$*$	$*$	$*$				
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$				
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$				
v_4	∞	$(v_1, 4)$	$*$	$*$				
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$				
v_6	∞	∞	∞	$(v_2, 5)$				
v_7	∞	∞	$v_4, 7$	$v_4, 7$				
v_8	∞	∞	∞	∞				

b)



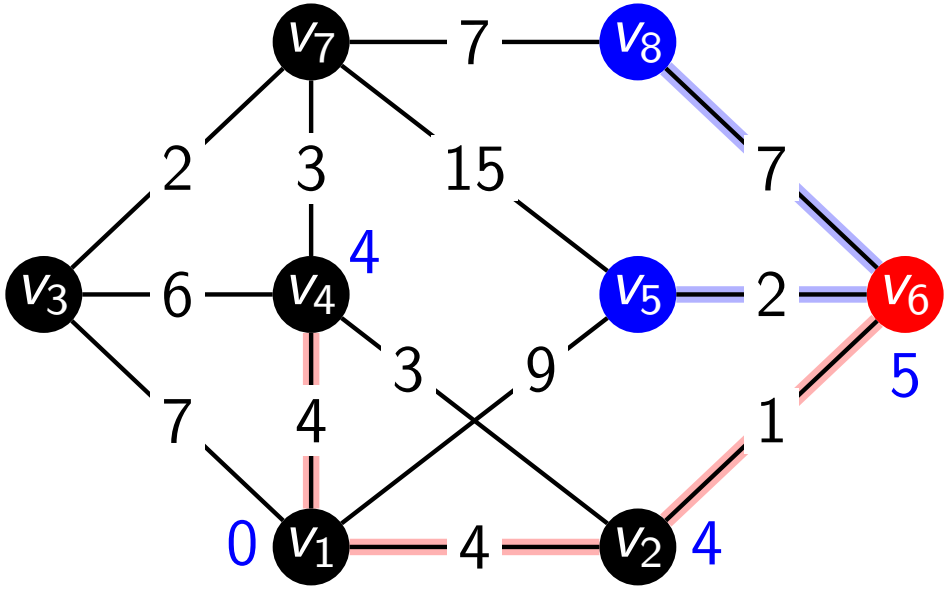
	0	1	2	3	4			
v_1	$(-, 0)$	$*$	$*$	$*$				
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$				
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$				
v_4	∞	$(v_1, 4)$	$*$	$*$				
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$				
v_6	∞	∞	∞	$(v_2, 5)$				
v_7	∞	∞	$v_4, 7$	$v_4, 7$				
v_8	∞	∞	∞	∞				

b)



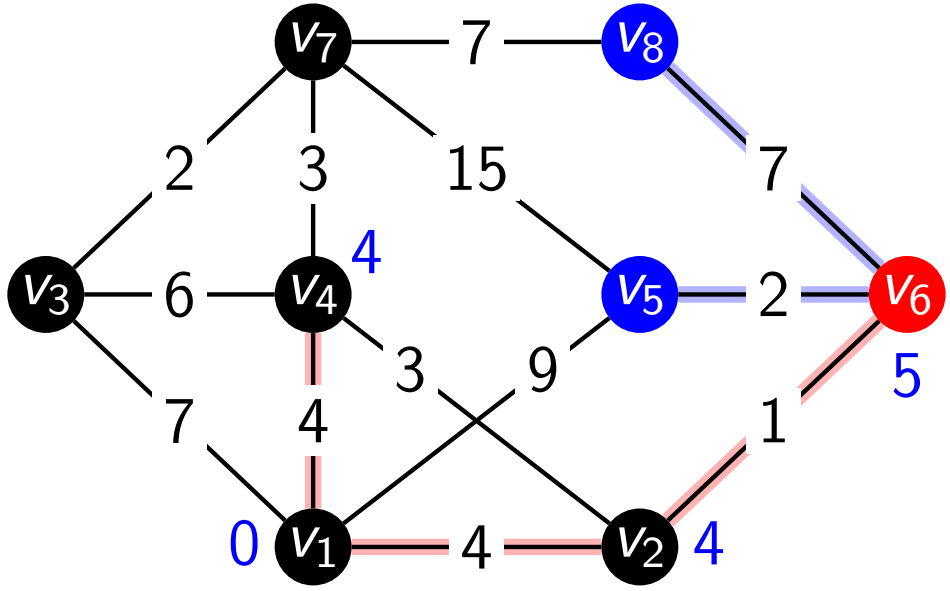
	0	1	2	3	4			
v_1	$(-,0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$			
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$				
v_4	∞	$(v_1,4)$	$*$	$*$	$*$			
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$				
v_6	∞	∞	∞	$(v_2,5)$	$*$			
v_7	∞	∞	$v_4,7$	$v_4,7$				
v_8	∞	∞	∞	∞				

b)



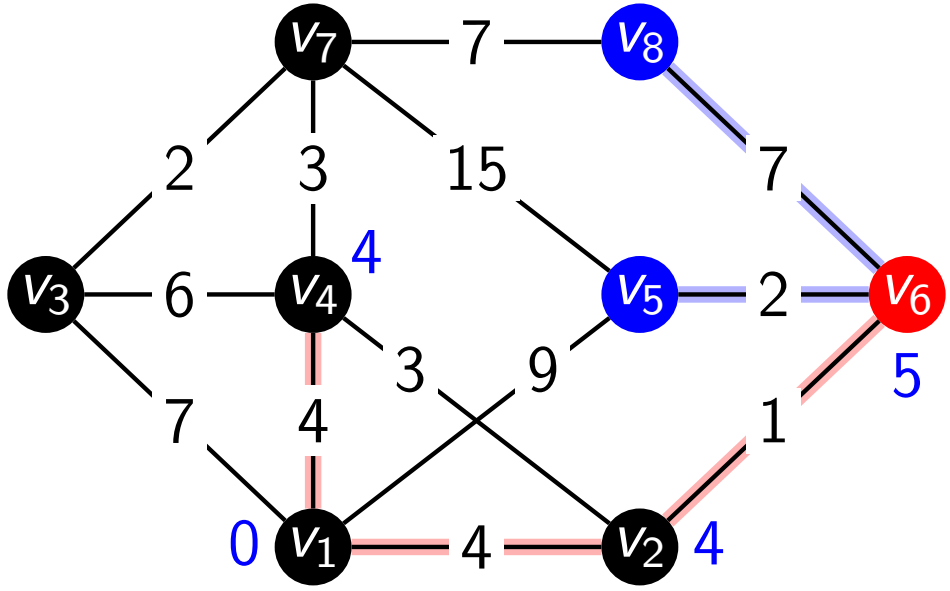
	0	1	2	3	4			
v_1	$(-,0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$			
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$				
v_4	∞	$(v_1,4)$	$*$	$*$	$*$			
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$				
v_6	∞	∞	∞	$(v_2,5)$	$*$			
v_7	∞	∞	$v_4,7$	$v_4,7$				
v_8	∞	∞	∞	∞				

b)



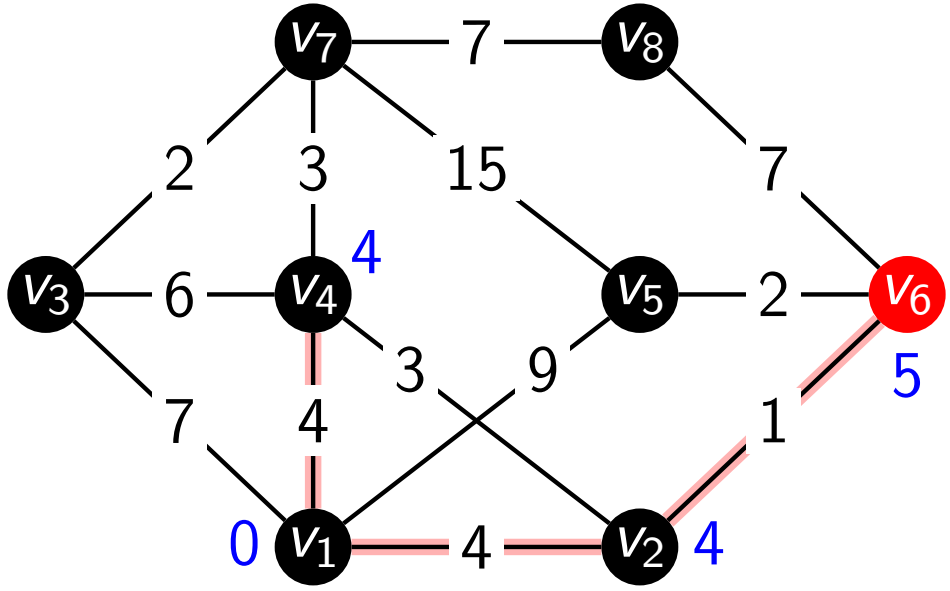
	0	1	2	3	4			
v_1	$(-,0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$			
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$				
v_4	∞	$(v_1,4)$	$*$	$*$	$*$			
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$			
v_6	∞	∞	∞	$(v_2,5)$	$*$			
v_7	∞	∞	$v_4,7$	$v_4,7$				
v_8	∞	∞	∞	∞				

b)



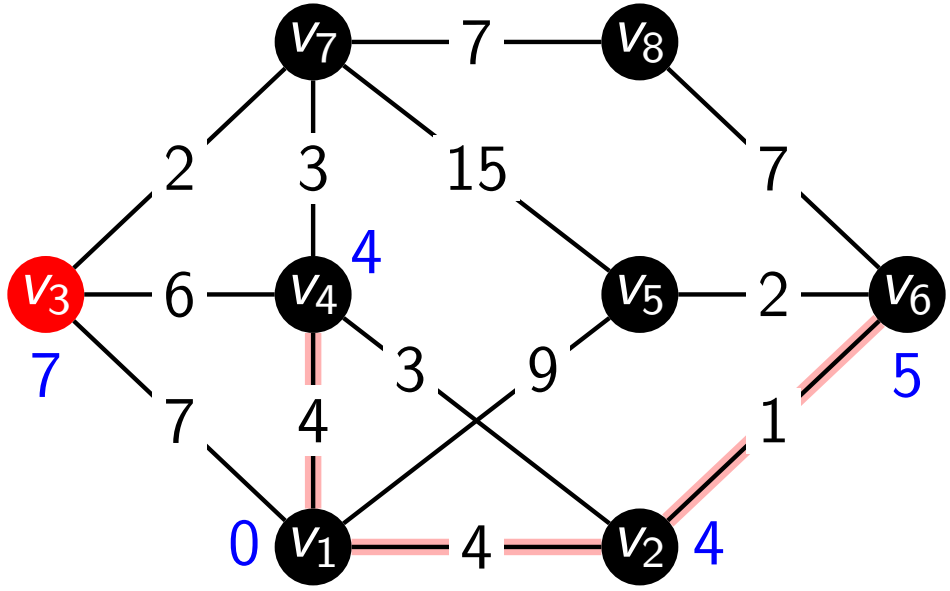
	0	1	2	3	4			
v_1	$(-,0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$			
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$				
v_4	∞	$(v_1,4)$	$*$	$*$	$*$			
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$			
v_6	∞	∞	∞	$(v_2,5)$	$*$			
v_7	∞	∞	$v_4,7$	$v_4,7$				
v_8	∞	∞	∞	∞	$v_6,12$			

b)



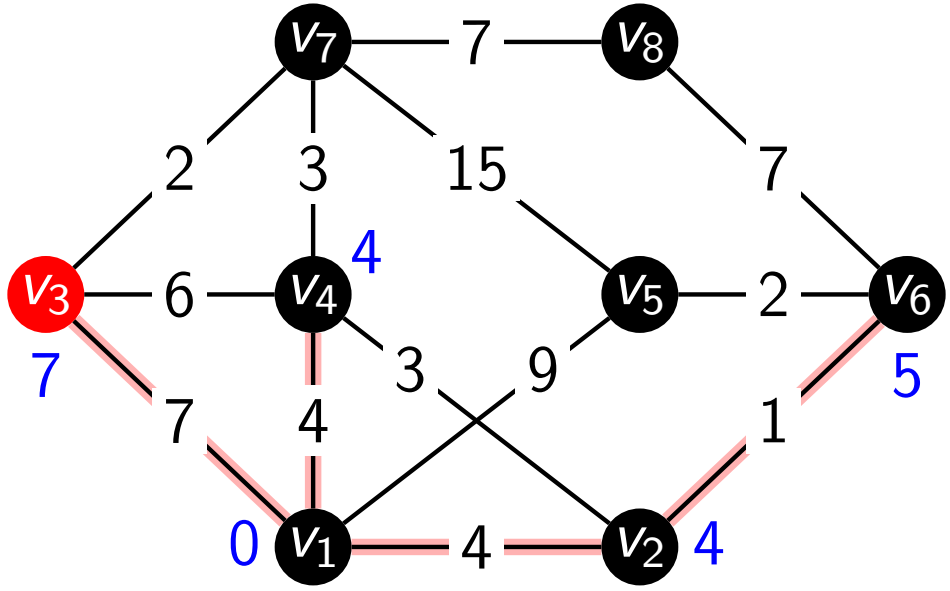
	0	1	2	3	4			
v_1	$(-, 0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$			
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$v_1, 7$			
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$			
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$			
v_6	∞	∞	∞	$(v_2, 5)$	$*$			
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$			
v_8	∞	∞	∞	∞	$v_6, 12$			

b)



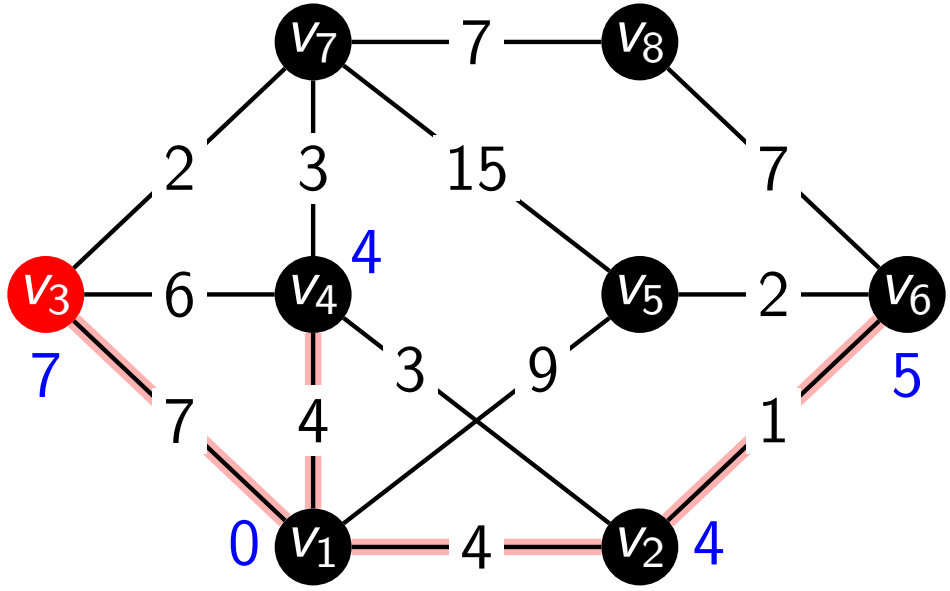
	0	1	2	3	4			
v_1	$(-, 0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$			
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$(v_1, 7)$			
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$			
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$			
v_6	∞	∞	∞	$(v_2, 5)$	$*$			
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$			
v_8	∞	∞	∞	∞	$v_6, 12$			

b)



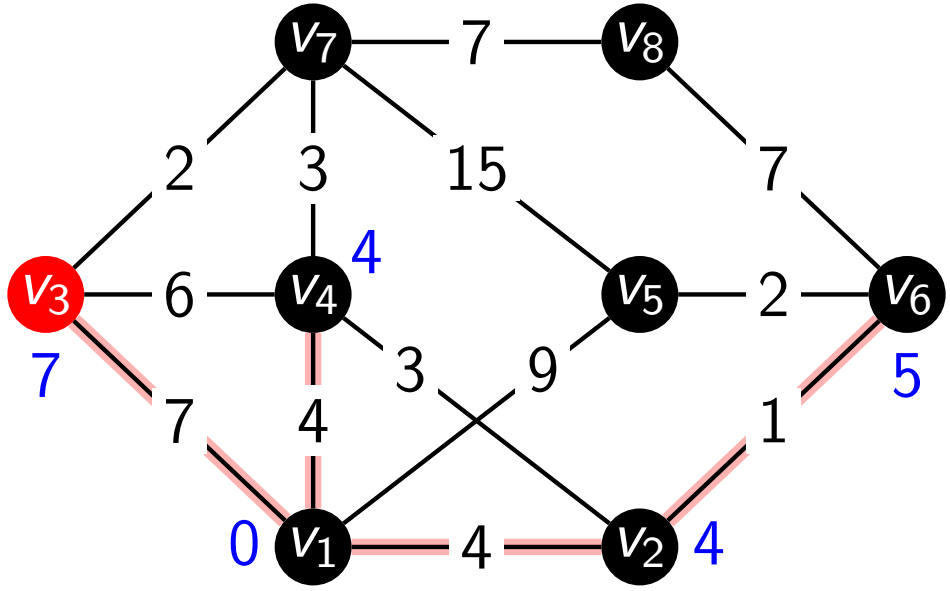
	0	1	2	3	4			
v_1	$(-, 0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$			
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$(v_1, 7)$			
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$			
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$			
v_6	∞	∞	∞	$(v_2, 5)$	$*$			
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$			
v_8	∞	∞	∞	∞	$v_6, 12$			

b)



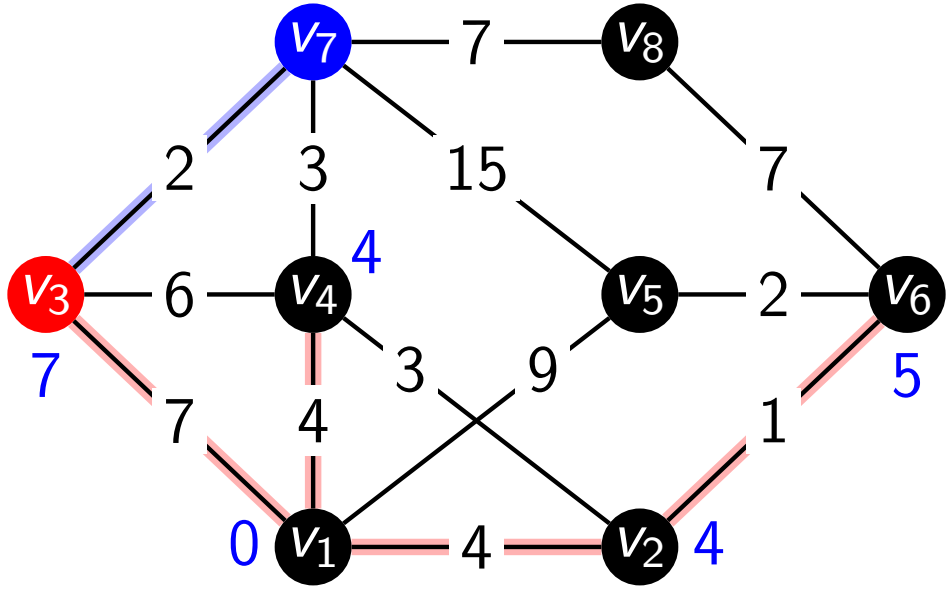
	0	1	2	3	4	5		
v_1	$(-, 0)$	$*$	$*$	$*$	$*$			
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$			
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$(v_1, 7)$			
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$			
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$			
v_6	∞	∞	∞	$(v_2, 5)$	$*$			
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$			
v_8	∞	∞	∞	∞	$v_6, 12$			

b)



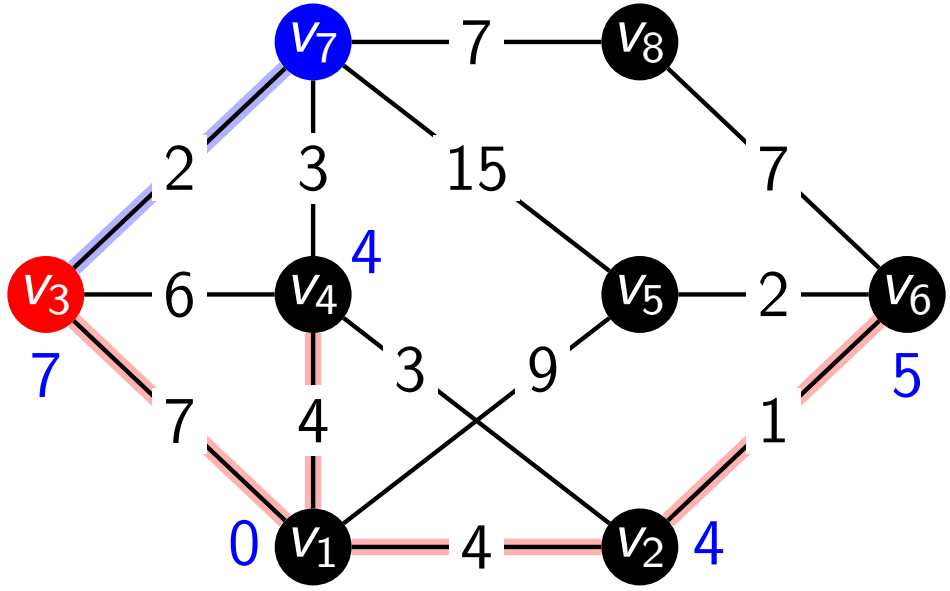
	0	1	2	3	4	5		
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$		
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$		
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$			
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$		
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$			
v_8	∞	∞	∞	∞	$v_6,12$			

b)



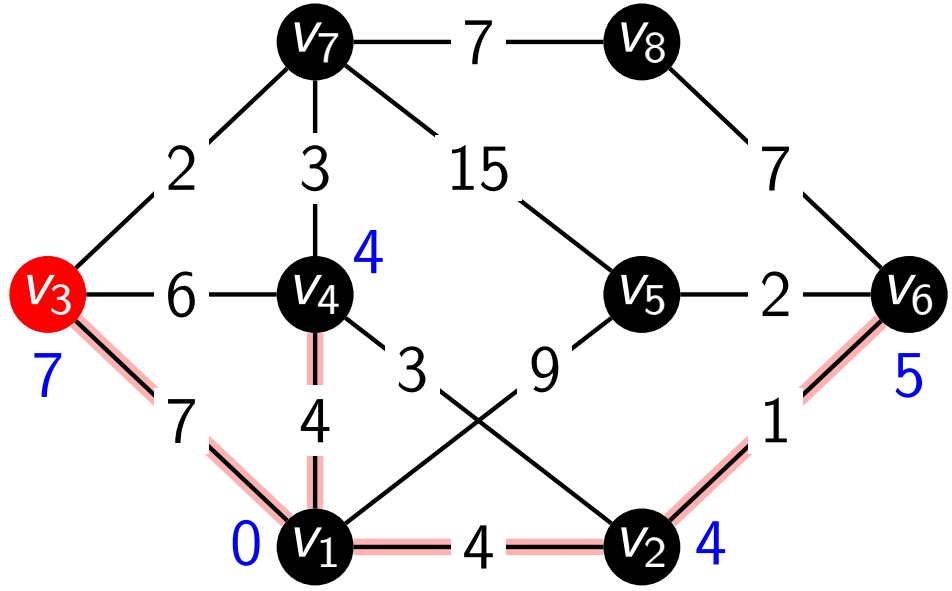
	0	1	2	3	4	5		
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$		
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$		
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$			
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$		
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$			
v_8	∞	∞	∞	∞	$v_6,12$			

b)



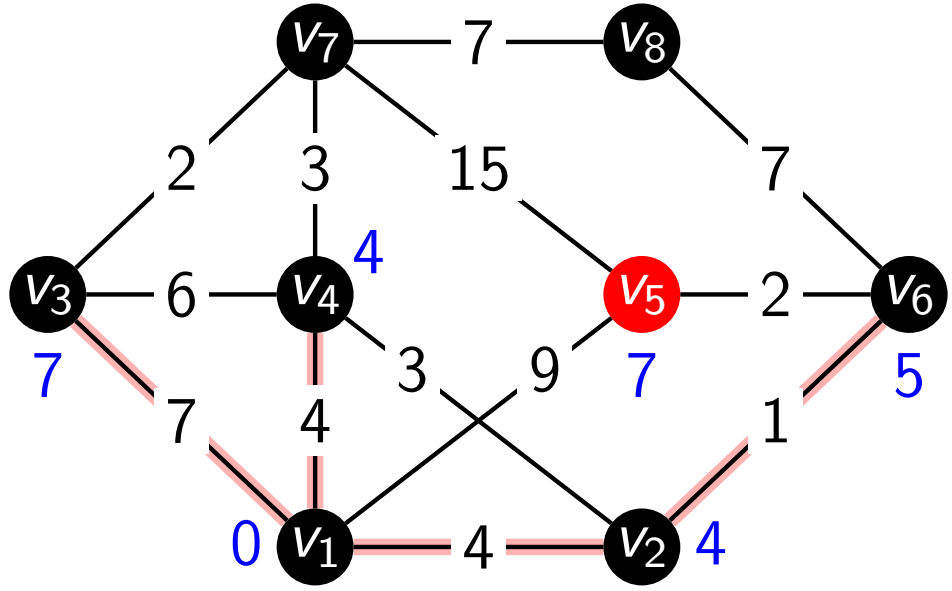
	0	1	2	3	4	5		
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$		
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$		
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$			
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$		
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$		
v_8	∞	∞	∞	∞	$v_6,12$			

b)



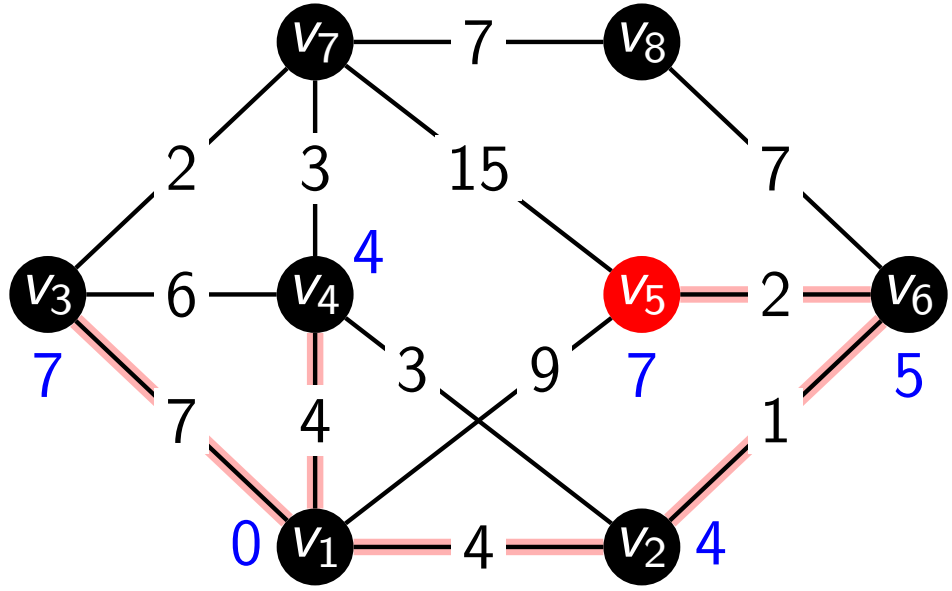
	0	1	2	3	4	5		
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$		
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$		
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$v_6,7$		
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$		
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$		
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$		

b)



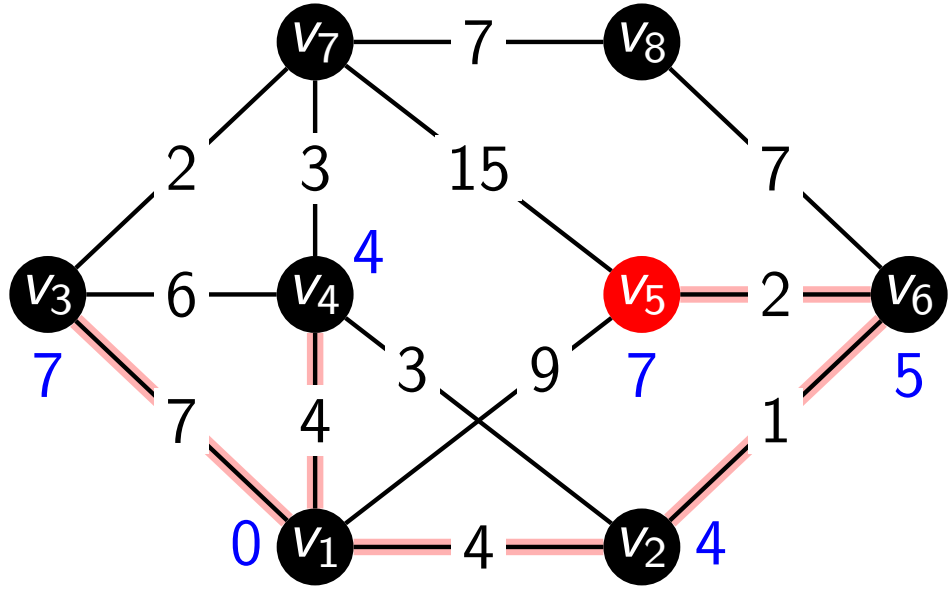
	0	1	2	3	4	5		
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$		
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$		
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$		
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$		
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$		
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$		

b)



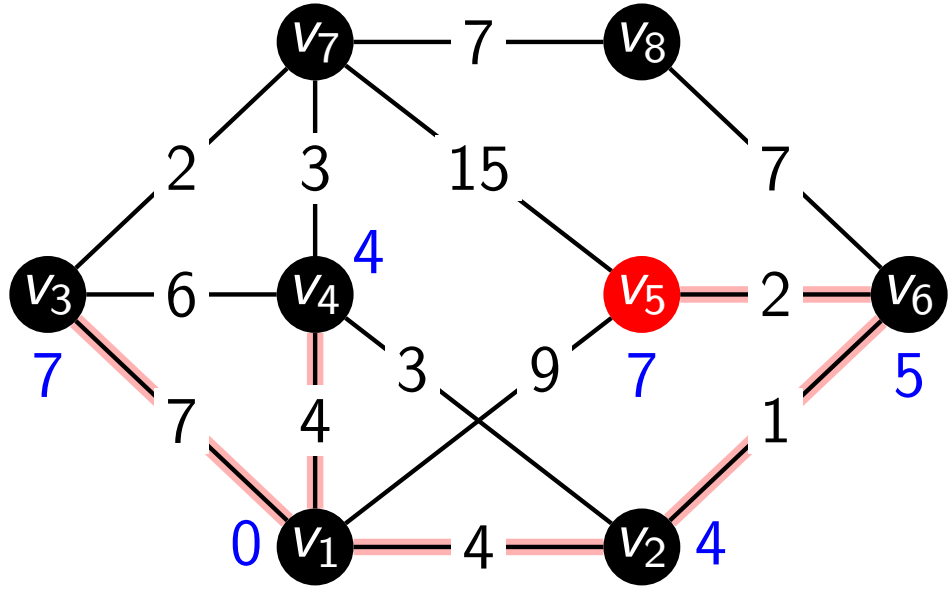
	0	1	2	3	4	5		
v_1	$(-, 0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$	$*$		
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$(v_1, 7)$	$*$		
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$	$(v_6, 7)$		
v_6	∞	∞	∞	$(v_2, 5)$	$*$	$*$		
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$	$v_4, 7$		
v_8	∞	∞	∞	∞	$v_6, 12$	$v_6, 12$		

b)



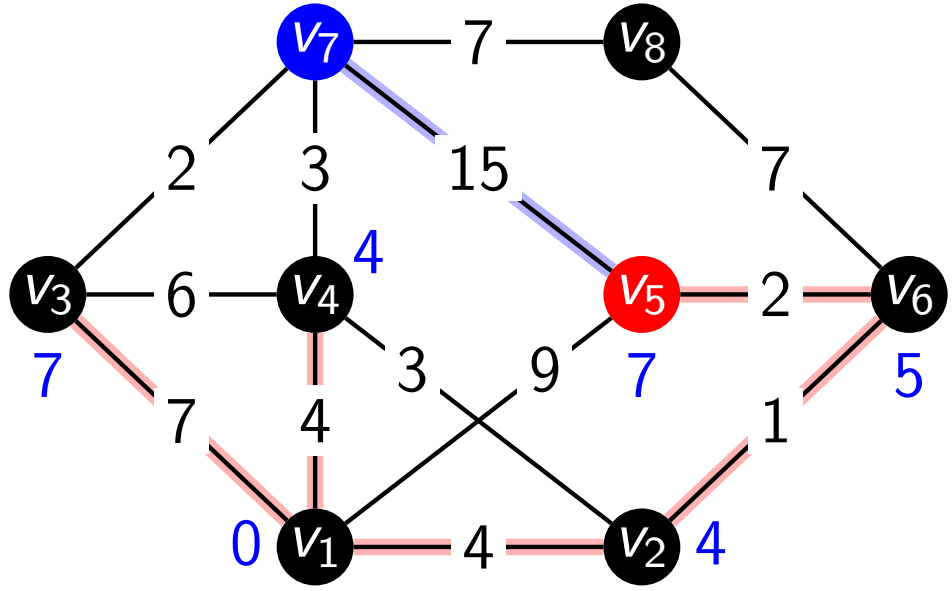
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$		
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$		
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$		
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$		
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$		
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$		
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$		
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$		

b)



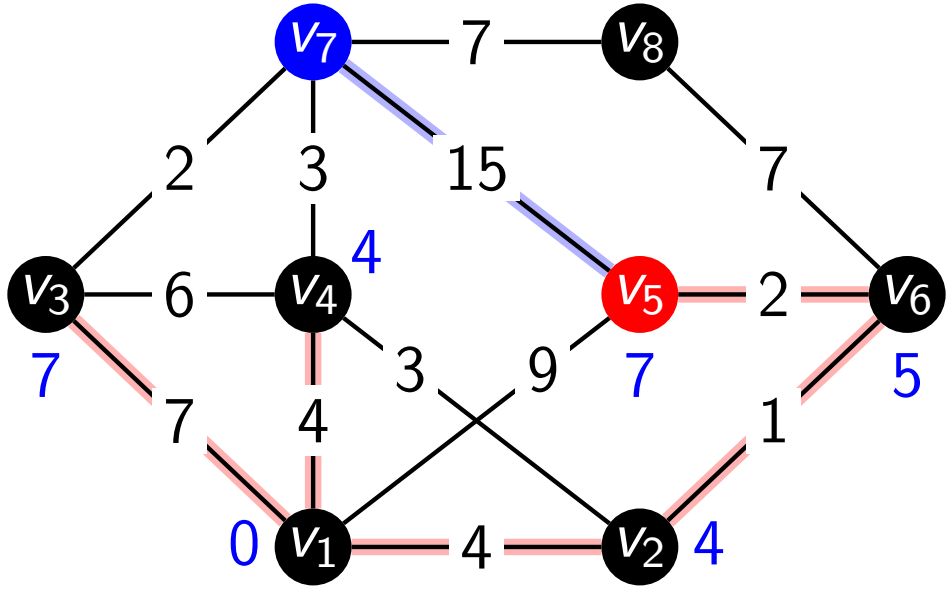
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$		
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$		

b)



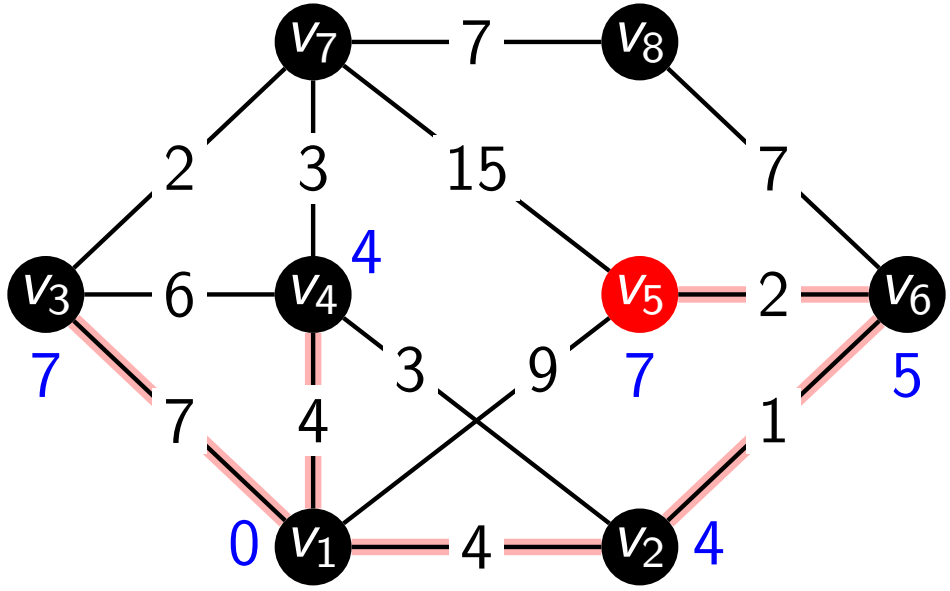
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$		
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$		

b)



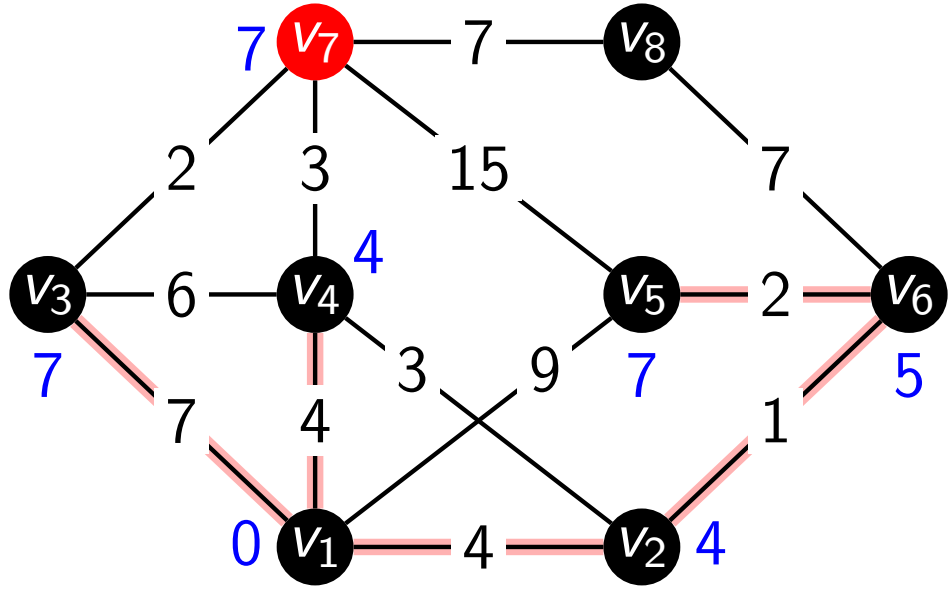
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$		

b)



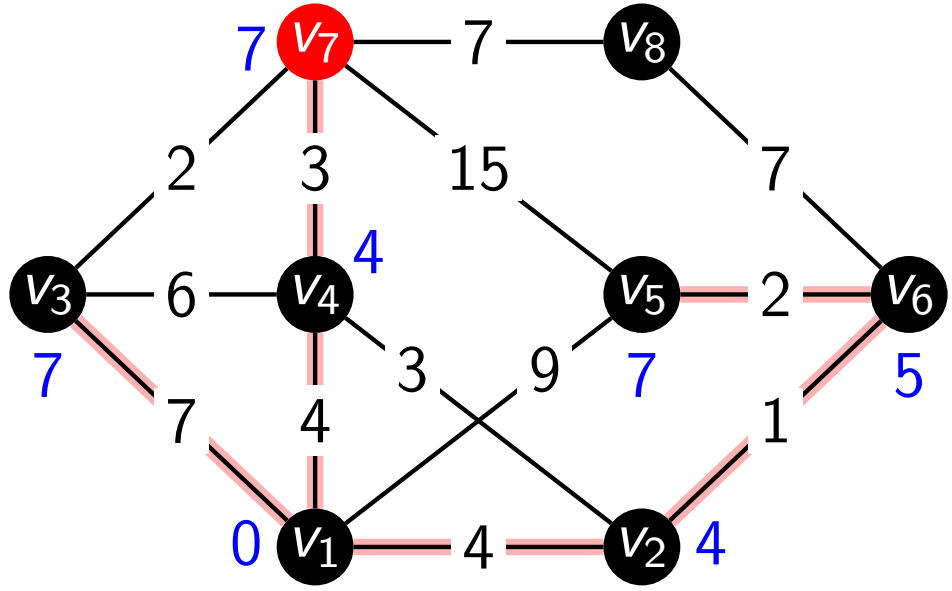
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	

b)



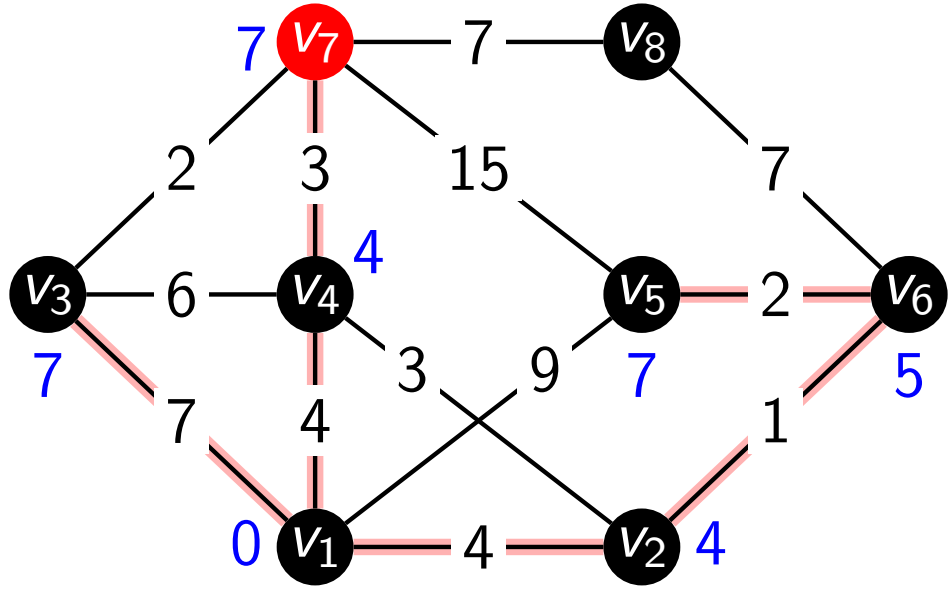
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	

b)



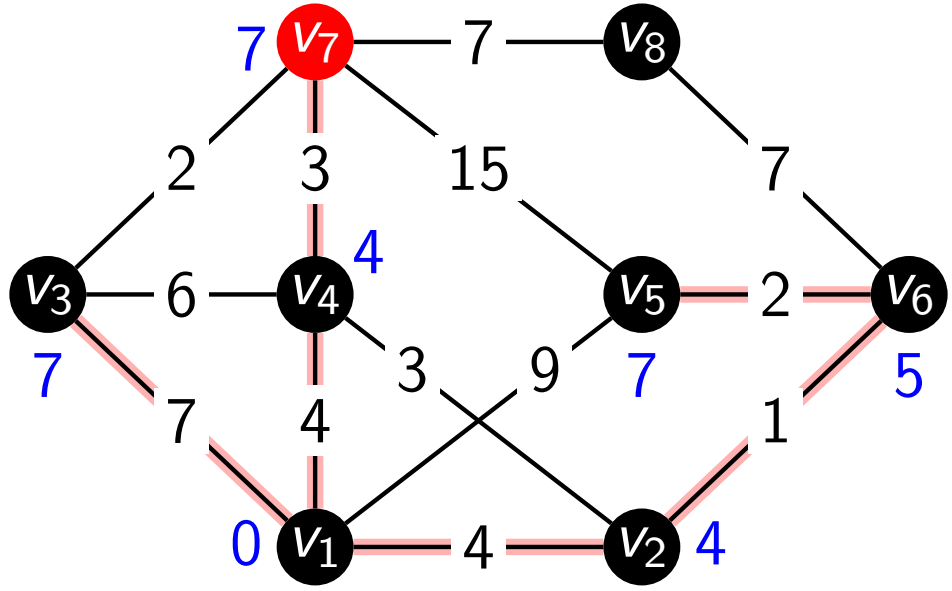
	0	1	2	3	4	5	6	
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	

b)



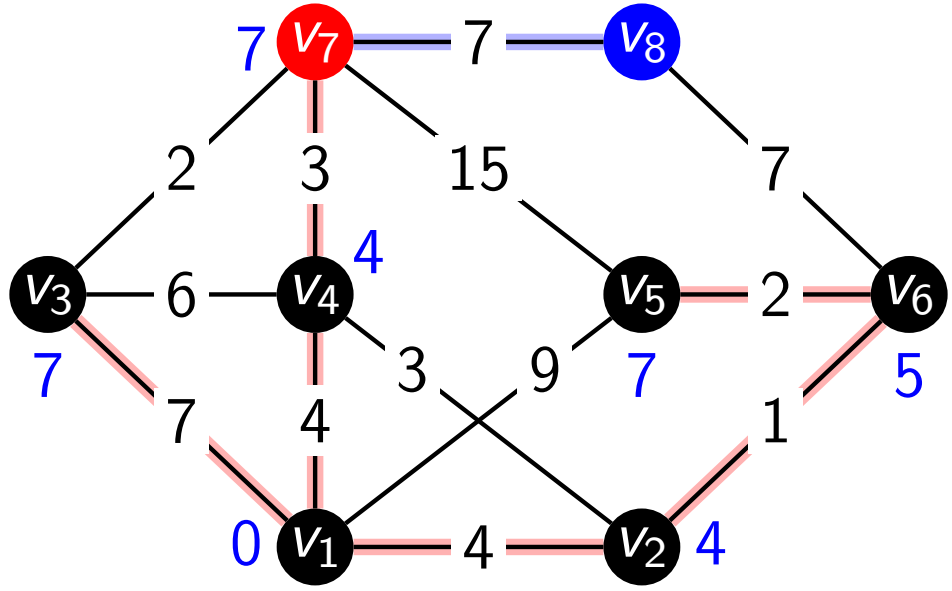
	0	1	2	3	4	5	6	7
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	

b)



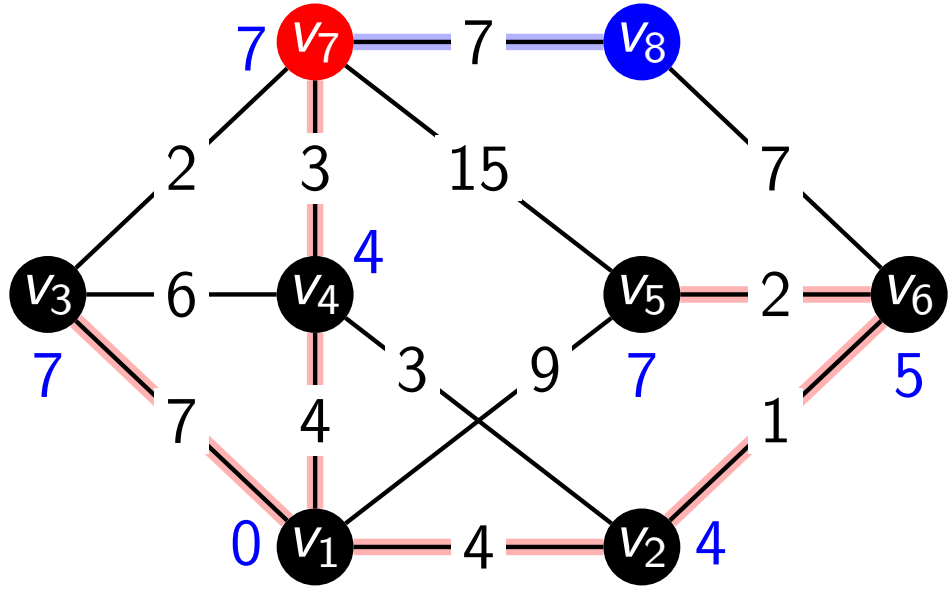
	0	1	2	3	4	5	6	7
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	$*$
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	$*$
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	$*$
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	$*$
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	$*$
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	

b)



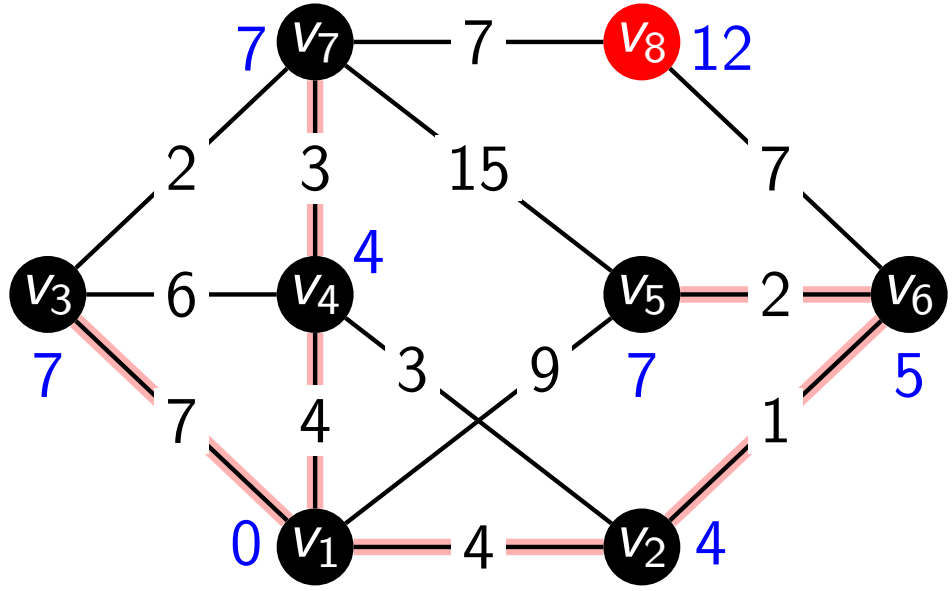
	0	1	2	3	4	5	6	7
v_1	$(-, 0)$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$	$*$	$*$	$*$
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$(v_1, 7)$	$*$	$*$	$*$
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$	$*$	$*$	$*$
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$	$(v_6, 7)$	$*$	$*$
v_6	∞	∞	∞	$(v_2, 5)$	$*$	$*$	$*$	$*$
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$	$v_4, 7$	$(v_4, 7)$	$*$
v_8	∞	∞	∞	∞	$v_6, 12$	$v_6, 12$	$v_6, 12$	

b)



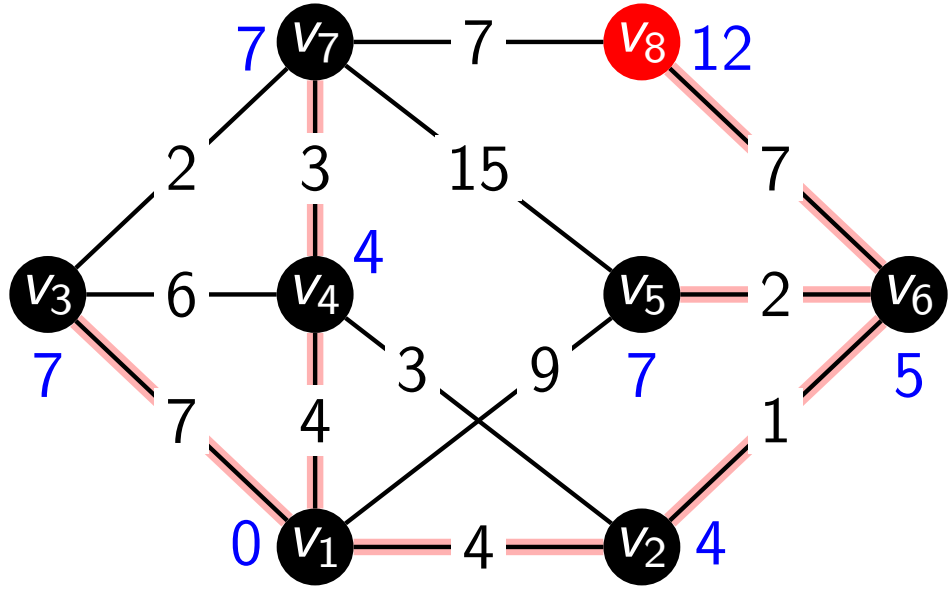
	0	1	2	3	4	5	6	7
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	$*$
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	$*$
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	$*$
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	$*$
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	$*$
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	$v_6,12$

b)



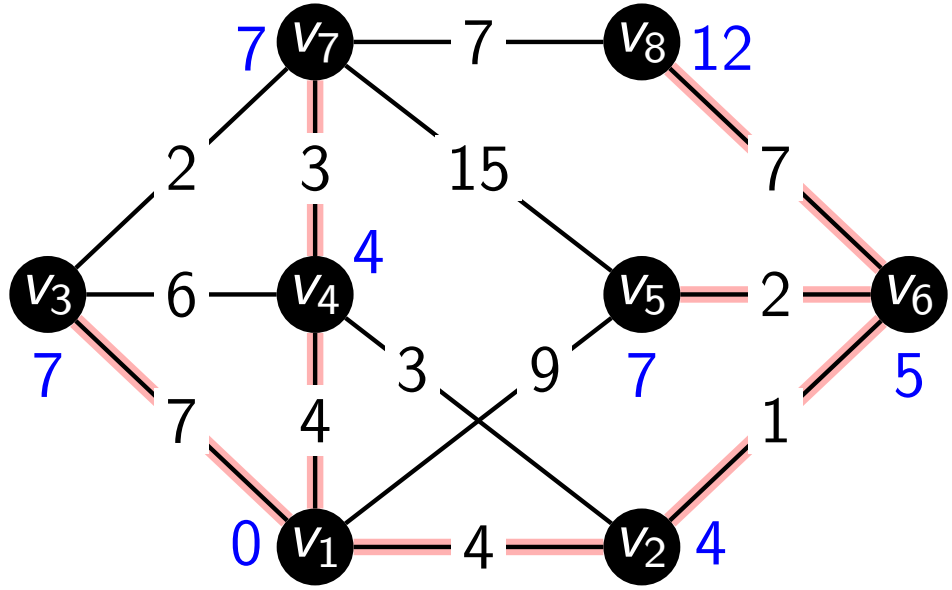
	0	1	2	3	4	5	6	7
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	$*$
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	$*$
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	$*$
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	$*$
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	$*$
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	$(v_6,12)$

b)



	0	1	2	3	4	5	6	7
v_1	$(-,0)$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
v_2	∞	$v_1,4$	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$
v_3	∞	$v_1,7$	$v_1,7$	$v_1,7$	$(v_1,7)$	$*$	$*$	$*$
v_4	∞	$(v_1,4)$	$*$	$*$	$*$	$*$	$*$	$*$
v_5	∞	$v_1,9$	$v_1,9$	$v_1,9$	$v_6,7$	$(v_6,7)$	$*$	$*$
v_6	∞	∞	∞	$(v_2,5)$	$*$	$*$	$*$	$*$
v_7	∞	∞	$v_4,7$	$v_4,7$	$v_4,7$	$v_4,7$	$(v_4,7)$	$*$
v_8	∞	∞	∞	∞	$v_6,12$	$v_6,12$	$v_6,12$	$(v_6,12)$

b)

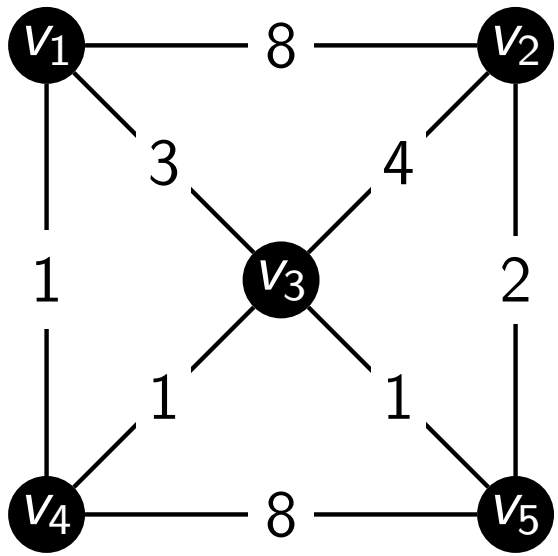


	0	1	2	3	4	5	6	7
v_1	$(-, 0)$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
v_2	∞	$v_1, 4$	$(v_1, 4)$	$*$	$*$	$*$	$*$	$*$
v_3	∞	$v_1, 7$	$v_1, 7$	$v_1, 7$	$(v_1, 7)$	$*$	$*$	$*$
v_4	∞	$(v_1, 4)$	$*$	$*$	$*$	$*$	$*$	$*$
v_5	∞	$v_1, 9$	$v_1, 9$	$v_1, 9$	$v_6, 7$	$(v_6, 7)$	$*$	$*$
v_6	∞	∞	∞	$(v_2, 5)$	$*$	$*$	$*$	$*$
v_7	∞	∞	$v_4, 7$	$v_4, 7$	$v_4, 7$	$v_4, 7$	$(v_4, 7)$	$*$
v_8	∞	∞	∞	∞	$v_6, 12$	$v_6, 12$	$v_6, 12$	$(v_6, 12)$

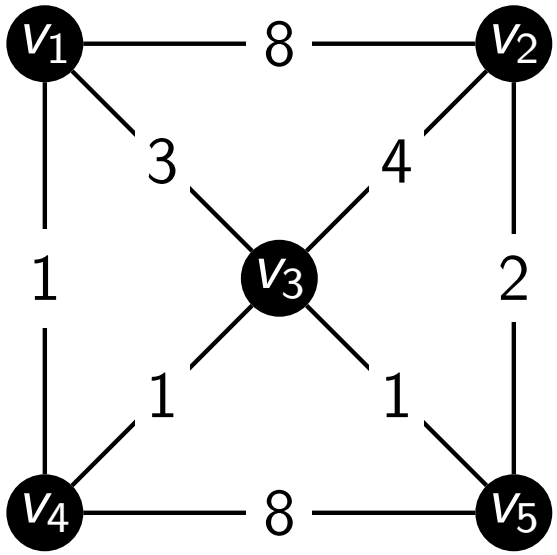
deseti zadatak

Zadatak 10

Pomoću Floyd-Warshallovog algoritma odredite najkraće udaljenosti između svaka dva vrha u težinskom grafu

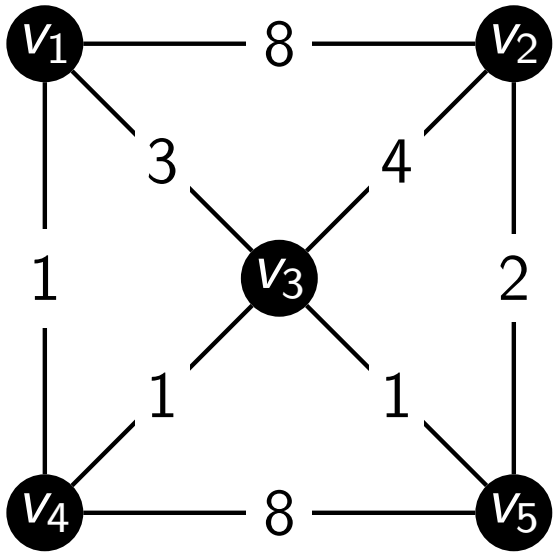


Rješenje



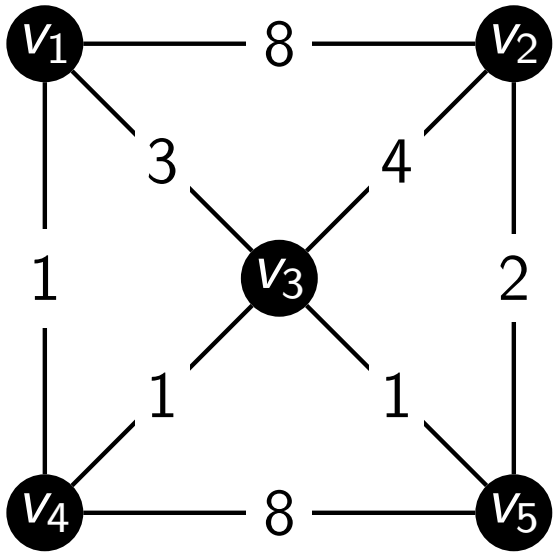
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

Rješenje



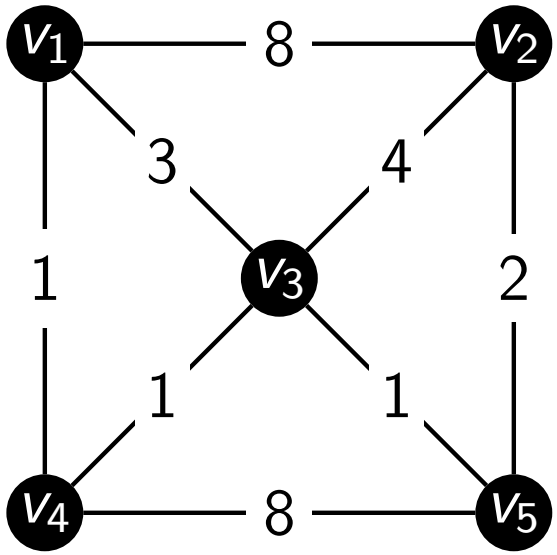
$k = 0$	v_1	v_2	v_3	v_4	v_5
v_1	0				
v_2					
v_3					
v_4					
v_5					

Rješenje



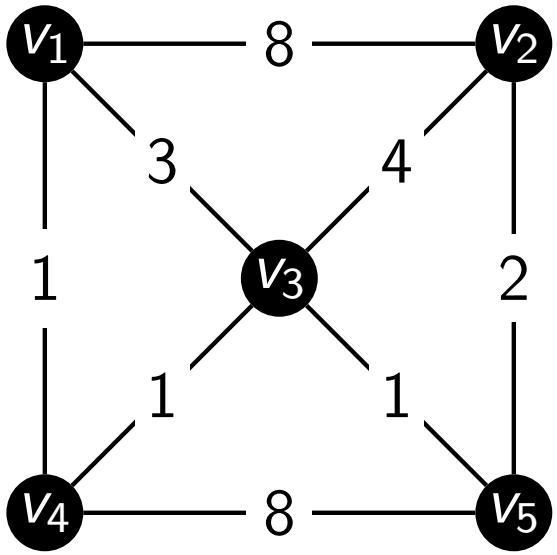
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8			
v_2					
v_3					
v_4					
v_5					

Rješenje



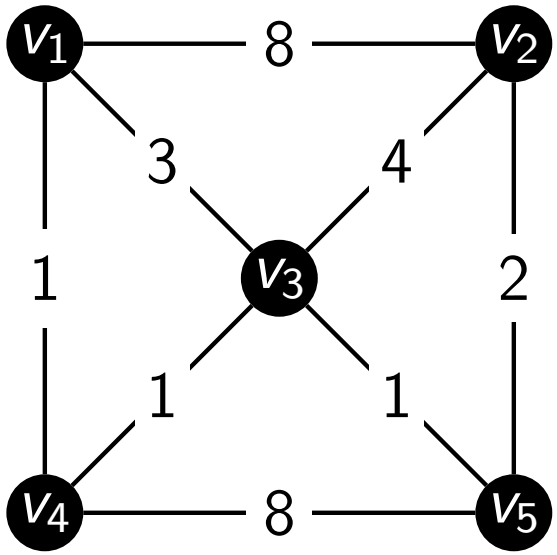
$k = 0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3		
v_2					
v_3					
v_4					
v_5					

Rješenje



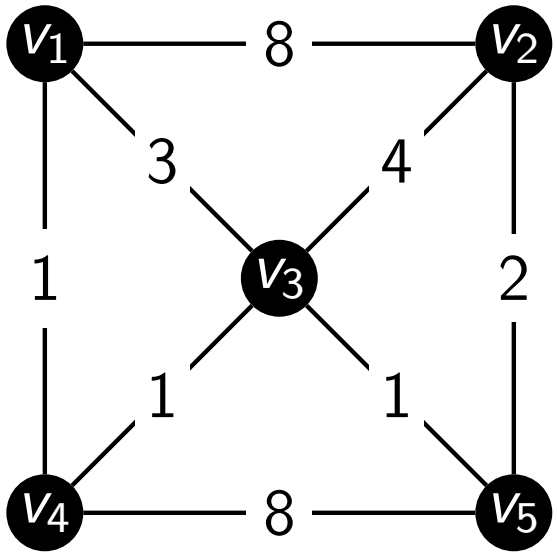
$k = 0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	
v_2					
v_3					
v_4					
v_5					

Rješenje



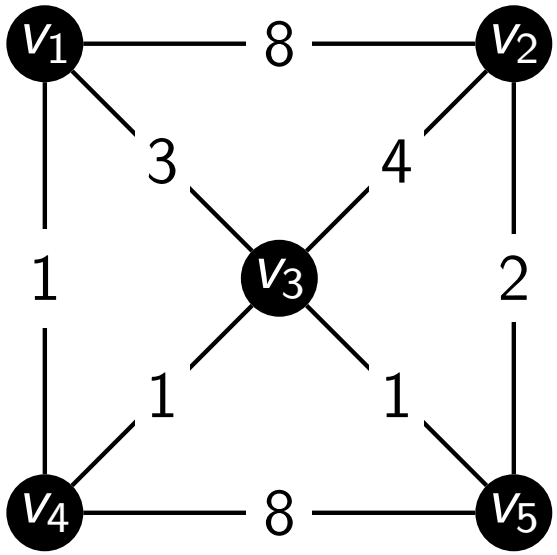
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2					
v_3					
v_4					
v_5					

Rješenje



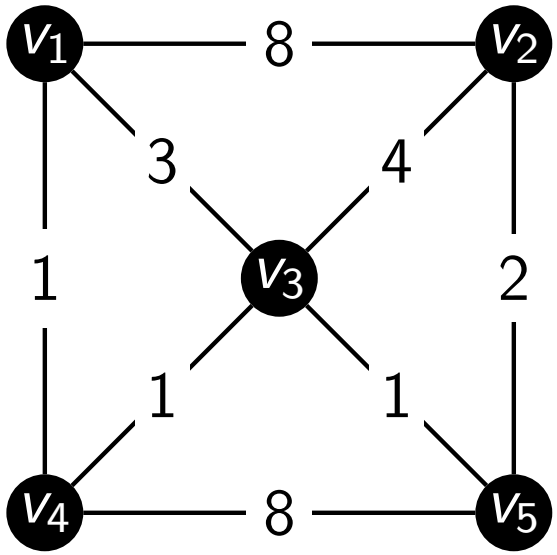
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8				
v_3	3				
v_4	1				
v_5	∞				

Rješenje



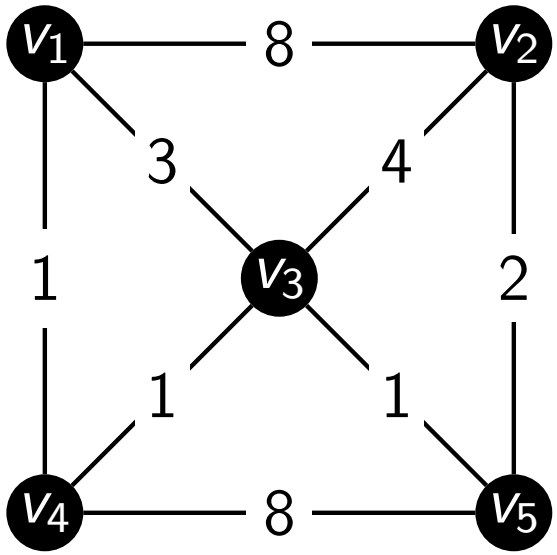
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0			
v_3	3				
v_4	1				
v_5	∞				

Rješenje



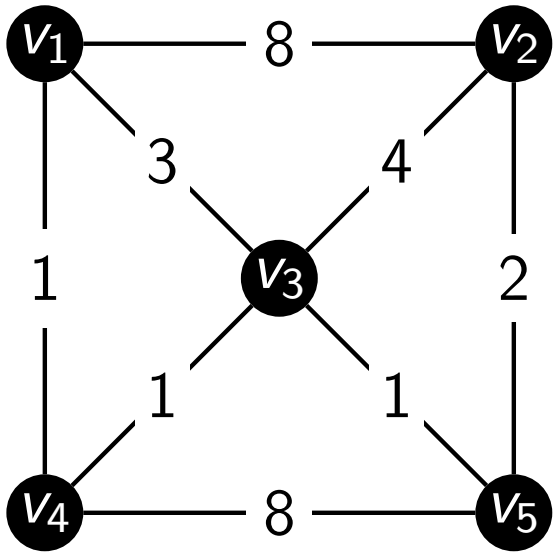
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4		
v_3	3				
v_4	1				
v_5	∞				

Rješenje



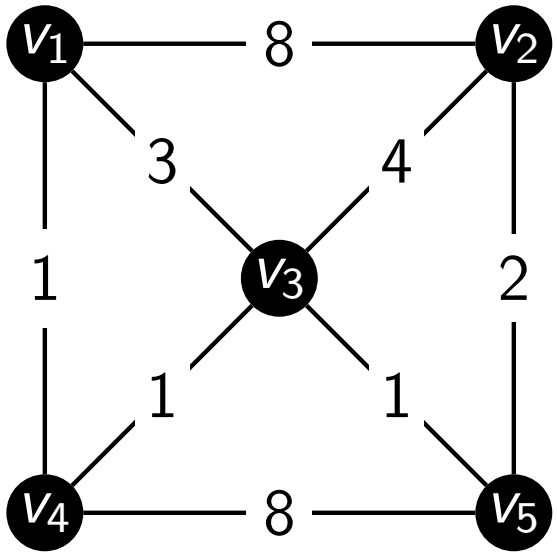
$k = 0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	
v_3	3				
v_4	1				
v_5	∞				

Rješenje



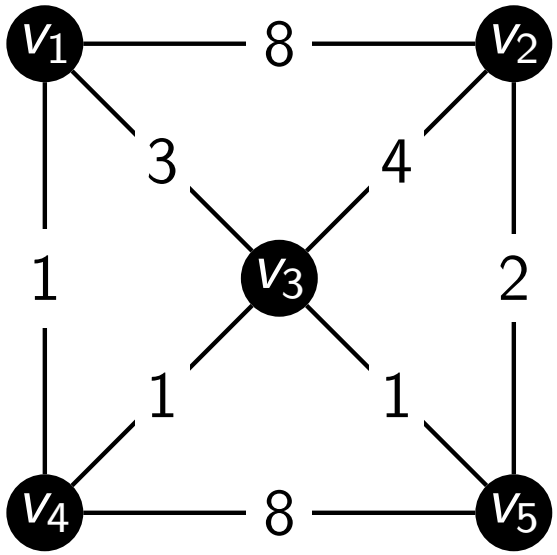
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3				
v_4	1				
v_5	∞				

Rješenje



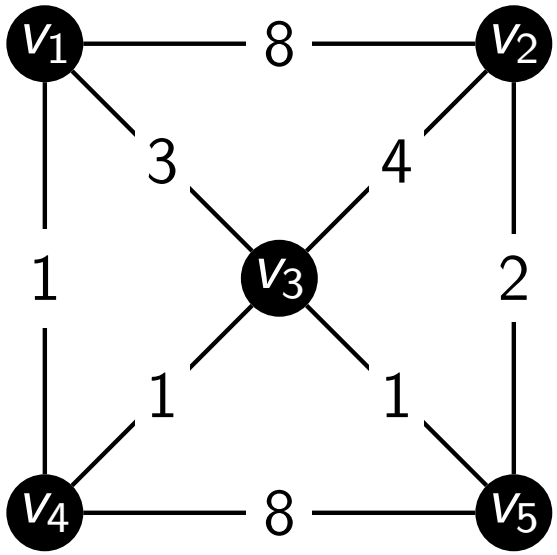
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4			
v_4	1	∞			
v_5	∞	2			

Rješenje



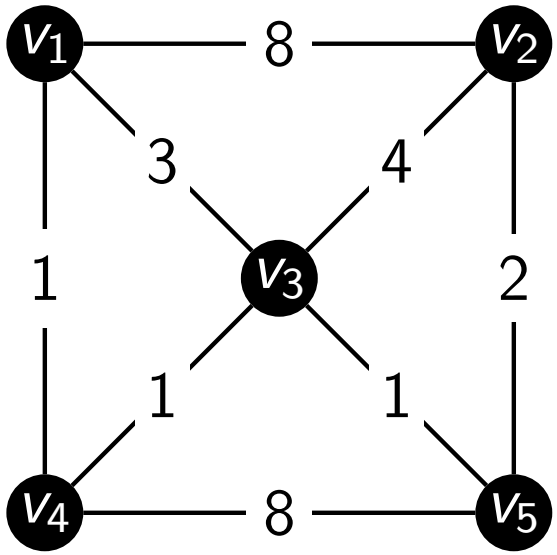
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0		
v_4	1	∞			
v_5	∞	2			

Rješenje



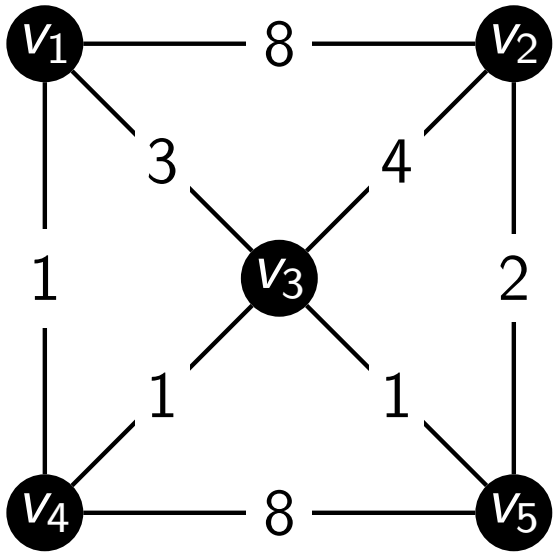
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	
v_4	1	∞			
v_5	∞	2			

Rješenje



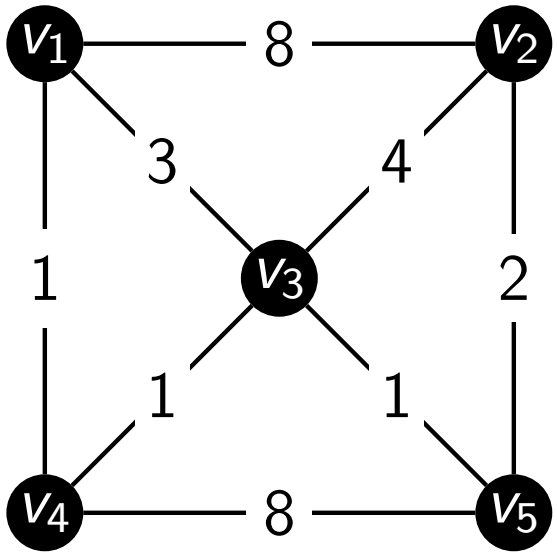
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞			
v_5	∞	2			

Rješenje



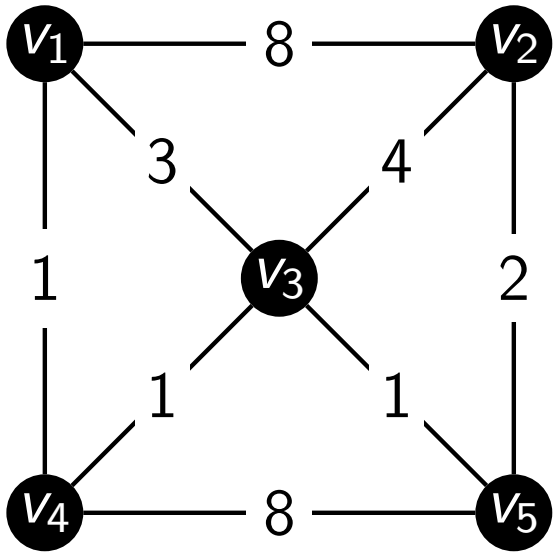
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1		
v_5	∞	2	1		

Rješenje



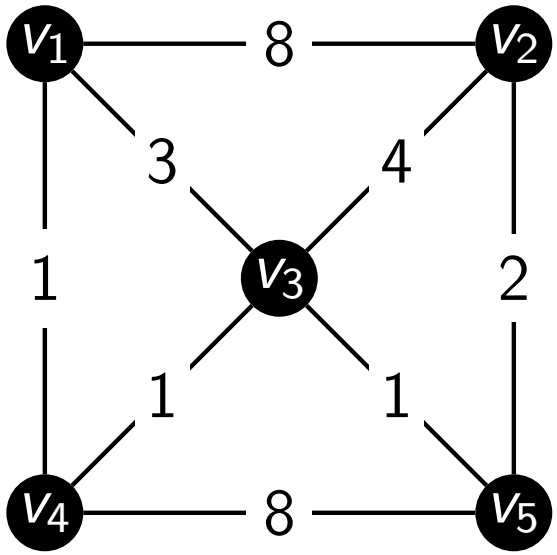
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	
v_5	∞	2	1		

Rješenje



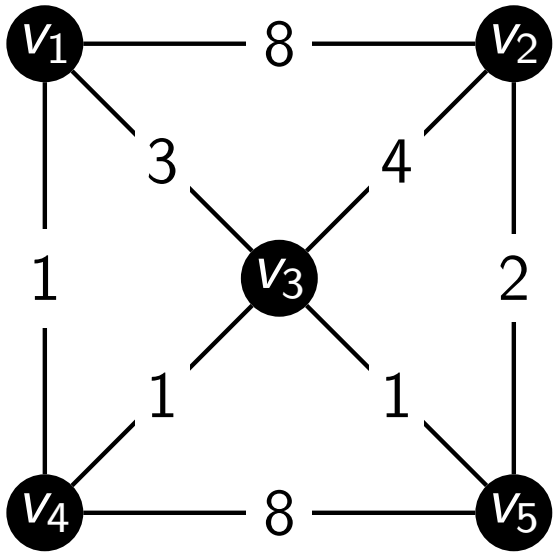
$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1		

Rješenje



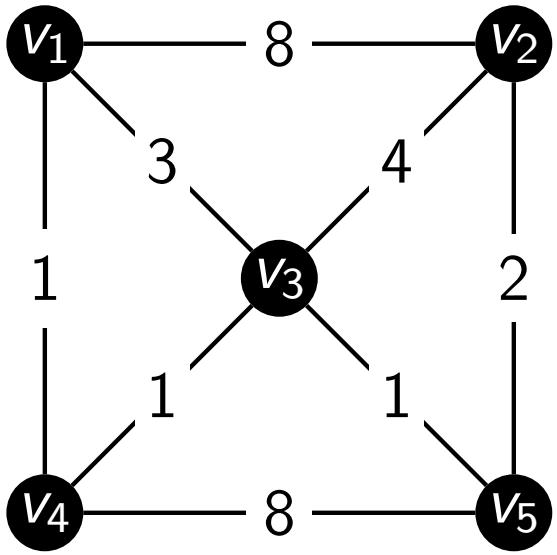
$k = 0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	

Rješenje



$k = 0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

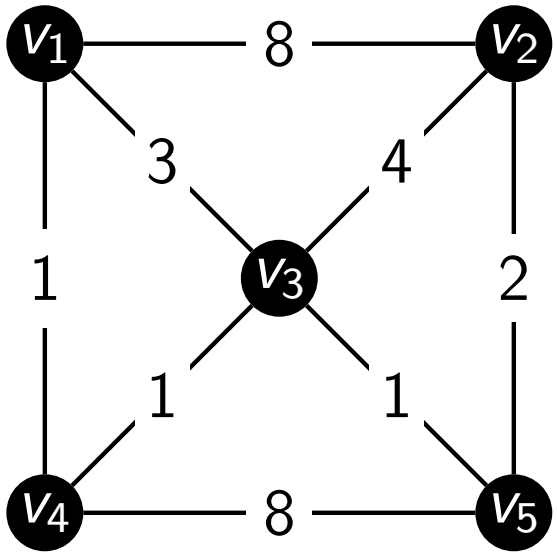
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

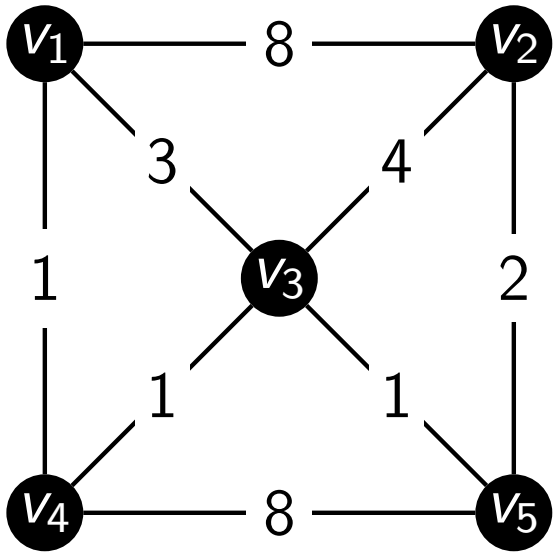
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

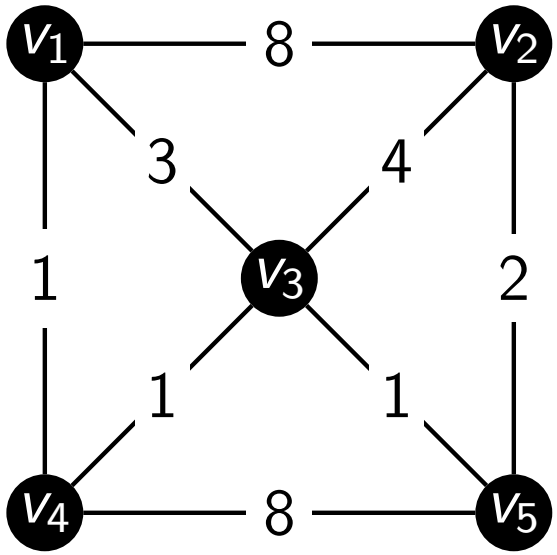
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2					
v_3					
v_4					
v_5					

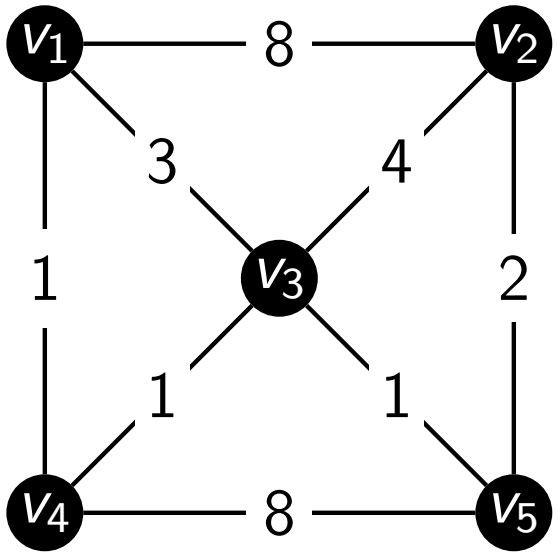
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8				
v_3	3				
v_4	1				
v_5	∞				

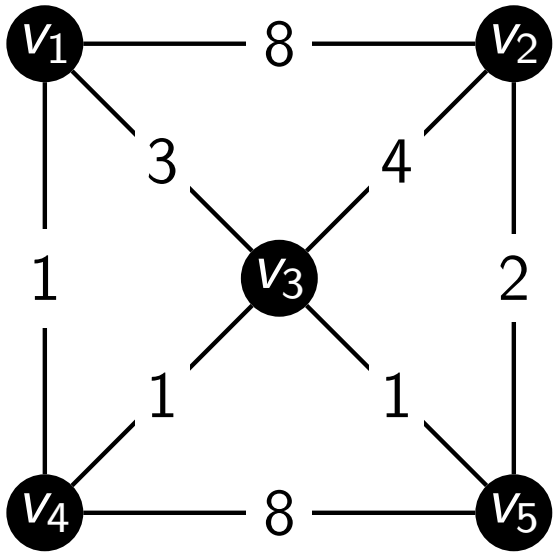
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0			
v_3	3				
v_4	1				
v_5	∞				

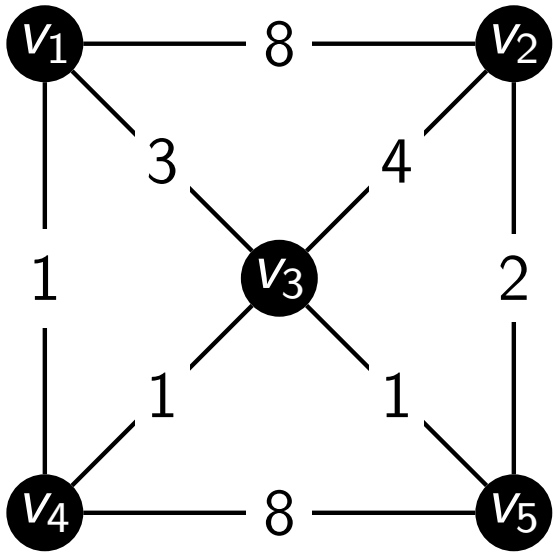
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4		
v_3	3				
v_4	1				
v_5	∞				

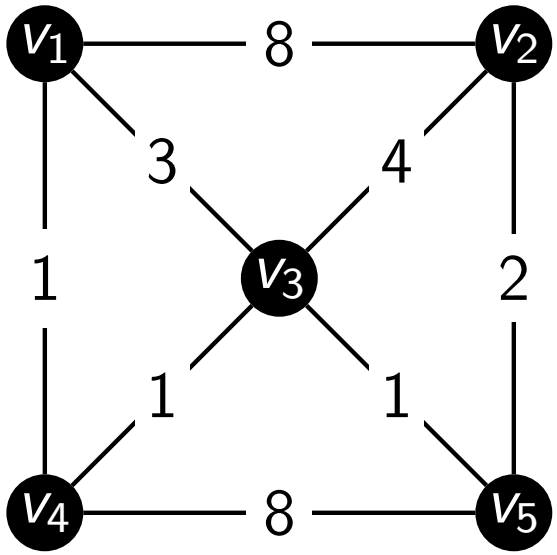
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	
v_3	3				
v_4	1				
v_5	∞				

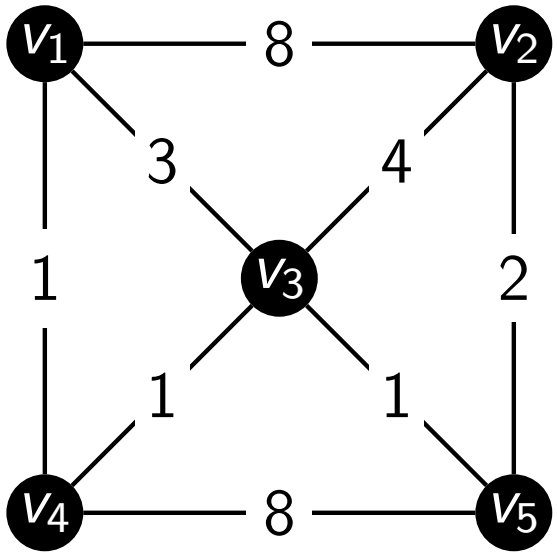
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3				
v_4	1				
v_5	∞				

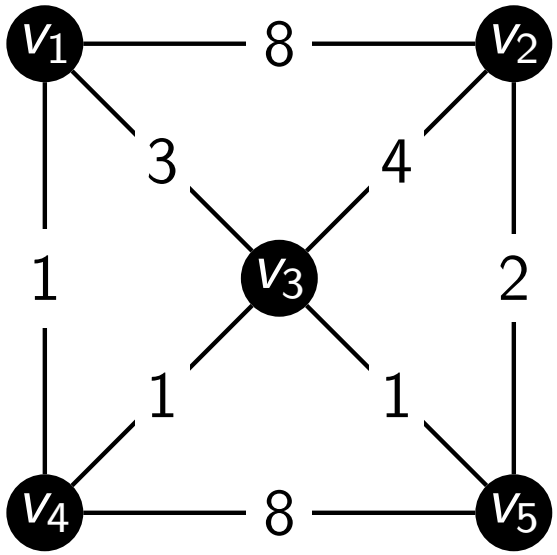
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4			
v_4	1	9			
v_5	∞	2			

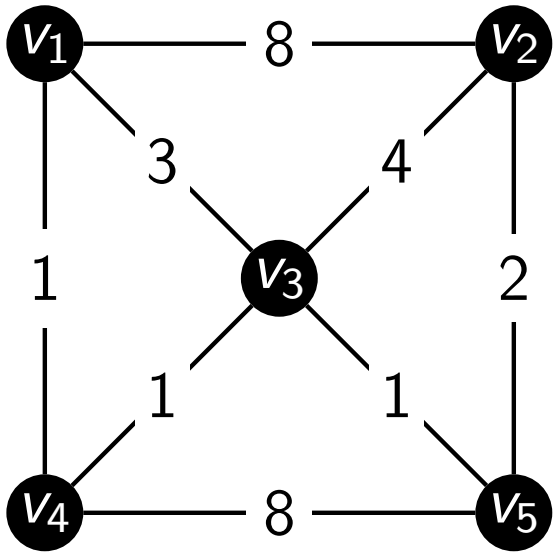
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0		
v_4	1	9			
v_5	∞	2			

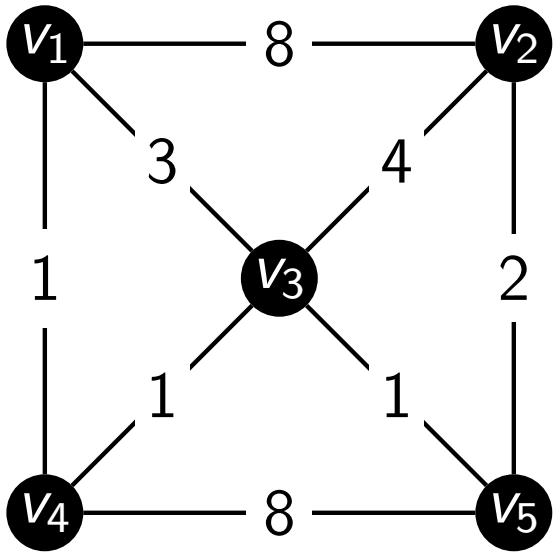
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	
v_4	1	9			
v_5	∞	2			

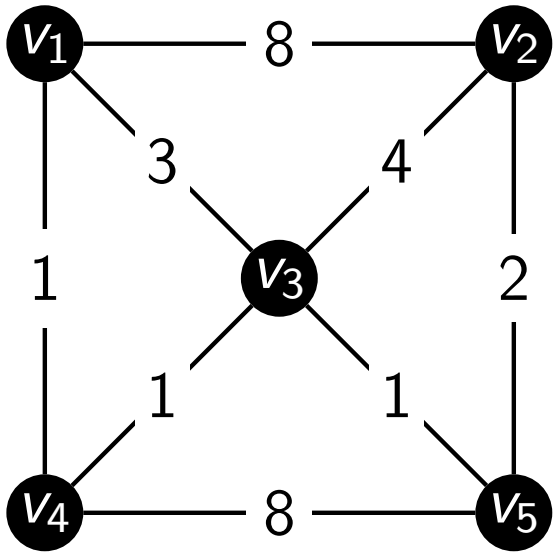
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9			
v_5	∞	2			

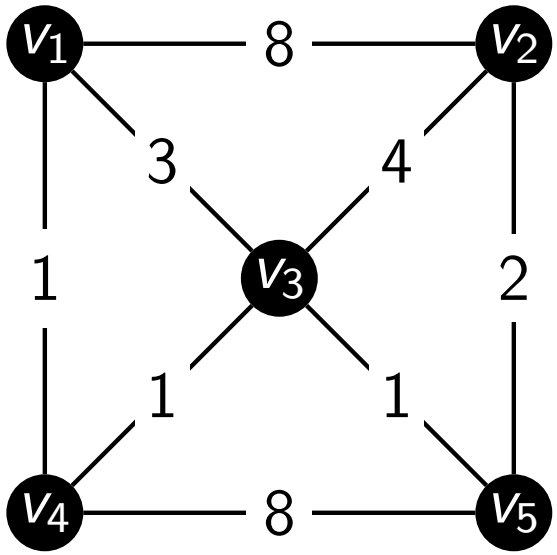
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1		
v_5	∞	2	1		

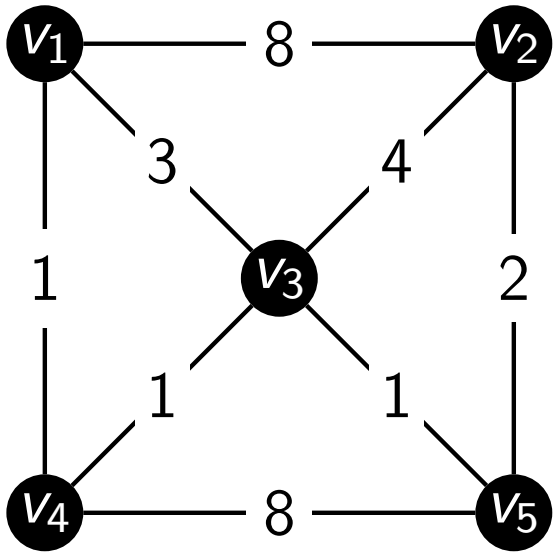
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	
v_5	∞	2	1		

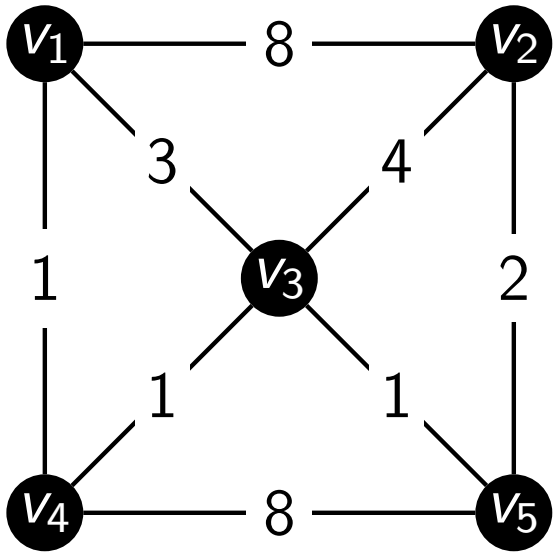
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1		

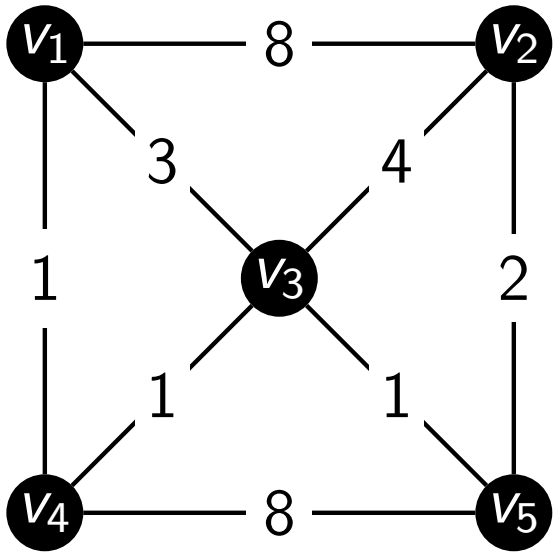
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	

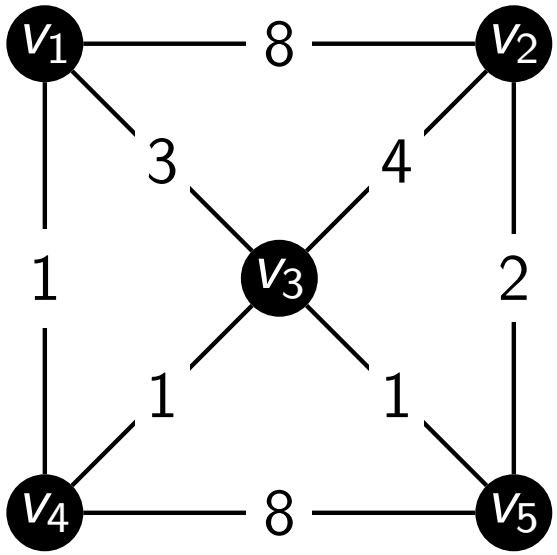
Rješenje



$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

Rješenje

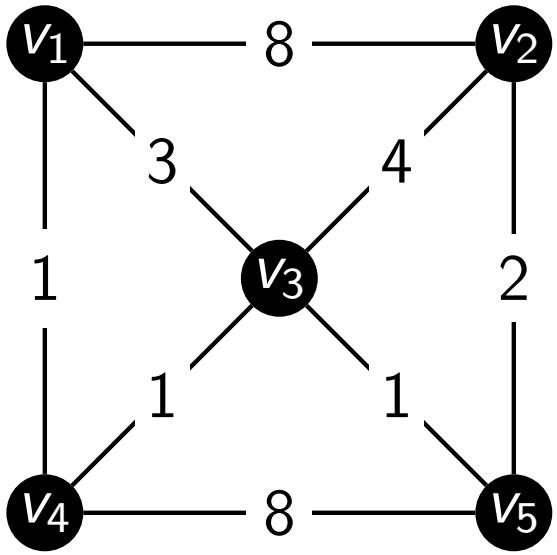


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

Rješenje

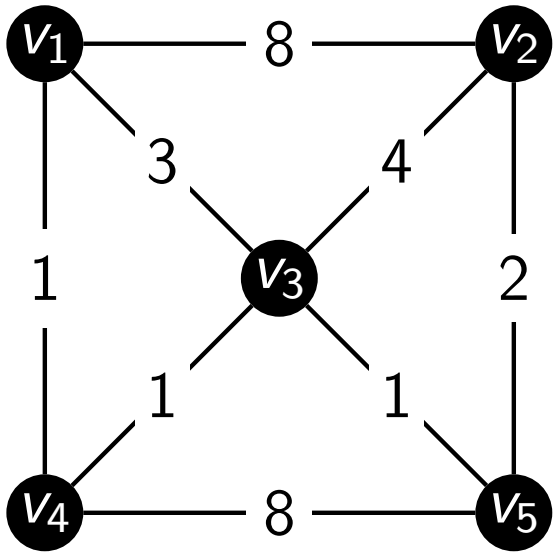


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

Rješenje

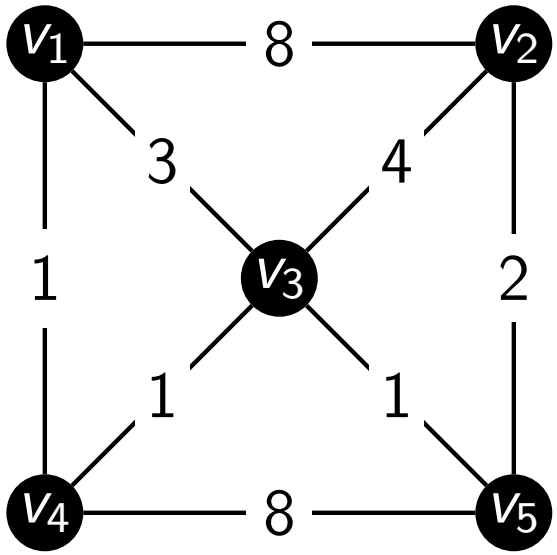


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2	8	0	4	9	2
v_3					
v_4					
v_5					

Rješenje

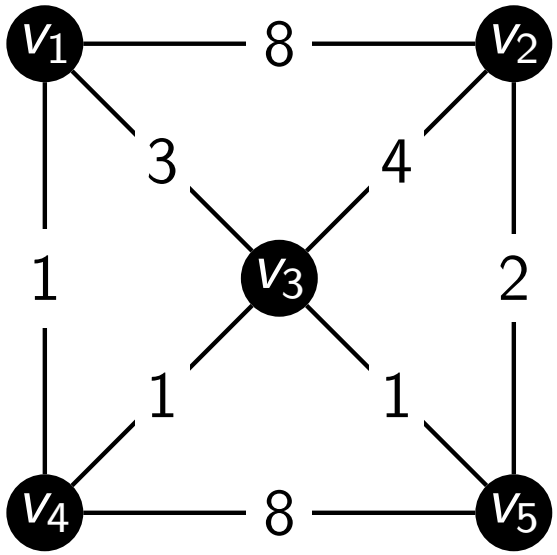


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1		8			
v_2	8	0	4	9	2
v_3		4			
v_4		9			
v_5		2			

Rješenje

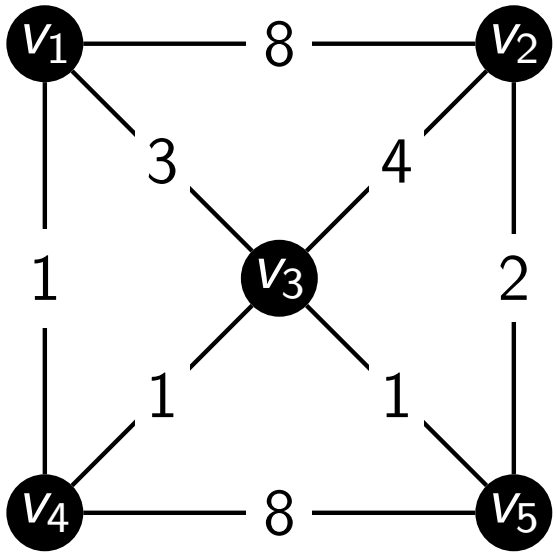


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8			
v_2	8	0	4	9	2
v_3		4			
v_4		9			
v_5		2			

Rješenje

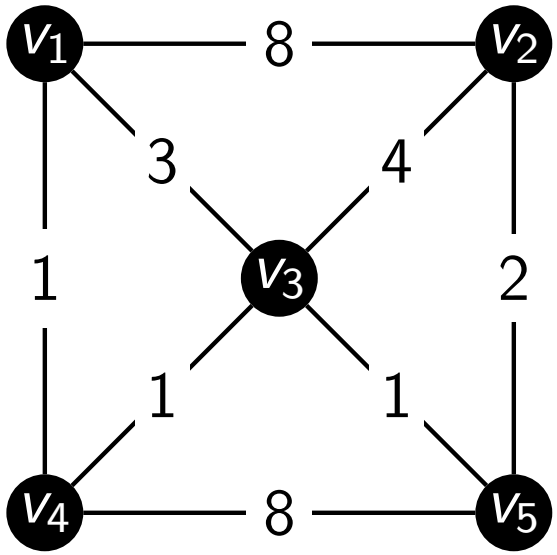


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3		
v_2	8	0	4	9	2
v_3		4			
v_4		9			
v_5		2			

Rješenje

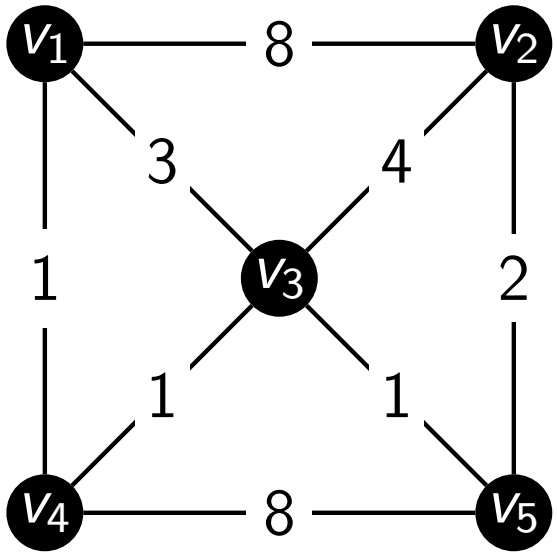


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	
v_2	8	0	4	9	2
v_3		4			
v_4		9			
v_5		2			

Rješenje

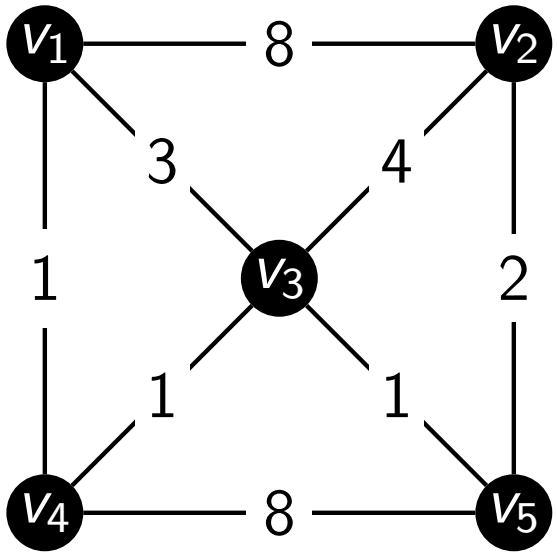


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3		4			
v_4		9			
v_5		2			

Rješenje

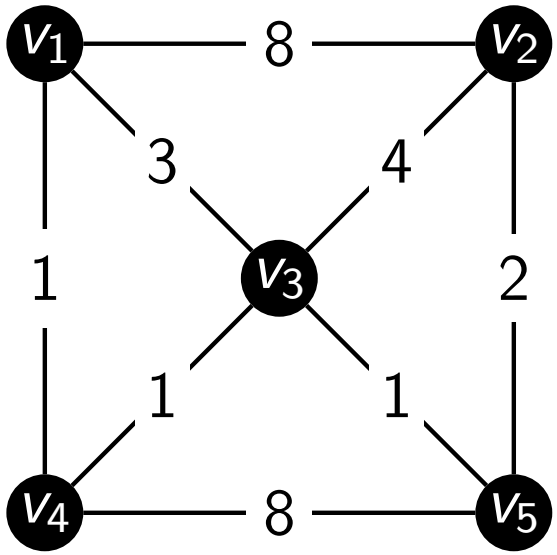


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4			
v_4	1	9			
v_5	10	2			

Rješenje

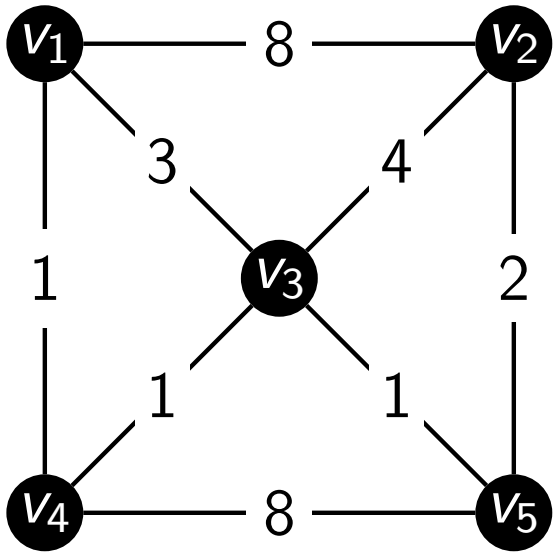


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0		
v_4	1	9			
v_5	10	2			

Rješenje

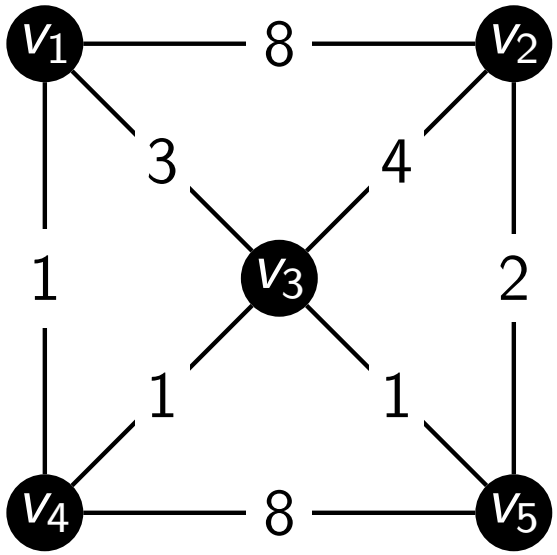


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	
v_4	1	9			
v_5	10	2			

Rješenje

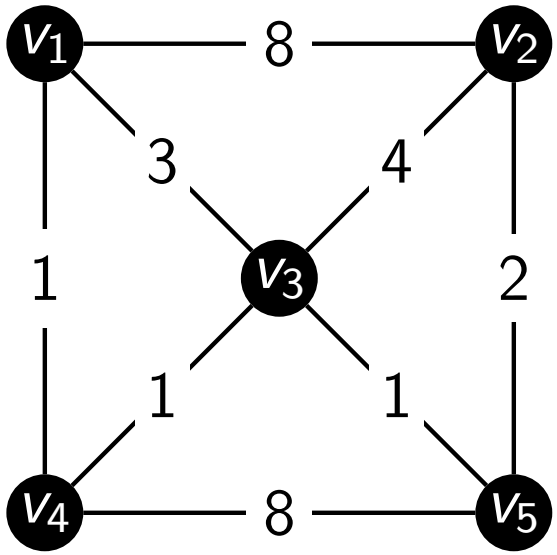


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9			
v_5	10	2			

Rješenje

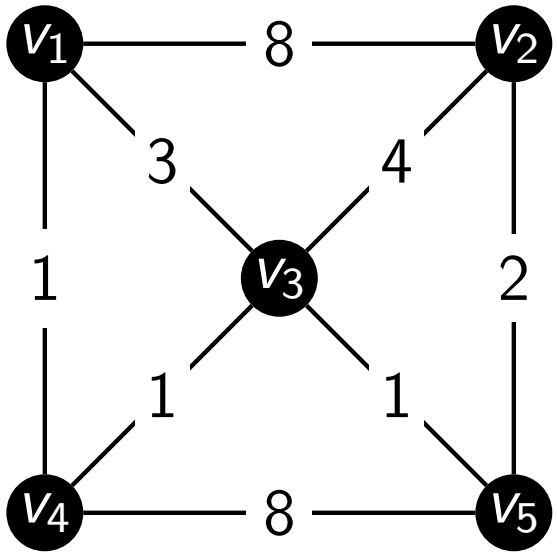


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1		
v_5	10	2	1		

Rješenje

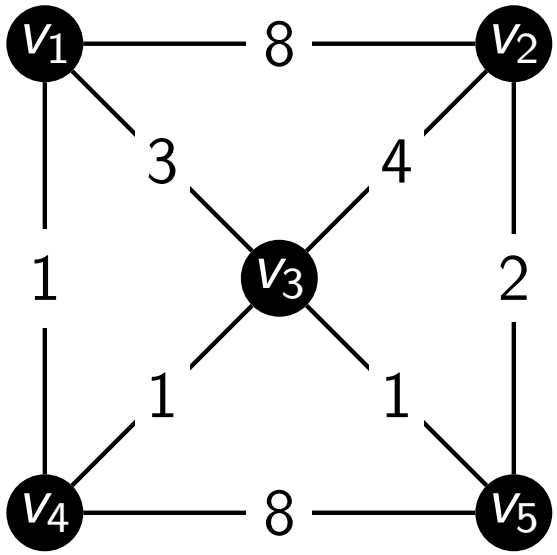


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	
v_5	10	2	1		

Rješenje

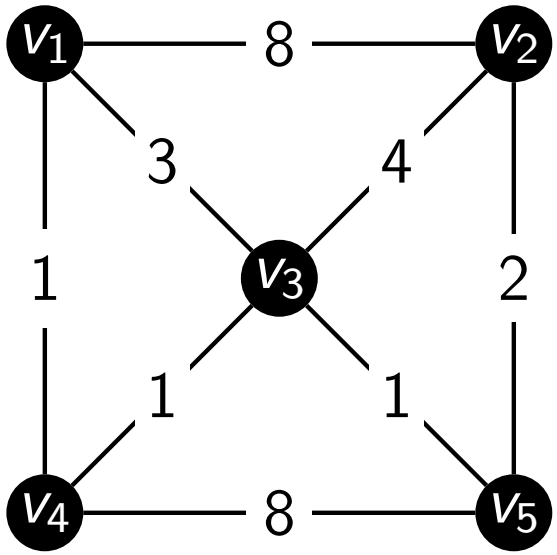


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1		

Rješenje

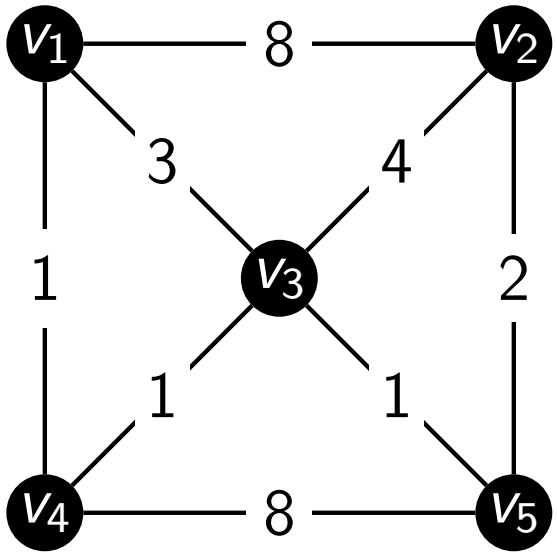


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	

Rješenje

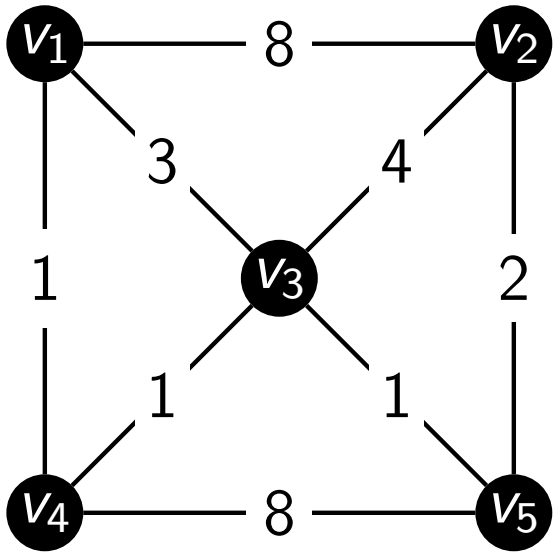


$k=0$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	∞	2
v_3	3	4	0	1	1
v_4	1	∞	1	0	8
v_5	∞	2	1	8	0

$k=1$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	∞
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	∞	2	1	8	0

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

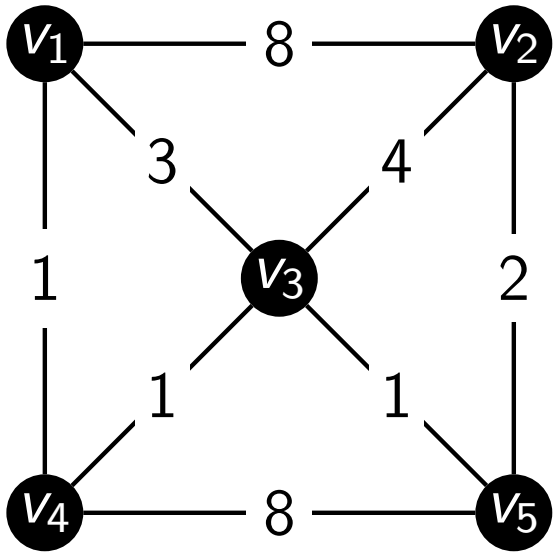
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

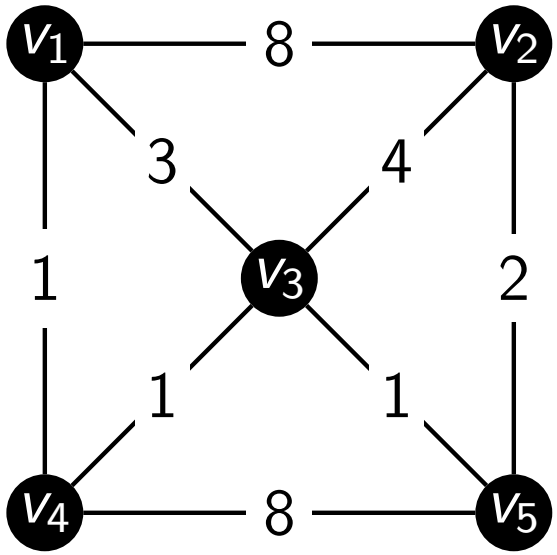
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

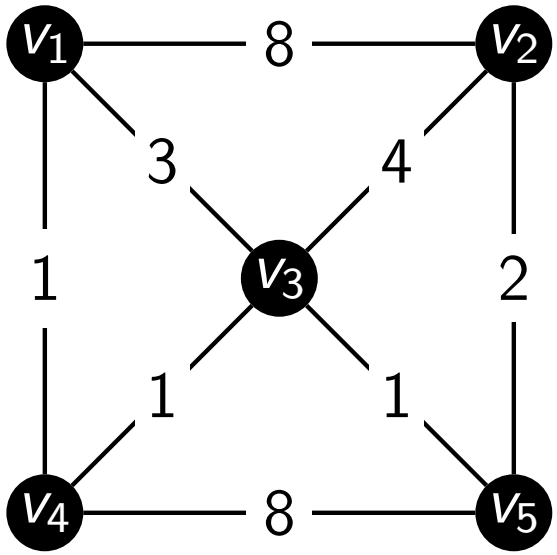
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3	3	4	0	1	1
v_4					
v_5					

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

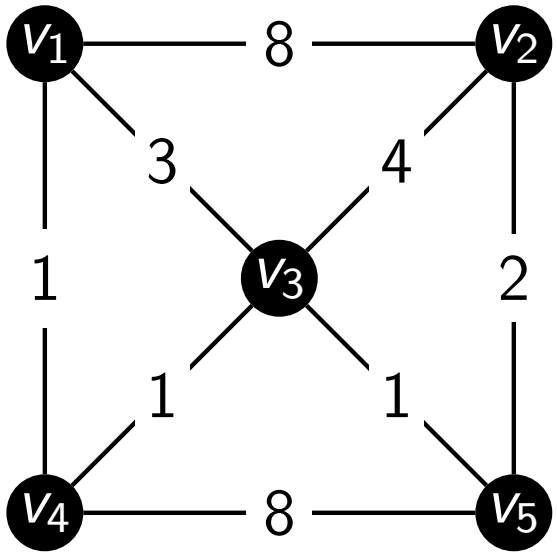
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1			3		
v_2			4		
v_3	3	4	0	1	1
v_4			1		
v_5			1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

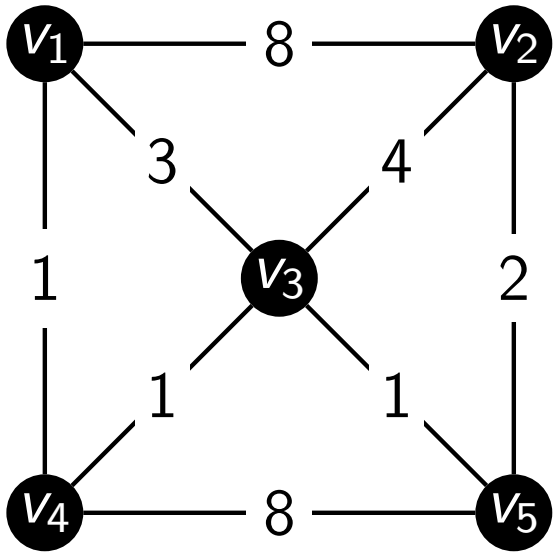
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0		3		
v_2			4		
v_3	3	4	0	1	1
v_4			1		
v_5			1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

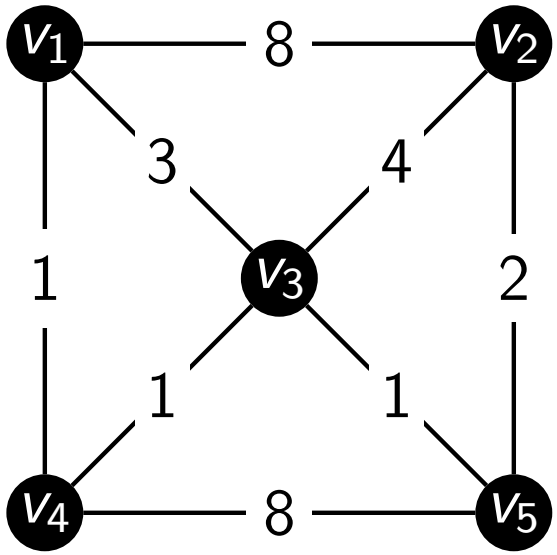
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3		
v_2			4		
v_3	3	4	0	1	1
v_4			1		
v_5			1		

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

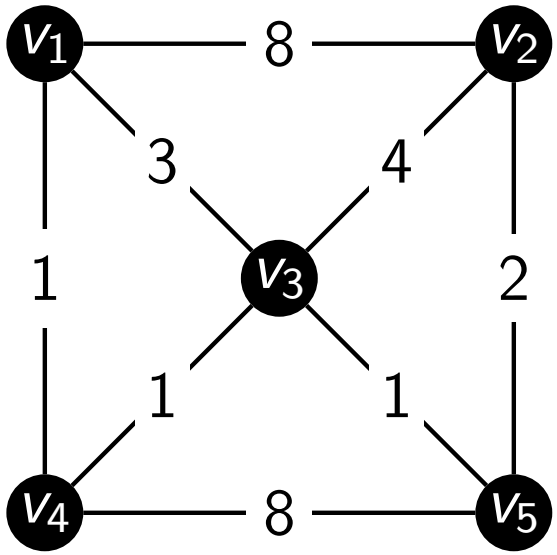
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	
v_2			4		
v_3	3	4	0	1	1
v_4			1		
v_5			1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

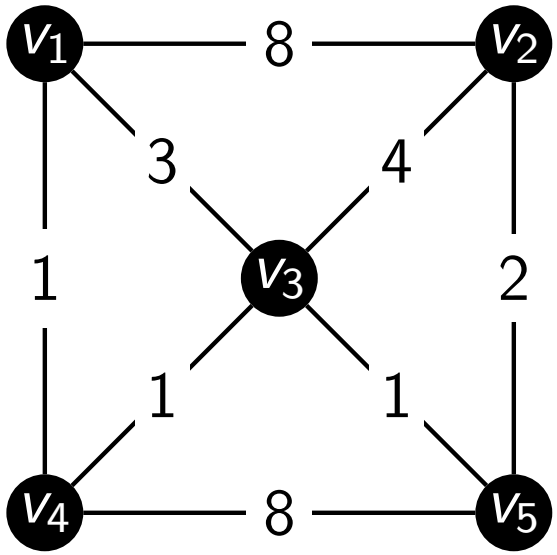
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2			4		
v_3	3	4	0	1	1
v_4			1		
v_5			1		

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

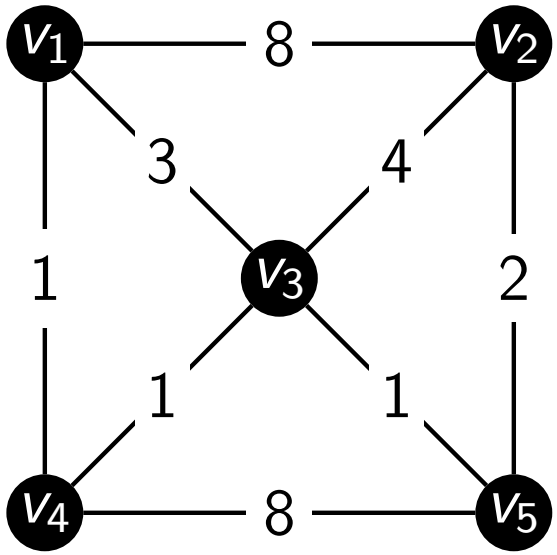
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7		4		
v_3	3	4	0	1	1
v_4	1		1		
v_5	4		1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

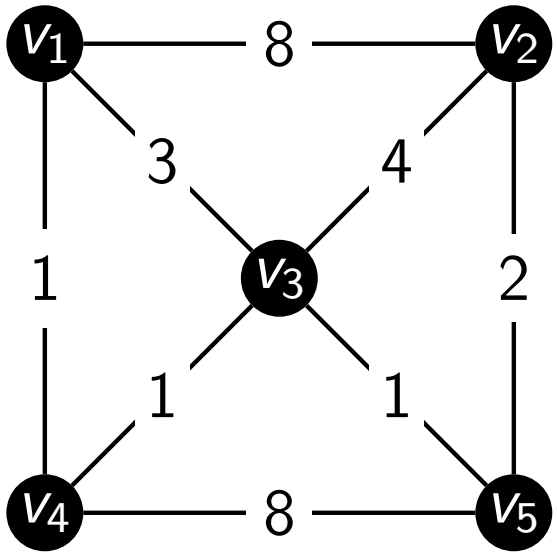
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4		
v_3	3	4	0	1	1
v_4	1		1		
v_5	4		1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

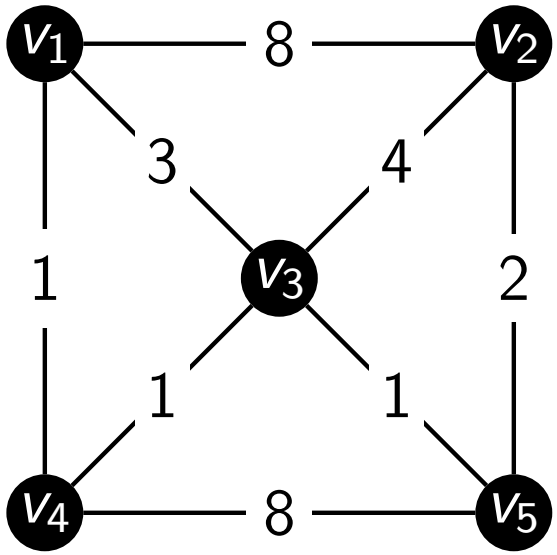
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	
v_3	3	4	0	1	1
v_4	1		1		
v_5	4		1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

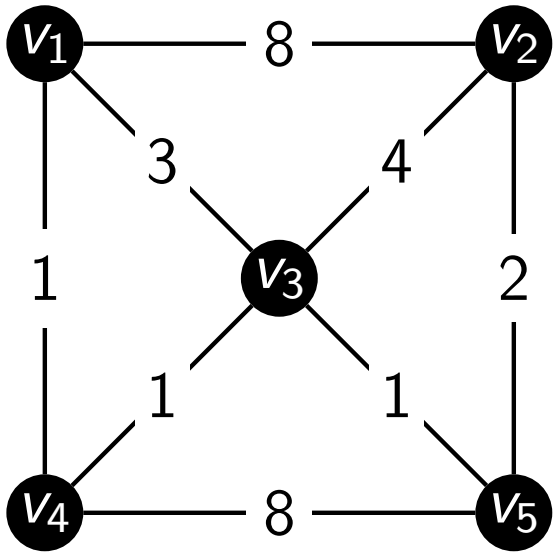
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1		1		
v_5	4		1		

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

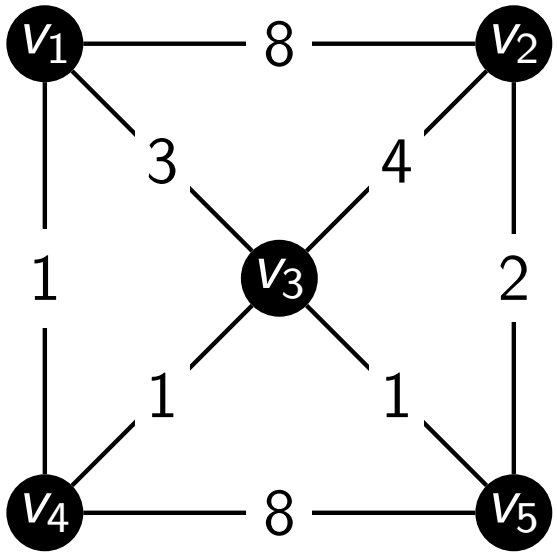
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1		
v_5	4	2	1		

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

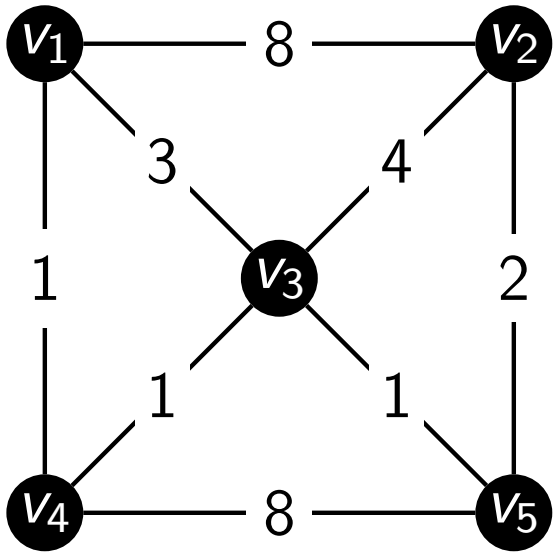
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	
v_5	4	2	1		

$k=2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

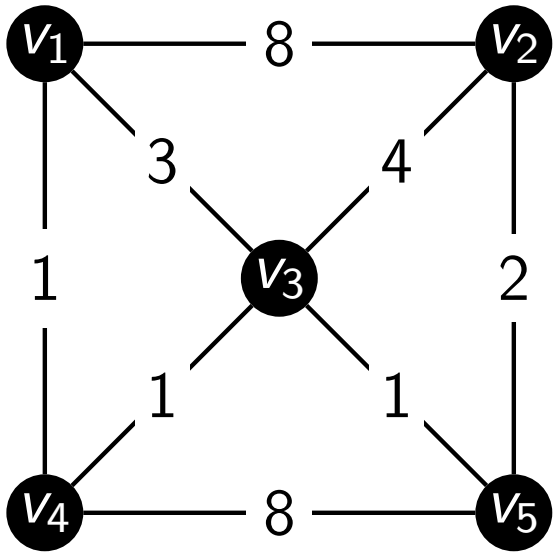
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1		

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

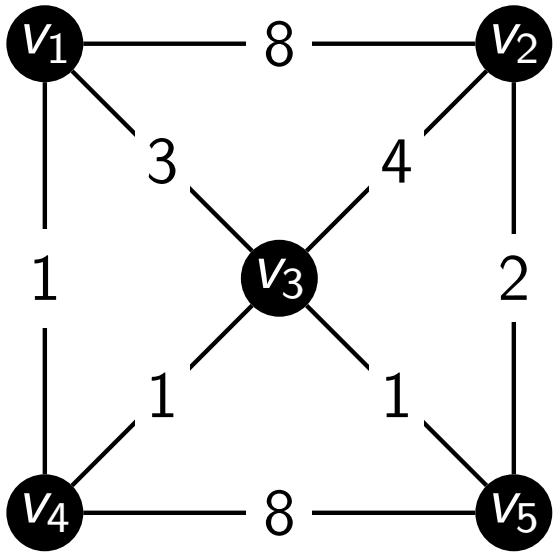
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

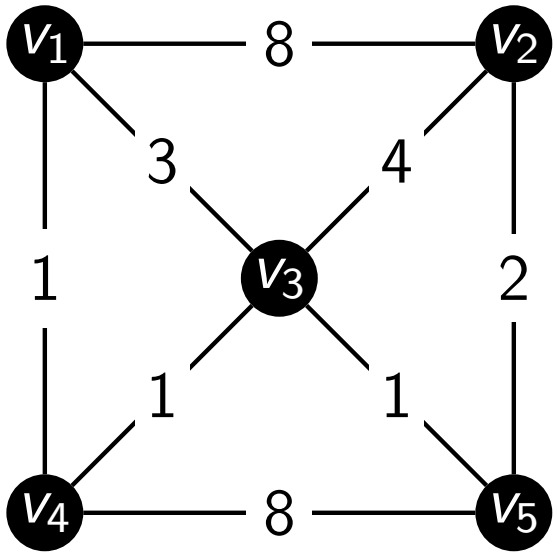
Rješenje



$k = 3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k = 2$	v_1	v_2	v_3	v_4	v_5
v_1	0	8	3	1	10
v_2	8	0	4	9	2
v_3	3	4	0	1	1
v_4	1	9	1	0	8
v_5	10	2	1	8	0

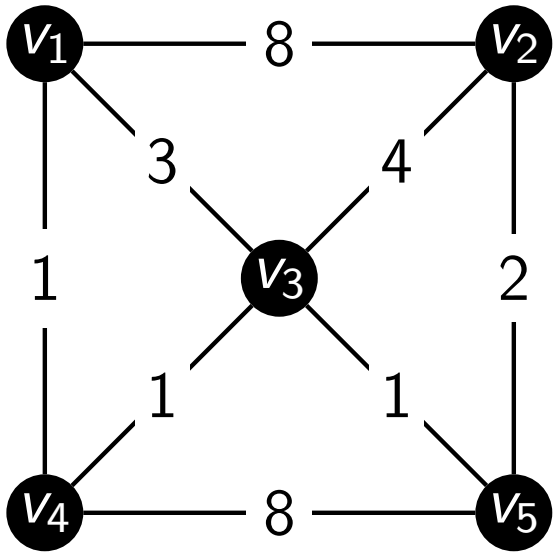
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

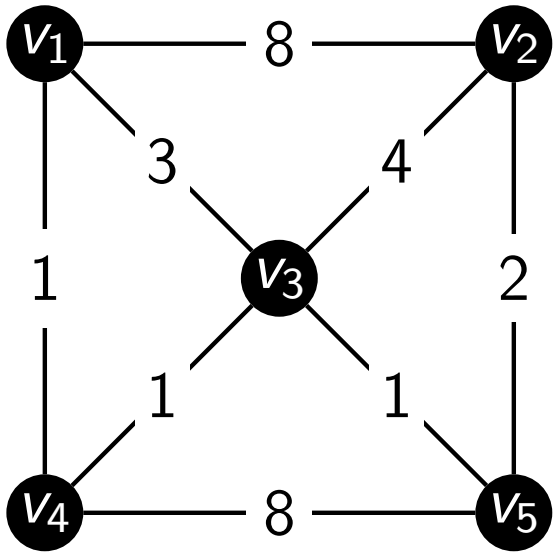
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

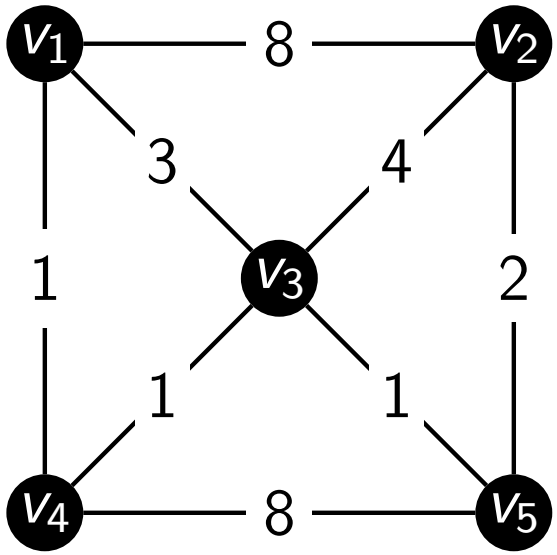
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4	1	5	1	0	2
v_5					

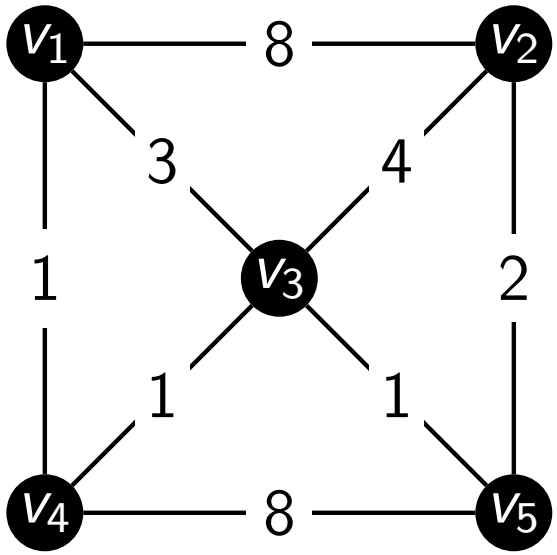
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1				1	
v_2				5	
v_3				1	
v_4	1	5	1	0	2
v_5				2	

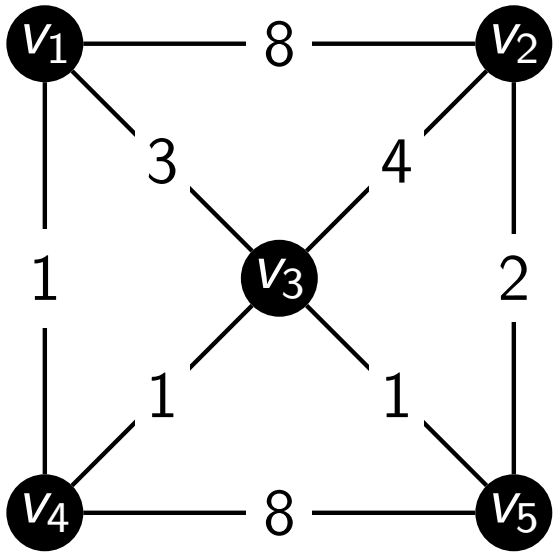
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0			1	
v_2				5	
v_3				1	
v_4	1	5	1	0	2
v_5				2	

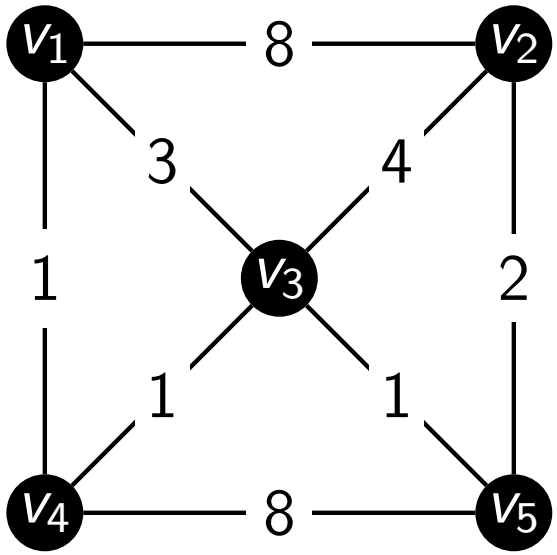
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6		1	
v_2				5	
v_3				1	
v_4	1	5	1	0	2
v_5				2	

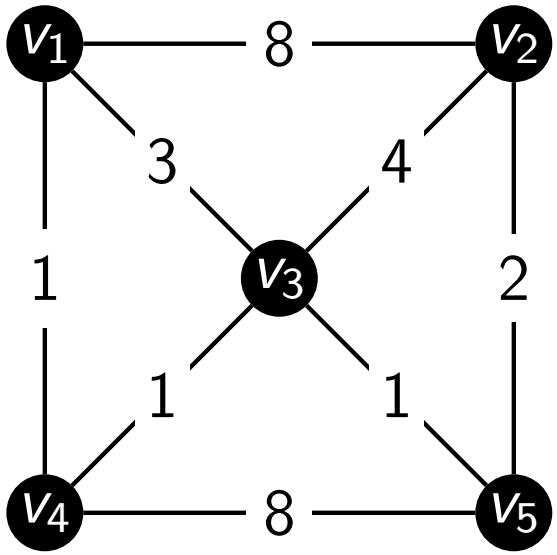
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	
v_2				5	
v_3				1	
v_4	1	5	1	0	2
v_5				2	

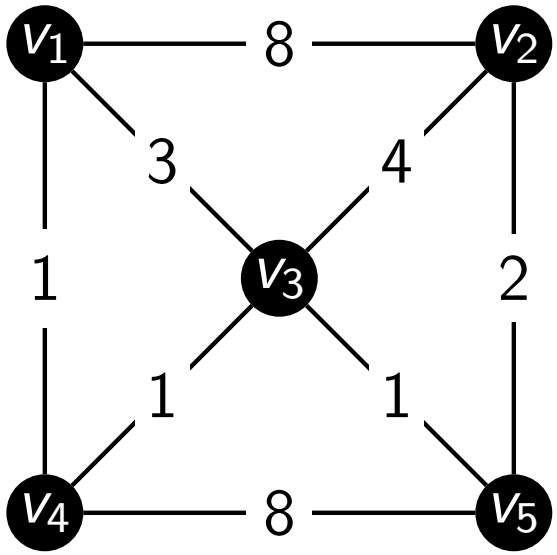
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2				5	
v_3				1	
v_4	1	5	1	0	2
v_5				2	

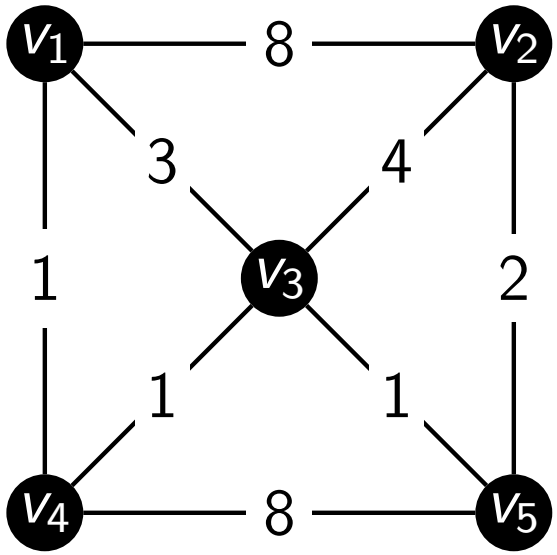
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6			5	
v_3	2			1	
v_4	1	5	1	0	2
v_5	3			2	

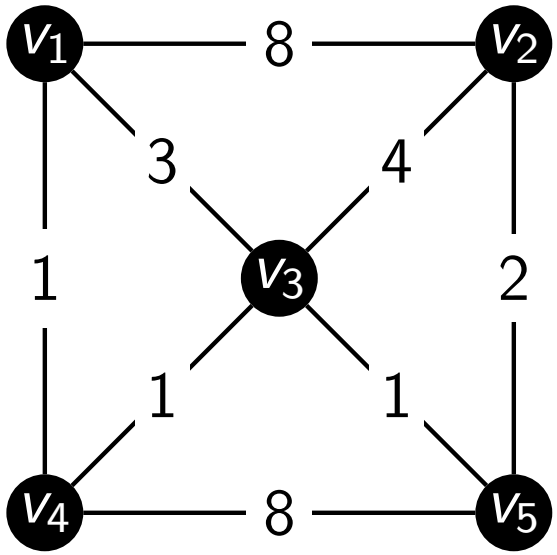
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0		5	
v_3	2			1	
v_4	1	5	1	0	2
v_5	3			2	

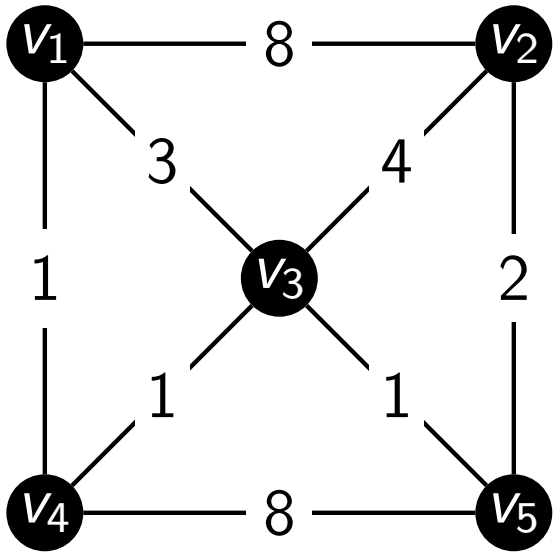
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	
v_3	2			1	
v_4	1	5	1	0	2
v_5	3			2	

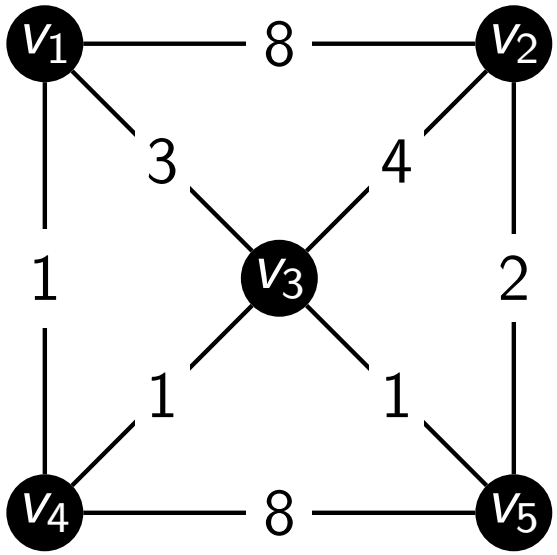
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2			1	
v_4	1	5	1	0	2
v_5	3			2	

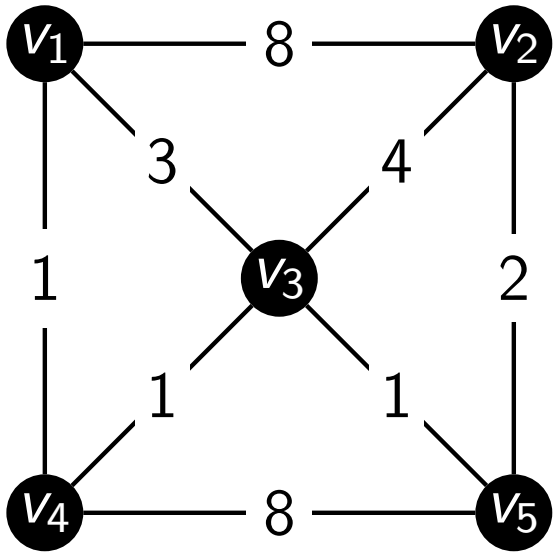
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4		1	
v_4	1	5	1	0	2
v_5	3	2		2	

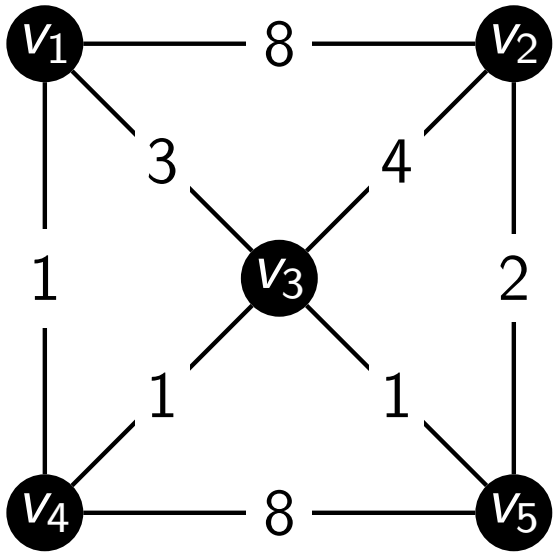
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	
v_4	1	5	1	0	2
v_5	3	2		2	

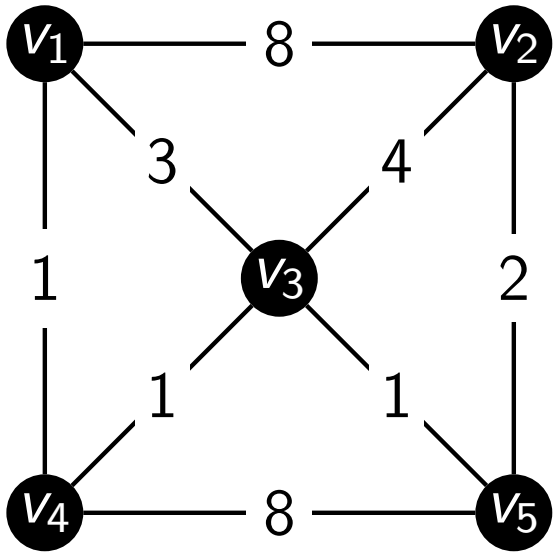
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2		2	

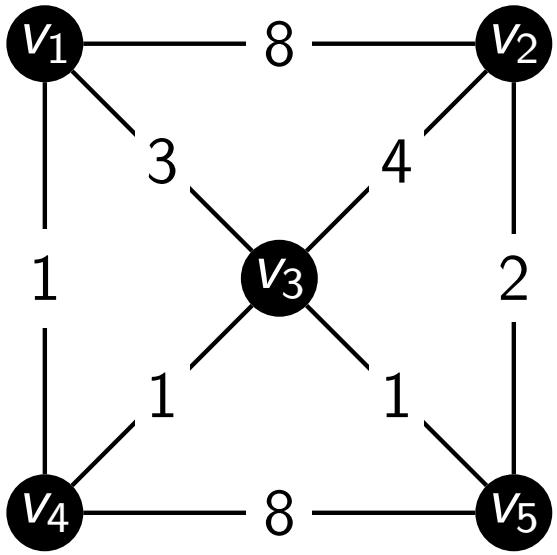
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	

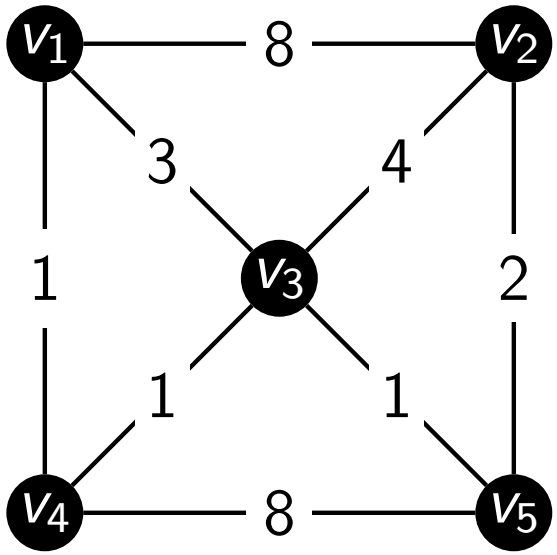
Rješenje



$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

Rješenje

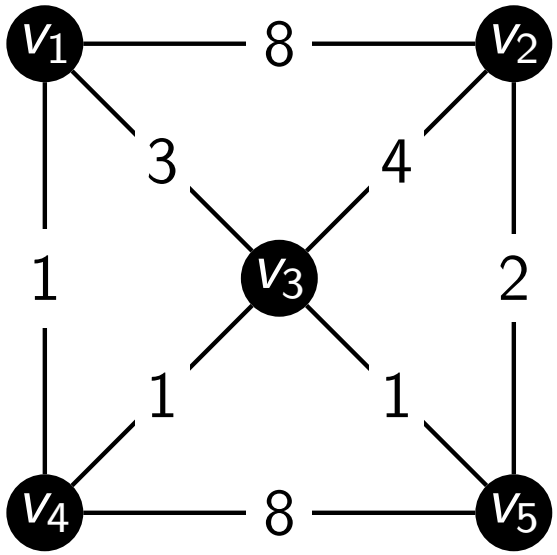


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

Rješenje

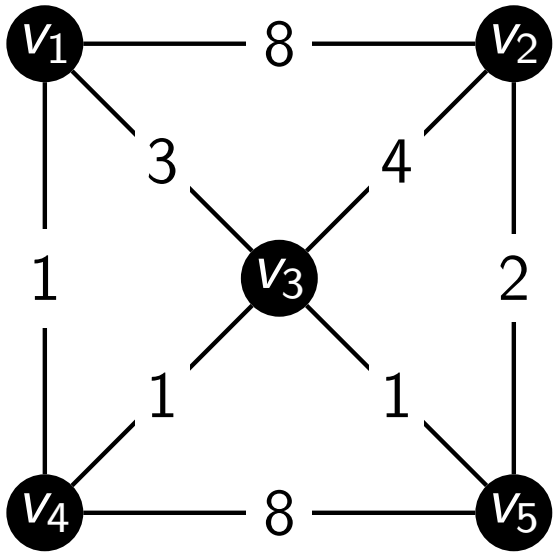


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5					

Rješenje

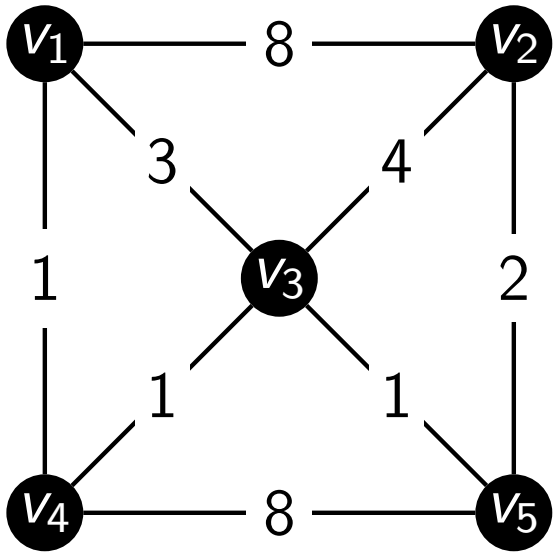


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1					
v_2					
v_3					
v_4					
v_5	3	2	1	2	0

Rješenje

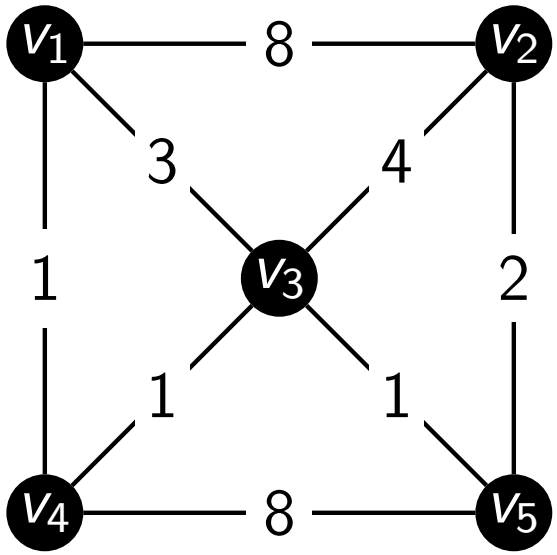


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1					3
v_2					2
v_3					1
v_4					2
v_5	3	2	1	2	0

Rješenje

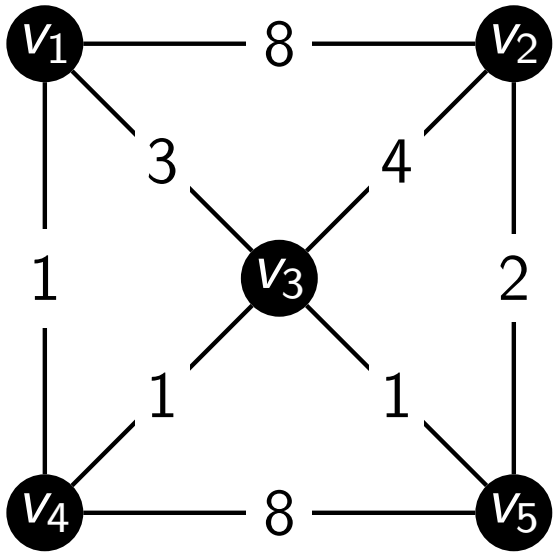


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0				3
v_2					2
v_3					1
v_4					2
v_5	3	2	1	2	0

Rješenje

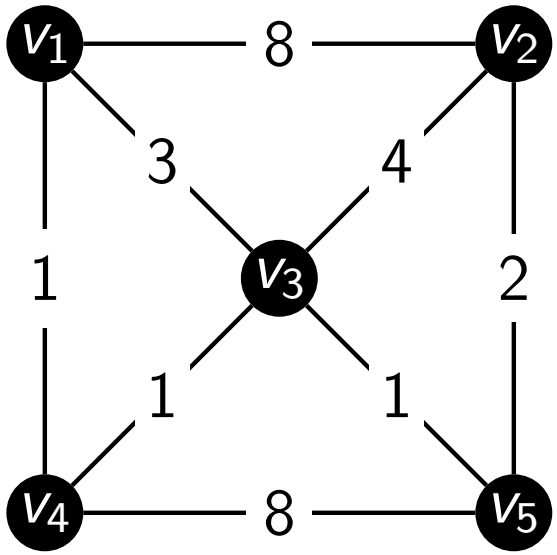


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5			3
v_2					2
v_3					1
v_4					2
v_5	3	2	1	2	0

Rješenje

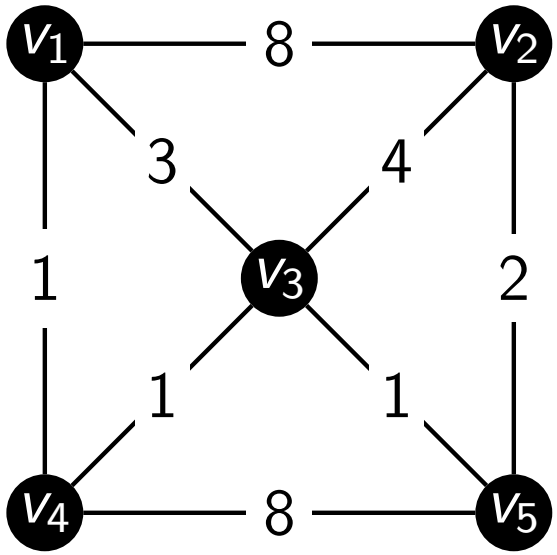


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2		3
v_2					2
v_3					1
v_4					2
v_5	3	2	1	2	0

Rješenje

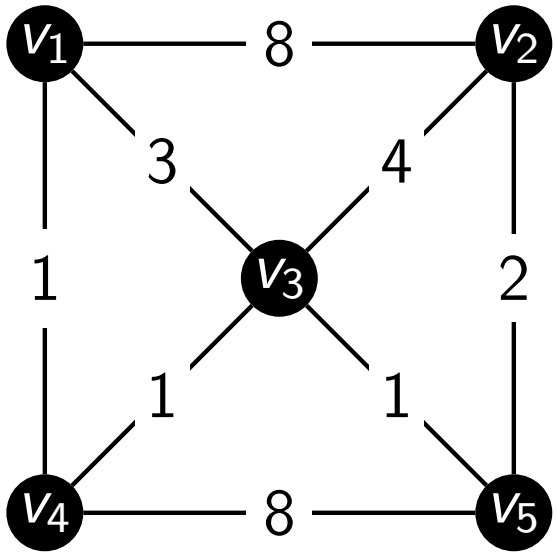


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2					2
v_3					1
v_4					2
v_5	3	2	1	2	0

Rješenje

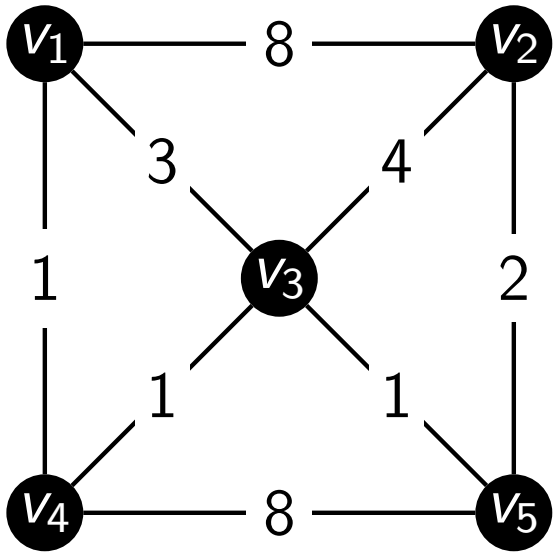


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5				2
v_3	2				1
v_4	1				2
v_5	3	2	1	2	0

Rješenje

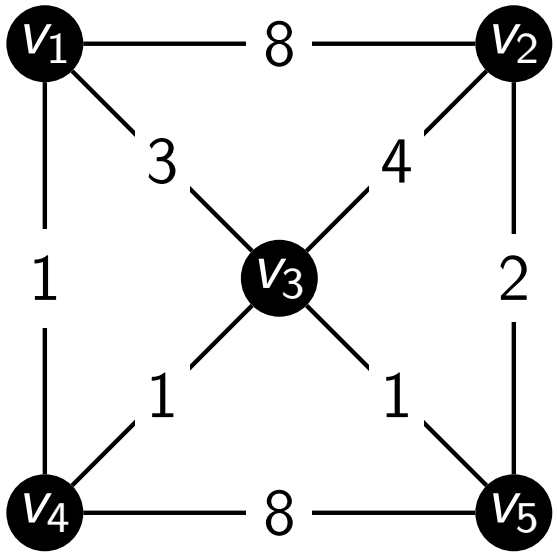


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0			2
v_3	2				1
v_4	1				2
v_5	3	2	1	2	0

Rješenje

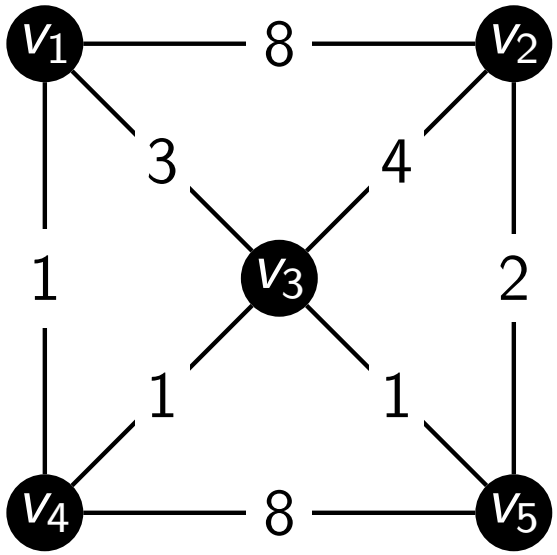


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3		2
v_3	2				1
v_4	1				2
v_5	3	2	1	2	0

Rješenje

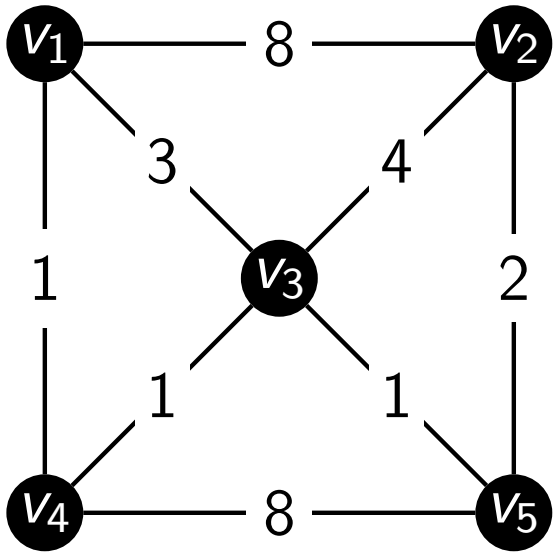


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2				1
v_4	1				2
v_5	3	2	1	2	0

Rješenje

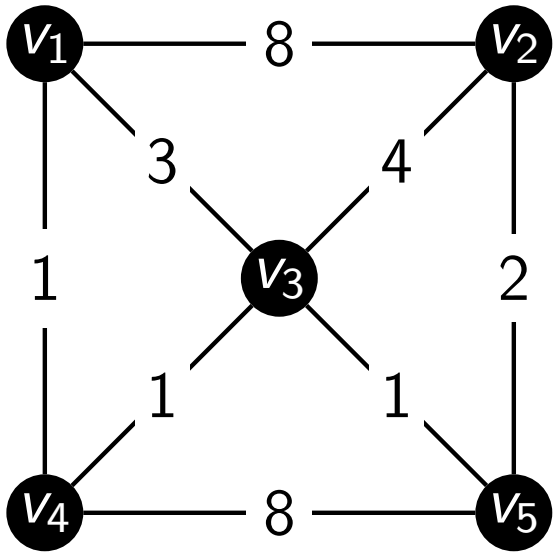


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2	3			1
v_4	1	4			2
v_5	3	2	1	2	0

Rješenje

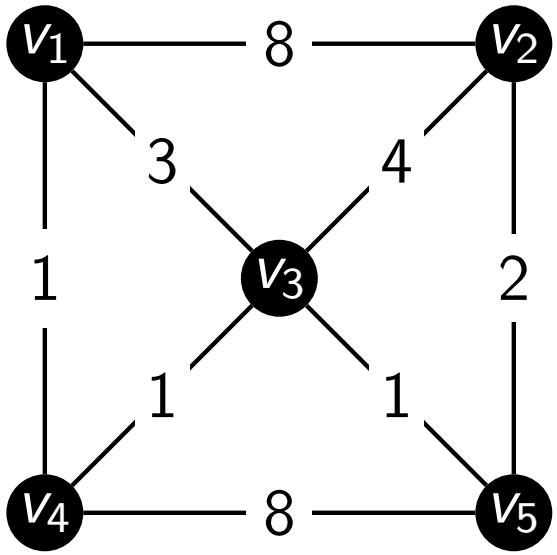


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2	3	0		1
v_4	1	4			2
v_5	3	2	1	2	0

Rješenje

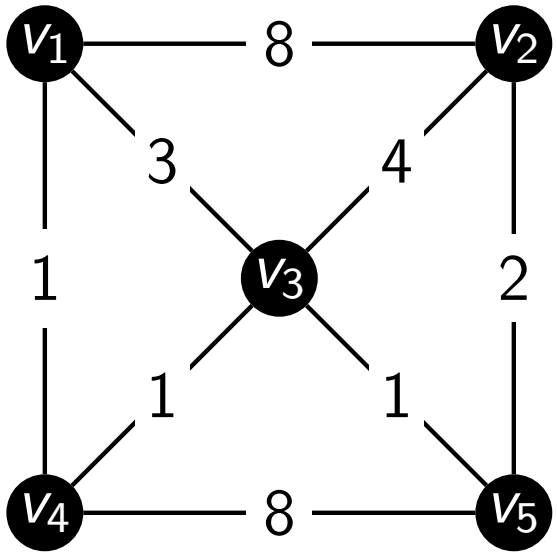


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2	3	0	1	1
v_4	1	4			2
v_5	3	2	1	2	0

Rješenje

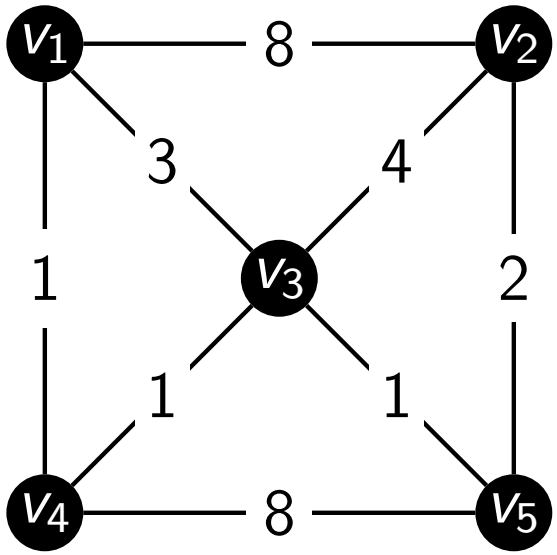


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2	3	0	1	1
v_4	1	4	1		2
v_5	3	2	1	2	0

Rješenje

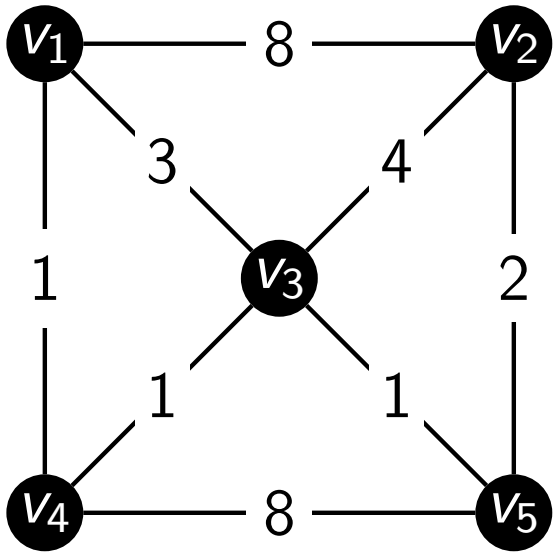


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2	3	0	1	1
v_4	1	4	1	0	2
v_5	3	2	1	2	0

Rješenje

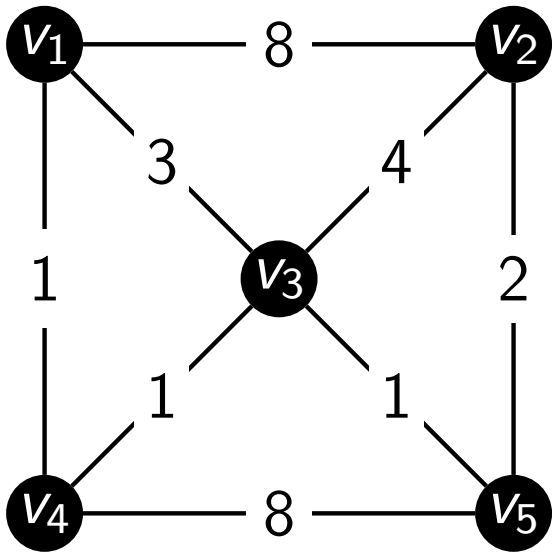


$k=3$	v_1	v_2	v_3	v_4	v_5
v_1	0	7	3	1	4
v_2	7	0	4	5	2
v_3	3	4	0	1	1
v_4	1	5	1	0	2
v_5	4	2	1	2	0

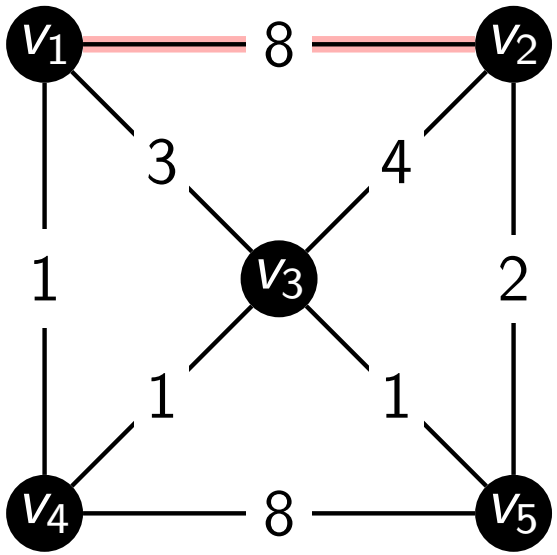
$k=4$	v_1	v_2	v_3	v_4	v_5
v_1	0	6	2	1	3
v_2	6	0	4	5	2
v_3	2	4	0	1	1
v_4	1	5	1	0	2
v_5	3	2	1	2	0

$k=5$	v_1	v_2	v_3	v_4	v_5
v_1	0	5	2	1	3
v_2	5	0	3	4	2
v_3	2	3	0	1	1
v_4	1	4	1	0	2
v_5	3	2	1	2	0

Vizualizacija Floyd-Warshallovog algoritma

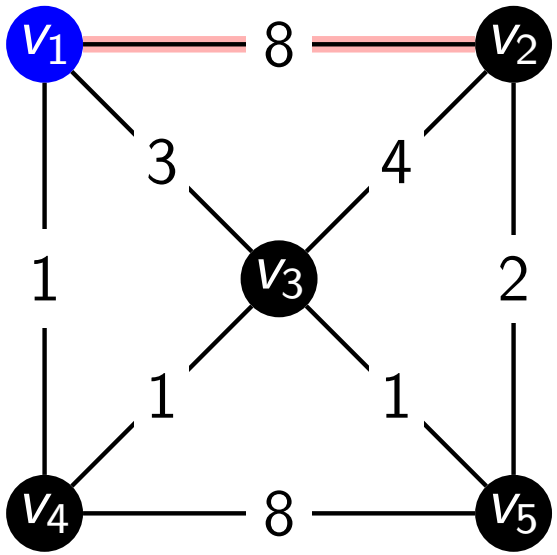


Vizualizacija Floyd-Warshallovog algoritma



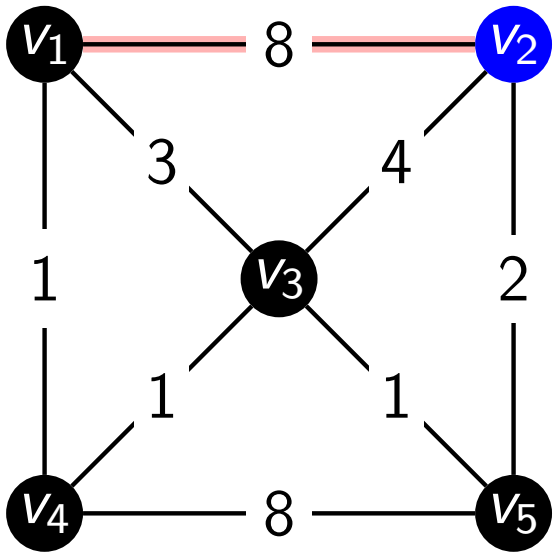
- Korak 0: $d(v_1, v_2) = 8$

Vizualizacija Floyd-Warshallovog algoritma



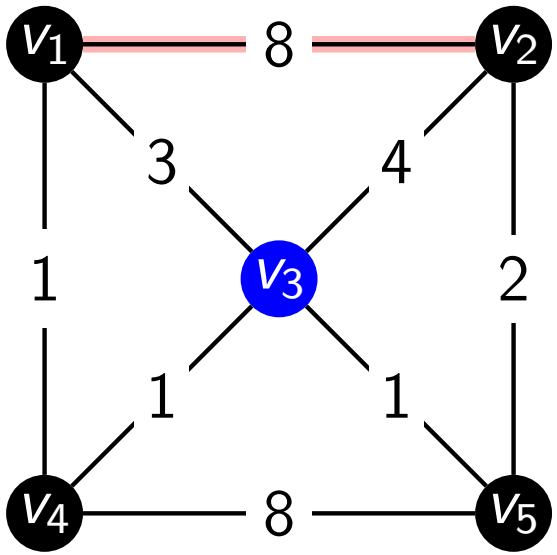
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$

Vizualizacija Floyd-Warshallovog algoritma



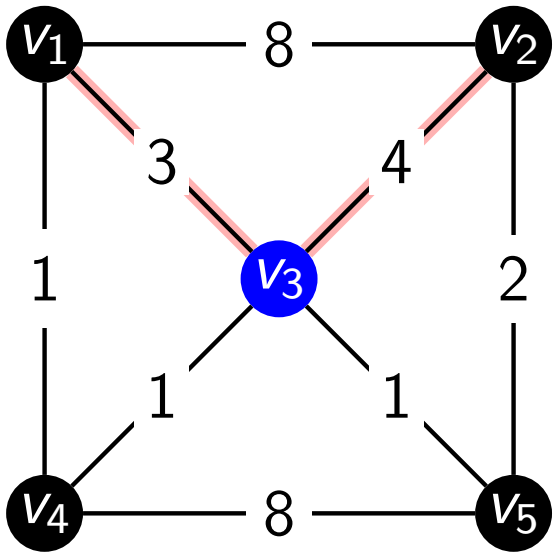
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$

Vizualizacija Floyd-Warshallovog algoritma



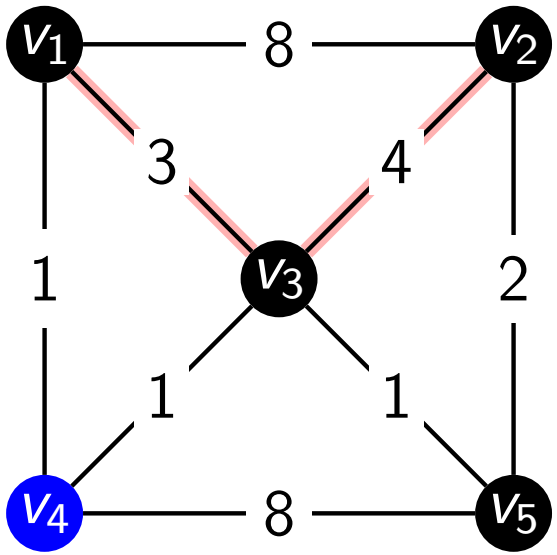
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3:

Vizualizacija Floyd-Warshallovog algoritma



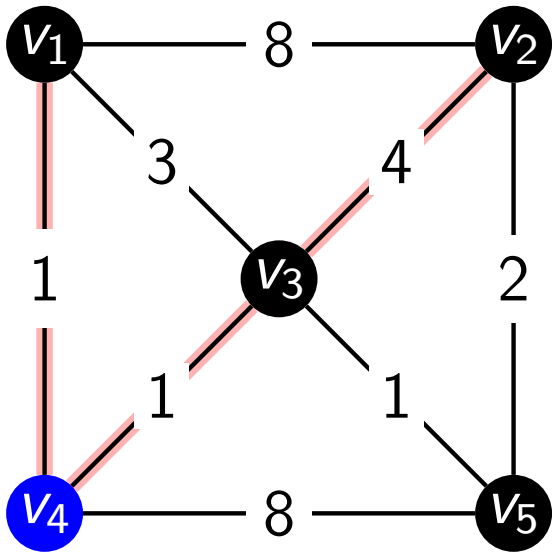
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3: $d(v_1, v_2) = 7$

Vizualizacija Floyd-Warshallovog algoritma



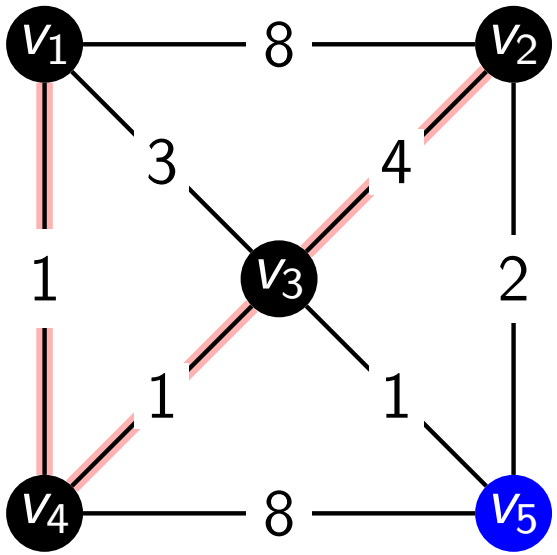
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3: $d(v_1, v_2) = 7$
- Korak 4:

Vizualizacija Floyd-Warshallovog algoritma



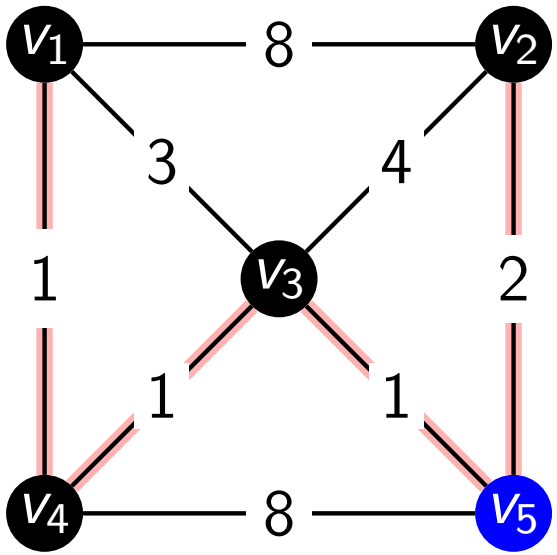
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3: $d(v_1, v_2) = 7$
- Korak 4: $d(v_1, v_2) = 6$

Vizualizacija Floyd-Warshallovog algoritma



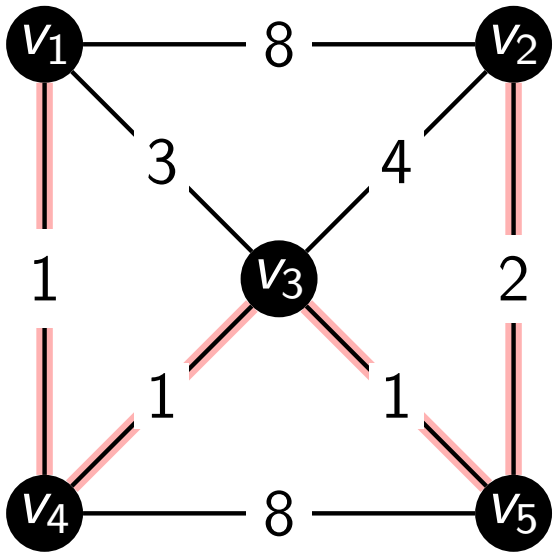
- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3: $d(v_1, v_2) = 7$
- Korak 4: $d(v_1, v_2) = 6$
- Korak 5:

Vizualizacija Floyd-Warshallovog algoritma



- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3: $d(v_1, v_2) = 7$
- Korak 4: $d(v_1, v_2) = 6$
- Korak 5: $d(v_1, v_2) = 5$

Vizualizacija Floyd-Warshallovog algoritma

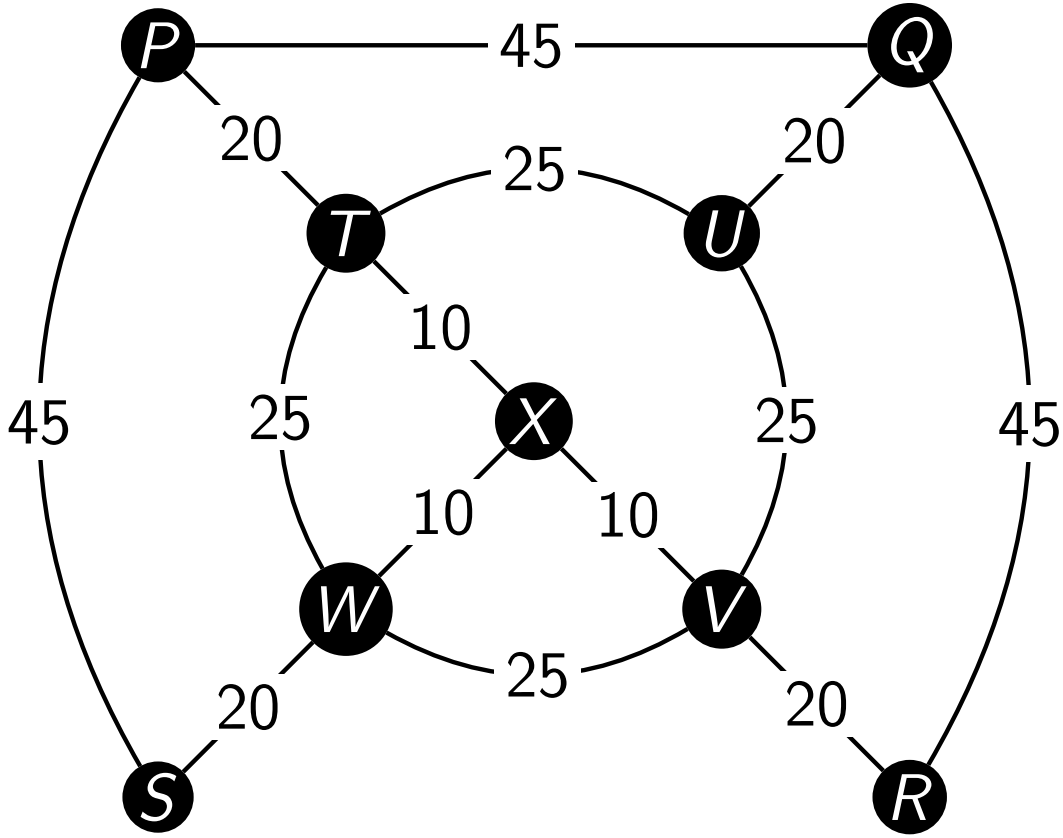


- Korak 0: $d(v_1, v_2) = 8$
- Korak 1: $d(v_1, v_2) = 8$
- Korak 2: $d(v_1, v_2) = 8$
- Korak 3: $d(v_1, v_2) = 7$
- Korak 4: $d(v_1, v_2) = 6$
- Korak 5: $d(v_1, v_2) = 5$

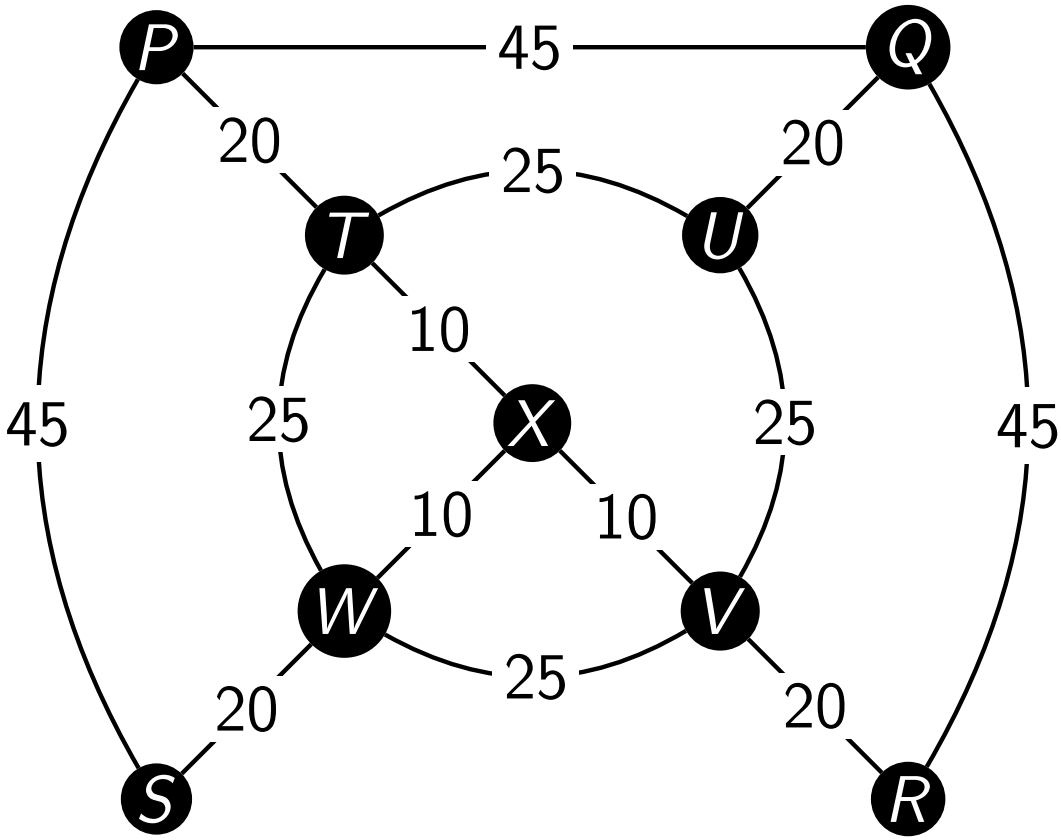
jedanaesti zadatak

Zadatak 11

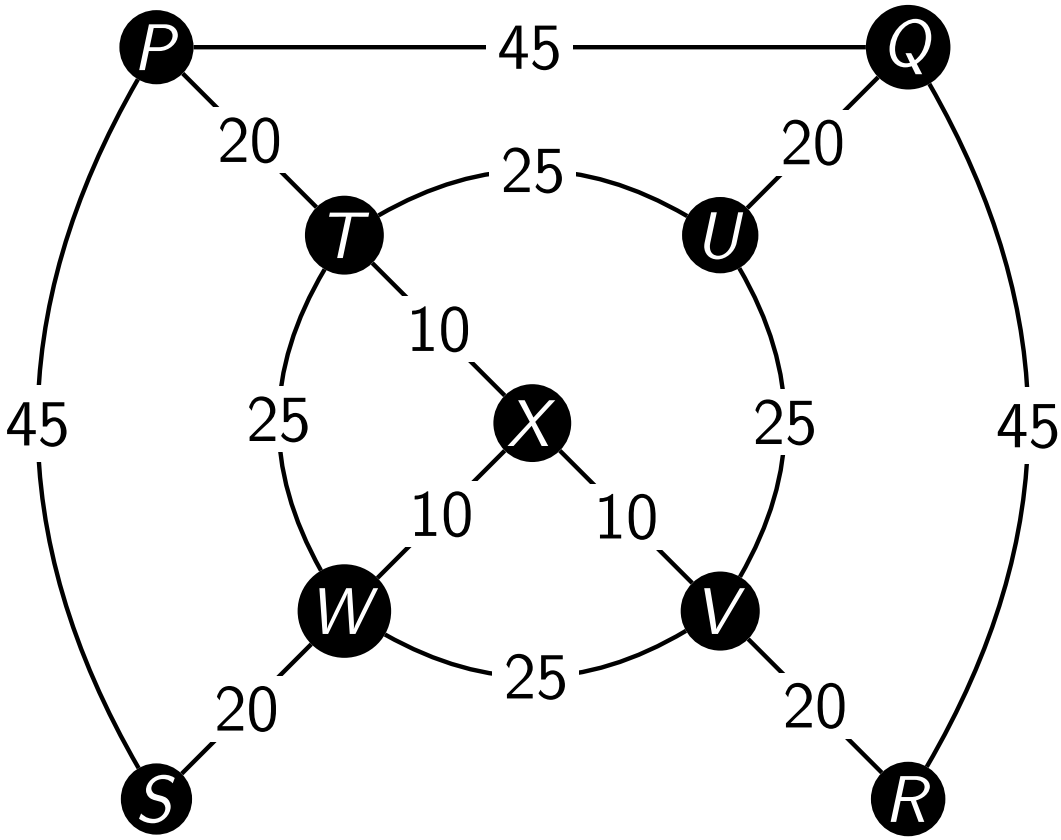
Riješite problem kineskog poštara za težinski graf G.



Rješenje

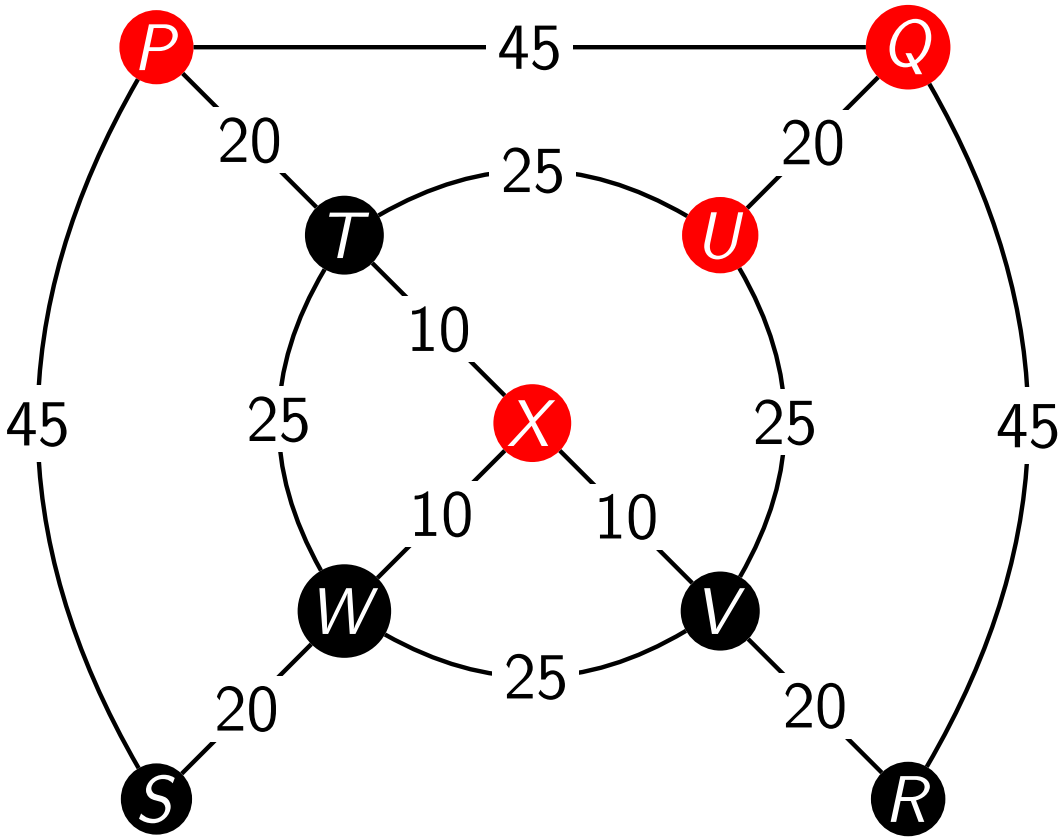


Rješenje



1) Vrhovi neparnog stupnja

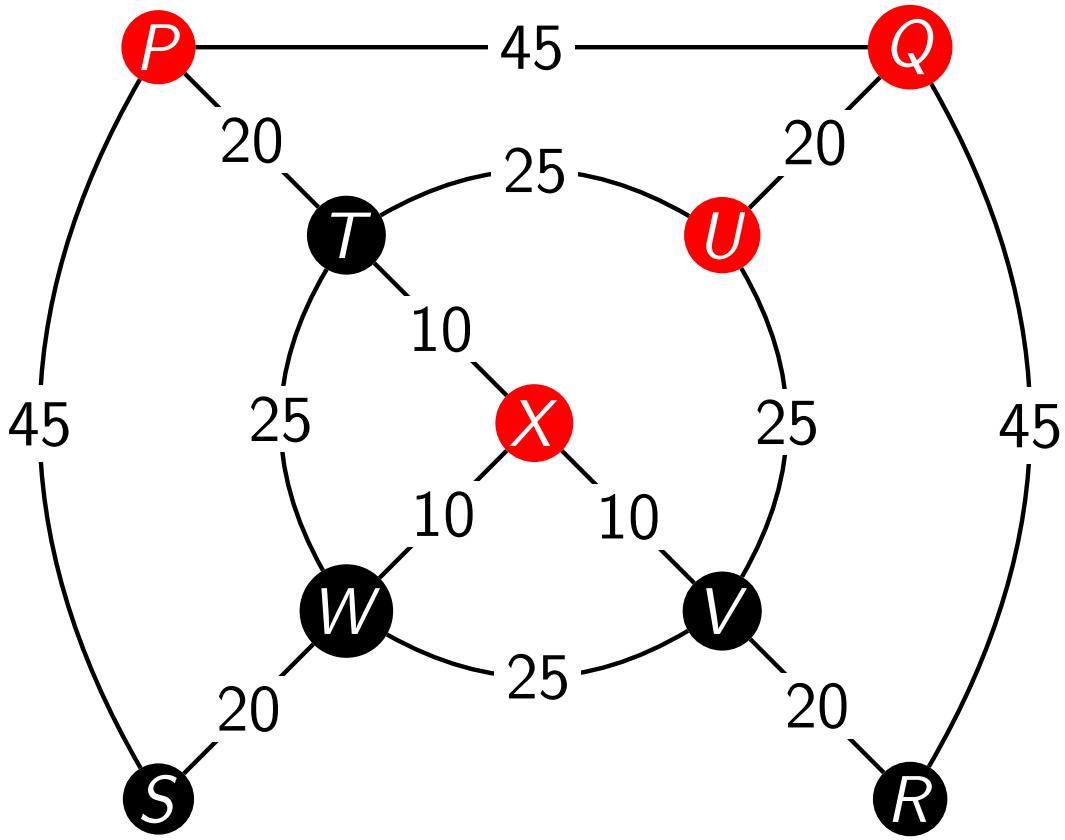
Rješenje



1) Vrhovi neparnog stupnja

P, Q, U, X

Rješenje

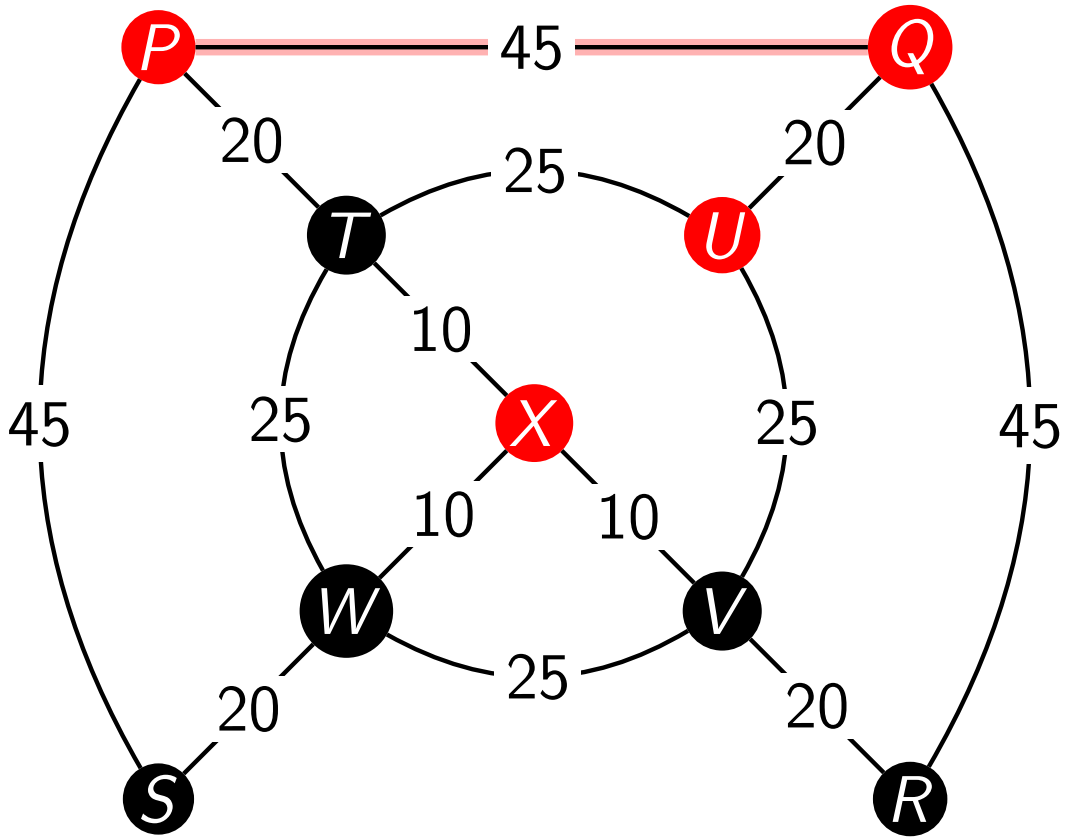


1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

Rješenje



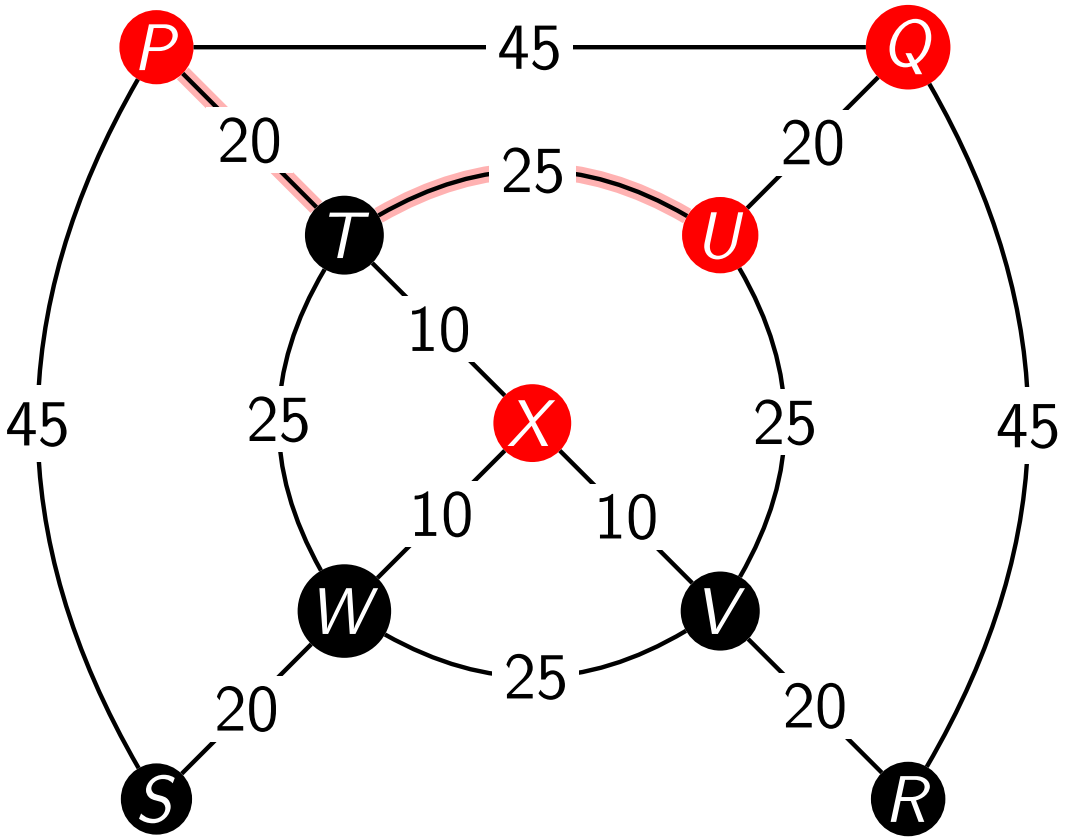
1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

$PQ \leftarrow 45$,

Rješenje



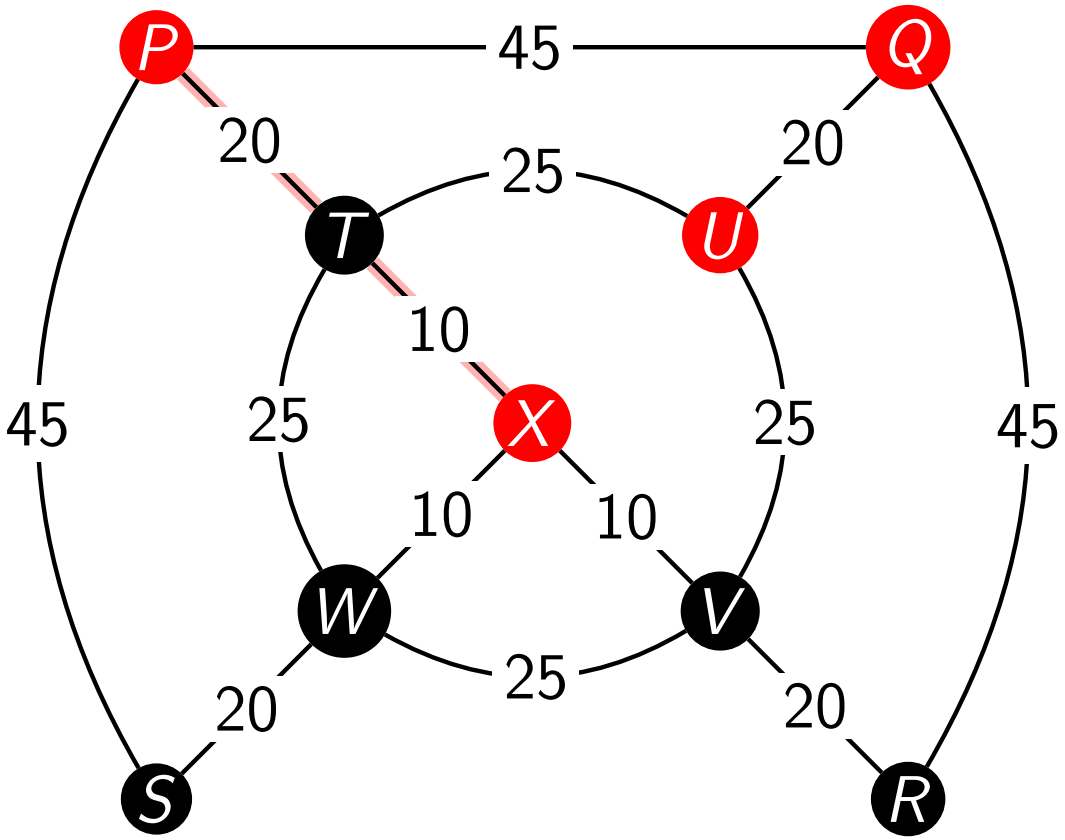
1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$

Rješenje



1) Vrhovi neparnog stupnja

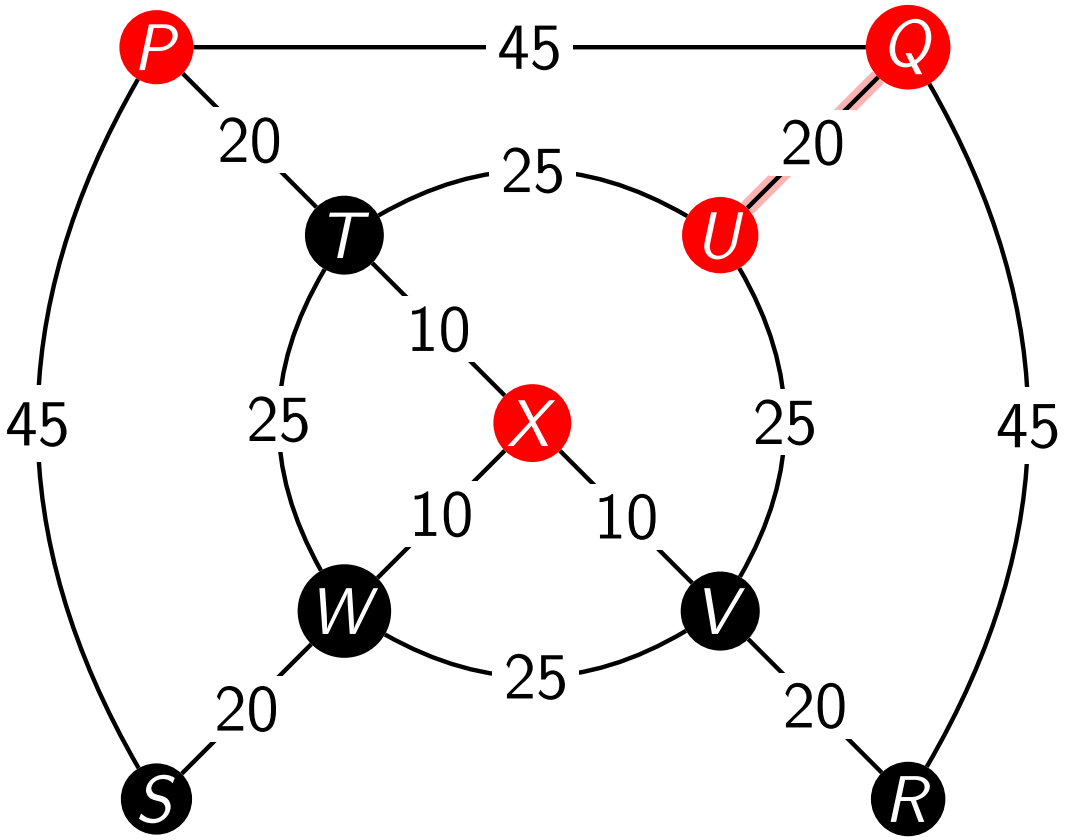
$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

$PQ \leftarrow 45$, $PU \leftarrow 45$,

$PX \leftarrow 30$,

Rješenje



1) Vrhovi neparnog stupnja

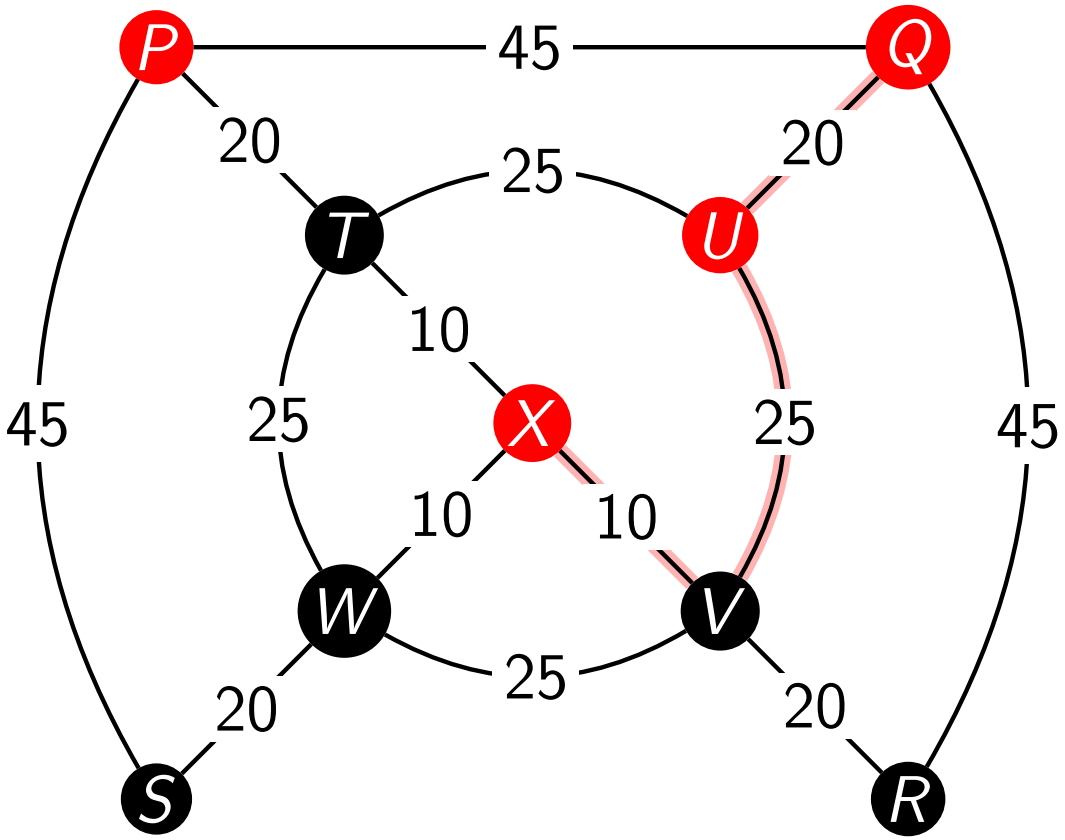
$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$

$$PX \leftarrow 30, \quad QU \leftarrow 20,$$

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

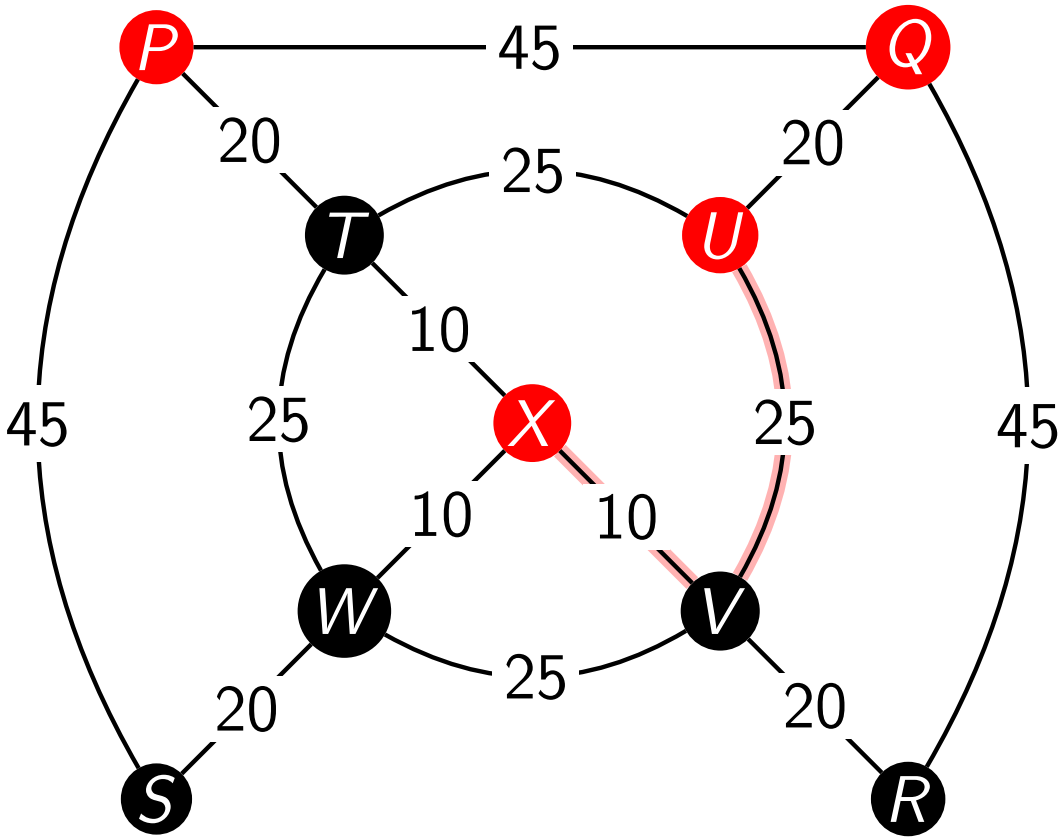
2) Udaljenosti između vrhova neparnog stupnja

$PQ \leftarrow 45$, $PU \leftarrow 45$,

$PX \leftarrow 30$, $QU \leftarrow 20$,

$QX \leftarrow 55$,

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

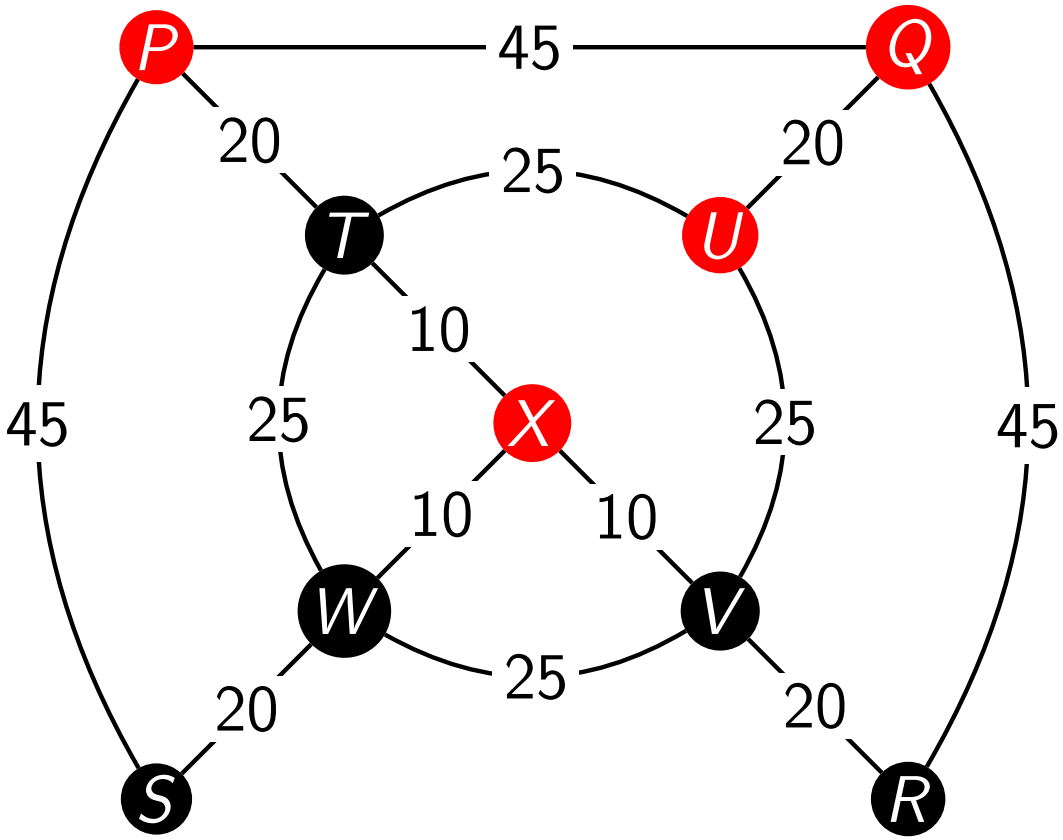
2) Udaljenosti između vrhova neparnog stupnja

$PQ \leftarrow 45$, $PU \leftarrow 45$,

$PX \leftarrow 30$, $QU \leftarrow 20$,

$QX \leftarrow 55$, $UX \leftarrow 35$

Rješenje



1) Vrhovi neparnog stupnja

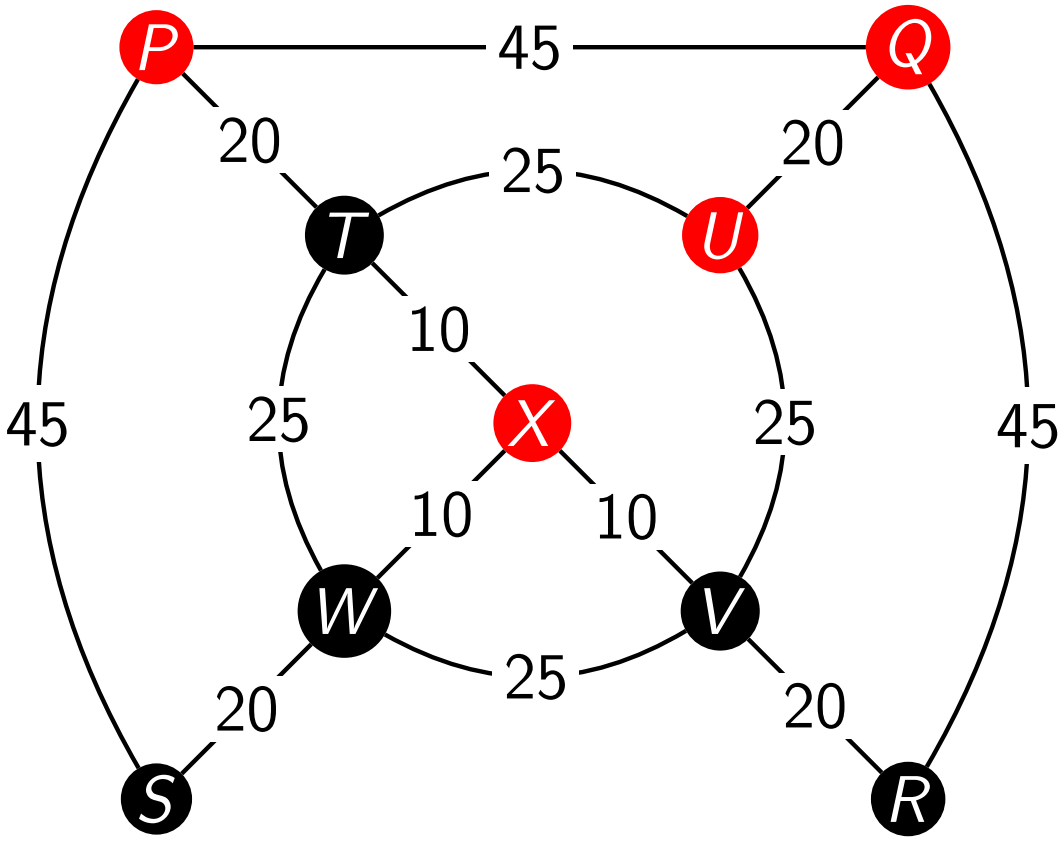
$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

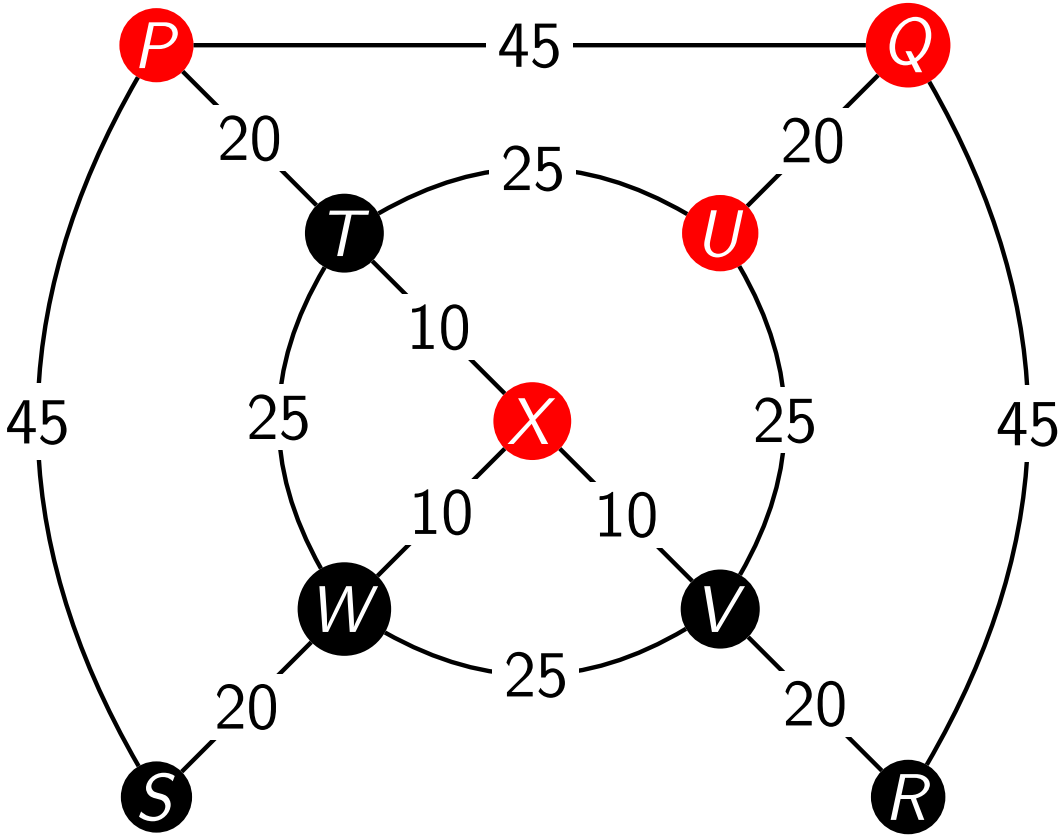
2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80,$$

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

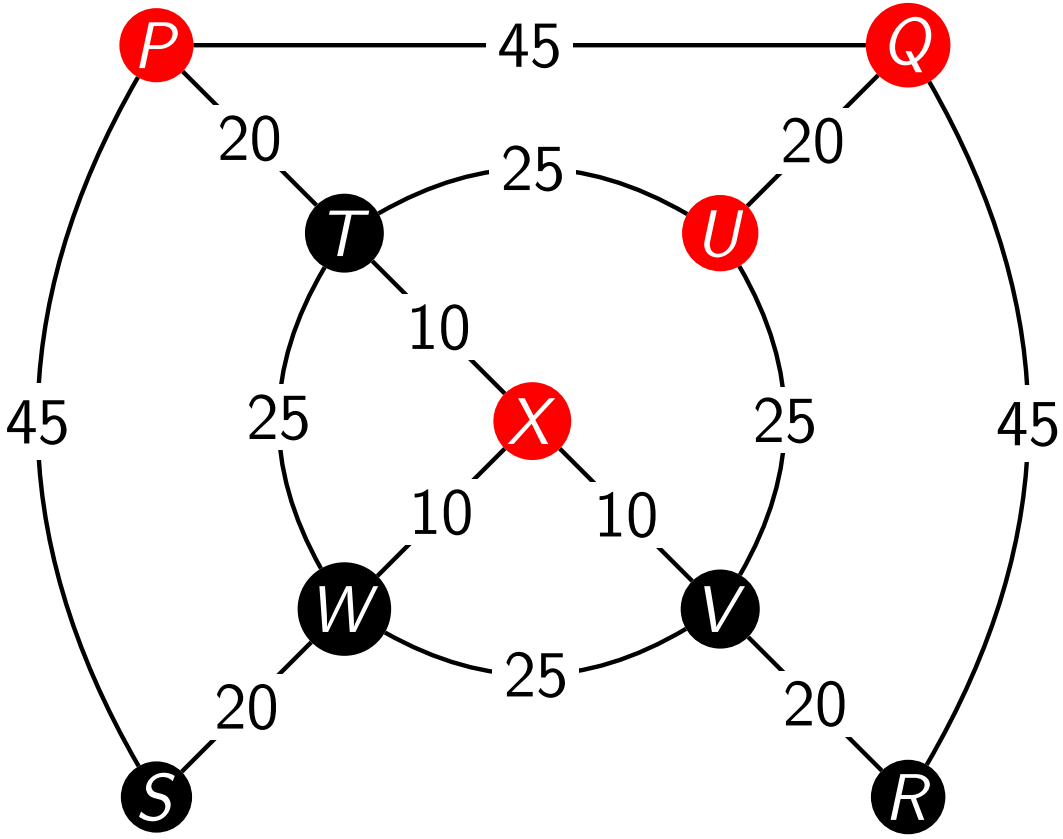
2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100,$$

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

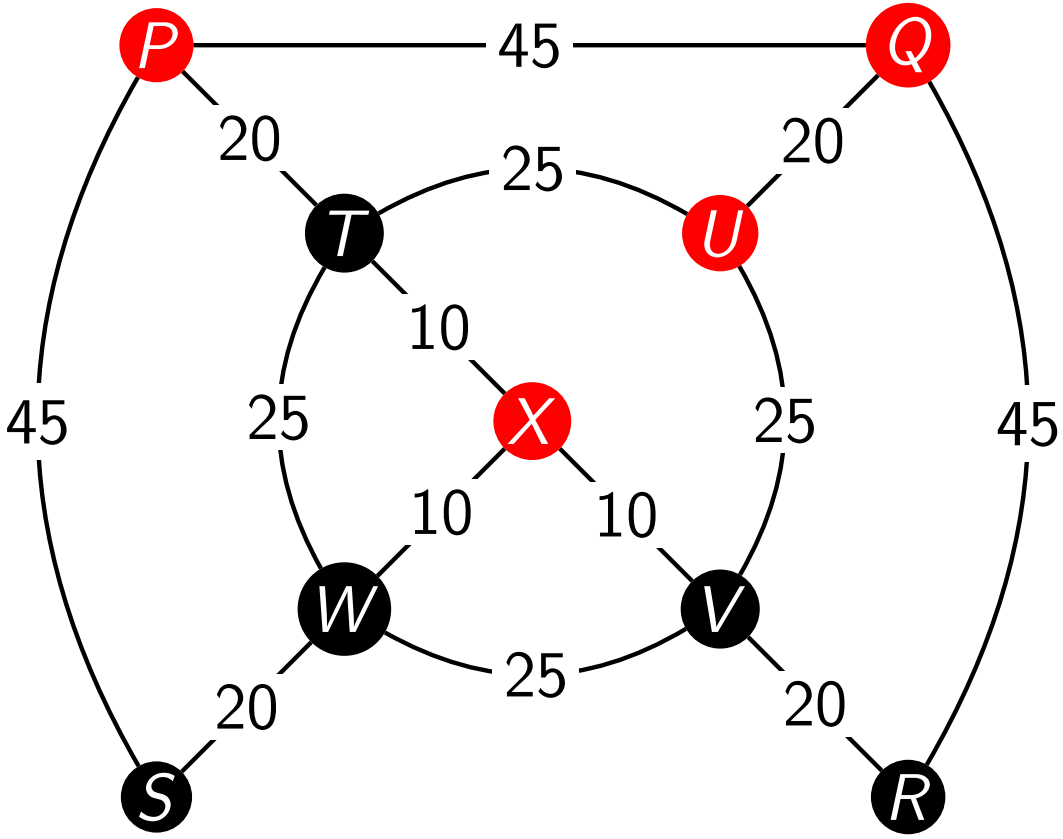
2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad PX + QU \leftarrow 50$$

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

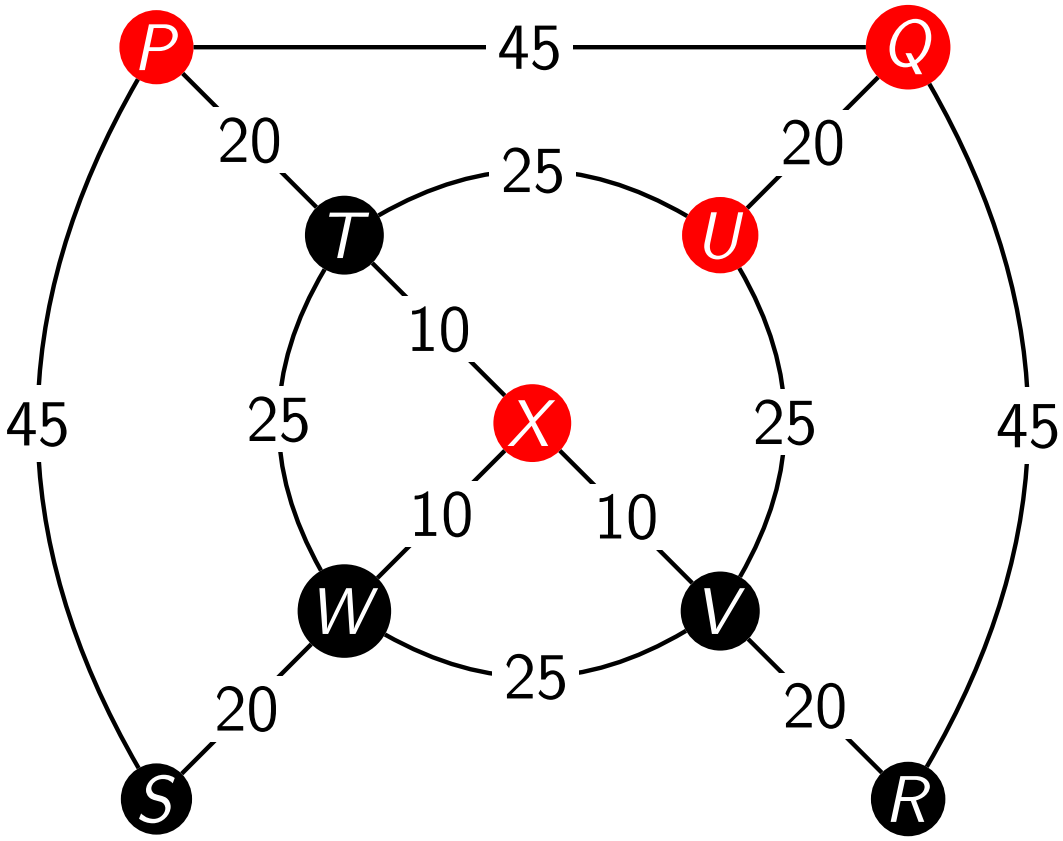
2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad \boxed{PX + QU \leftarrow 50}$$

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

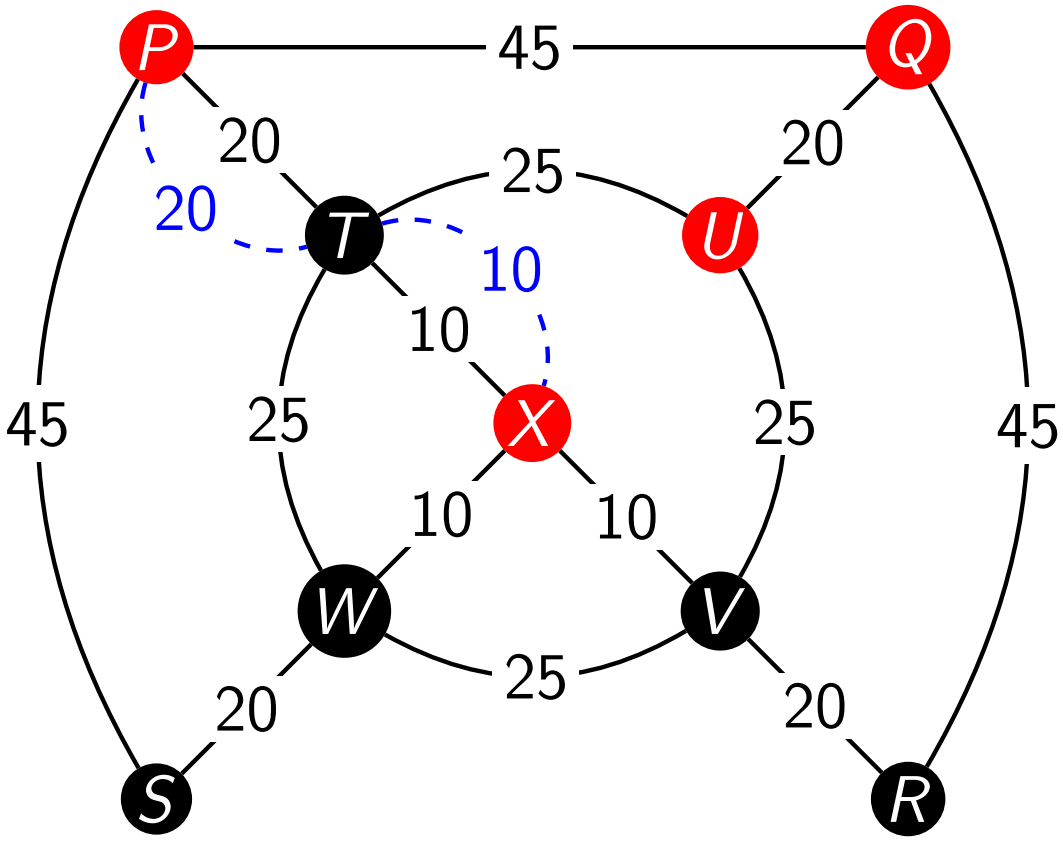
$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad \boxed{PX + QU \leftarrow 50}$$

4) Udvostručimo najkraći (P, X)-put

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

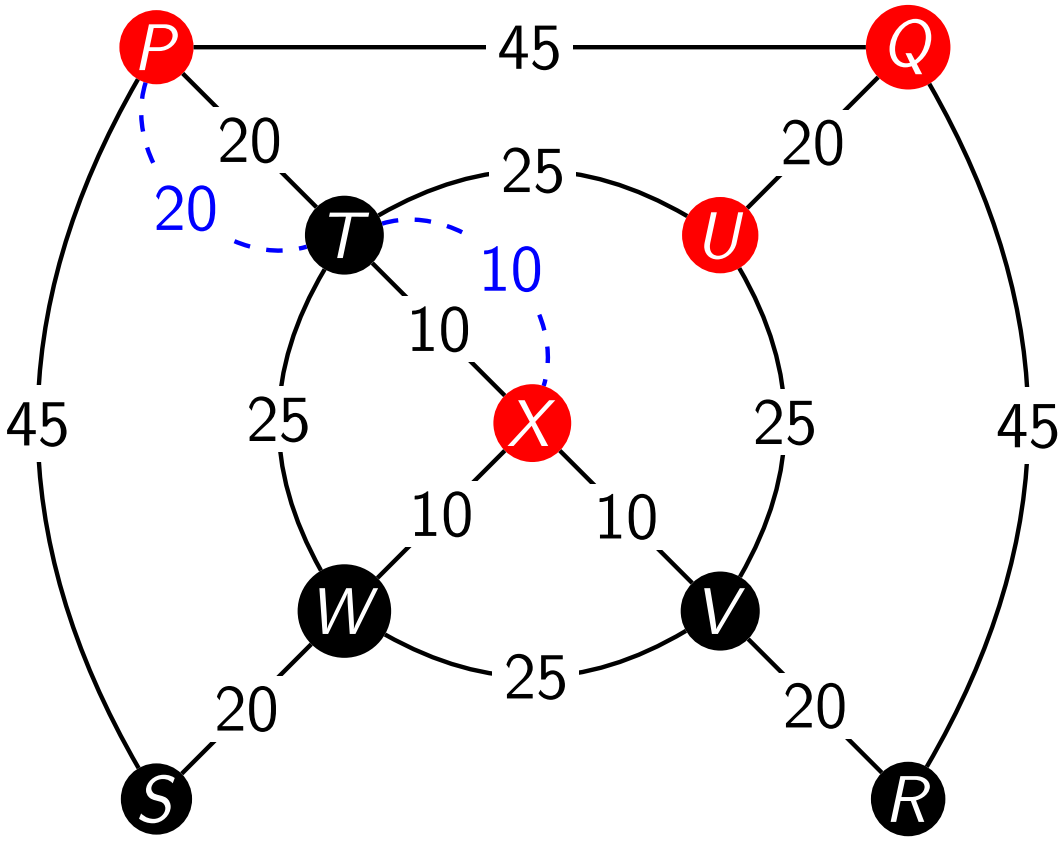
$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad \boxed{PX + QU \leftarrow 50}$$

4) Udvostručimo najkraći (P, X)-put

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

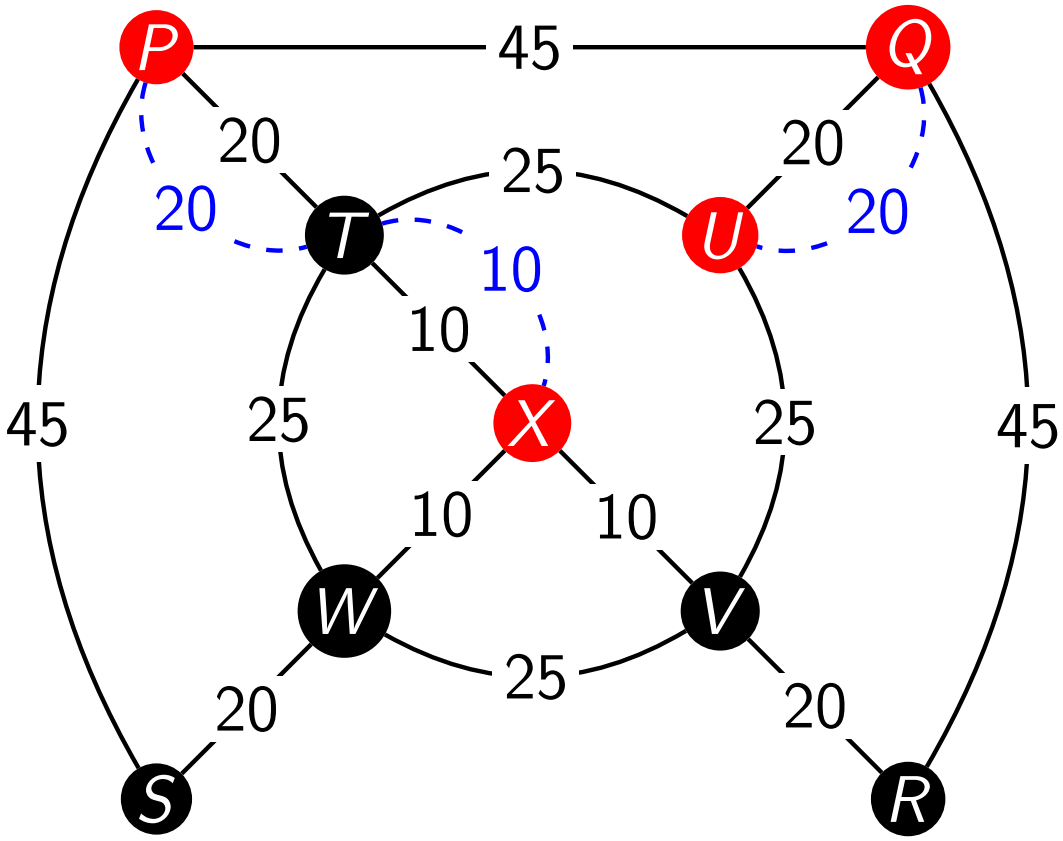
$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad \boxed{PX + QU \leftarrow 50}$$

4) Udvostručimo najkraći (P, X)-put i najkraći (Q, U)-put.

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

2) Udaljenosti između vrhova neparnog stupnja

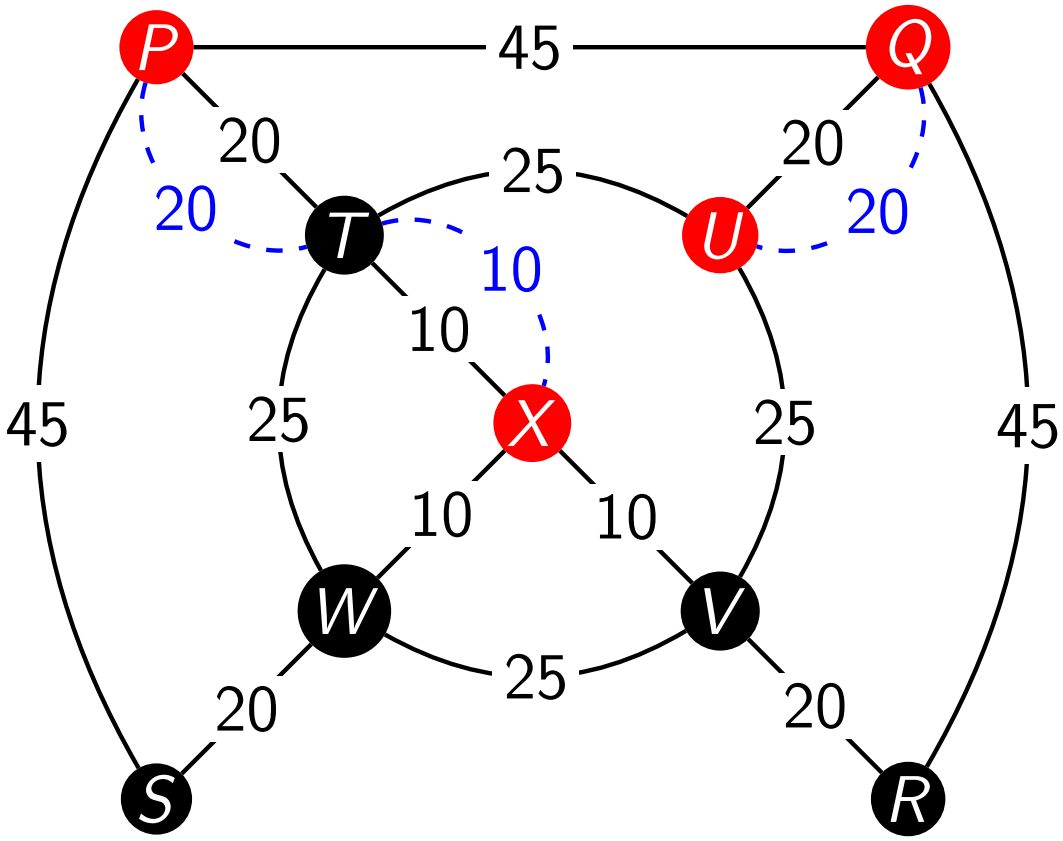
$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad \boxed{PX + QU \leftarrow 50}$$

4) Udvostručimo najkraći (P, X)-put i najkraći (Q, U)-put.

Rješenje



1) Vrhovi neparnog stupnja

$$P, Q, U, X$$

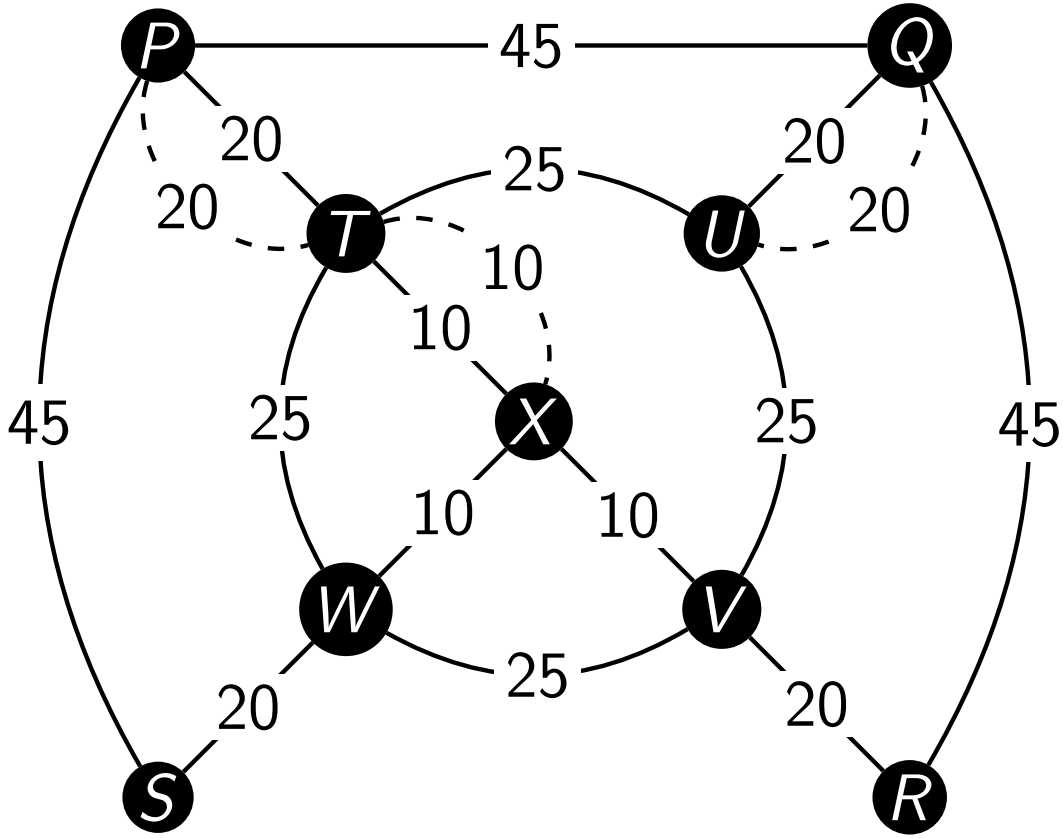
2) Udaljenosti između vrhova neparnog stupnja

$$PQ \leftarrow 45, \quad PU \leftarrow 45,$$
$$PX \leftarrow 30, \quad QU \leftarrow 20,$$
$$QX \leftarrow 55, \quad UX \leftarrow 35$$

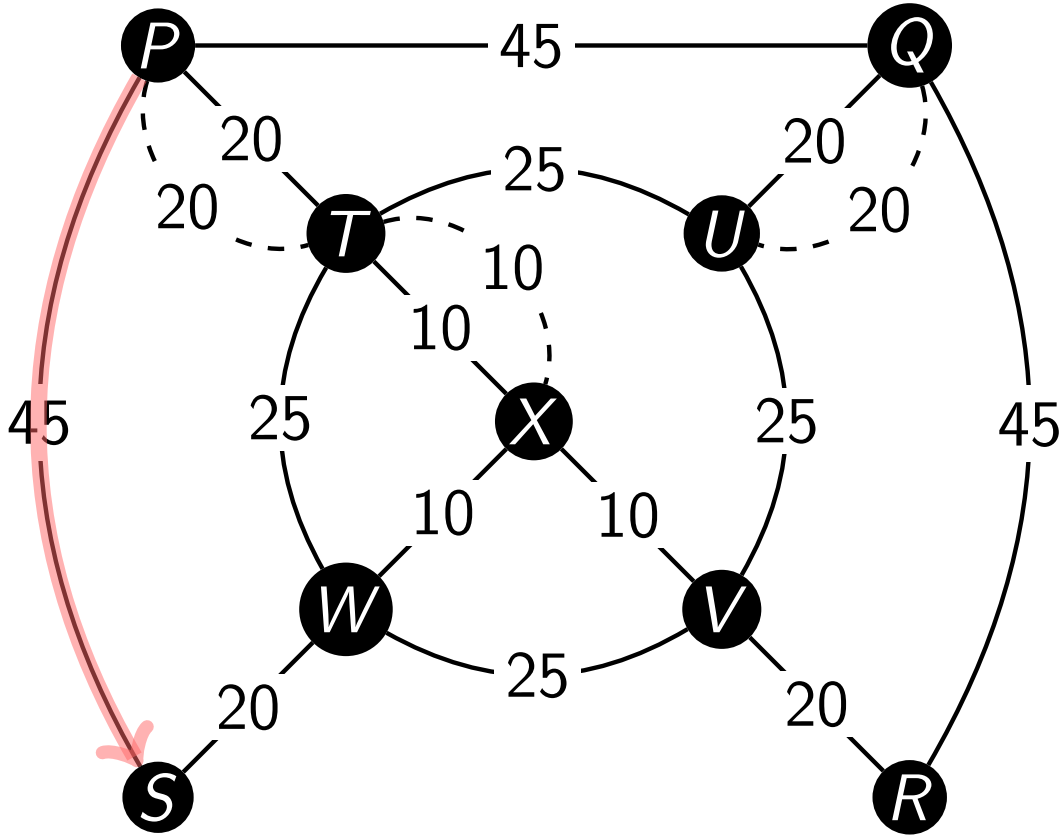
3) Uparivanje vrhova neparnog stupnja:

$$PQ + UX \leftarrow 80, \quad PU + QX \leftarrow 100, \quad \boxed{PX + QU \leftarrow 50}$$

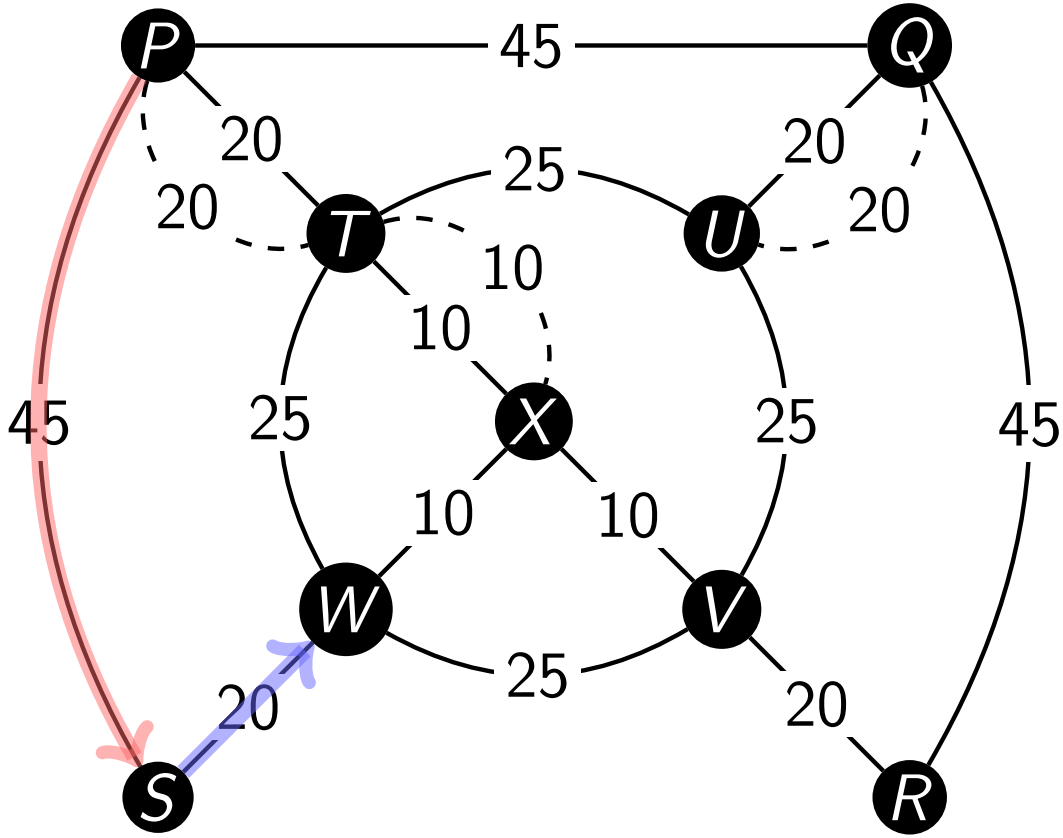
4) Udvostručimo najkraći (P, X)-put i najkraći (Q, U)-put. Dobivamo pseudograf G'.



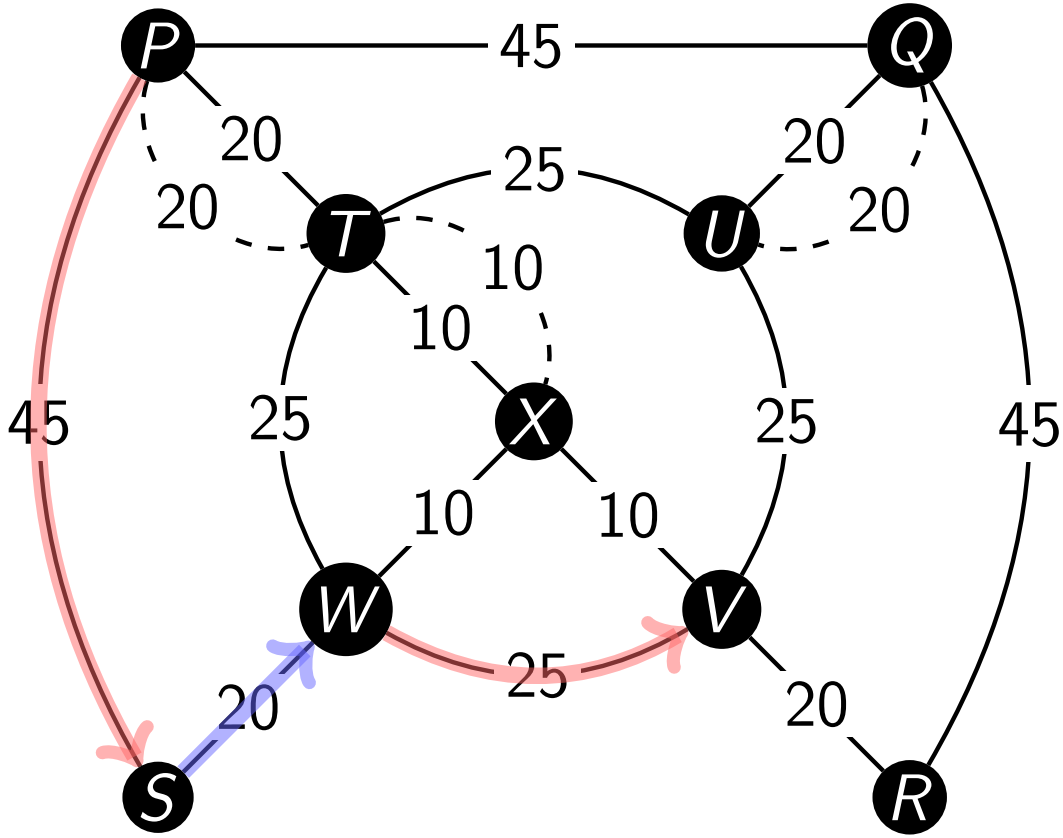
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G':



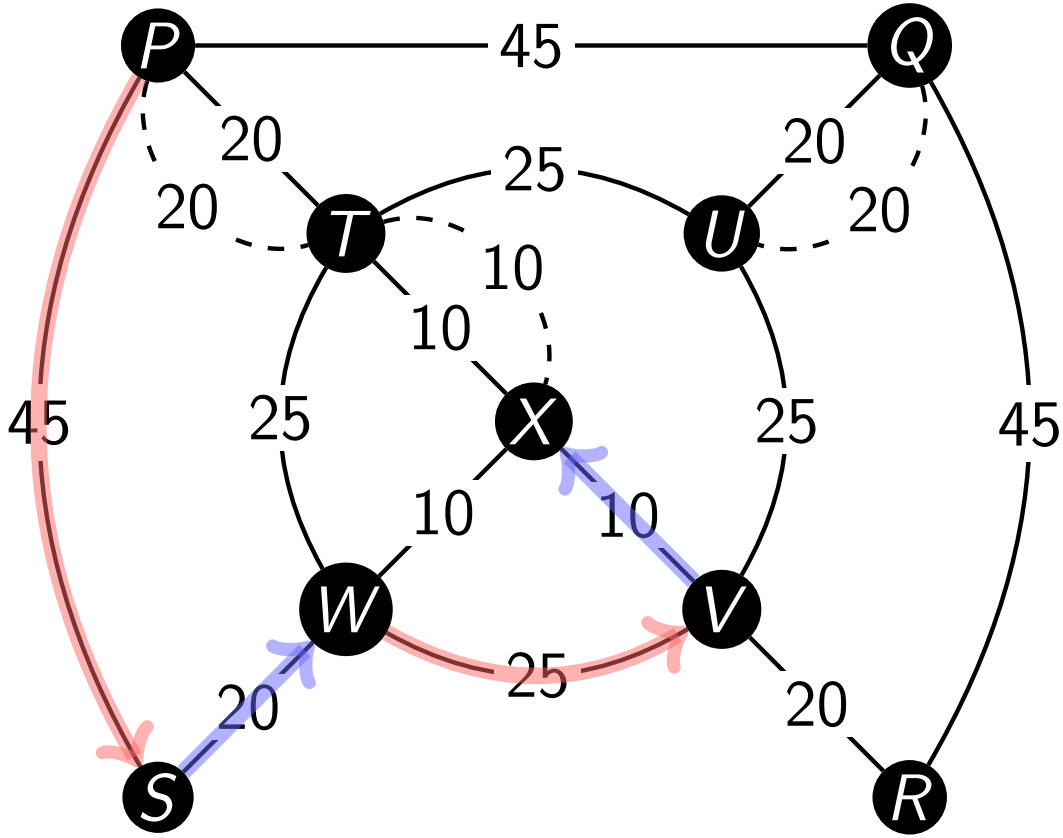
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': PS



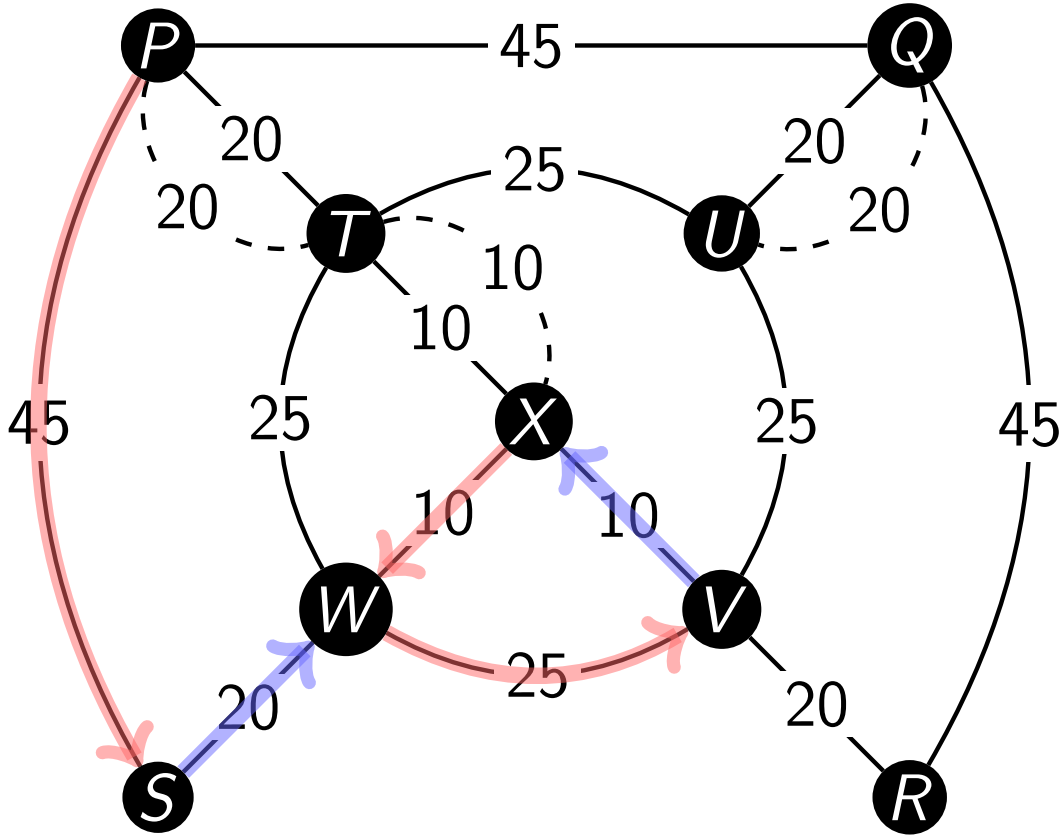
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': PSW



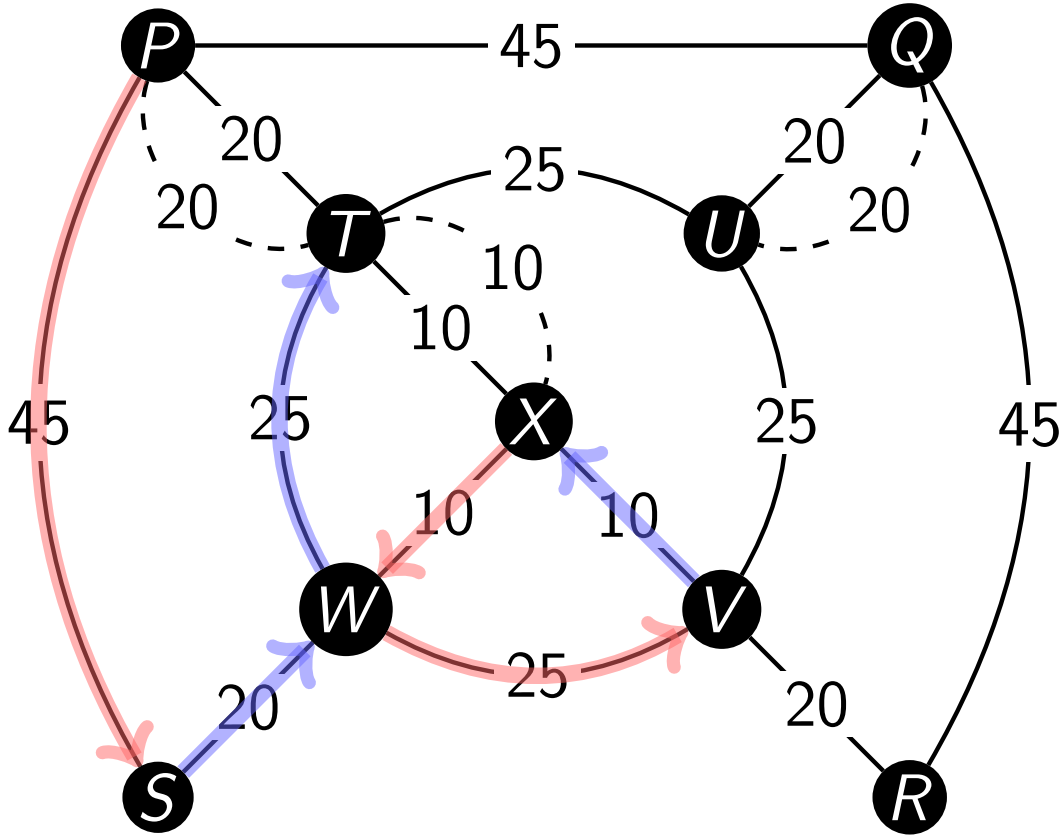
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWV$



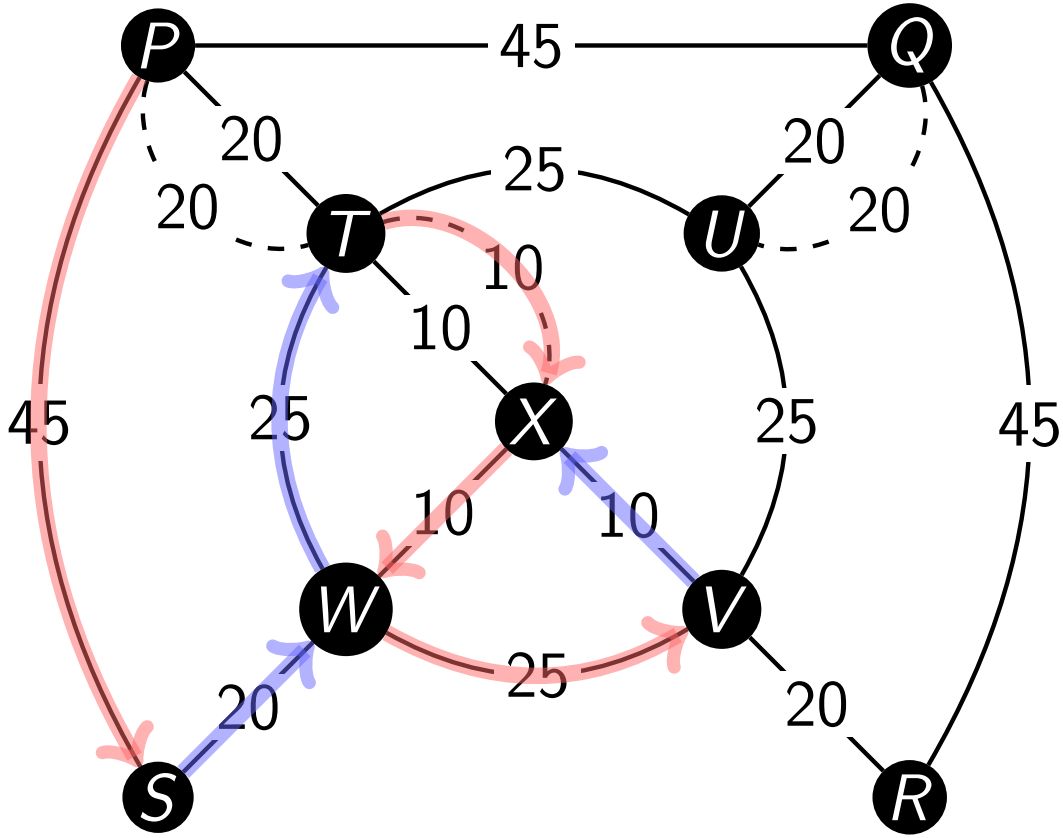
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVX$



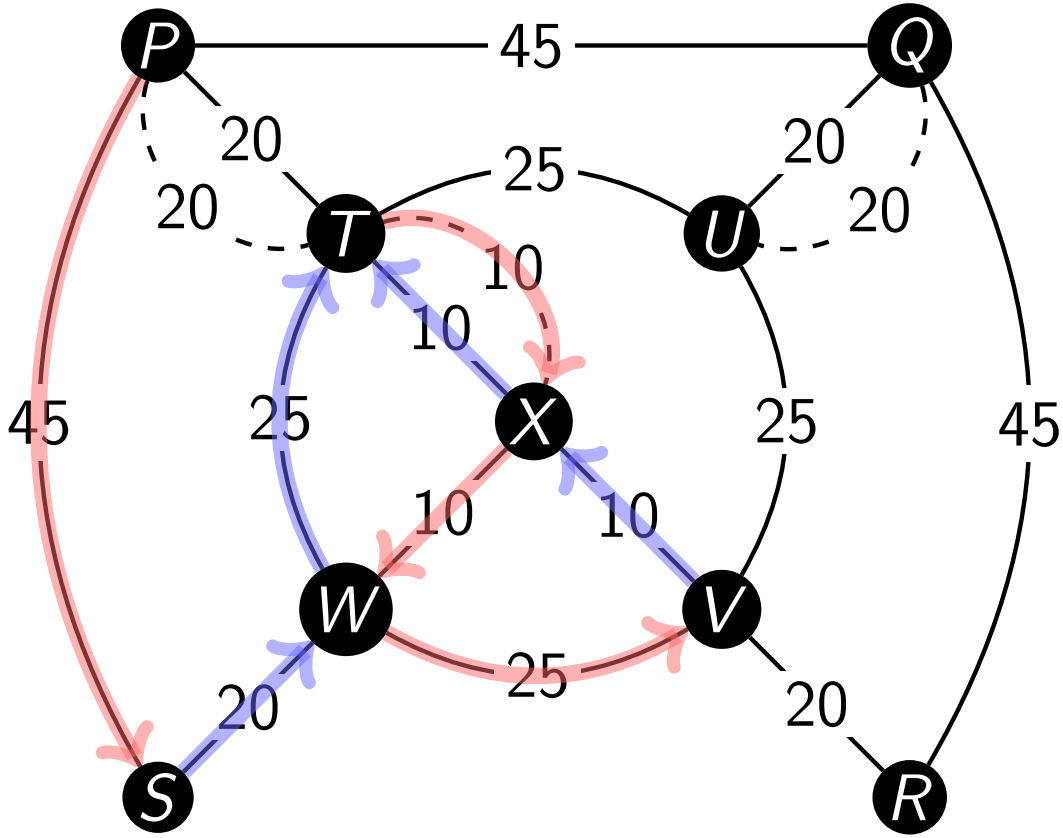
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXW$



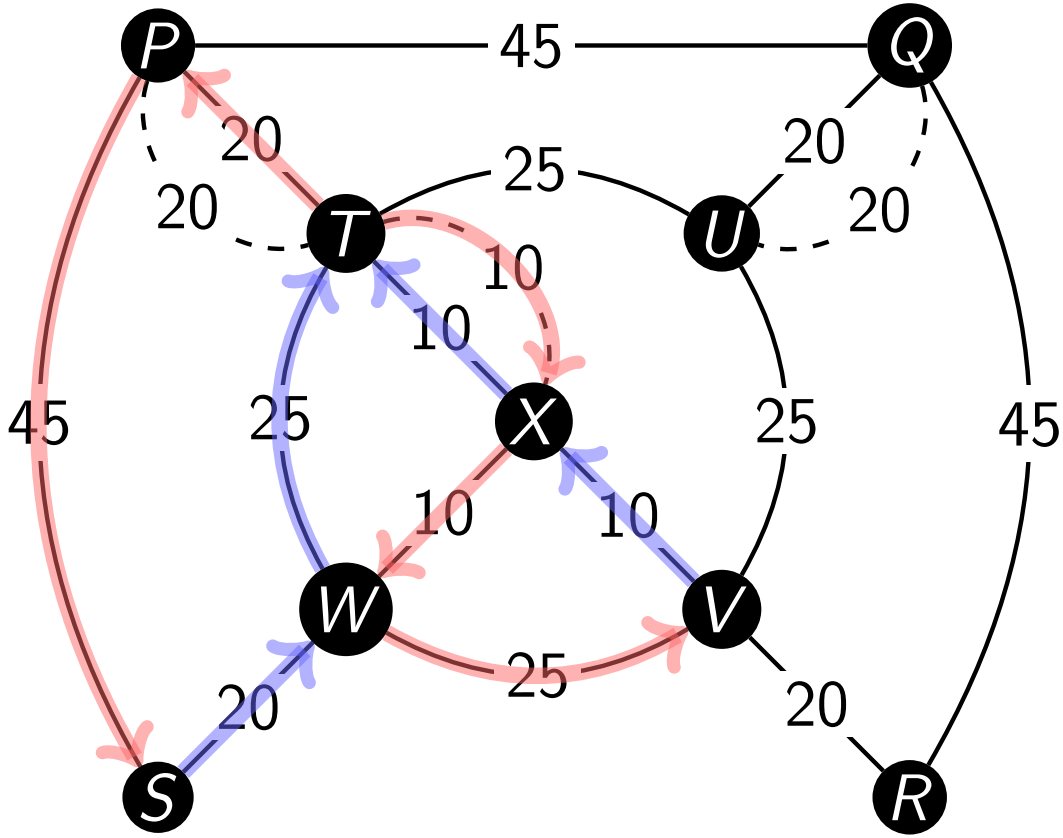
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $P\,S\,W\,V\,X\,W\,T$



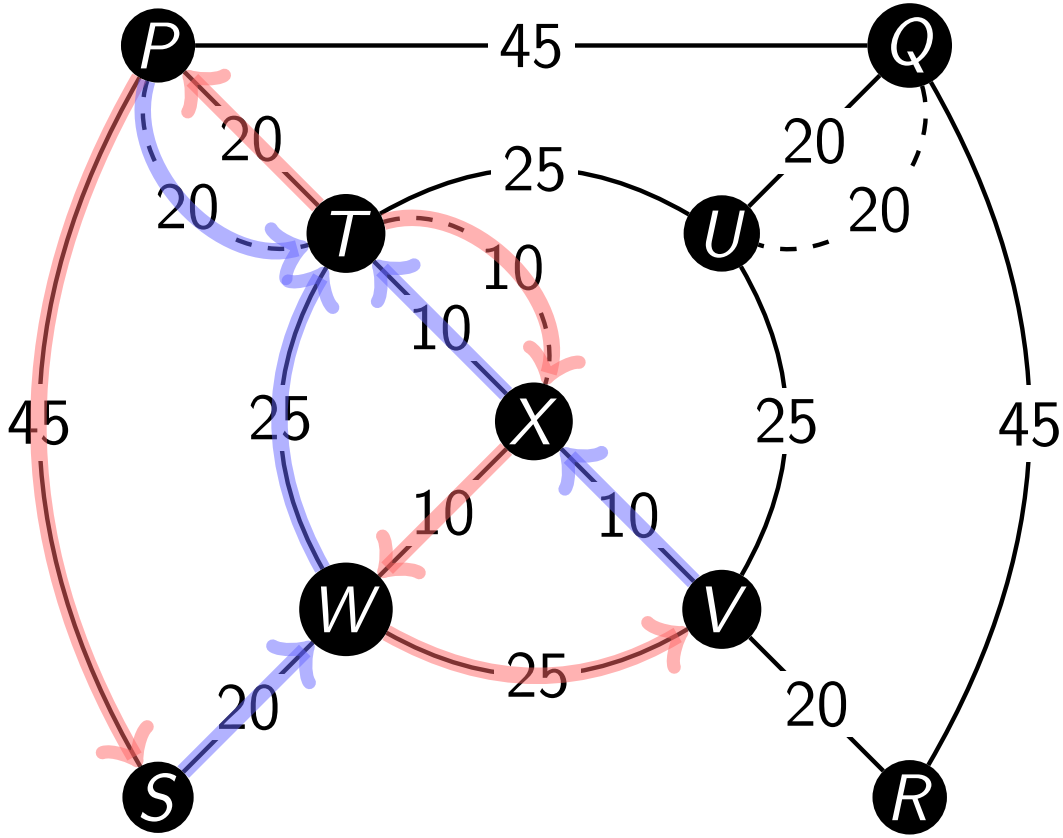
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTX$



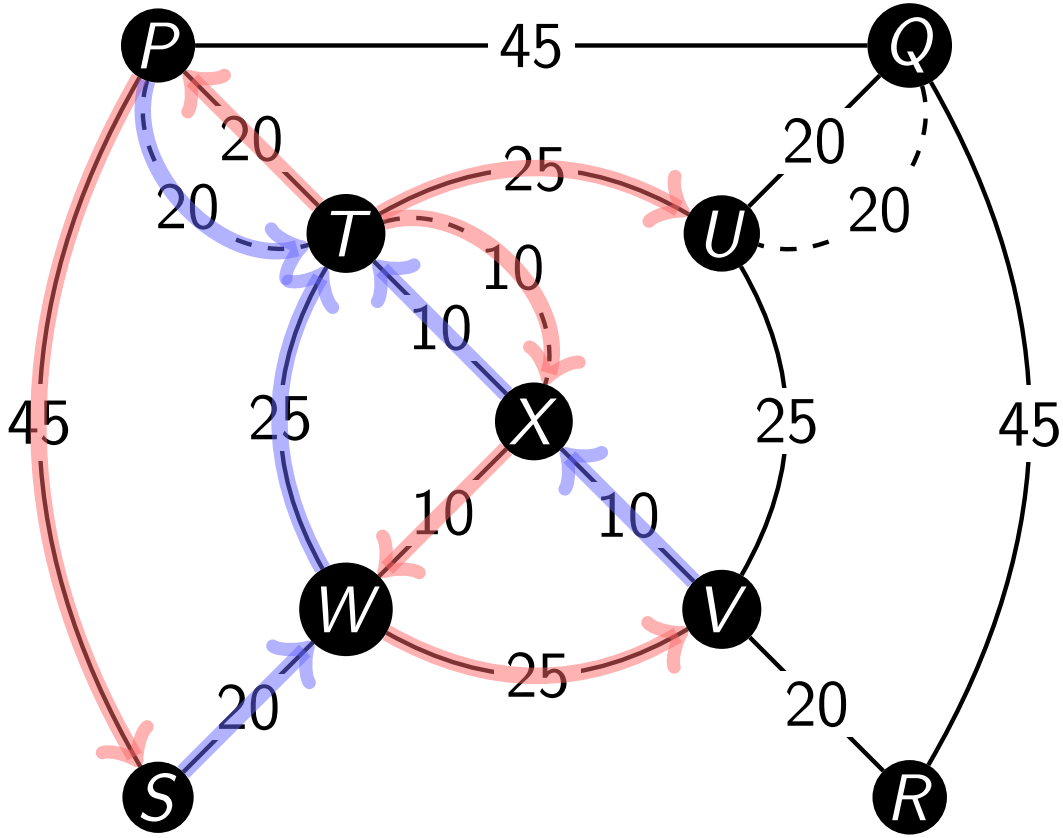
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $P\,S\,W\,V\,X\,W\,T\,X\,T$



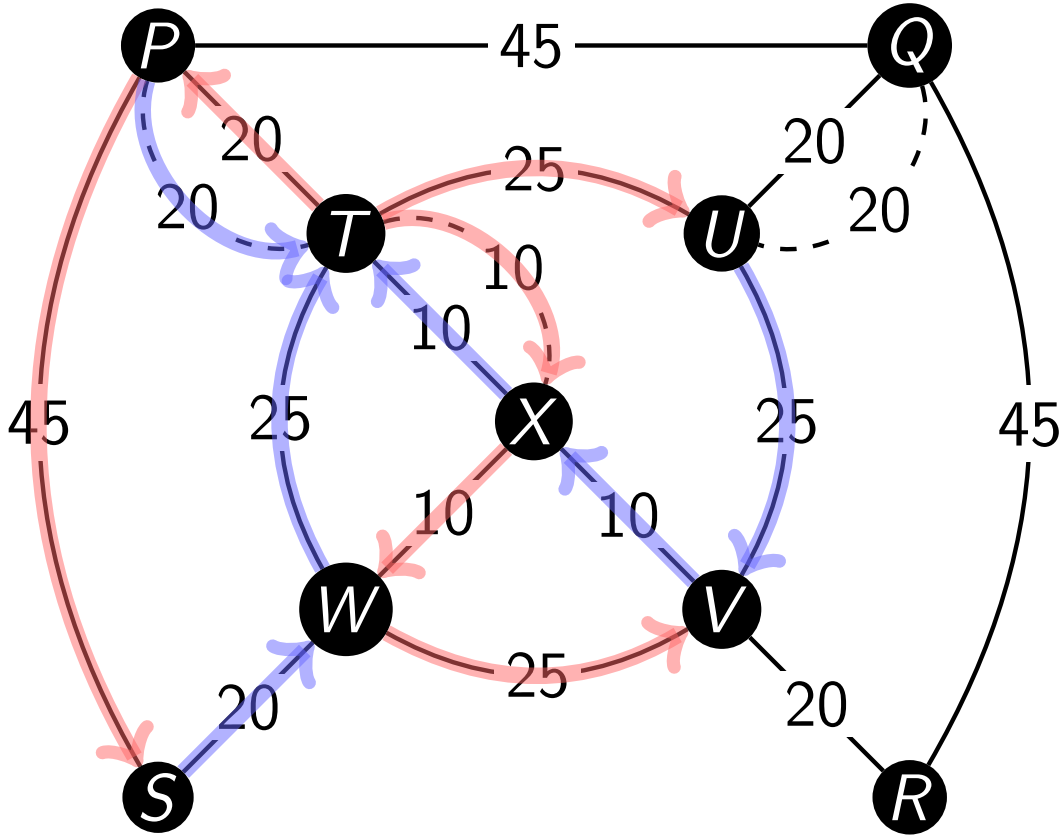
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTP$



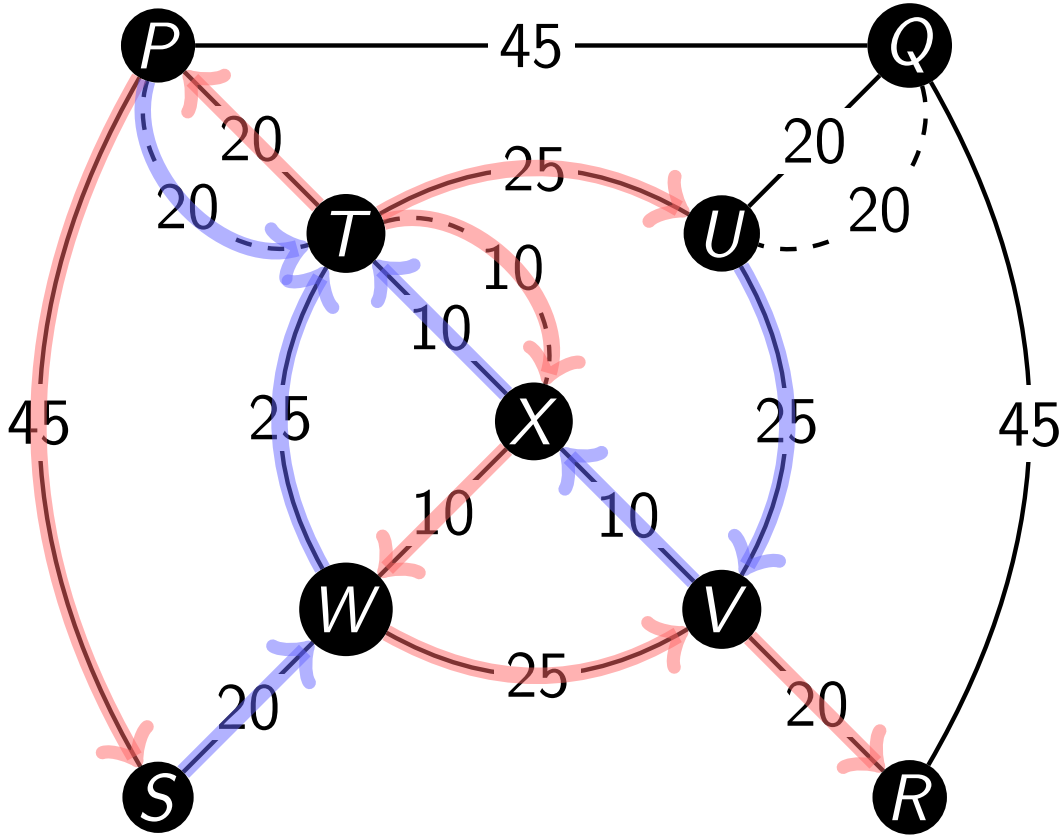
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPT$



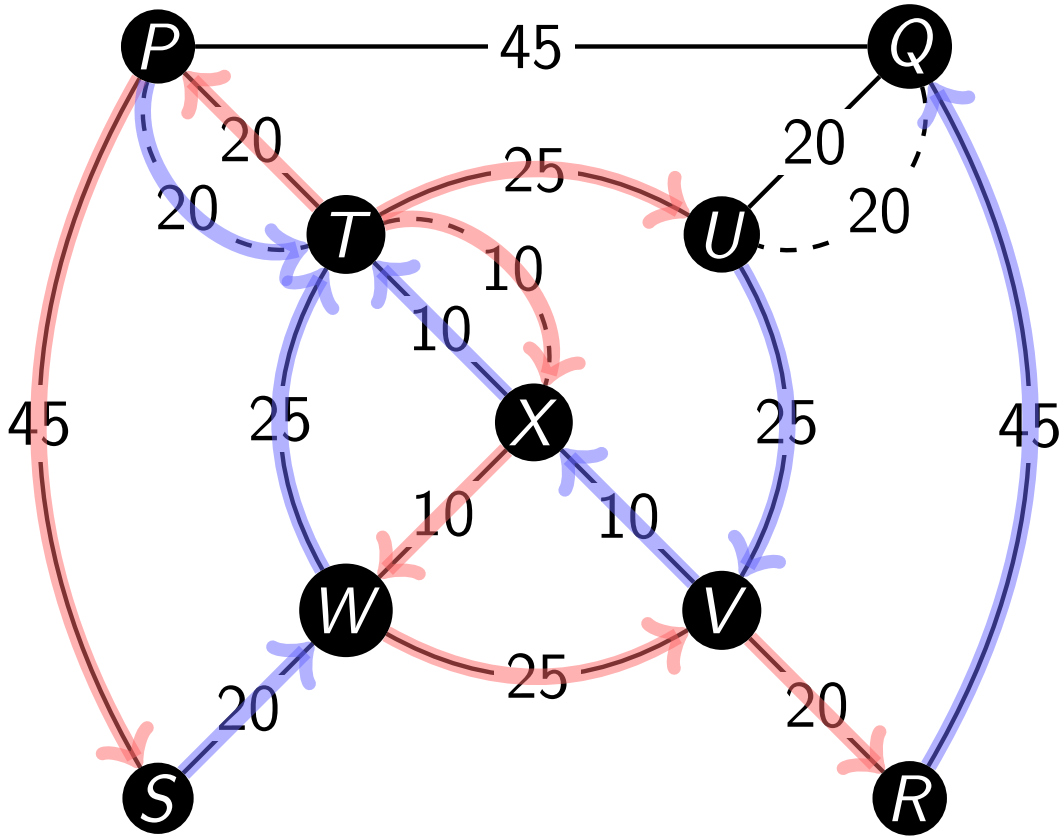
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTU$



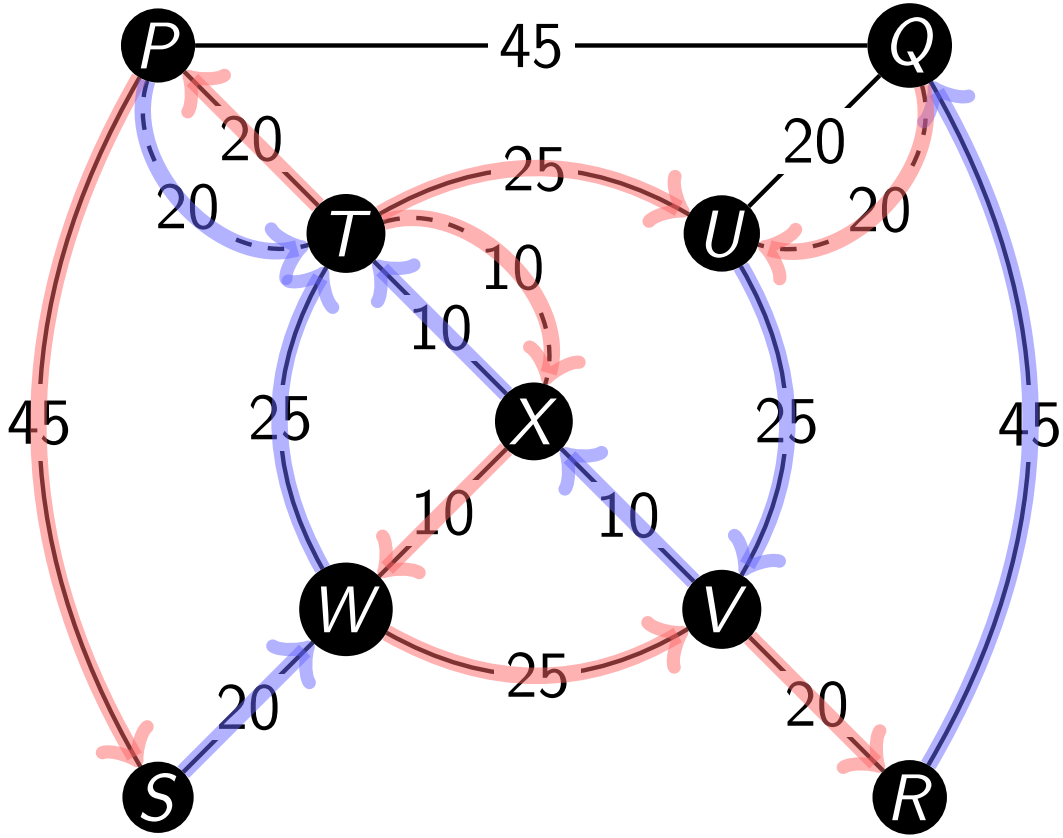
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUV$



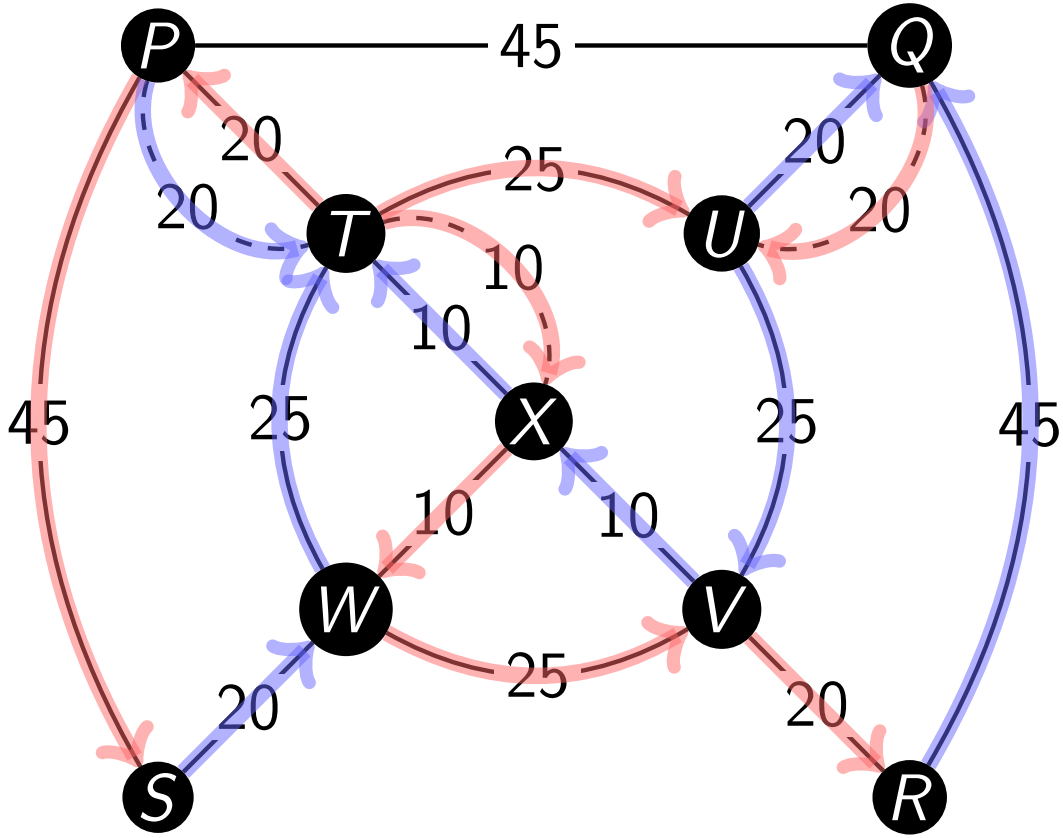
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVR$



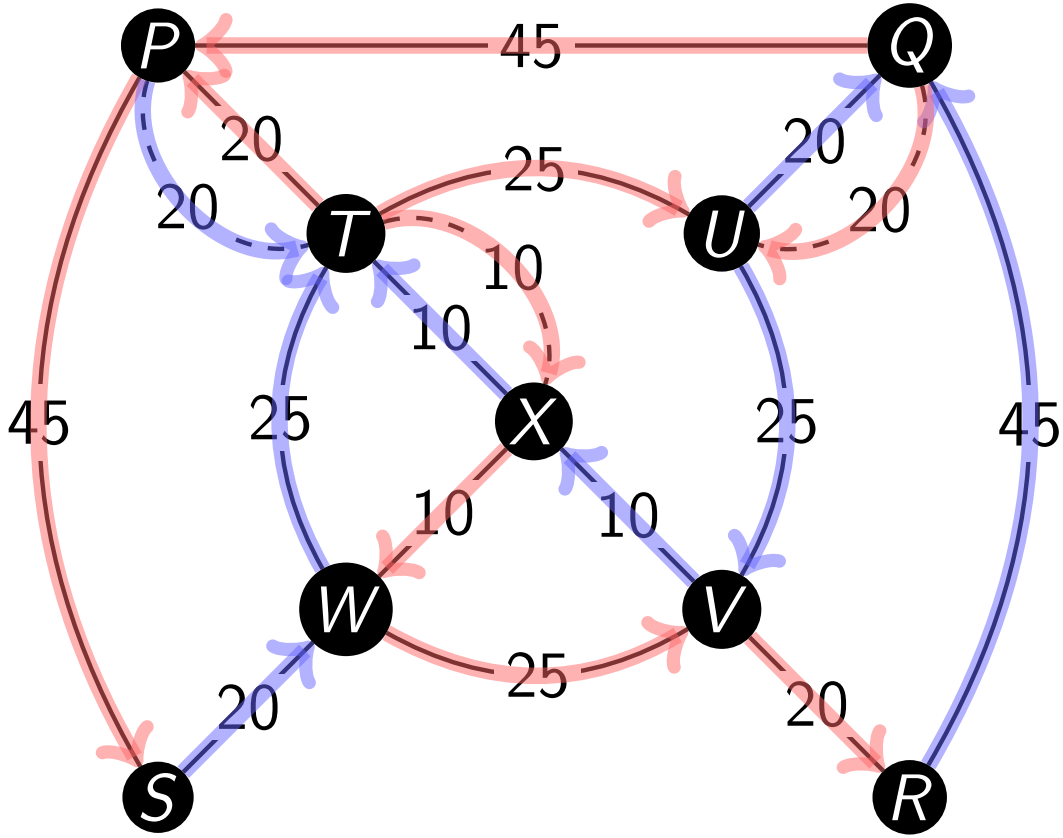
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQ$



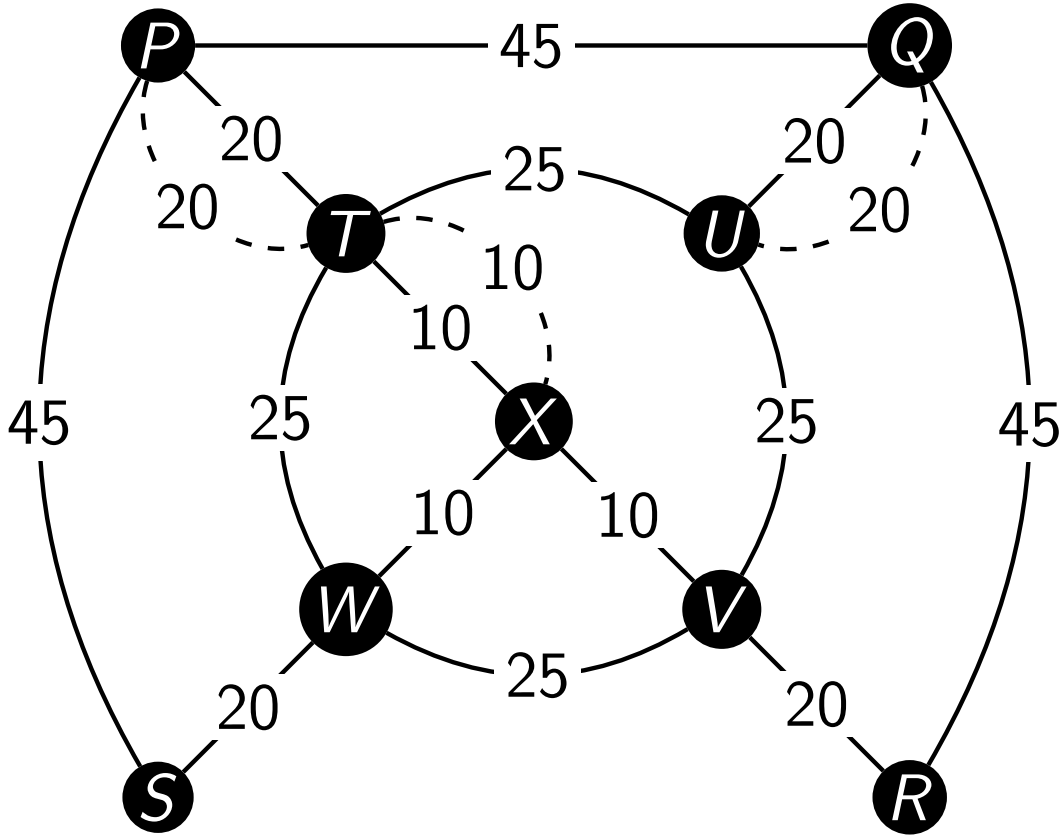
5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQU$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQ$

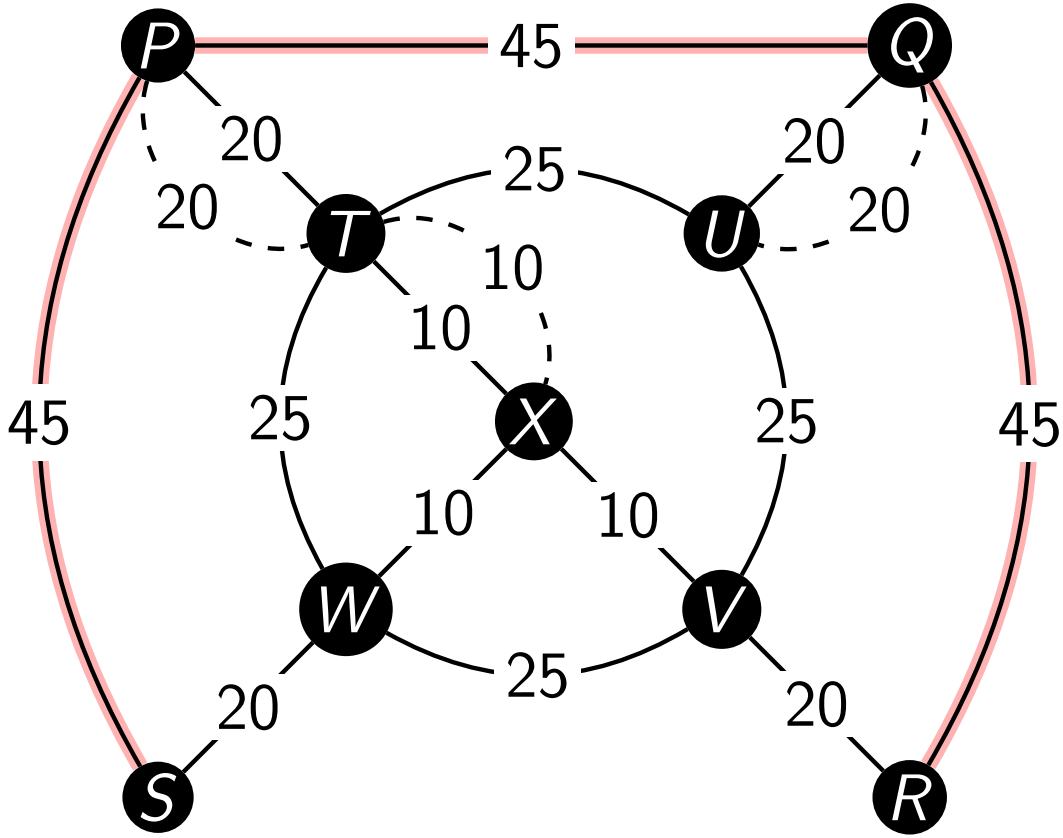


5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

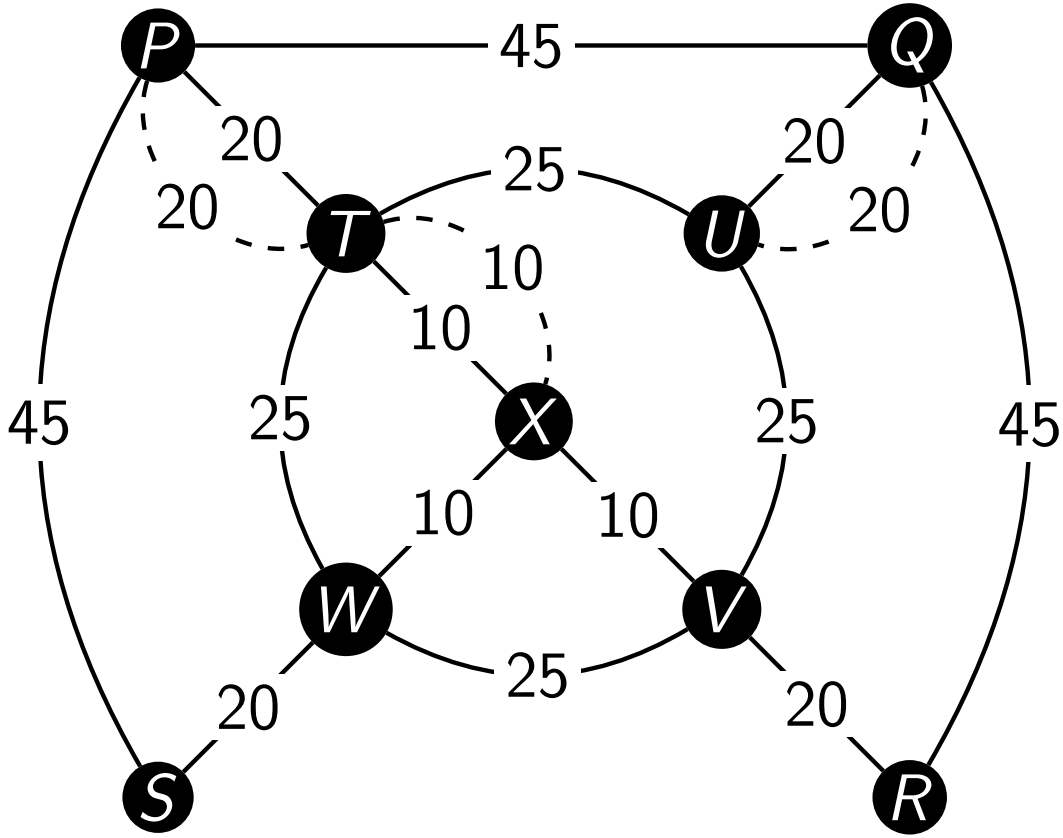
   Težina optimalne ture:



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

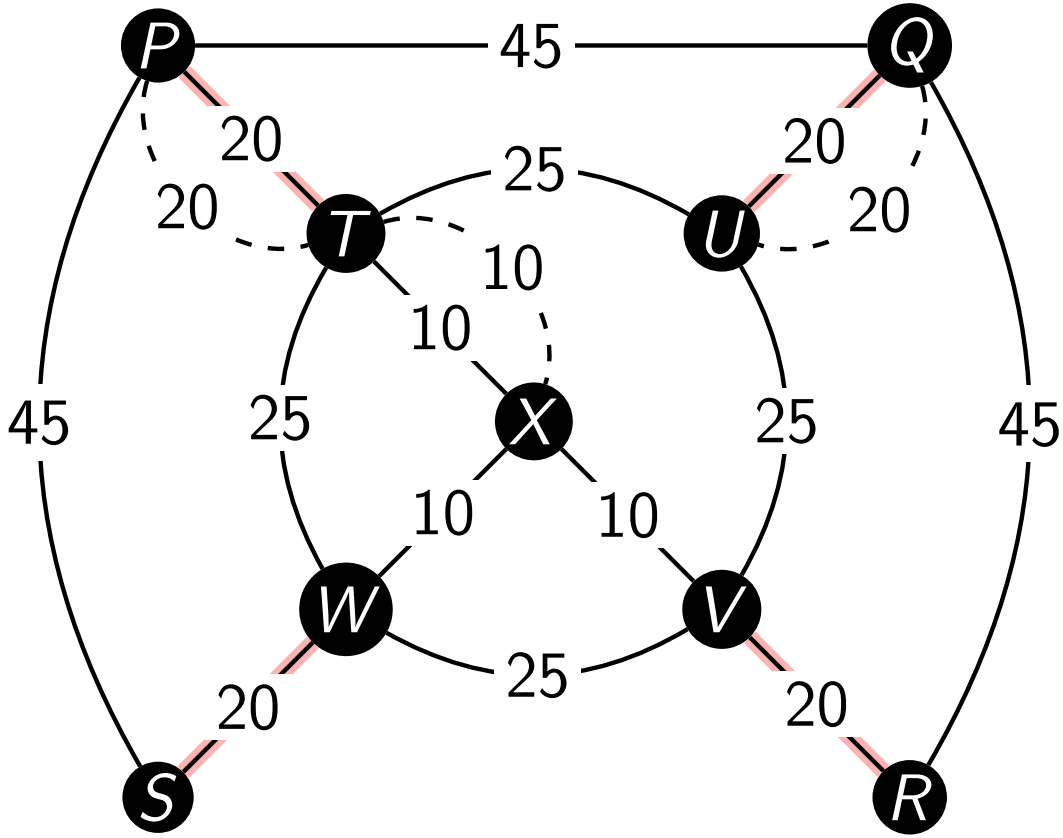
$3 \cdot 45$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

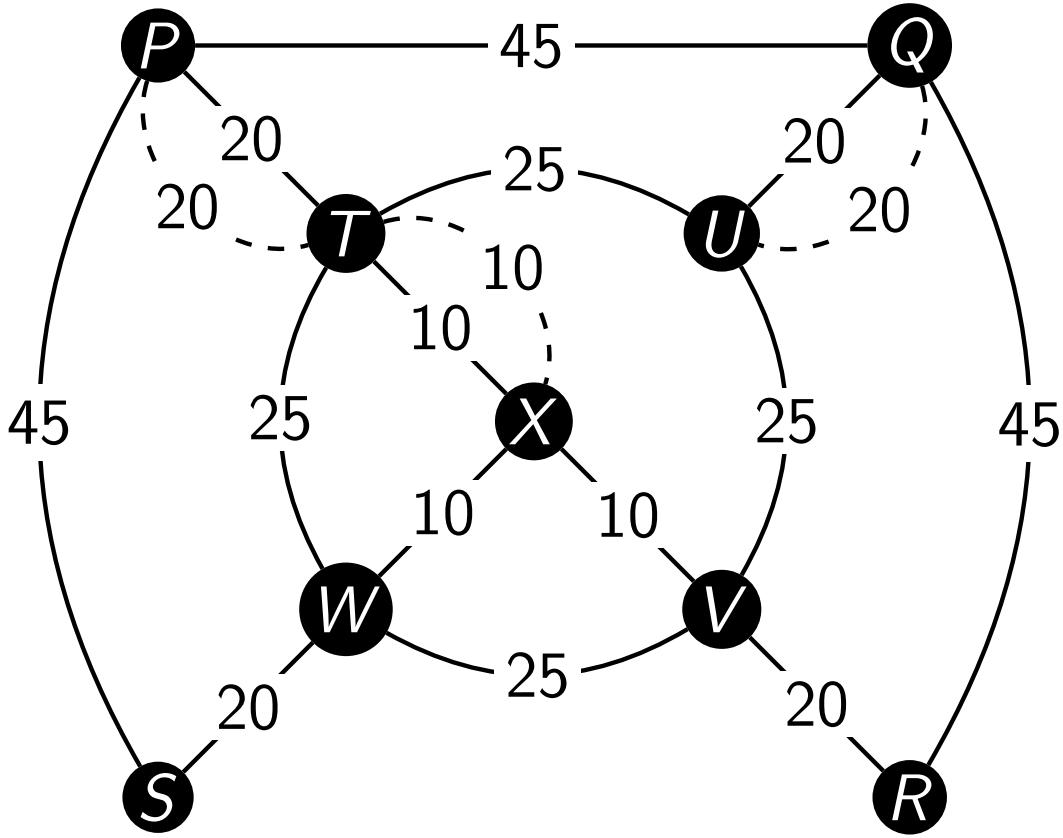
$3 \cdot 45 +$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

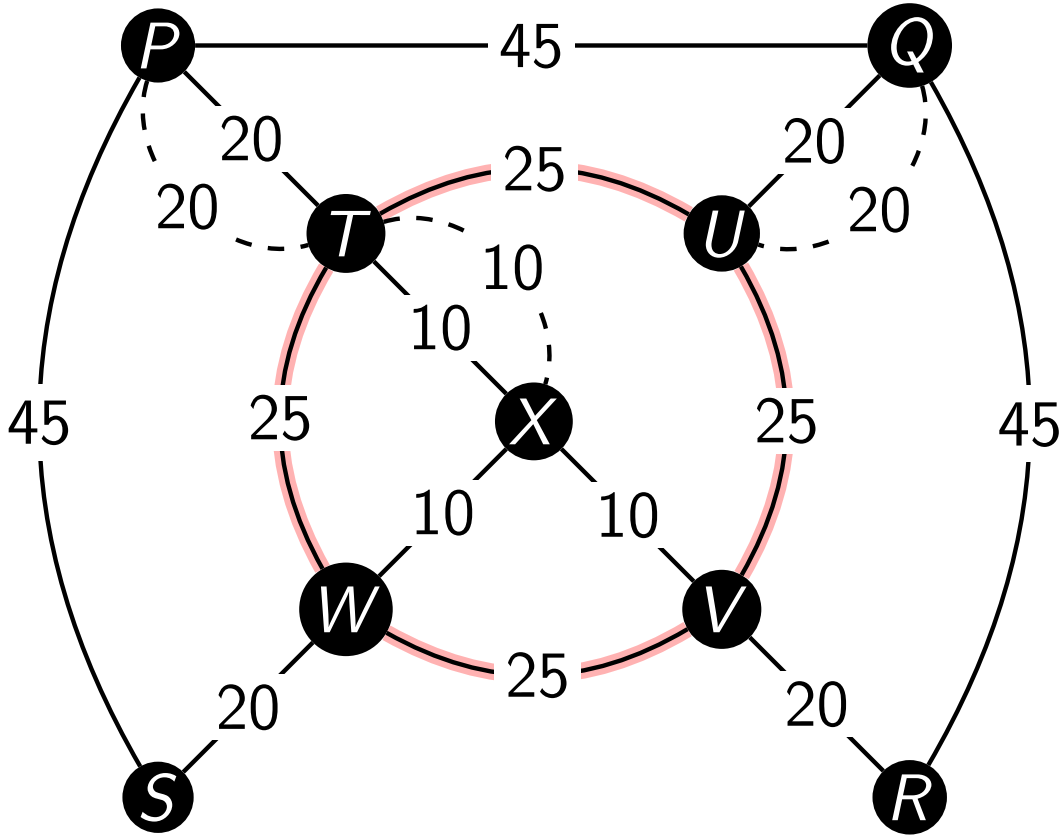
$3 \cdot 45 + 4 \cdot 20$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

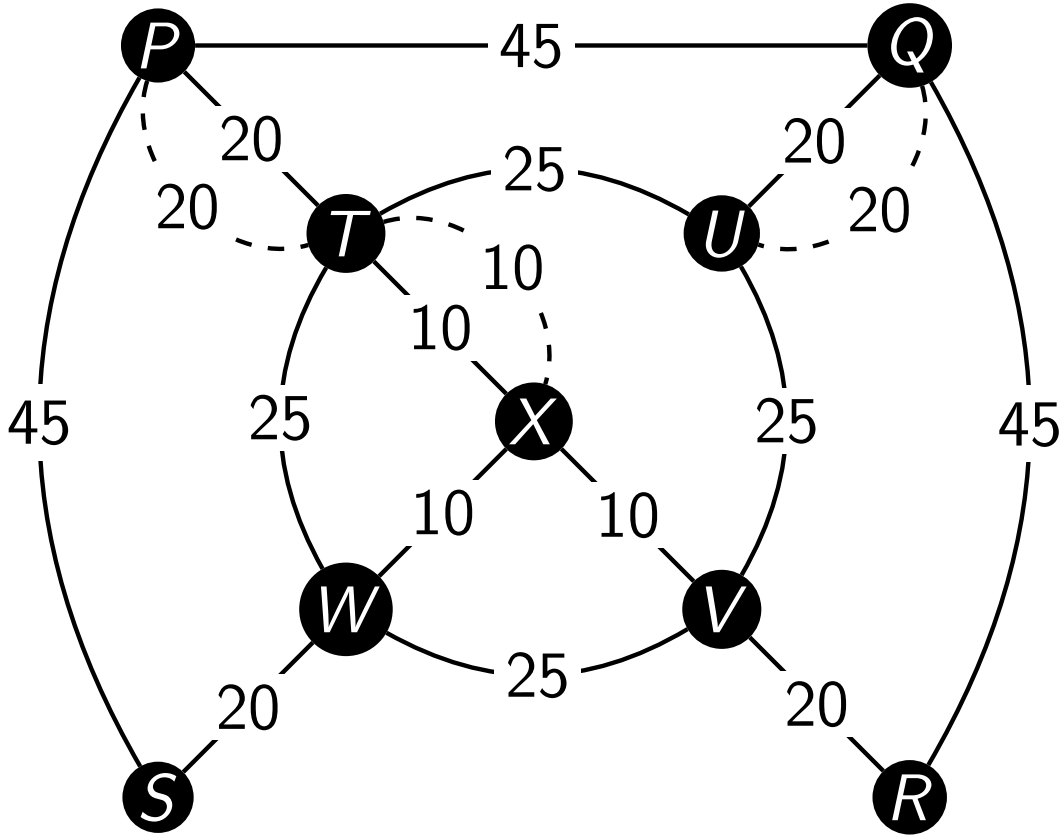
$3 \cdot 45 + 4 \cdot 20 +$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

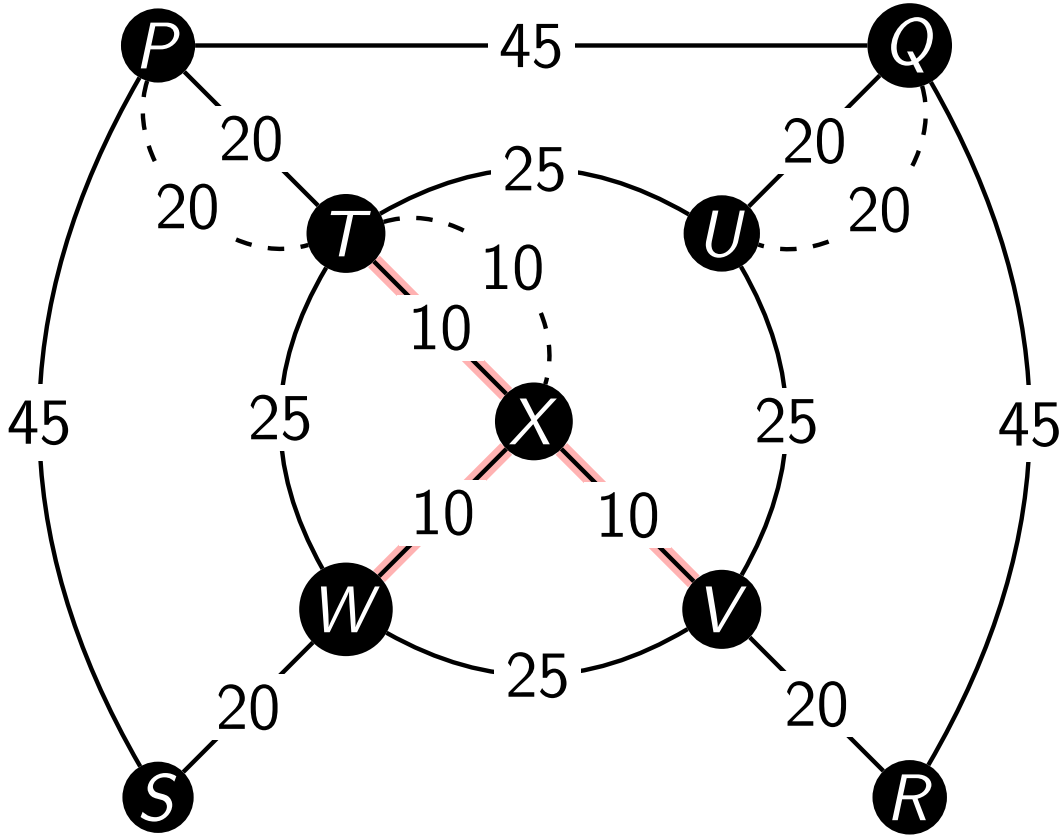
$3 \cdot 45 + 4 \cdot 20 + 4 \cdot 25$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

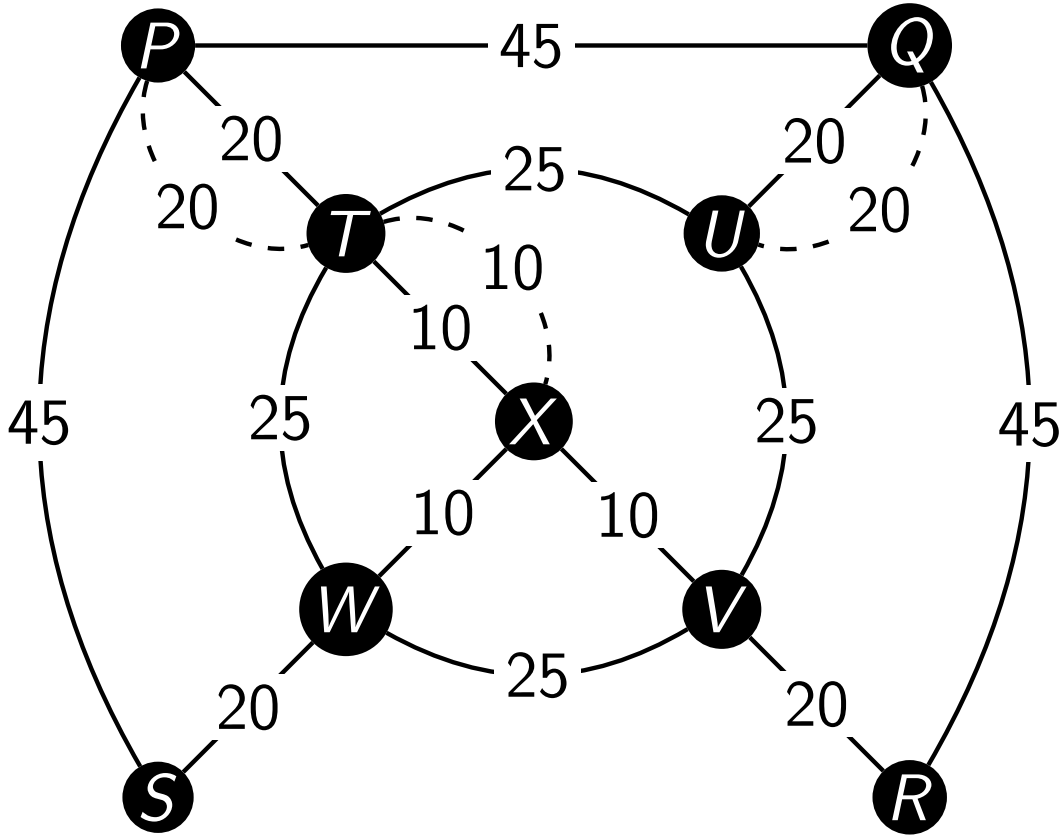
$3 \cdot 45 + 4 \cdot 20 + 4 \cdot 25 +$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

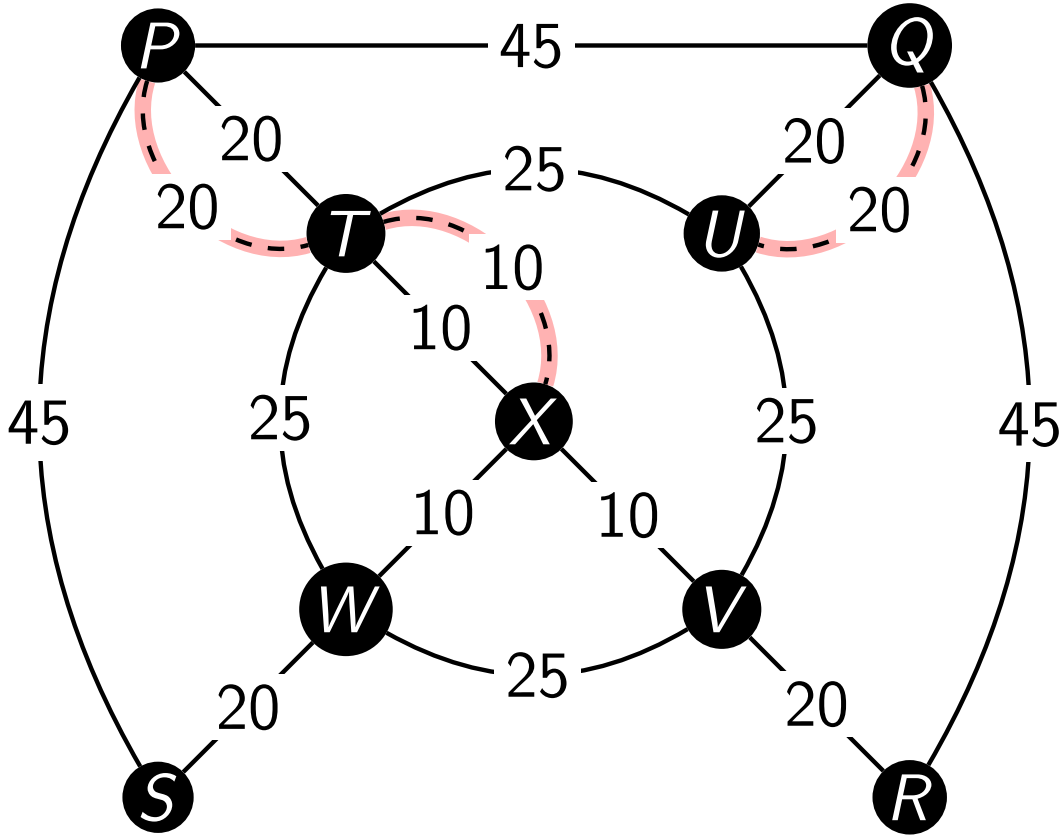
$3 \cdot 45 + 4 \cdot 20 + 4 \cdot 25 + 3 \cdot 10$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

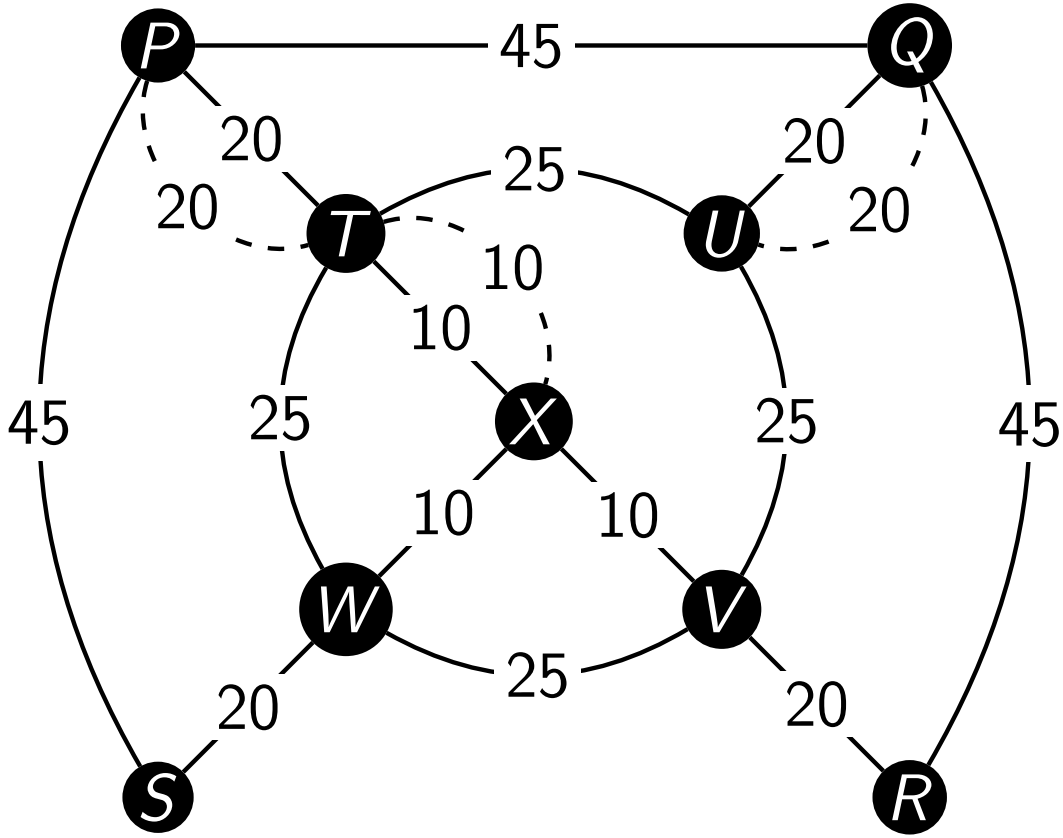
$3 \cdot 45 + 4 \cdot 20 + 4 \cdot 25 + 3 \cdot 10 +$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

$3 \cdot 45 + 4 \cdot 20 + 4 \cdot 25 + 3 \cdot 10 + (20 + 10 + 20)$



5) Pomoću Fleuryjevog algoritma pronađemo Eulerovu turu u pseudografu G': $PSWVXWTXTPTUVRQUQP$

Težina optimalne ture:

$3 \cdot 45 + 4 \cdot 20 + 4 \cdot 25 + 3 \cdot 10 + (20 + 10 + 20) = 395$