Osnovni pojmovi iz teorije grafova

Diskretne strukture s teorijom grafova

Damir Horvat

FOI, Varaždin

Sadržaj

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

peti zadatak

šesti zadatak

sedmi zadatak

osmi zadatak

deveti zadatak

deseti zadatak

jedanaesti zadatak

Teorem

U svakom grafu $G = \big(V, E\big)$ zbroj stupnjeva svih vrhova jednak je dvostrukom broju bridova, tj.

$$\sum_{v \in V} d(v) = 2\varepsilon$$

Lema o rukovanju

U svakom grafu je broj vrhova neparnog stupnja paran broj.

prvi zadatak

Zadatak 1

a) *Nacrtajte primjer grafa sa* 6 *vrhova u kojemu su barem* 3 *vrha neparnog stupnja i barem* 2 *vrha parnog stupnja.*

b) *Nacrtajte primjer grafa sa* 6 *vrhova u kojemu su najviše* 3 *vrha neparnog stupnja i barem* 2 *vrha parnog stupnja.*

c) *Postoji li graf koji istovremeno zadovoljava sve uvjete iz a) i b) dijela zadatka? Obrazložite svoj odgovor. Ukoliko postoji takav graf, navedite jedan primjer.*

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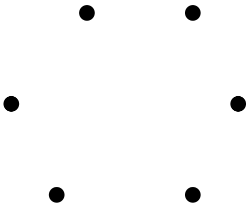

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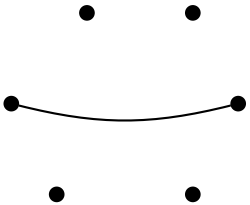

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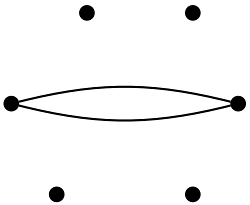

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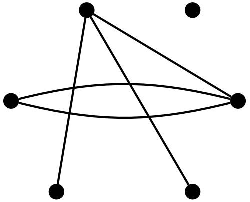

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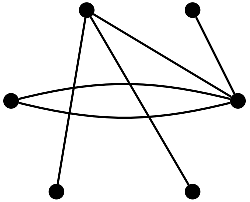

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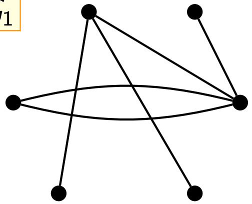

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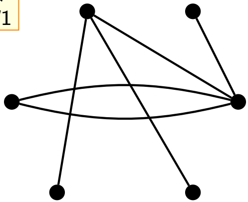

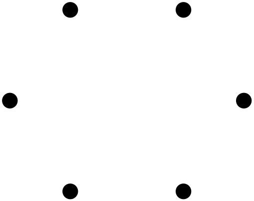

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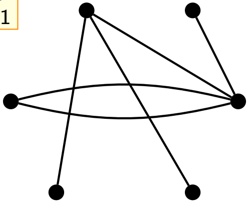

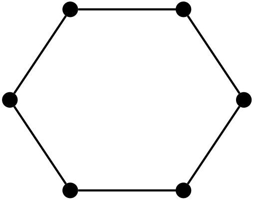

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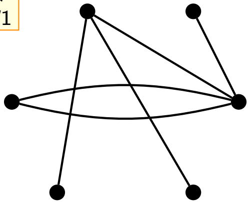

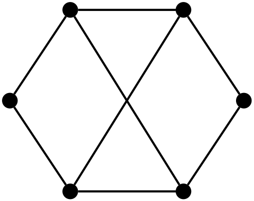

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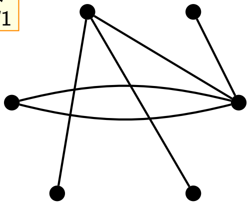

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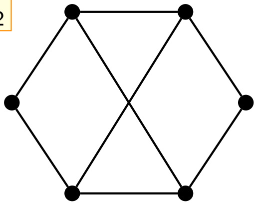

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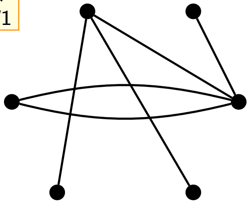
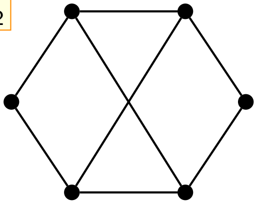


pseudograf

G_2



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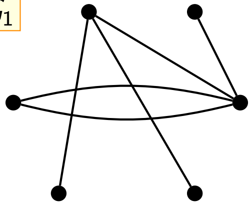

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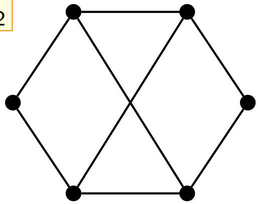

jednostavni graf

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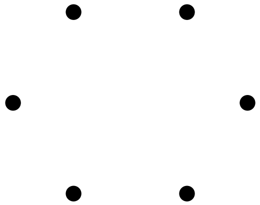

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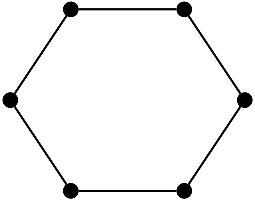

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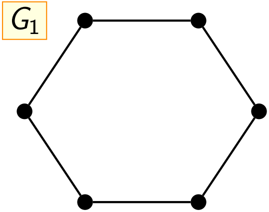

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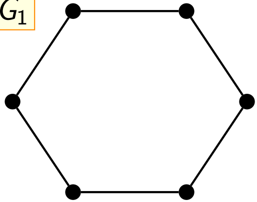

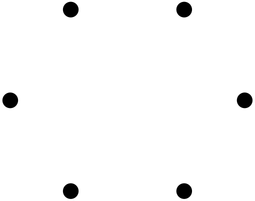

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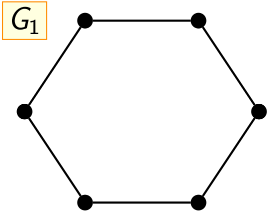

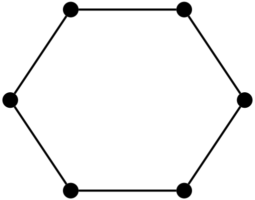

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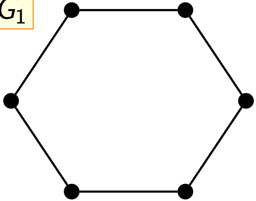

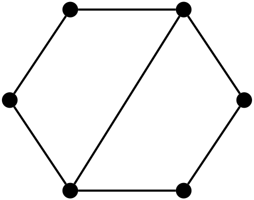

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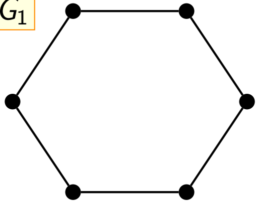
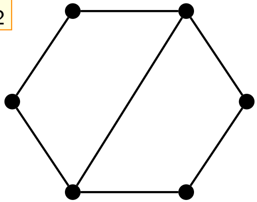


G_2



c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

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a) dio zadatka

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G ima 4 vrha neparnog stupnja.

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a) dio zadatka

G ima 4 vrha neparnog stupnja.

G ima 2 vrha parnog stupnja.

c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

a) dio zadatka

G ima 4 vrha neparnog stupnja.

G ima 2 vrha parnog stupnja.

b) dio zadatka

c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

a) dio zadatka

G ima 4 vrha neparnog stupnja.

G ima 2 vrha parnog stupnja.

b) dio zadatka

G ima najviše 2 vrha neparnog stupnja.

c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

a) dio zadatka

G ima 4 vrha neparnog stupnja.

G ima 2 vrha parnog stupnja.

b) dio zadatka

G ima najviše 2 vrha neparnog stupnja.

G ima barem 2 vrha parnog stupnja.

c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

a) dio zadatka

G ima 4 vrha neparnog stupnja.

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b) dio zadatka

G ima najviše 2 vrha neparnog stupnja.

G ima barem 2 vrha parnog stupnja.

c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

a) dio zadatka

moraju vrijediti

oba uvjeta

$\boxed{G \text{ ima 4 vrha neparnog stupnja.}}$

G ima 2 vrha parnog stupnja.

b) dio zadatka

$\boxed{G \text{ ima najviše 2 vrha neparnog stupnja.}}$

G ima barem 2 vrha parnog stupnja.

c) Neka je G graf sa 6 vrhova, tj. $\nu(G) = 6$.

a) dio zadatka

G ima 4 vrha neparnog stupnja.

G ima 2 vrha parnog stupnja.

moraju vrijediti oba uvjeta

Kontradikcija

b) dio zadatka

G ima najviše 2 vrha neparnog stupnja.

G ima barem 2 vrha parnog stupnja.

drugi zadatak

Zadatak 2

Nađite primjer grafa koji ima četiri vrha međusobno različitih stupnjeva.

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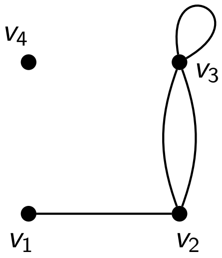

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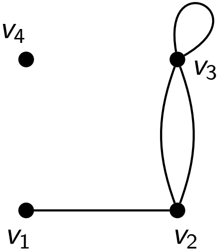

$$d(v_1) = 1$$

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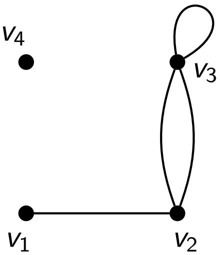

$$d(v_1) = 1$$
$$d(v_2) = 3$$

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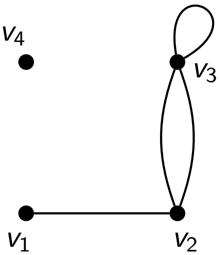

$$d(v_1) = 1$$
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$$d(v_3) = 4$$

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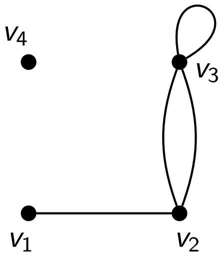
Rješenje



$$d(v_1) = 1$$
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Nađite primjer grafa koji ima četiri vrha međusobno različitih stupnjeva.

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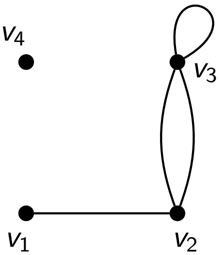

$$d(v_1) = 1$$
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$$d(v_3) = 4$$
$$d(v_4) = 0$$

Postoji li takav jednostavni graf?

treći zadatak

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Dokažite da u svakoj grupi ljudi postoji dvoje ljudi s istim brojem prijatelja iz te grupe.

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Definiramo graf G u kojemu vrhovi predstavljaju ljude, a bridovi prijateljstva.

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vrhovi $\longleftrightarrow$ ljudi   bridovi $\longleftrightarrow$ prijateljstva

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$$\text{vrhovi} \leftrightsquigarrow \text{ljudi} \qquad \text{bridovi} \leftrightsquigarrow \text{prijateljstva}$$

- G je jednostavni graf.

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- G je jednostavni graf.
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- Nema višestrukih bridova: ne treba prijateljstvo više puta isticati.
- Nema petlji: pod prijateljstvom podrazumijevamo prijateljstvo s drugim osobama, a ne sa samim sobom.

$\boxed{\text{Tvrdimo}}$ U svakom jednostavnom grafu postoje barem dva vrha istog stupnja.

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Pretpostavimo suprotno, tj. da postoji jednostavni graf G s n vrhova u kojemu svi vrhovi imaju međusobno različite stupnjeve.

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Kako je G jednostavni graf, mogući stupnjevi vrhova su

$$0, 1, 2, 3, \ldots, n-2, n-1.$$

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  Naime, u tom slučaju u jednostavnom grafu u svaki vrh stavimo petlju, a petlje će dvama vrhovima istog stupnja samo povećati njihove stupnjeve za 2 pa će u novom grafu oni opet biti istog stupnja.

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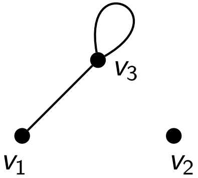
- *Tvrdnja ne mora vrijediti ukoliko postoje osobe koje nisu same sebi prijatelji.*



četvrti zadatak

Zadatak 4

Graf G ima 35 bridova i svaki vrh ima stupanj barem 3, tj. $\delta(G) = 3$. Koliko najviše vrhova može imati graf G?

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Graf G ima najviše 23 vrha.

peti zadatak

Zadatak 5

Postoji li grupa od 11 *ljudi u kojoj svaka osoba poznaje točno troje ljudi iz te grupe? Obrazložite svoj odgovor jezikom teorije grafova.*

Zadatak 5

Postoji li grupa od 11 *ljudi u kojoj svaka osoba poznaje točno troje ljudi iz te grupe? Obrazložite svoj odgovor jezikom teorije grafova.*

Rješenje

Pitanje je postoji li 3-regularni graf s 11 vrhova.

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pa je $2\varepsilon = 33$.

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pa je $2\varepsilon = 33$. Međutim, to je kontradikcija pa takav graf ne postoji.

šesti zadatak

Zadatak 6

Dokažite da u svakoj grupi od 6 *osoba postoji troje ljudi koji se međusobno poznaju ili postoji troje ljudi koji se međusobno ne poznaju.*

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Dokažite da u svakoj grupi od 6 osoba postoji troje ljudi koji se međusobno poznaju ili postoji troje ljudi koji se međusobno ne poznaju.

Rješenje

Neka je $\{O_1, O_2, O_3, O_4, O_5, O_6\}$ skup od 6 osoba.

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Neka je $\{O_1, O_2, O_3, O_4, O_5, O_6\}$ skup od 6 osoba.

Neka je G graf u kojemu su vrhovi navedene osobe, a dva vrha O_i i O_j su susjedni ako se pripadne osobe O_i i O_j poznaju.

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Promotrimo vrh O_1 u grafu G.

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Promotrimo vrh O_1 u grafu G. Tada je sigurno istinit jedan od sljedeća dva slučaja: $d(O_1) \geqslant 3$ ili $d(O_1) < 3$.

Zadatak 6

Dokažite da u svakoj grupi od 6 osoba postoji troje ljudi koji se međusobno poznaju ili postoji troje ljudi koji se međusobno ne poznaju.

Rješenje

Neka je $\{O_1, O_2, O_3, O_4, O_5, O_6\}$ skup od 6 osoba.

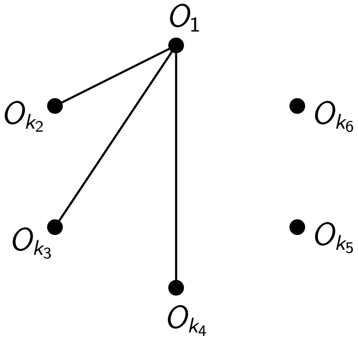
Neka je G graf u kojemu su vrhovi navedene osobe, a dva vrha O_i i O_j su susjedni ako se pripadne osobe O_i i O_j poznaju.

Promotrimo vrh O_1 u grafu G. Tada je sigurno istinit jedan od sljedeća dva slučaja: $d(O_1) \geqslant 3$ ili $d(O_1) < 3$.

$$\big(O_{k_2}, O_{k_3}, O_{k_4}, O_{k_5}, O_{k_6}\big) \leftsquigarrow \text{neka permutacija od } \big(O_2, O_3, O_4, O_5, O_6\big)$$

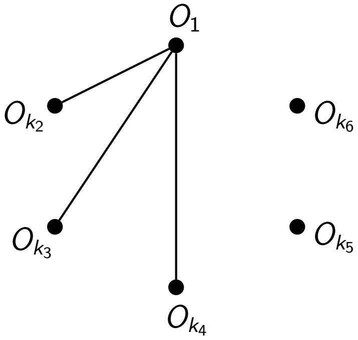
$\boxed{d(O_1) \geqslant 3}$

U ovom slučaju osoba O_1 poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



$\boxed{d(O_1) \geqslant 3}$

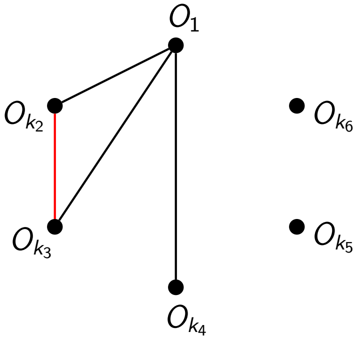
U ovom slučaju osoba O_1 poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



Ukoliko u skupu poznanika od O_1 postoje dvije osobe koje se poznaju,

$\boxed{d(O_1) \geqslant 3}$

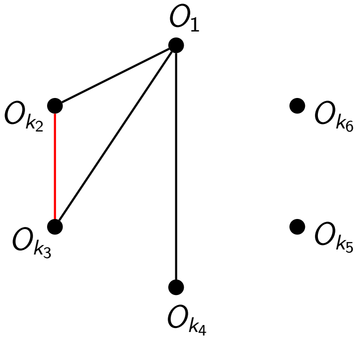
U ovom slučaju osoba O_1 poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



Ukoliko u skupu poznanika od O_1 postoje dvije osobe koje se poznaju, npr. osobe O_{k_2} i O_{k_3},

$\boxed{d(O_1) \geqslant 3}$

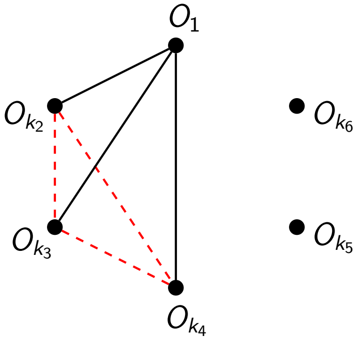
U ovom slučaju osoba O_1 poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



Ukoliko u skupu poznanika od O_1 postoje dvije osobe koje se poznaju, npr. osobe O_{k_2} i O_{k_3}, tada je $\{O_1, O_{k_2}, O_{k_3}\}$ skup od troje ljudi koji se međusobno poznaju.

$\boxed{d(O_1) \geqslant 3}$

U ovom slučaju osoba O_1 poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.

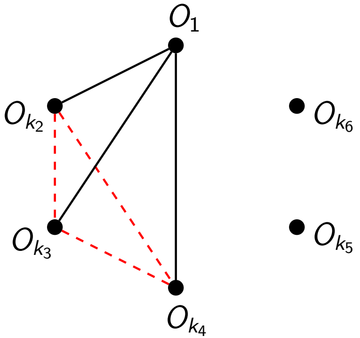


Ukoliko u skupu poznanika od O_1 postoje dvije osobe koje se poznaju, npr. osobe O_{k_2} i O_{k_3}, tada je $\{O_1, O_{k_2}, O_{k_3}\}$ skup od troje ljudi koji se međusobno poznaju.

Ako se među poznanicima od O_1 nikoje od tih osoba međusobno ne poznaju,

$\boxed{d(O_1) \geqslant 3}$

U ovom slučaju osoba O_1 poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.

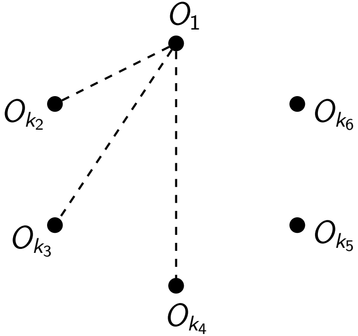


Ukoliko u skupu poznanika od O_1 postoje dvije osobe koje se poznaju, npr. osobe O_{k_2} i O_{k_3}, tada je $\{O_1, O_{k_2}, O_{k_3}\}$ skup od troje ljudi koji se međusobno poznaju.

Ako se među poznanicima od O_1 nikoje od tih osoba međusobno ne poznaju, tada je $\{O_{k_2}, O_{k_3}, O_{k_4}\}$ skup od troje ljudi koji se međusobno ne poznaju.

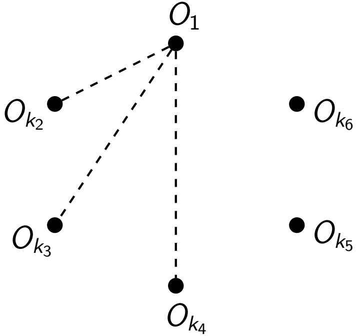
$\boxed{d(O_1) < 3}$

U ovom slučaju osoba O_1 ne poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



$\boxed{d(O_1) < 3}$

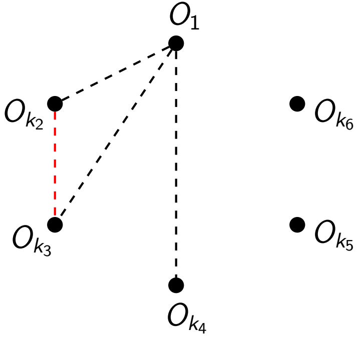
U ovom slučaju osoba O_1 ne poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



Ukoliko u skupu osoba koje O_1 ne poznaje postoje dvije osobe koje se ne poznaju,

$\boxed{d(O_1) < 3}$

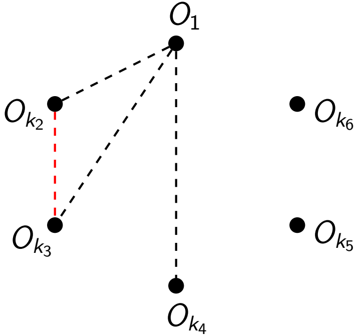
U ovom slučaju osoba O_1 ne poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



Ukoliko u skupu osoba koje O_1 ne poznaje postoje dvije osobe koje se ne poznaju, npr. osobe O_{k_2} i O_{k_3},

$\boxed{d(O_1) < 3}$

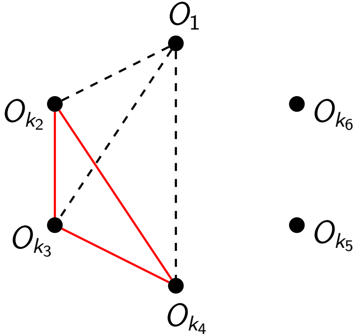
U ovom slučaju osoba O_1 ne poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.



Ukoliko u skupu osoba koje O_1 ne poznaje postoje dvije osobe koje se ne poznaju, npr. osobe O_{k_2} i O_{k_3}, tada je $\{O_1, O_{k_2}, O_{k_3}\}$ skup od troje ljudi koji se međusobno ne poznaju.

$\boxed{d(O_1) < 3}$

U ovom slučaju osoba O_1 ne poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.

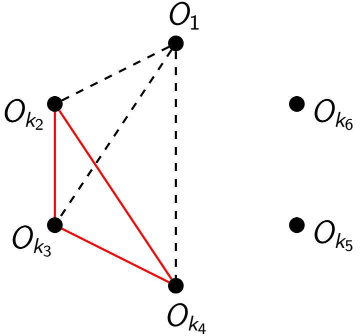


Ukoliko u skupu osoba koje O_1 ne poznaje postoje dvije osobe koje se ne poznaju, npr. osobe O_{k_2} i O_{k_3}, tada je $\{O_1, O_{k_2}, O_{k_3}\}$ skup od troje ljudi koji se međusobno ne poznaju.

Ukoliko se svake dvije osobe međusobno poznaju u skupu osoba koje O_1 ne poznaje,

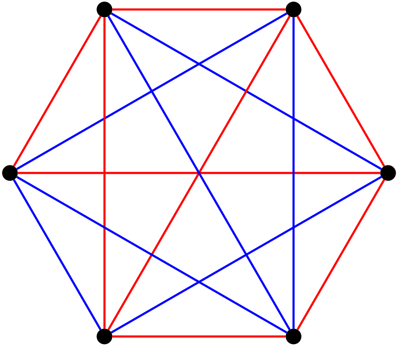
$\boxed{d(O_1) < 3}$

U ovom slučaju osoba O_1 ne poznaje barem troje ljudi $O_{k_2}, O_{k_3}, O_{k_4}$.

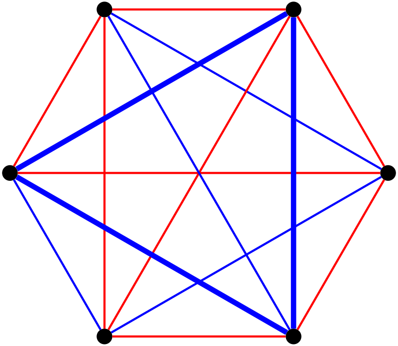


Ukoliko u skupu osoba koje O_1 ne poznaje postoje dvije osobe koje se ne poznaju, npr. osobe O_{k_2} i O_{k_3}, tada je $\{O_1, O_{k_2}, O_{k_3}\}$ skup od troje ljudi koji se međusobno ne poznaju.

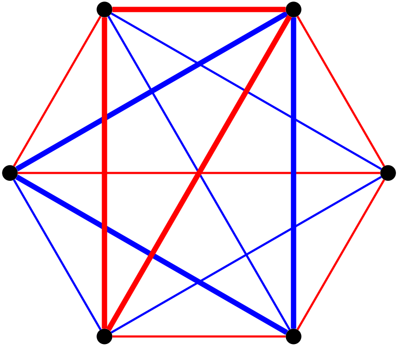
Ukoliko se svake dvije osobe međusobno poznaju u skupu osoba koje O_1 ne poznaje, tada je $\{O_{k_2}, O_{k_3}, O_{k_4}\}$ skup od troje ljudi koji se međusobno poznaju.



Ako bojamo bridove grafa K_6 s dvije boje, tada postoji jednobojni trokut.

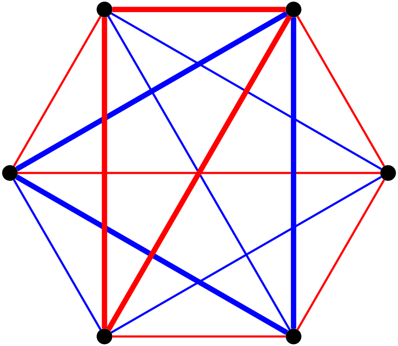


Ako bojamo bridove grafa K_6 s dvije boje, tada postoji jednobojni trokut.



Ako bojamo bridove grafa K_6 s dvije boje, tada postoji jednobojni trokut.

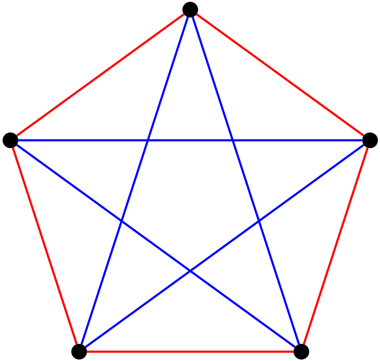
Zapravo postoje barem dva jednobojna trokuta.



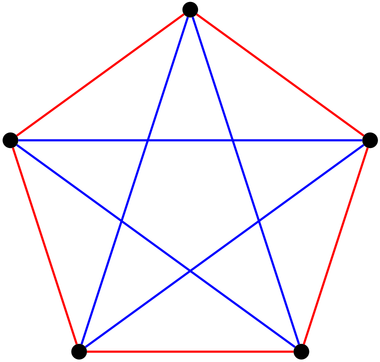
Ako bojamo bridove grafa K_6 s dvije boje, tada postoji jednobojni trokut.

Daje li nam prethodni dokaz algoritam za brzo pronalaženje jednobojnog trokuta?

Zapravo postoje barem dva jednobojna trokuta.



Ako bojamo bridove grafa K_5 s dvije boje, tada ne mora postojati jednobojni trokut.



Ako bojamo bridove grafa K_5 s dvije boje, tada ne mora postojati jednobojni trokut.

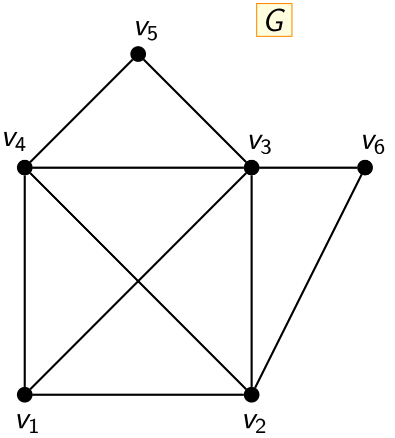
U grupi od 5 osoba ne mora postojati troje ljudi koji se međusobno poznaju niti troje ljudi koji se međusobno ne poznaju.

Klika

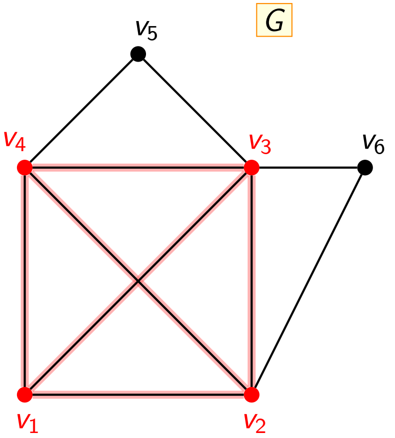
Klika reda m u jednostavnom grafu $G = (V, E)$ je m-člani podskup $S \subseteq V$ takav da je inducirani podgraf $G[S]$ potpuni graf K_m.

Nezavisni skup

Nezavisni skup reda m u jednostavnom grafu $G = (V, E)$ je m-člani podskup $S \subseteq V$ takav da je inducirani podgraf $G[S]$ prazan graf.

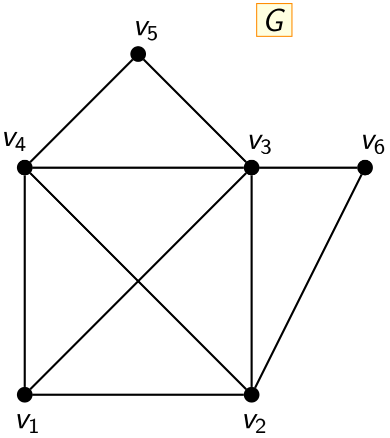


klika reda 4 u grafu G



klika reda 4 u grafu G

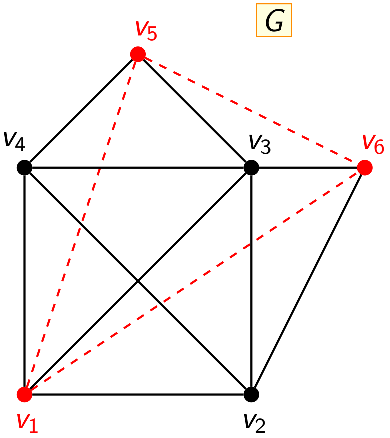
$\{v_1, v_2, v_3, v_4\}$



klika reda 4 u grafu G

$$\{v_1, v_2, v_3, v_4\}$$

nezavisni skup reda 3 u grafu G

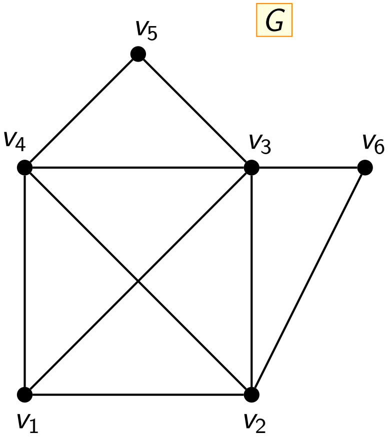


klika reda 4 u grafu G

$\{v_1, v_2, v_3, v_4\}$

nezavisni skup reda 3 u grafu G

$\{v_1, v_5, v_6\}$



klika reda 4 u grafu G

$\{v_1, v_2, v_3, v_4\}$

nezavisni skup reda 3 u grafu G

$\{v_1, v_5, v_6\}$

Navedite još neke klike i nezavisne skupove u grafu G.

Ramseyev broj – prva definicija

Ramseyev broj $R(s,t)$ je najmanji $n \in \mathbb{N}$ takav da svaki jednostavni graf s n vrhova sadrži kliku reda s ili nezavisni skup reda t.

Ramseyev broj – druga definicija

Ramseyev broj $R(s,t)$ je najmanji $n \in \mathbb{N}$ takav da svako bojanje bridova potpunog grafa K_n s dvije boje sadrži kliku reda s u prvoj boji ili kliku reda t u drugoj boji.

$$R(s,t) \leqslant \binom{s+t-2}{s-1}$$

- Mi smo dokazali da je $R(3,3) = 6$.
- Jasno je da vrijedi $R(t,2) = t$.

Ramseyev broj – poopćenje na više boja

Ramseyev broj $R(s_1, s_2, \dots, s_k)$ je najmanji $n \in \mathbb{N}$ takav da svako bojanje bridova potpunog grafa K_n s k boja sadrži kliku reda s_i u boji i za neki $i \in \{1, 2, \dots, k\}$.

Ramseyev broj – poopćenje na bojanje m-članih podskupova

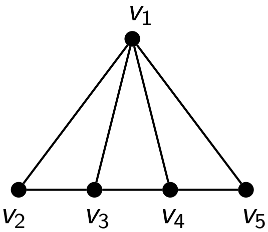
Ramseyev broj $R(s_1, s_2, \ldots, s_k; m)$ je najmanji $n \in \mathbb{N}$ takav da svako bojanje m-članih podskupova n-članog skupa s k boja sadrži s_i-člani podskup čiji su svi m-člani podskupovi obojani bojom i za neki $i \in \{1, 2, \ldots, k\}$.

- U potpunom grafu K_n bojamo klike reda m s k boja.
- Zaključak je da postoji klika reda s_i čije su sve klike reda m obojane istom bojom.
- Za $m = 2$ dobivamo bojanje bridova, a to su zapravo klike reda 2.
- $R(s_1, s_2, \ldots, s_k; 2) = R(s_1, s_2, \ldots, s_k)$

sedmi zadatak

Zadatak 7

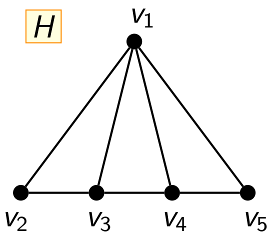
a) *Nacrtajte komplementarni graf grafa H prikazanog na slici.*



b) *Neka je G jednostavni graf s 15 bridova. Ako graf G^c ima 13 bridova, koliko vrhova ima graf G?*

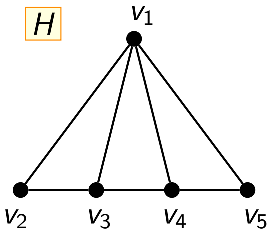
Rješenje

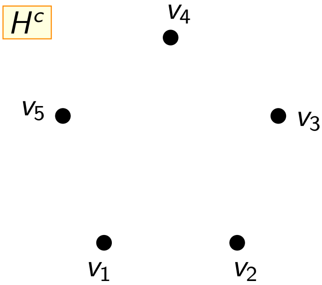
a)



Rješenje

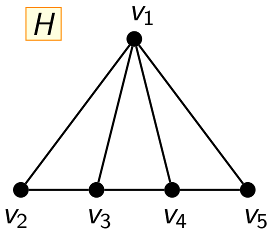
a)

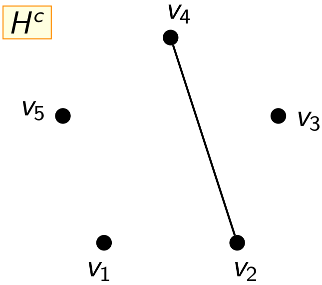




Rješenje

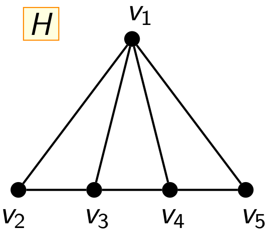
a)

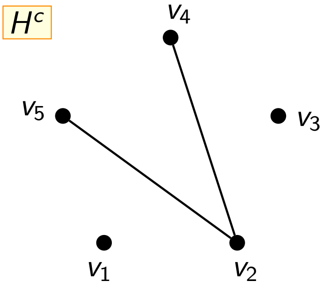




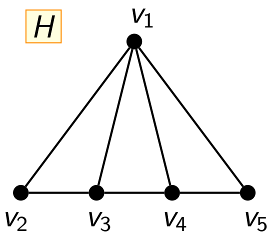
Rješenje

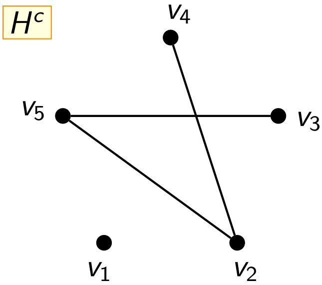
a)





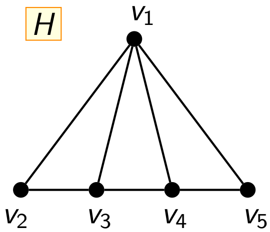
Rješenje

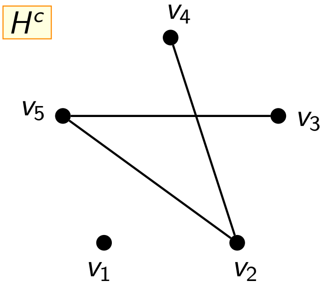
a) 



Rješenje

a)

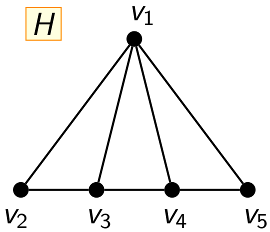


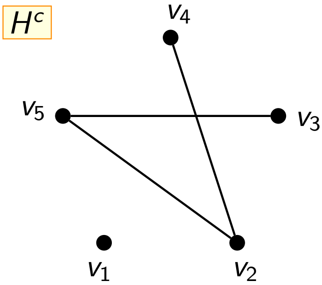


b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

Rješenje

a)



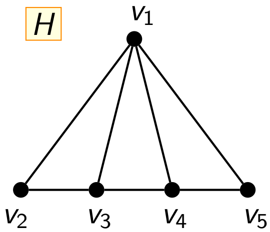


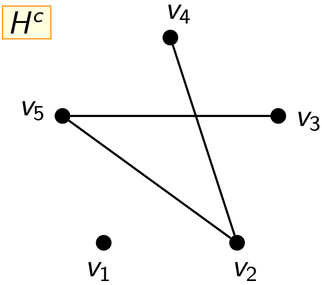
b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

$\varepsilon(G) + \varepsilon(G^c) =$

Rješenje

a)



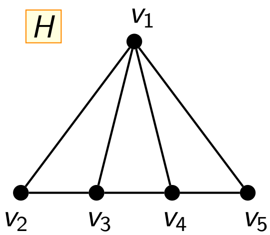


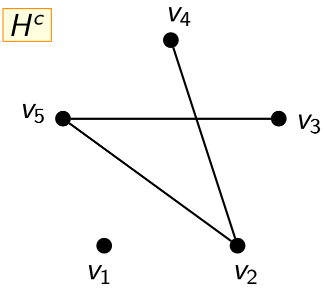
b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

$$\varepsilon(G) + \varepsilon(G^c) = \varepsilon(K_\nu)$$

Rješenje

a)





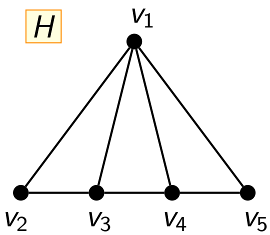
b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

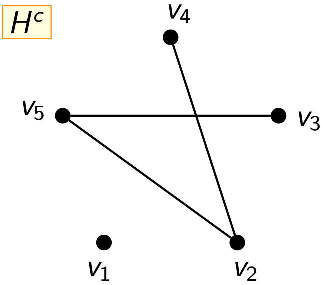
$$\varepsilon(G) + \varepsilon(G^c) = \varepsilon(K_\nu)$$

$$15 + 13 =$$

Rješenje

a)





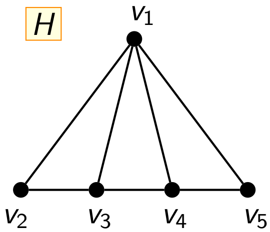
b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

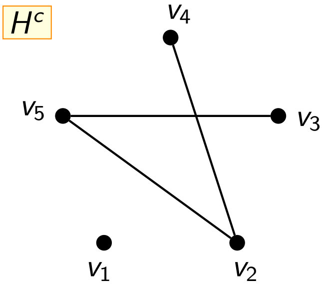
$$\varepsilon(G) + \varepsilon(G^c) = \varepsilon(K_\nu)$$

$$15 + 13 = \binom{\nu}{2}$$

Rješenje

a)





b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

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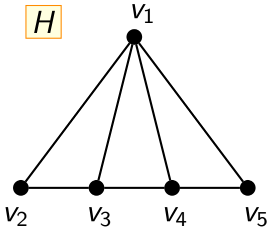
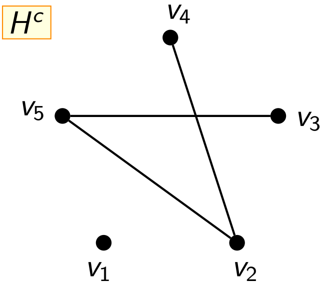
$$15 + 13 = \binom{\nu}{2}$$

$$\frac{\nu(\nu - 1)}{2} = 28$$

Rješenje

a)





b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

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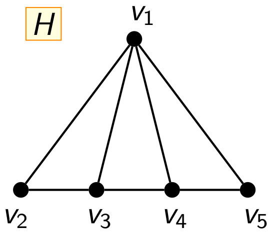
$$15 + 13 = \binom{\nu}{2}$$

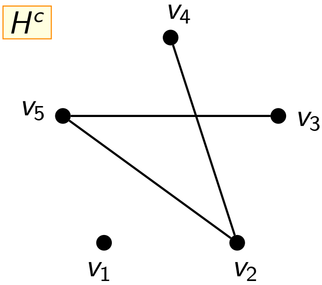
$$\frac{\nu(\nu-1)}{2} = 28$$

$$\nu^2 - \nu - 56 = 0$$

Rješenje

a)





b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

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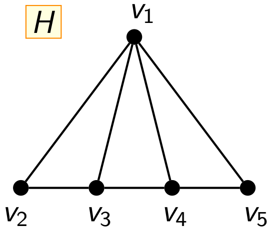
$$\frac{\nu(\nu - 1)}{2} = 28$$

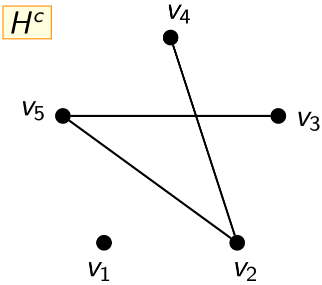
$$\nu^2 - \nu - 56 = 0$$

$$\nu_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 56}}{2}$$

Rješenje

a)





b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

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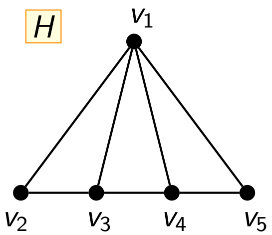
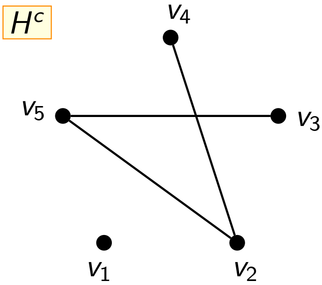
$$\nu_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 56}}{2}$$

$$\nu_1 = 8 \qquad \nu_2 = -7$$

Rješenje

a)





b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

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$$\frac{\nu(\nu-1)}{2} = 28$$

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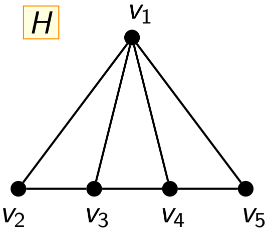
$$\nu_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 56}}{2}$$

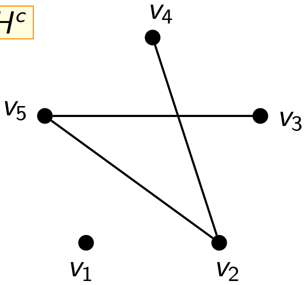
$$\boxed{\nu_1 = 8} \qquad \nu_2 = -7$$

Rješenje

a) H



H^c



b) $\varepsilon(G) = 15, \quad \varepsilon(G^c) = 13, \quad \nu(G) = ?$

$$\boxed{\nu(G) = 8}$$

$$\varepsilon(G) + \varepsilon(G^c) = \varepsilon(K_\nu)$$

$$15 + 13 = \binom{\nu}{2}$$

$$\frac{\nu(\nu - 1)}{2} = 28$$

$$\nu^2 - \nu - 56 = 0$$

$$\nu_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 56}}{2}$$

$$\boxed{\nu_1 = 8} \qquad \nu_2 = -7$$

osmi zadatak

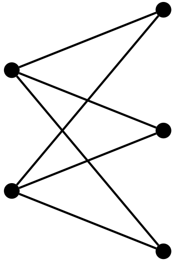
Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

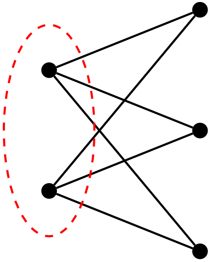
Rješenje



Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

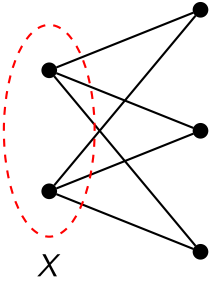
Rješenje



Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

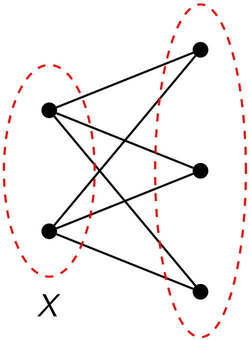
Rješenje



Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

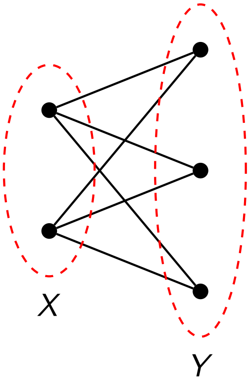
Rješenje



Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

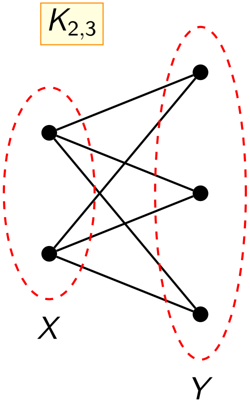
Rješenje



Zadatak 8

Potpuni bipartitni graf ima ukupno 456 bridova i 12 vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?

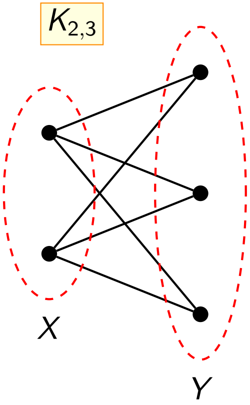
Rješenje



Zadatak 8

Potpuni bipartitni graf ima ukupno 456 bridova i 12 vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?

Rješenje

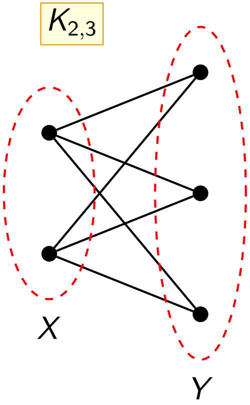


$$\boxed{\varepsilon(K_{m,n}) = mn}$$

Zadatak 8

Potpuni bipartitni graf ima ukupno 456 bridova i 12 vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?

Rješenje



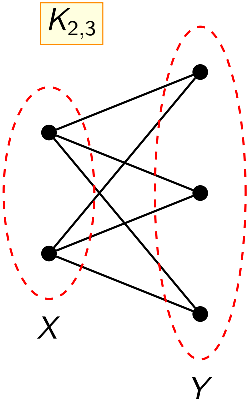
$$\boxed{\varepsilon(K_{m,n}) = mn}$$

$$\varepsilon = 456, \; m = 12$$

Zadatak 8

Potpuni bipartitni graf ima ukupno 456 *bridova i* 12 *vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?*

Rješenje



$$\boxed{\varepsilon(K_{m,n}) = mn}$$

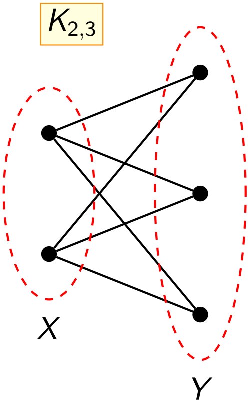
$$\varepsilon = 456, \;\; m = 12$$

$$n = \frac{456}{12}$$

Zadatak 8

Potpuni bipartitni graf ima ukupno 456 bridova i 12 vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?

Rješenje



$$\boxed{\varepsilon(K_{m,n}) = mn}$$

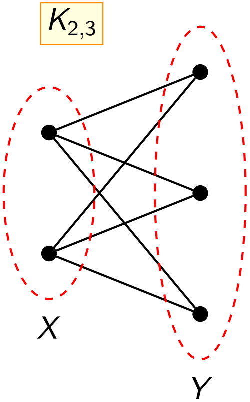
$$\varepsilon = 456, \;\; m = 12$$

$$n = \frac{456}{12} = 38$$

Zadatak 8

Potpuni bipartitni graf ima ukupno 456 bridova i 12 vrhova u jednom elementu particije. Koliko ima vrhova u drugom elementu particije?

Rješenje



$$\boxed{\varepsilon(K_{m,n}) = mn}$$

$$\varepsilon = 456, \ m = 12$$

$$n = \frac{456}{12} = 38$$

U drugom elementu particije ima 38 vrhova.

deveti zadatak

Zadatak 9

Ispitajte postoje li jednostavni grafovi sa sljedećim nizovima stupnjeva vrhova.

a) $3, 3, 3, 3, 2$

b) $0, 1, 2, 2, 3$

c) $1, 1, 1, 1, 1$

d) $1, 2, 3, 4, 4$

U slučaju da takav graf postoji navedite jedan primjer, a u protivnom obrazložite zbog čega ne postoji.

Rješenje

a) $3, 3, 3, 3, 2$

Rješenje

a) $3, 3, 3, 3, 2$

broj bridova

Rješenje

a) 3, 3, 3, 3, 2

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v)$$

Rješenje

a) 3, 3, 3, 3, 2

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 3 + 3 + 3 + 3 + 2$$

Rješenje

a) 3, 3, 3, 3, 2

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 3 + 3 + 3 + 3 + 2 = 14$$

Rješenje

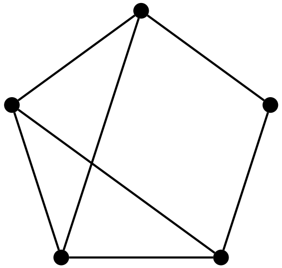
a) $3, 3, 3, 3, 2$

broj bridova $\quad \varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 3 + 3 + 3 + 3 + 2 = 14$$

Rješenje

a) 3, 3, 3, 3, 2

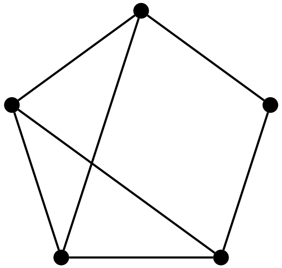


broj bridova

$\varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 3 + 3 + 3 + 3 + 2 = 14$$

Rješenje



a) 3, 3, 3, 3, 2

broj bridova $\qquad \varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 3 + 3 + 3 + 3 + 2 = 14$$

Takav jednostavni graf G postoji.

b) $0, 1, 2, 2, 3$

b) $0, 1, 2, 2, 3$

broj bridova

b) $0, 1, 2, 2, 3$

$\boxed{\text{broj bridova}}$

$$2\varepsilon = \sum_{v \in V(G)} d(v)$$

b) $0, 1, 2, 2, 3$

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 0 + 1 + 2 + 2 + 3$$

b) $0, 1, 2, 2, 3$

broj bridova

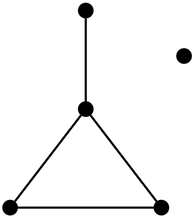
$$2\varepsilon = \sum_{v \in V(G)} d(v) = 0 + 1 + 2 + 2 + 3 = 8$$

b) $0, 1, 2, 2, 3$

broj bridova $\qquad \varepsilon = 4$

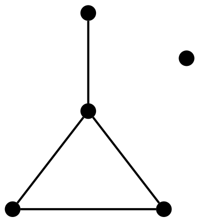
$$2\varepsilon = \sum_{v \in V(G)} d(v) = 0 + 1 + 2 + 2 + 3 = 8$$

b) $0, 1, 2, 2, 3$



$\boxed{\text{broj bridova}}$ $\quad \varepsilon = 4$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 0 + 1 + 2 + 2 + 3 = 8$$



b) $0, 1, 2, 2, 3$

broj bridova $\qquad \varepsilon = 4$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 0 + 1 + 2 + 2 + 3 = 8$$

Takav jednostavni graf G postoji.

c) $1, 1, 1, 1, 1$

c) $1, 1, 1, 1, 1$

broj bridova

c) $1, 1, 1, 1, 1$

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v)$$

c) $1, 1, 1, 1, 1$

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 1 + 1 + 1 + 1$$

c) $1, 1, 1, 1, 1$

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 1 + 1 + 1 + 1 = 5$$

c) $1, 1, 1, 1, 1$

$\boxed{\text{broj bridova}}$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 1 + 1 + 1 + 1 = 5$$

Ne postoji takav graf G, niti jednostavni niti pseudograf.

c) $1, 1, 1, 1, 1$

$\boxed{\text{broj bridova}}$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 1 + 1 + 1 + 1 = 5$$

Ne postoji takav graf G, niti jednostavni niti pseudograf.

U svakom grafu broj vrhova neparnog stupnja mora biti paran broj.

d) $1, 2, 3, 4, 4$

d) $1, 2, 3, 4, 4$

broj bridova

d) $1, 2, 3, 4, 4$

$\boxed{\text{broj bridova}}$

$$2\varepsilon = \sum_{v \in V(G)} d(v)$$

d) $1, 2, 3, 4, 4$

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4$$

d) $1, 2, 3, 4, 4$

broj bridova

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

d) $1, 2, 3, 4, 4$

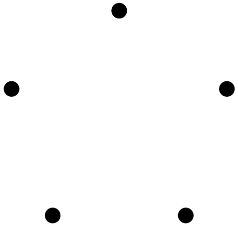
broj bridova $\varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

d) $1, 2, 3, 4, 4$

broj bridova $\quad \varepsilon = 7$

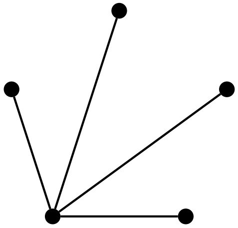
$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$



d) $1, 2, 3, 4, 4$

broj bridova $\quad \varepsilon = 7$

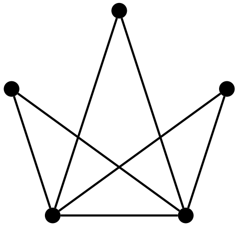
$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$



d) $1, 2, 3, 4, 4$

broj bridova $\qquad \varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

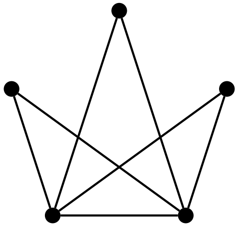


d) $1, 2, 3, 4, 4$

broj bridova $\varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

Ako je G jednostavni graf s 5 vrhova pri čemu dva vrha imaju stupanj 4, tada preostala tri vrha imaju stupanj barem 2.



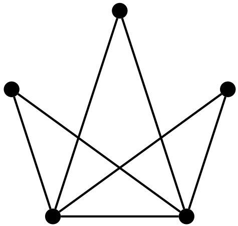
d) $1, 2, 3, 4, 4$

broj bridova $\quad \varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

Ako je G jednostavni graf s 5 vrhova pri čemu dva vrha imaju stupanj 4, tada preostala tri vrha imaju stupanj barem 2.

Stoga ne postoji jednostavni graf G sa zadanim nizom stupnjeva vrhova.



d) $1, 2, 3, 4, 4$

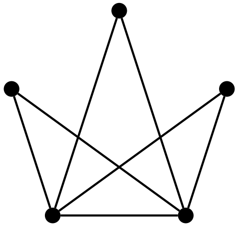
broj bridova $\quad \varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

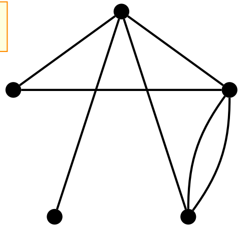
Ako je G jednostavni graf s 5 vrhova pri čemu dva vrha imaju stupanj 4, tada preostala tri vrha imaju stupanj barem 2.

Stoga ne postoji jednostavni graf G sa zadanim nizom stupnjeva vrhova.

Međutim, postoji takav pseudograf G.



pseudograf G



d) $1, 2, 3, 4, 4$

broj bridova $\quad \varepsilon = 7$

$$2\varepsilon = \sum_{v \in V(G)} d(v) = 1 + 2 + 3 + 4 + 4 = 14$$

Ako je G jednostavni graf s 5 vrhova pri čemu dva vrha imaju stupanj 4, tada preostala tri vrha imaju stupanj barem 2.

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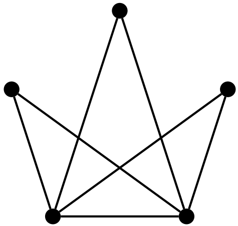
Međutim, postoji takav pseudograf G.



deseti zadatak

Zadatak 10

Dokažite da je niz nenegativnih cijelih brojeva $d_1, d_2, \ldots, d_n$ *niz stupnjeva vrhova nekog grafa ako i samo ako je* $\sum_{i=1}^{n} d_i$ *parni broj.*

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Rješenje

$\boxed{\Rightarrow}$

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Rješenje

$\boxed{\Rightarrow}$

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$\boxed{\Leftarrow}$

Neka je $d_1, d_2, \ldots, d_n$ niz nenegativnih cijelih brojeva i $\sum_{i=1}^{n} d_i$ parni broj.

Kako je suma svih članova niza parni broj, zaključujemo da u tom nizu postoji parni broj članova koji su neparni.

Tražimo graf G sa skupom vrhova $V = \{v_1, v_2, \ldots, v_n\}$ za koji vrijedi

$$d(v_i) = d_i, \quad i = 1, 2, \ldots, n.$$

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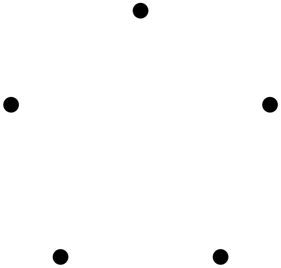
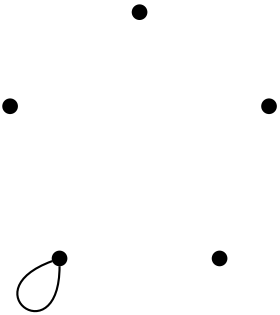
Primjer

- $1, 2, 3, 4, 4$



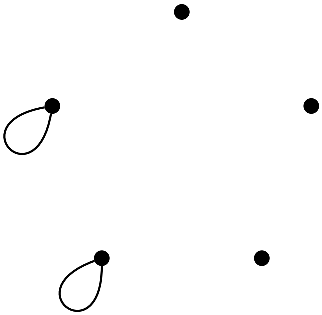
Primjer

- $1, 2, 3, 4, 4$



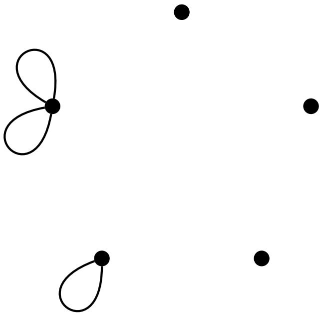
Primjer

- $1, 2, 3, 4, 4$



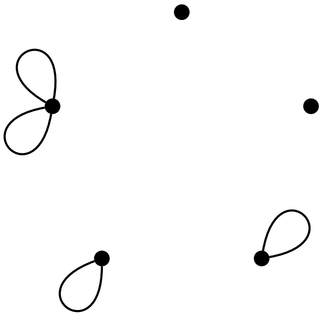
Primjer

- $1, 2, 3, 4, 4$



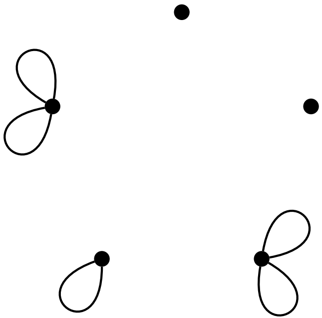
Primjer

- $1, 2, 3, 4, 4$



Primjer

- $1, 2, 3, 4, 4$



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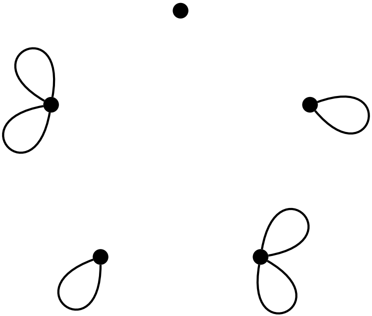
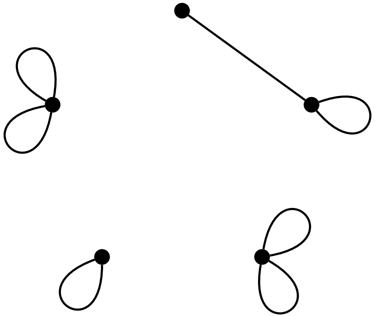
- $1, 2, 3, 4, 4$



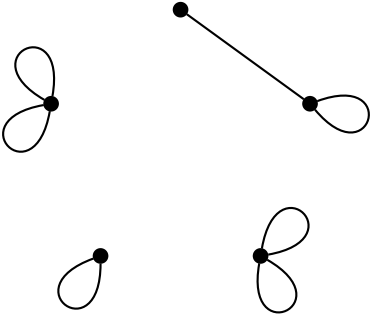
Primjer

- $1, 2, 3, 4, 4$

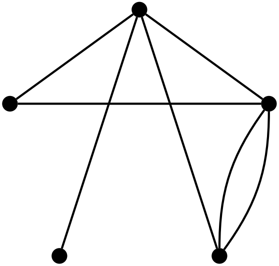


Primjer

- $1, 2, 3, 4, 4$



primjer iz prethodnog zadatka



Teorem (Erdős, Gallai, 1960.)

Niz $d_1 \geqslant d_2 \geqslant \cdots \geqslant d_n$ nenegativnih cijelih brojeva je niz stupnjeva vrhova nekog jednostavnog grafa ako i samo ako vrijedi

- $\sum\limits_{i=1}^{n} d_i$ je parni broj.
- $\sum\limits_{i=1}^{k} d_i \leqslant k(k-1) + \sum\limits_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n.$

Postoji više različitih kratkih i jednostavnih dokaza ovog teorema.

- Dokaz matematičkom indukcijom po sumi stupnjeva svih vrhova
- Konstruktivni dokaz

- $1, 2, 3, 4, 4$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

- $1, 2, 3, 4, 4$

Silazno sortirani niz $\quad 4, 4, 3, 2, 1$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\begin{array}{ccccc} \boxed{d_1} & \boxed{d_2} & \boxed{d_3} & \boxed{d_4} & \boxed{d_5} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4, & 4, & 3, & 2, & 1 \end{array}$$

- $1, 2, 3, 4, 4$

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz $4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\underset{4,}{d_1}\ \underset{4,}{d_2}\ \underset{3,}{d_3}\ \underset{2,}{d_4}\ \underset{1}{d_5}$$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $1, 2, 3, 4, 4$

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz $\quad 4, 4, 3, 2, 1$

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$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

- $1, 2, 3, 4, 4$

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz

$4, 4, 3, 2, 1$

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$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

$4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\overset{d_1}{4}, \overset{d_2}{4}, \overset{d_3}{3}, \overset{d_4}{2}, \overset{d_5}{1}$$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\}$$

- $1, 2, 3, 4, 4$

d_1 d_2 d_3 d_4 d_5

Silazno sortirani niz

$4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\boxed{\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n}$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\}$$

- 1, 2, 3, 4, 4

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz $\quad 4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\boxed{\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n}$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\}$$

- 1, 2, 3, 4, 4

$$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$$

Silazno sortirani niz

$$4, 4, 3, 2, 1$$

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$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$$

$$4, 4, 3, 2, 1$$

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$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\begin{array}{ccccc} d_1 & d_2 & d_3 & d_4 & d_5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4, & 4, & 3, & 2, & 1 \end{array}$$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

$$4 \leqslant 4$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\underbrace{4}_{d_1}, \underbrace{4}_{d_2}, \underbrace{3}_{d_3}, \underbrace{2}_{d_4}, \underbrace{1}_{d_5}$$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

$$4 \leqslant 4$$

- $k = 2$

- 1, 2, 3, 4, 4

Silazno sortirani niz

$$\begin{array}{ccccc} \boxed{d_1} & \boxed{d_2} & \boxed{d_3} & \boxed{d_4} & \boxed{d_5} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4, & 4, & 3, & 2, & 1 \end{array}$$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\boxed{\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n}$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

$$4 \leqslant 4$$

- $k = 2$

$$4 + 4 \leqslant$$

- 1, 2, 3, 4, 4

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz

4, 4, 3, 2, 1

- Suma svih članova niza je parni broj: $4+4+3+2+1=14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k=1$

$$4 \leqslant 1 \cdot 0 + \min\{1,4\} + \min\{1,3\} + \min\{1,2\} + \min\{1,1\}$$

$$4 \leqslant 0+1+1+1+1$$

$$4 \leqslant 4$$

- $k=2$

$$4+4 \leqslant 2 \cdot 1$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

$4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

$$4 \leqslant 4$$

- $k = 2$

$$4 + 4 \leqslant 2 \cdot 1 + \min\{2, 3\}$$

- 1, 2, 3, 4, 4

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz

4, 4, 3, 2, 1

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1,4\} + \min\{1,3\} + \min\{1,2\} + \min\{1,1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

$$4 \leqslant 4$$

- $k = 2$

$$4 + 4 \leqslant 2 \cdot 1 + \min\{2,3\} + \min\{2,2\}$$

- $1, 2, 3, 4, 4$

$\boxed{\text{Silazno sortirani niz}}$

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

$4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\boxed{\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n}$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$

$$4 \leqslant 0 + 1 + 1 + 1 + 1$$

$$4 \leqslant 4$$

- $k = 2$

$$4 + 4 \leqslant 2 \cdot 1 + \min\{2, 3\} + \min\{2, 2\} + \min\{2, 1\}$$

- 1, 2, 3, 4, 4

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

Silazno sortirani niz

4, 4, 3, 2, 1

- Suma svih članova niza je parni broj: $4+4+3+2+1=14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k=1$

$$4 \leqslant 1 \cdot 0 + \min\{1,4\} + \min\{1,3\} + \min\{1,2\} + \min\{1,1\}$$

$$4 \leqslant 0+1+1+1+1$$

$$4 \leqslant 4$$

- $k=2$

$$4+4 \leqslant 2 \cdot 1 + \min\{2,3\} + \min\{2,2\} + \min\{2,1\}$$

$$8 \leqslant 2+2+2+1$$

- 1, 2, 3, 4, 4

$$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$$

Silazno sortirani niz

$$4, 4, 3, 2, 1$$

- Suma svih članova niza je parni broj: $4+4+3+2+1=14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k=1$

$$4 \leqslant 1 \cdot 0 + \min\{1,4\} + \min\{1,3\} + \min\{1,2\} + \min\{1,1\}$$

$$4 \leqslant 0+1+1+1+1$$

$$4 \leqslant 4$$

- $k=2$

$$4+4 \leqslant 2 \cdot 1 + \min\{2,3\} + \min\{2,2\} + \min\{2,1\}$$

$$8 \leqslant 2+2+2+1$$

$$8 \leqslant 7$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$\boxed{d_1}\ \boxed{d_2}\ \boxed{d_3}\ \boxed{d_4}\ \boxed{d_5}$

$4, 4, 3, 2, 1$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1,4\} + \min\{1,3\} + \min\{1,2\} + \min\{1,1\}$$
$$4 \leqslant 0 + 1 + 1 + 1 + 1$$
$$4 \leqslant 4$$

- $k = 2$

$$4 + 4 \leqslant 2 \cdot 1 + \min\{2,3\} + \min\{2,2\} + \min\{2,1\}$$
$$8 \leqslant 2 + 2 + 2 + 1$$
$$8 \leqslant 7 \longleftarrow \text{ne vrijedi}$$

- $1, 2, 3, 4, 4$

Silazno sortirani niz

$$\begin{array}{ccccc} d_1 & d_2 & d_3 & d_4 & d_5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4, & 4, & 3, & 2, & 1 \end{array}$$

- Suma svih članova niza je parni broj: $4 + 4 + 3 + 2 + 1 = 14$

$$\sum_{i=1}^{k} d_i \leqslant k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}, \quad 1 \leqslant k \leqslant n$$

- $k = 1$

$$4 \leqslant 1 \cdot 0 + \min\{1, 4\} + \min\{1, 3\} + \min\{1, 2\} + \min\{1, 1\}$$
$$4 \leqslant 0 + 1 + 1 + 1 + 1$$
$$4 \leqslant 4$$

Ne postoji jednostavni graf s nizom stupnjeva vrhova $1, 2, 3, 4, 4$.

- $k = 2$

$$4 + 4 \leqslant 2 \cdot 1 + \min\{2, 3\} + \min\{2, 2\} + \min\{2, 1\}$$
$$8 \leqslant 2 + 2 + 2 + 1$$
$$8 \leqslant 7 \leftarrow \text{ne vrijedi}$$

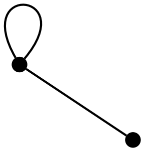
Propozicija

Niz $d_1 \geqslant d_2 \geqslant \cdots \geqslant d_n$ nenegativnih cijelih brojeva je niz stupnjeva vrhova nekog grafa bez petlji ako i samo ako vrijedi

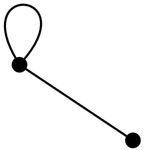
- $\sum\limits_{i=1}^{n} d_i$ je parni broj.
- $d_1 \leqslant \sum\limits_{i=2}^{n} d_i$.

- $3, 1$

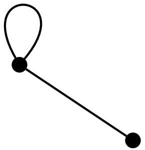
- $3, 1$



- $3, 1$ Ne postoji graf bez petlji

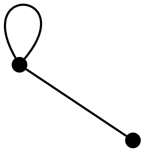


- $3, 1$ Ne postoji graf bez petlji

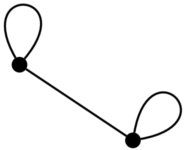


- $3, 3$

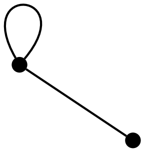
- $3, 1$ Ne postoji graf bez petlji



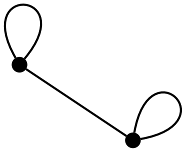
- $3, 3$



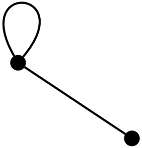
- $3, 1 \leftsquigarrow$ Ne postoji graf bez petlji



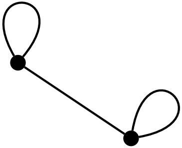
- $3, 3 \leftsquigarrow$ Postoji graf bez petlji

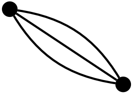


- $3, 1$ ← Ne postoji graf bez petlji



- $3, 3$ ← Postoji graf bez petlji





graf bez petlji

Teorem (Havel-Hakimi)

Neka je D niz nenegativnih cijelih brojeva

$$d_1, d_2, \ldots, d_{n-1}, d_n$$

pri čemu je $d_1 \geqslant d_2 \geqslant \cdots \geqslant d_{n-1} \geqslant d_n$, $d_1 < n$ i $n \geqslant 2$.

Neka je D' niz od $n-1$ brojeva

$$d_2 - 1, d_3 - 1, \ldots, d_k - 1, d_{k+1} - 1, d_{k+2}, \ldots, d_{n-1}, d_n$$

pri čemu je $k = d_1$. Tada je D niz stupnjeva vrhova nekog jednostavnog grafa ako i samo ako je D' niz stupnjeva vrhova nekog jednostavnog grafa.

Primjer 1 – Havel-Hakimi

3 3 3 2 1

Primjer 1 – Havel-Hakimi

$\boxed{3}$ 3 3 2 1

Primjer 1 – Havel-Hakimi

obriši $\rightsquigarrow$ $\boxed{3}$ 3 3 2 1

Primjer 1 – Havel-Hakimi

$$\begin{array}{cccccc} & -1 & -1 & -1 & & \\ & \downarrow & \downarrow & \downarrow & & \\ \text{obriši} \rightsquigarrow \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{cccccc} & -1 & -1 & -1 & & \\ & \downarrow & \downarrow & \downarrow & & \\ \text{obriši} \rightsquigarrow \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$2 \quad 2 \quad 1 \quad 1$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{cccccc} & & -1 & -1 & -1 & \\ & & \downarrow & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow & \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$\boxed{2} \quad 2 \quad 1 \quad 1$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{ccccccc} & & -1 & -1 & -1 & \\ & & \downarrow & \downarrow & \downarrow & \\ \boxed{\text{obriši}} \rightsquigarrow & \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$\boxed{\text{obriši}} \rightsquigarrow \boxed{2} \quad 2 \quad 1 \quad 1$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{cccccc} & & -1 & -1 & -1 & \\ & & \downarrow & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow & \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$\begin{array}{ccccc} & & -1 & -1 & \\ & & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow & \boxed{2} & 2 & 1 & 1 \end{array}$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{ccccc} & -1 & -1 & -1 & \\ & \downarrow & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$\begin{array}{cccc} & -1 & -1 & \\ & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow \boxed{2} & 2 & 1 & 1 \end{array}$$

$$1 \quad 0 \quad 1$$

Primjer 1 – Havel-Hakimi

obriši $\rightsquigarrow$
$$\begin{array}{ccccc} & -1 & -1 & -1 & \\ & \downarrow & \downarrow & \downarrow & \\ \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

obriši $\rightsquigarrow$
$$\begin{array}{cccc} & -1 & -1 & \\ & \downarrow & \downarrow & \\ \boxed{2} & 2 & 1 & 1 \end{array}$$

sortiraj silazno $\rightsquigarrow$
$$\begin{array}{ccc} 1 & 0 & 1 \end{array}$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{ccccc} & -1 & -1 & -1 & \\ & \downarrow & \downarrow & \downarrow & \\ \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

obriši

$$\begin{array}{cccc} & -1 & -1 & \\ & \downarrow & \downarrow & \\ \boxed{2} & 2 & 1 & 1 \end{array}$$

obriši

$$1 \quad 0 \quad 1$$

sortiraj silazno

$$1 \quad 1 \quad 0$$

Primjer 1 – Havel-Hakimi

$$\begin{array}{cccccc} & & -1 & -1 & -1 & \\ \text{obriši} \rightsquigarrow & \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$\begin{array}{ccccc} & & -1 & -1 & \\ \text{obriši} \rightsquigarrow & \boxed{2} & 2 & 1 & 1 \end{array}$$

$$\begin{array}{cccc} \text{sortiraj silazno} \rightsquigarrow & 1 & 0 & 1 \end{array}$$

$$\begin{array}{ccc} \boxed{1} & 1 & 0 \end{array}$$

Primjer 1 – Havel-Hakimi

obriši $\rightsquigarrow \boxed{3} \quad \overset{-1}{\underset{\downarrow}{}}3 \quad \overset{-1}{\underset{\downarrow}{}}3 \quad \overset{-1}{\underset{\downarrow}{}}2 \quad 1$

obriši $\rightsquigarrow \boxed{2} \quad \overset{-1}{\underset{\downarrow}{}}2 \quad \overset{-1}{\underset{\downarrow}{}}1 \quad 1$

sortiraj silazno $\rightsquigarrow 1 \quad 0 \quad 1$

obriši $\rightsquigarrow \boxed{1} \quad 1 \quad 0$

Primjer 1 – Havel-Hakimi

$$\begin{array}{cccccc} & & -1 & -1 & -1 & \\ & & \downarrow & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow & \boxed{3} & 3 & 3 & 2 & 1 \end{array}$$

$$\begin{array}{ccccc} & & -1 & -1 & \\ & & \downarrow & \downarrow & \\ \text{obriši} \rightsquigarrow & \boxed{2} & 2 & 1 & 1 \end{array}$$

$$\begin{array}{cccc} \text{sortiraj silazno} \rightsquigarrow & 1 & 0 & 1 \end{array}$$

$$\begin{array}{cccc} & & -1 & \\ & & \downarrow & \\ \text{obriši} \rightsquigarrow & \boxed{1} & 1 & 0 \end{array}$$

Primjer 1 – Havel-Hakimi

obriši $\rightsquigarrow \boxed{3} \quad \overset{-1}{\downarrow} \atop 3 \quad$

$$\text{obriši} \rightsquigarrow \boxed{3} \quad \overset{-1}{3} \quad \overset{-1}{3} \quad \overset{-1}{2} \quad 1$$

$$\text{obriši} \rightsquigarrow \boxed{2} \quad \overset{-1}{2} \quad \overset{-1}{1} \quad 1$$

$$\text{sortiraj silazno} \rightsquigarrow 1 \quad 0 \quad 1$$

$$\text{obriši} \rightsquigarrow \boxed{1} \quad \overset{-1}{1} \quad 0$$

$$0 \quad 0$$

Primjer 1 – Havel-Hakimi

obriši $\rightsquigarrow \boxed{3}\quad \overset{\overset{-1}{\downarrow}}{3}\quad \overset{\overset{-1}{\downarrow}}{3}\quad \overset{\overset{-1}{\downarrow}}{2}\quad 1$

obriši $\rightsquigarrow \boxed{2}\quad \overset{\overset{-1}{\downarrow}}{2}\quad \overset{\overset{-1}{\downarrow}}{1}\quad 1$

sortiraj silazno $\rightsquigarrow 1\quad 0\quad 1$

obriši $\rightsquigarrow \boxed{1}\quad \overset{\overset{-1}{\downarrow}}{1}\quad 0$

$0\quad 0 \leftsquigarrow$ jest niz stupnjeva vrhova nekog jednostavnog grafa

Primjer 1 – Havel-Hakimi

		-1	-1	-1	
obriši $\rightsquigarrow$	$\boxed{3}$	3	3	2	1
			-1	-1	
obriši $\rightsquigarrow$		$\boxed{2}$	2	1	1
sortiraj silazno $\rightsquigarrow$			1	0	1
				-1	
obriši $\rightsquigarrow$			$\boxed{1}$	1	0
				0	0 $\leftsquigarrow$ jest niz stupnjeva vrhova nekog jednostavnog grafa

Niz $3, 3, 3, 2, 1$ jest niz stupnjeva vrhova nekog jednostavnog grafa.

Primjer 2 – Havel-Hakimi

4 4 3 2 1

Primjer 2 – Havel-Hakimi

$\boxed{4}$ 4 3 2 1

Primjer 2 – Havel-Hakimi

obriši $\rightsquigarrow$ $\boxed{4}$ 4 3 2 1

Primjer 2 – Havel-Hakimi

$$\begin{array}{ccccc} & -1 & -1 & -1 & -1 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{obriši} \rightsquigarrow \boxed{4} & 4 & 3 & 2 & 1 \end{array}$$

Primjer 2 – Havel-Hakimi

$$\begin{array}{cccccc} & -1 & -1 & -1 & -1 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{obriši} \rightsquigarrow \boxed{4} & 4 & 3 & 2 & 1 \end{array}$$

3 2 1 0

Primjer 2 – Havel-Hakimi

$$\begin{array}{ccccc} & -1 & -1 & -1 & -1 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{obriši} \rightsquigarrow \boxed{4} & 4 & 3 & 2 & 1 \end{array}$$

$$\boxed{3} \quad 2 \quad 1 \quad 0$$

Primjer 2 – Havel-Hakimi

$$\begin{array}{cccccc} & & -1 & -1 & -1 & -1 \\ & & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{obriši} \rightsquigarrow & \boxed{4} & 4 & 3 & 2 & 1 \end{array}$$

$$\begin{array}{ccccc} \text{obriši} \rightsquigarrow & \boxed{3} & 2 & 1 & 0 \end{array}$$

Primjer 2 – Havel-Hakimi

$$\boxed{\text{obriši}} \rightsquigarrow \boxed{4} \quad \begin{array}{cccc} -1 & -1 & -1 & -1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 \end{array}$$

$$\boxed{\text{obriši}} \rightsquigarrow \boxed{3} \quad \begin{array}{ccc} -1 & -1 & -1 \\ \downarrow & \downarrow & \downarrow \\ 2 & 1 & 0 \end{array}$$

Primjer 2 – Havel-Hakimi

$$\begin{array}{ccccc} & -1 & -1 & -1 & -1 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{obriši} \rightsquigarrow \boxed{4} & 4 & 3 & 2 & 1 \end{array}$$

$$\begin{array}{cccc} & -1 & -1 & -1 \\ & \downarrow & \downarrow & \downarrow \\ \text{obriši} \rightsquigarrow \boxed{3} & 2 & 1 & 0 \end{array}$$

$$1 \quad 0 \quad -1$$

Primjer 2 – Havel-Hakimi

$$\begin{array}{rccccc}
 & & -1 & -1 & -1 & -1 \\
 & & \downarrow & \downarrow & \downarrow & \downarrow \\
\text{obriši} \rightsquigarrow & \boxed{4} & 4 & 3 & 2 & 1
\end{array}$$

$$\begin{array}{rcccc}
 & & -1 & -1 & -1 \\
 & & \downarrow & \downarrow & \downarrow \\
\text{obriši} \rightsquigarrow & \boxed{3} & 2 & 1 & 0
\end{array}$$

$$1 \quad 0 \quad -1 \leftsquigarrow$$

nije niz stupnjeva vrhova nekog jednostavnog grafa

Primjer 2 – Havel-Hakimi

obriši $\boxed{4}\quad \overset{-1}{\downarrow}{4}\quad \overset{-1}{\downarrow}{3}\quad \overset{-1}{\downarrow}{2}\quad \overset{-1}{\downarrow}{1}$

obriši $\boxed{3}\quad \overset{-1}{\downarrow}{2}\quad \overset{-1}{\downarrow}{1}\quad \overset{-1}{\downarrow}{0}$

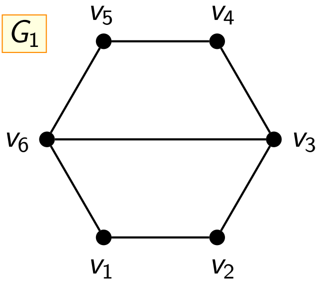
$1 \quad 0 \quad -1$ nije niz stupnjeva vrhova nekog jednostavnog grafa

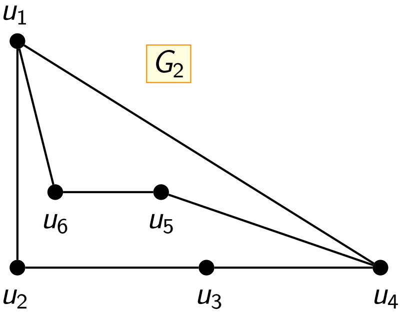
Niz $4, 4, 3, 2, 1$ nije niz stupnjeva vrhova nekog jednostavnog grafa.

jedanaesti zadatak

Zadatak 11

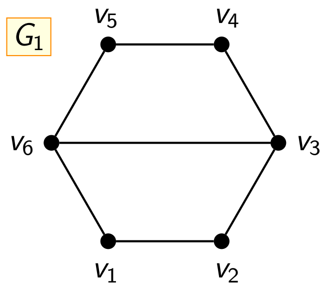
Ispitajte jesu li grafovi G_1 i G_2 izomorfni.

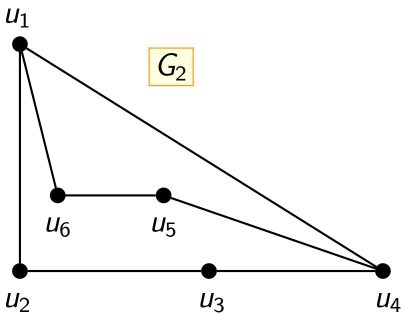




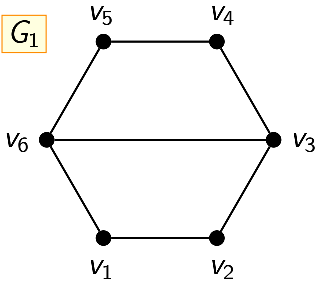
Ukoliko jesu, pronađite jedan izomorfizam između njih i pripadnu matricu permutacije koja povezuje njihove matrice susjedstva. U protivnom, objasnite zašto nisu izomorfni.

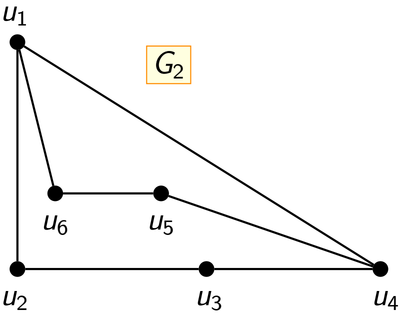
Rješenje





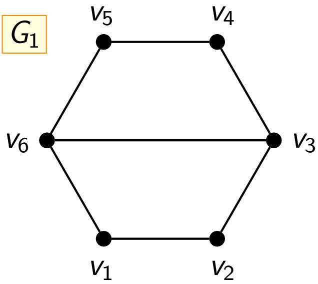
Rješenje

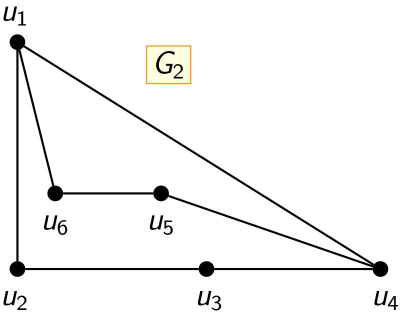




$$A_1 = \begin{bmatrix} \quad \end{bmatrix}$$

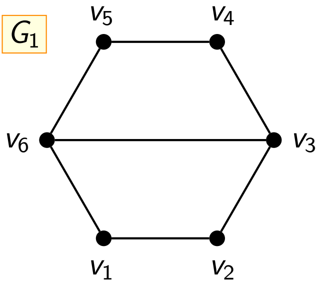
Rješenje

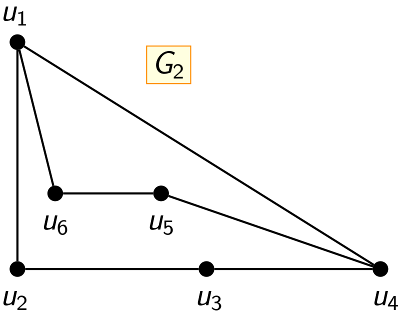




$$A_1 = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{bmatrix} & & & & & \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

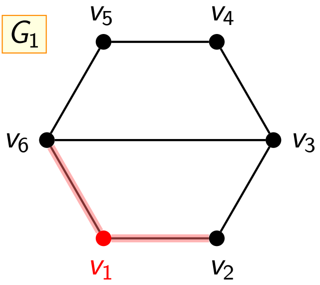
Rješenje

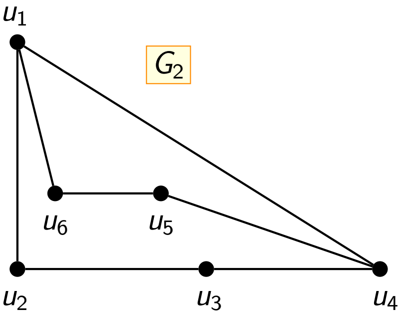




$$A_1 = \begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \left[\begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix}\right] \end{array}$$

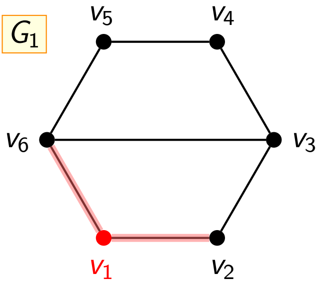
Rješenje

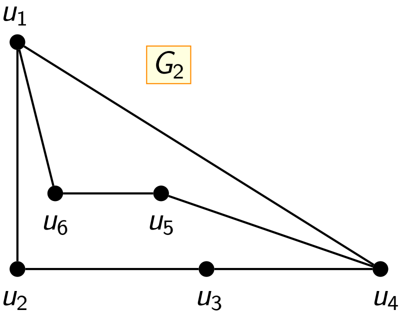




$$A_1 = \begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}$$

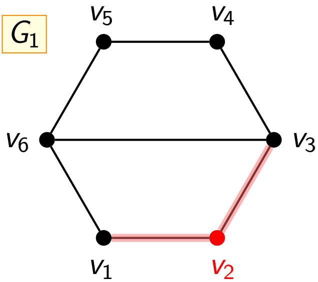
Rješenje

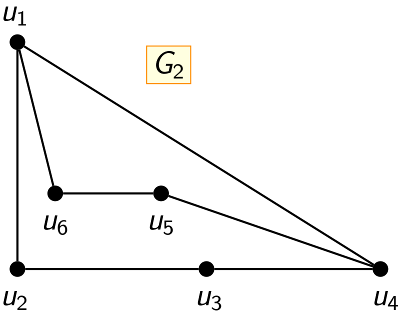




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ 1 & & & & & \end{bmatrix} \end{array}$$

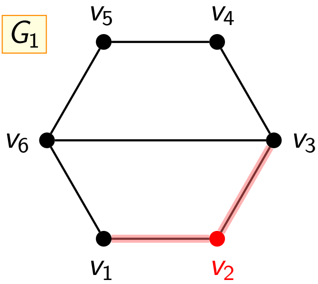
Rješenje

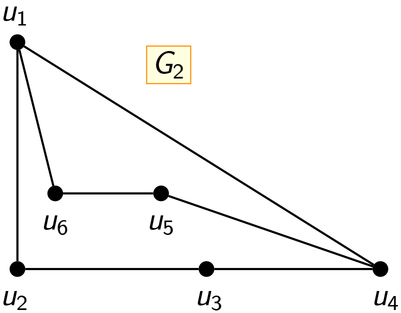




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ 1 & & & & & \end{bmatrix} \end{array}$$

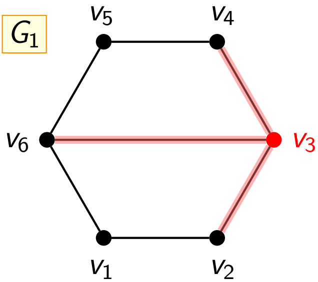
Rješenje

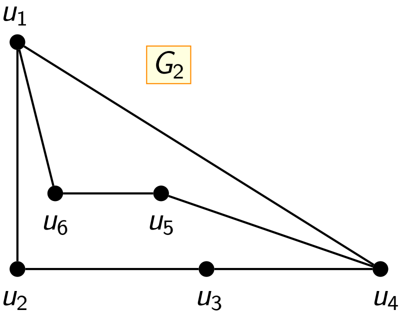




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & & & & \\ 0 & 0 & & & & \\ 0 & 0 & & & & \\ 1 & 0 & & & & \end{bmatrix} \end{array}$$

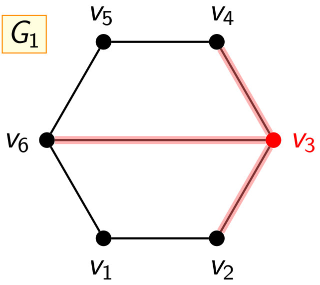
Rješenje

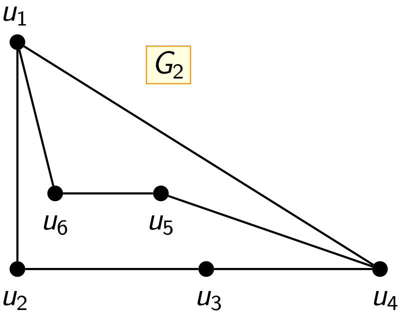




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & & & & \\ 0 & 0 & & & & \\ 1 & 0 & & & & \end{bmatrix} \end{array}$$

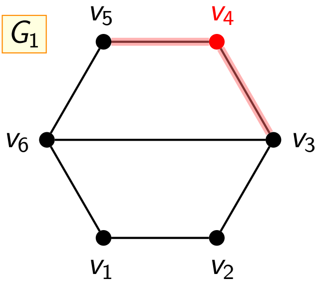
Rješenje

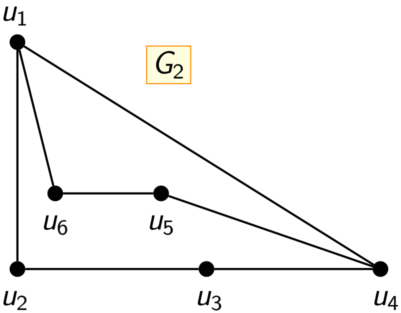




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & & & \\ 0 & 0 & 0 & & & \\ 1 & 0 & 1 & & & \end{bmatrix} \end{array}$$

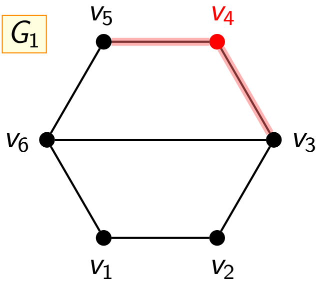
Rješenje

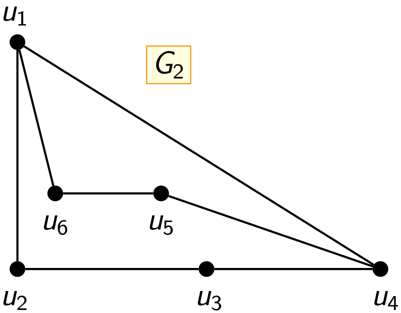




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & & & \\ 1 & 0 & 1 & & & \end{bmatrix} \end{array}$$

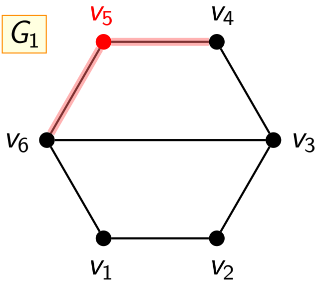
Rješenje

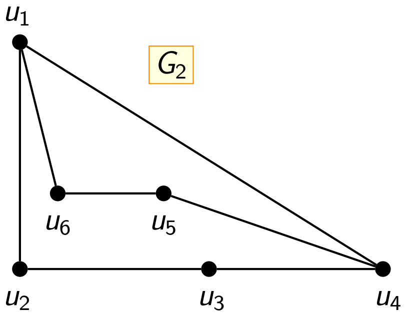




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & & \\ 1 & 0 & 1 & 0 & & \end{bmatrix} \end{array}$$

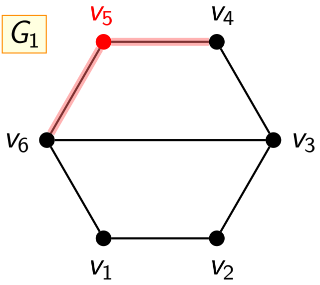
Rješenje

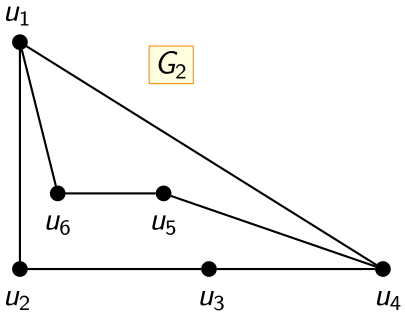




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & & \end{bmatrix} \end{array}$$

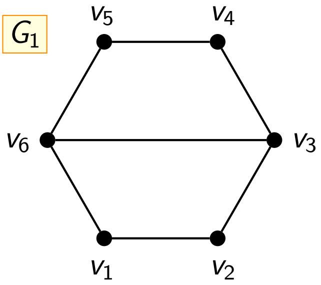
Rješenje

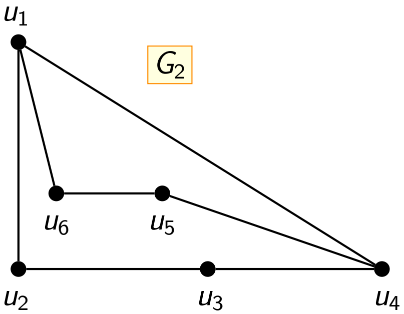




$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & \end{bmatrix} \end{array}$$

Rješenje





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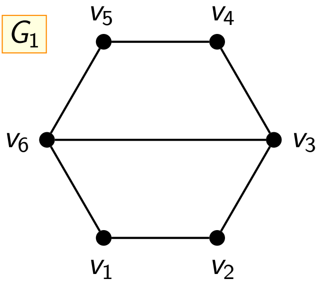
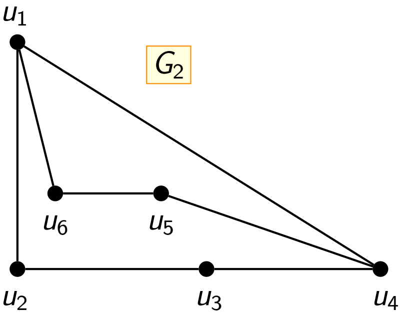
Rješenje

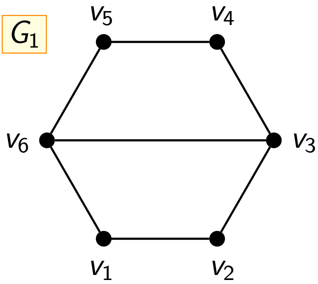


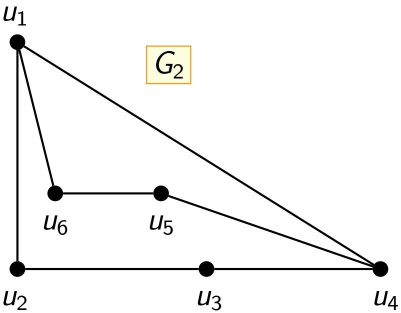


$$A_1 = \begin{array}{cc} & \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

Rješenje

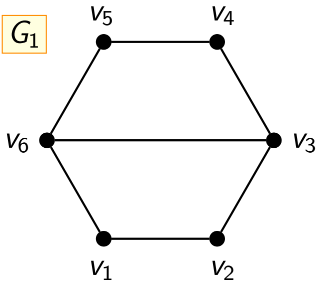


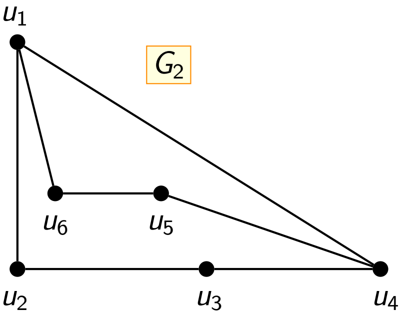


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

Rješenje

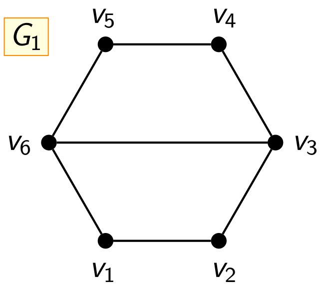


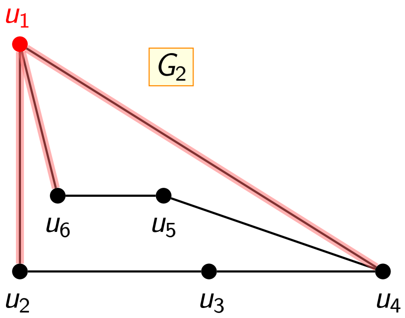


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} \ & \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ & \ \end{bmatrix} \end{array}$$

Rješenje

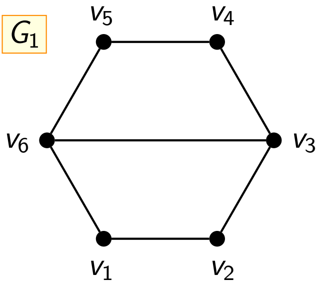


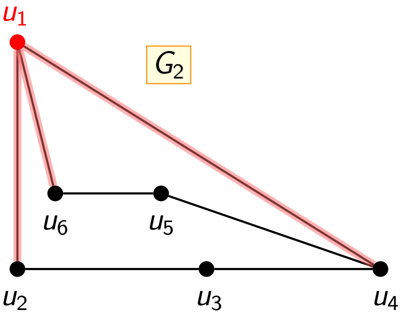


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}$$

Rješenje

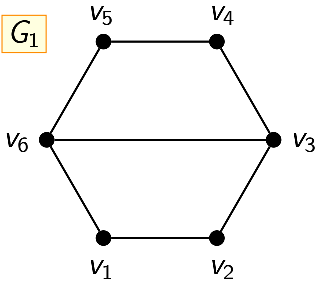


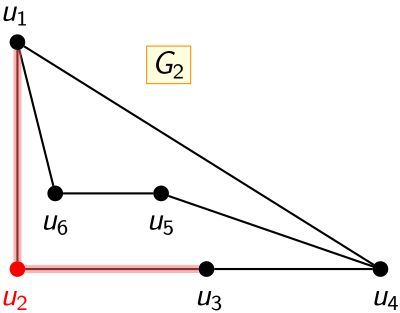


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & & & & & \\ 0 & & & & & \\ 1 & & & & & \\ 0 & & & & & \\ 1 & & & & & \end{bmatrix} \end{array}$$

Rješenje

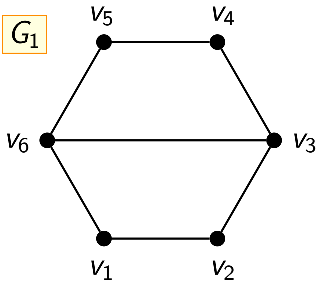


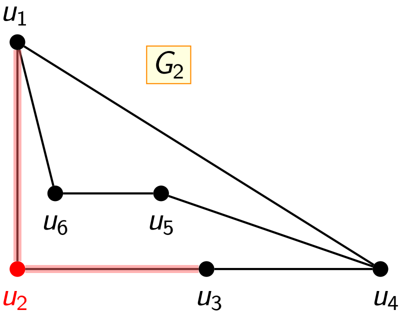


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & & & & & \\ 1 & & & & & \\ 0 & & & & & \\ 1 & & & & & \end{bmatrix} \end{array}$$

Rješenje

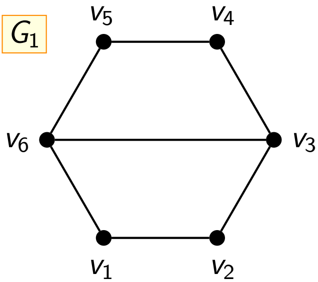


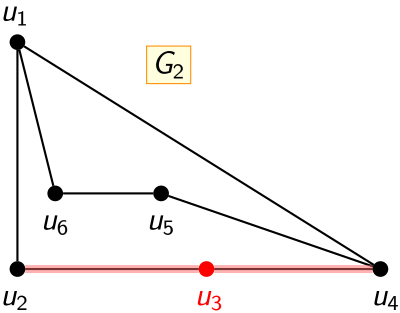


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & & & & \\ 1 & 0 & & & & \\ 0 & 0 & & & & \\ 1 & 0 & & & & \end{bmatrix} \end{array}$$

Rješenje

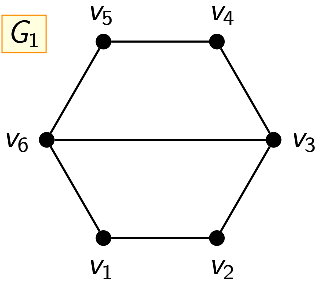


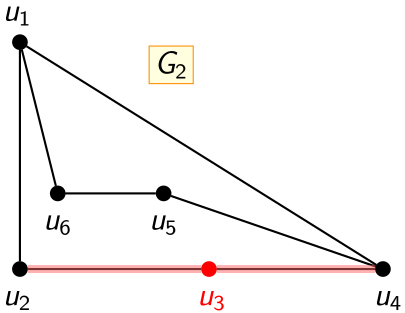


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & & & & \\ 0 & 0 & & & & \\ 1 & 0 & & & & \end{bmatrix} \end{array}$$

Rješenje

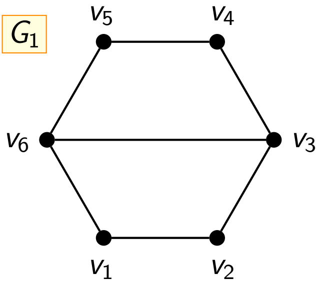


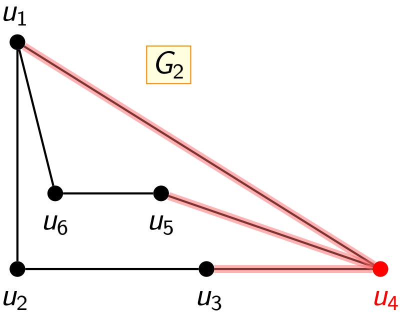


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & & & \\ 0 & 0 & 0 & & & \\ 1 & 0 & 0 & & & \end{bmatrix} \end{array}$$

Rješenje

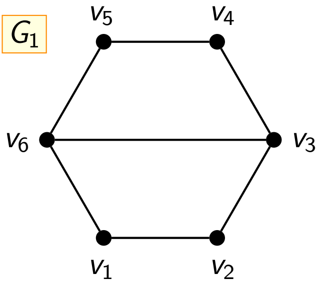


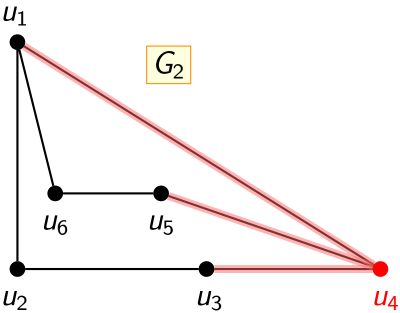


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & & & \\ 1 & 0 & 0 & & & \end{bmatrix} \end{array}$$

Rješenje

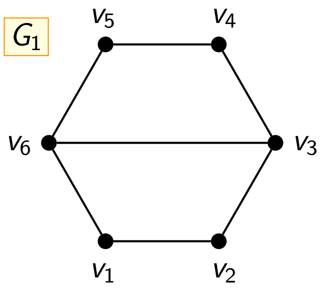


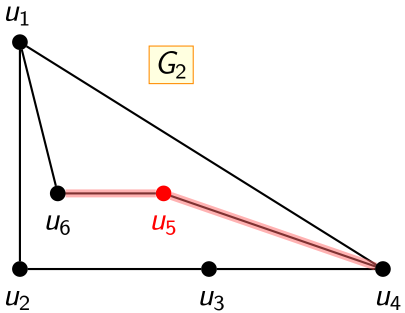


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & & \\ 1 & 0 & 0 & 0 & & \end{bmatrix} \end{array}$$

Rješenje

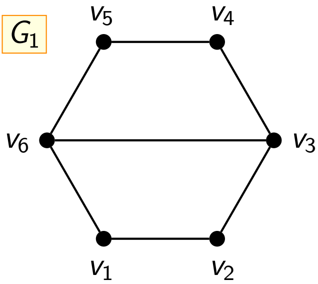


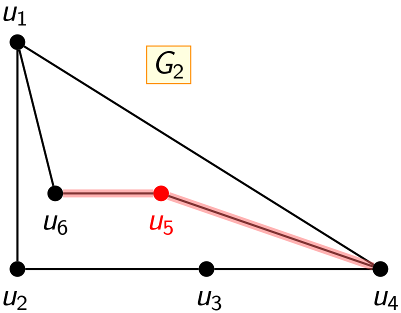


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & & \end{bmatrix} \end{array}$$

Rješenje

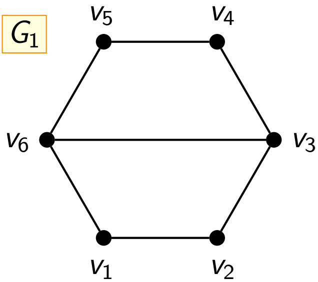


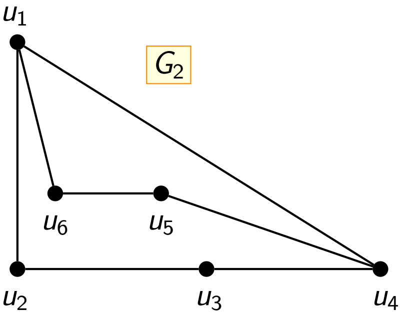


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & \end{bmatrix} \end{array}$$

Rješenje

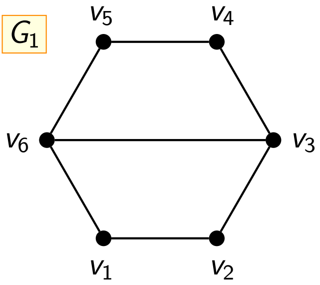


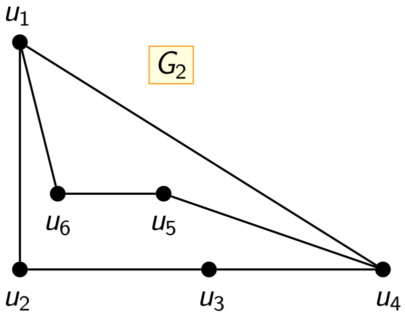


$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

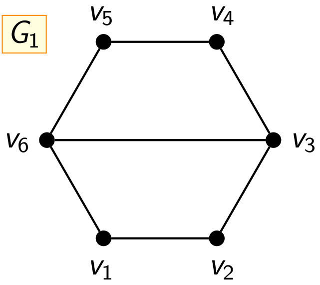
$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

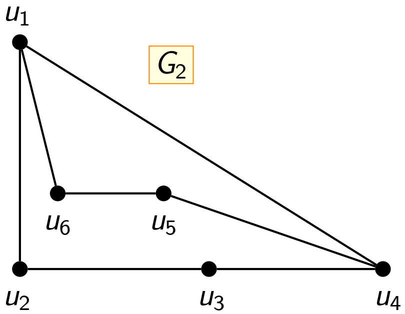
Rješenje





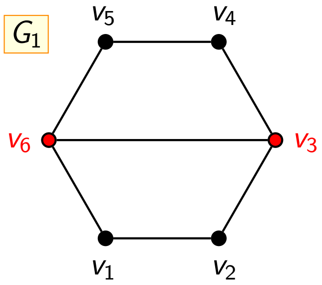
Rješenje

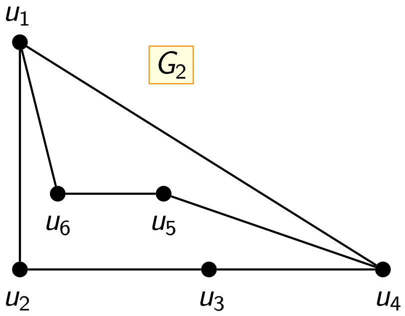




$v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6$

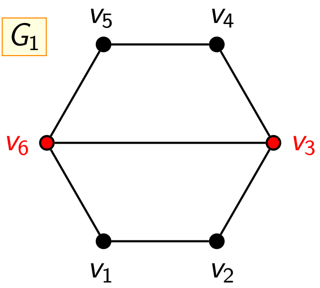
Rješenje

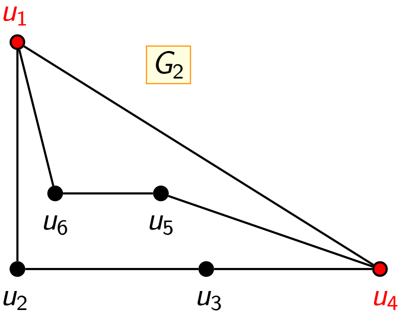




v_1 $\quad v_2$ $\quad v_3$ $\quad v_4$ $\quad v_5$ $\quad v_6$

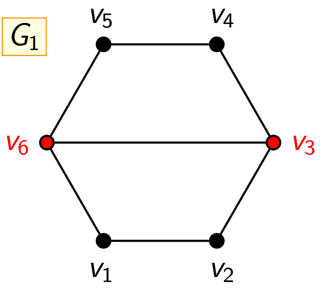
Rješenje

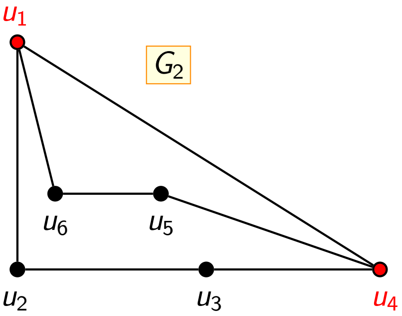




$v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6$

Rješenje

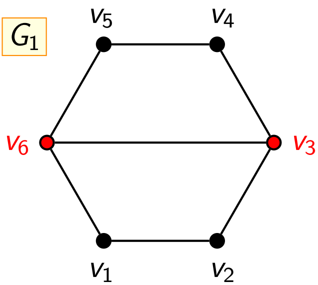


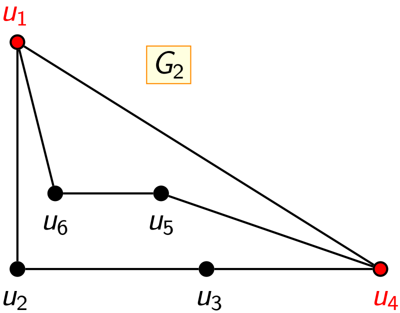


$\boxed{u_1, u_4}$

v_1 v_2 v_3 v_4 v_5 v_6

Rješenje

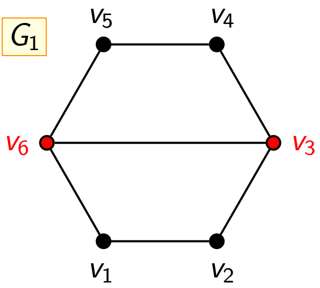




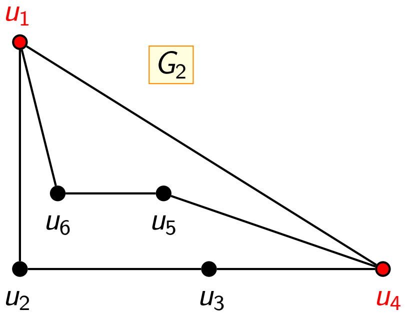
$\boxed{u_1, u_4}$ $\qquad$ $\boxed{u_1, u_4}$

$v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6$

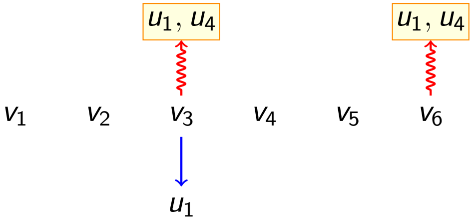
Rješenje



G_1
v_5
v_4
v_6
v_3
v_1
v_2

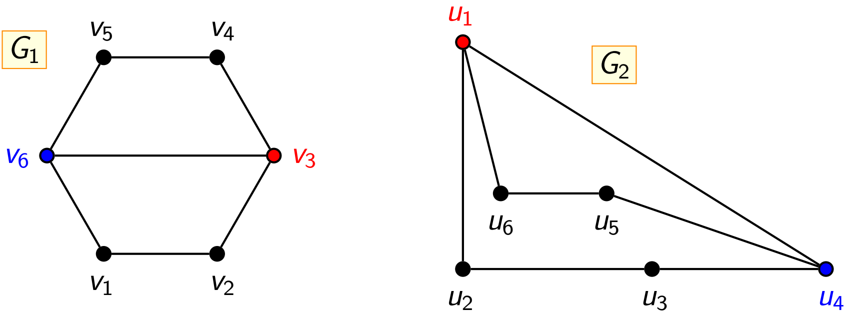


u_1
G_2
u_6
u_5
u_2
u_3
u_4

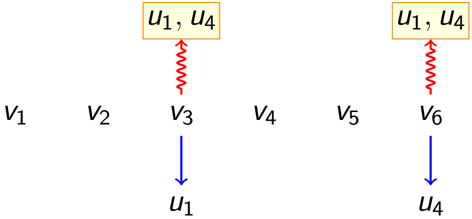


u_1, u_4
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_1

Rješenje

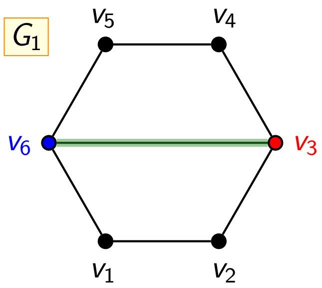


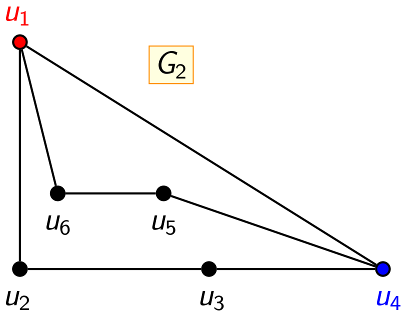
G_1
v_5
v_4
v_6
v_3
v_1
v_2
u_1
G_2
u_6
u_5
u_2
u_3
u_4

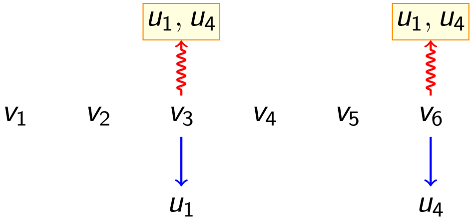


u_1, u_4
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_1
u_4

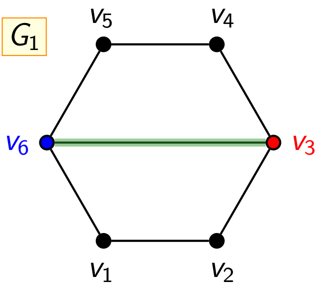
Rješenje


G_1
v_5
v_4
v_6
v_3
v_1
v_2

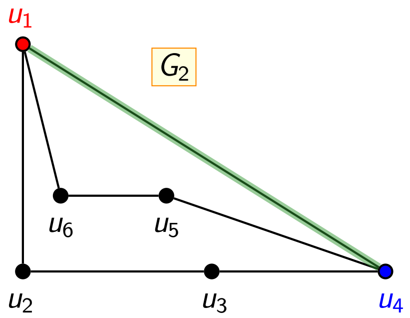

u_1
G_2
u_6
u_5
u_2
u_3
u_4


u_1, u_4
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_1
u_4

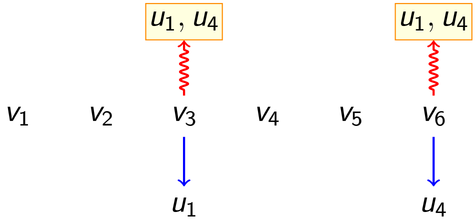
Rješenje



G_1
v_5
v_4
v_6
v_3
v_1
v_2

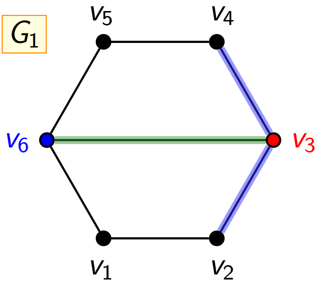


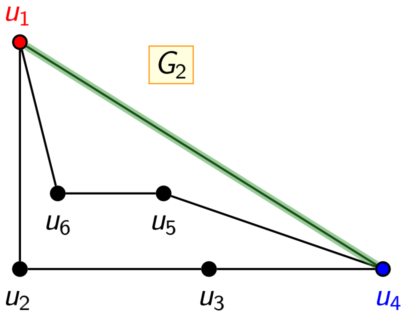
u_1
G_2
u_6
u_5
u_2
u_3
u_4

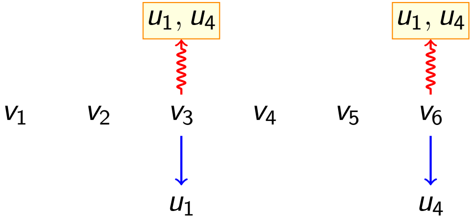


u_1, u_4
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_1
u_4

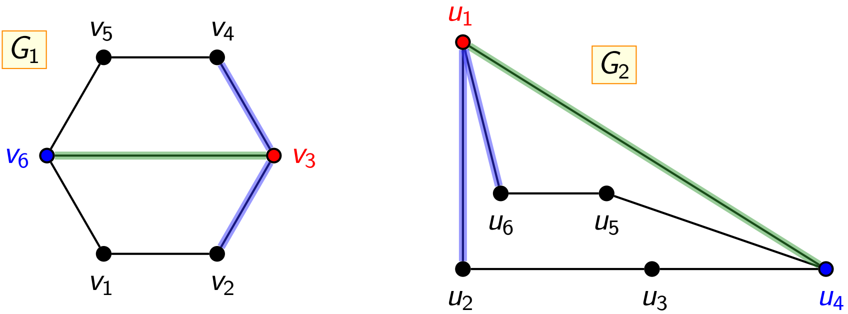
Rješenje


G_1
v_5
v_4
v_6
v_3
v_1
v_2

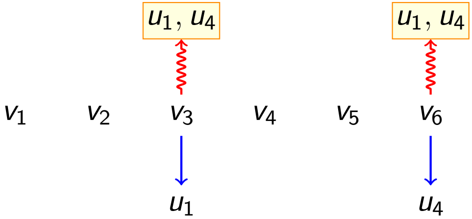

u_1
G_2
u_6
u_5
u_2
u_3
u_4


u_1, u_4
u_1, u_4
v_1
v_2
v_3
v_4
v_5
v_6
u_1
u_4

Rješenje

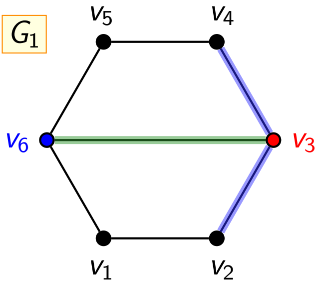


G_1
v_5
v_4
v_6
v_3
v_1
v_2
u_1
G_2
u_6
u_5
u_2
u_3
u_4

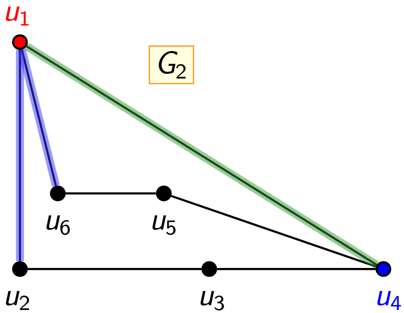


u_1, u_4
u_1, u_4
v_1
v_2
v_3
v_4
v_5
v_6
u_1
u_4

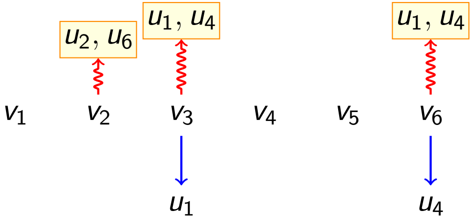
Rješenje



G1
v5
v4
v6
v3
v1
v2

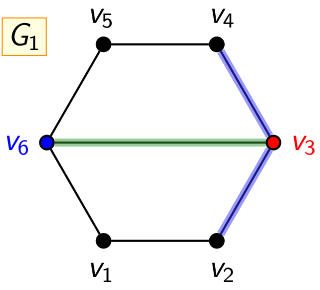


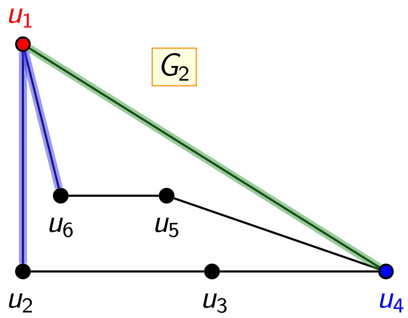
u1
G2
u6
u5
u2
u3
u4

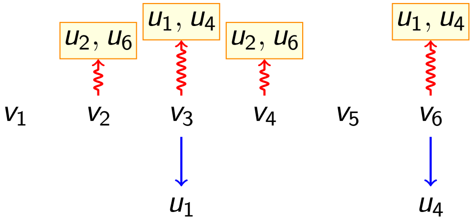


u2, u6
u1, u4
u1, u4
v1
v2
v3
v4
v5
v6
u1
u4

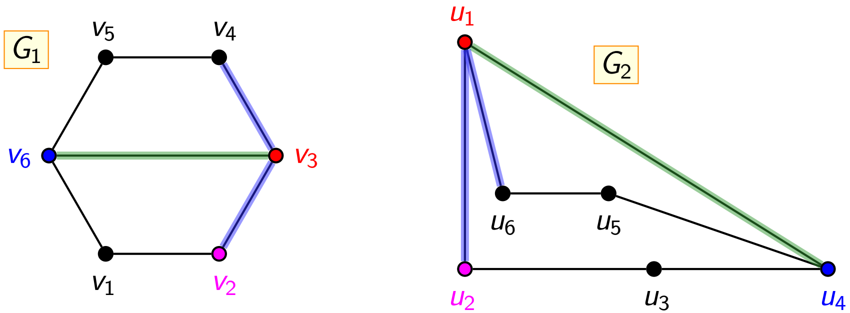
Rješenje

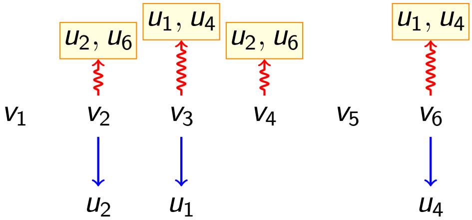

G_1
v_5
v_4
v_6
v_3
v_1
v_2


u_1
G_2
u_6
u_5
u_2
u_3
u_4

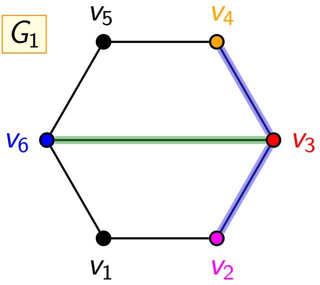

u_2, u_6
u_1, u_4
u_2, u_6
u_1, u_4
v_1
v_2
v_3
v_4
v_5
v_6
u_1
u_4

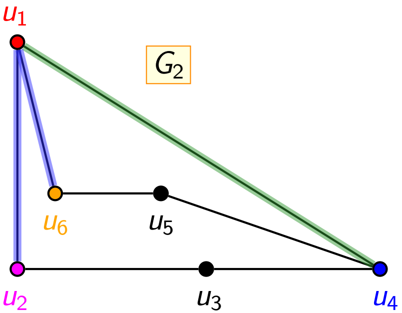
Rješenje

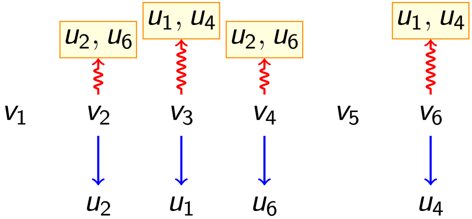

G1
v5
v4
v6
v3
v1
v2
u1
G2
u6
u5
u2
u3
u4


u2, u6
u1, u4
u2, u6
u1, u4
v1
v2
v3
v4
v5
v6
u2
u1
u4

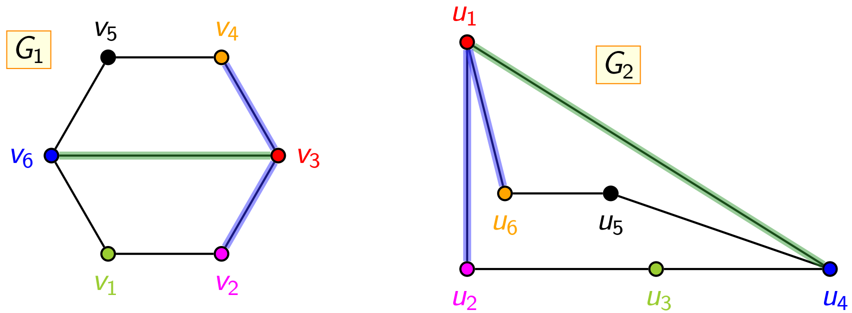
Rješenje


G_1
v_1
v_2
v_3
v_4
v_5
v_6

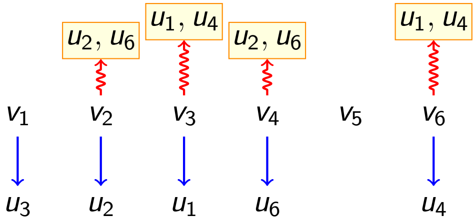

G_2
u_1
u_2
u_3
u_4
u_5
u_6


u_2, u_6
u_1, u_4
u_2, u_6
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_2 u_1 u_6 u_4

Rješenje

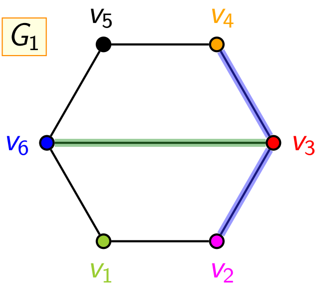


G_1
v_1 v_2 v_3 v_4 v_5 v_6
G_2
u_1 u_2 u_3 u_4 u_5 u_6

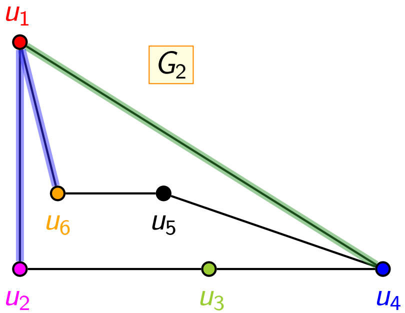


u_2, u_6
u_1, u_4
u_2, u_6
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_3 u_2 u_1 u_6 u_4

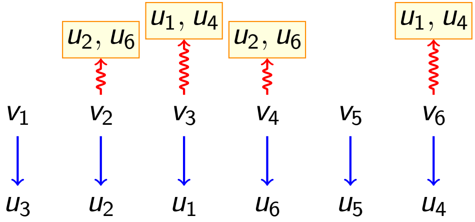
Rješenje



G_1
v_1
v_2
v_3
v_4
v_5
v_6

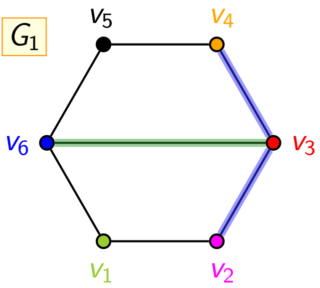


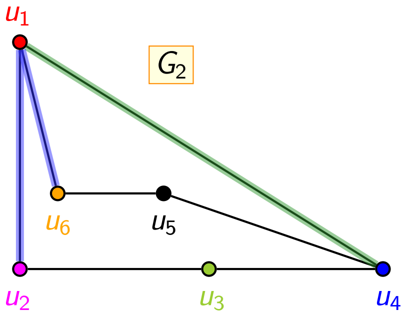
G_2
u_1
u_2
u_3
u_4
u_5
u_6



u_2, u_6
u_1, u_4
u_2, u_6
u_1, u_4
v_1 v_2 v_3 v_4 v_5 v_6
u_3 u_2 u_1 u_6 u_5 u_4

Rješenje

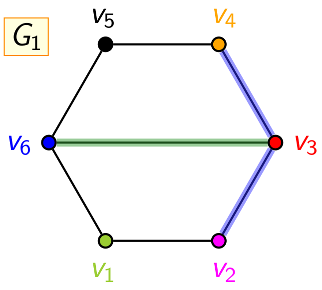


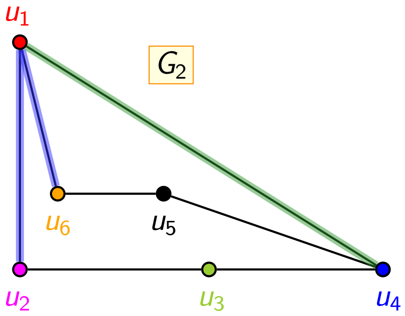


G_1 i G_2 su izomorfni grafovi.

$$
\begin{array}{cccccc}
 & \boxed{u_2, u_6} & \boxed{u_1, u_4} & \boxed{u_2, u_6} & & \boxed{u_1, u_4} \\
 & \uparrow & \uparrow & \uparrow & & \uparrow \\
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
u_3 & u_2 & u_1 & u_6 & u_5 & u_4
\end{array}
$$

Rješenje



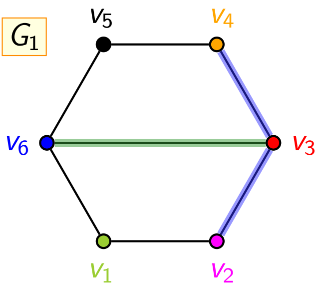


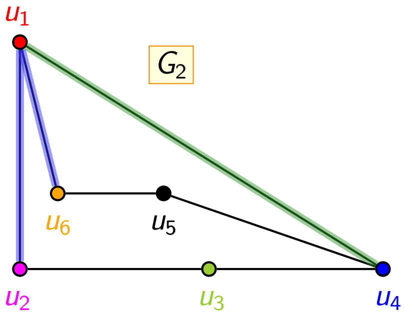
G_1 i G_2 su izomorfni grafovi.

$$\begin{array}{cccccc} & \boxed{u_2\,,\, u_6} & \boxed{u_1\,,\, u_4} & \boxed{u_2\,,\, u_6} & & \boxed{u_1\,,\, u_4} \\ & \uparrow & \uparrow & \uparrow & & \uparrow \\ v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ u_3 & u_2 & u_1 & u_6 & u_5 & u_4 \end{array}$$

$$P = \left[\qquad \right]$$

Rješenje



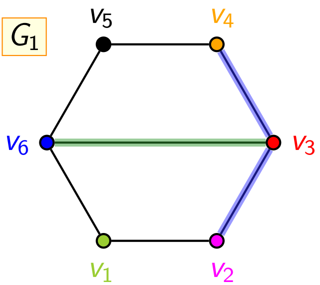


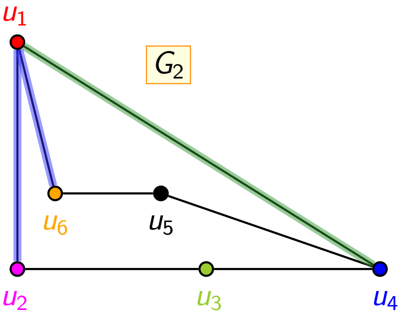
G_1 i G_2 su izomorfni grafovi.

	u_2, u_6	u_1, u_4	u_2, u_6		u_1, u_4
v_1	v_2	v_3	v_4	v_5	v_6
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
u_3	u_2	u_1	u_6	u_5	u_4

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & \\ & & & & & \end{bmatrix}$$

Rješenje



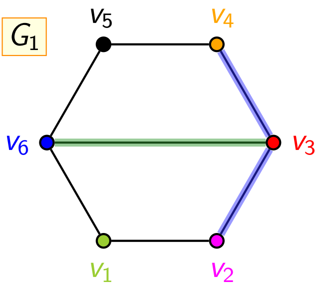


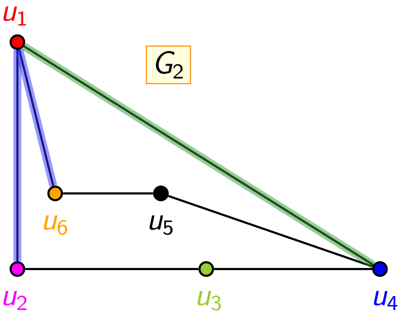
G_1 i G_2 su izomorfni grafovi.

$$\begin{array}{cccccc} & \boxed{u_2, u_6} & \boxed{u_1, u_4} & \boxed{u_2, u_6} & & \boxed{u_1, u_4} \\ & \uparrow & \uparrow & \uparrow & & \uparrow \\ v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ u_3 & u_2 & u_1 & u_6 & u_5 & u_4 \end{array}$$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

Rješenje



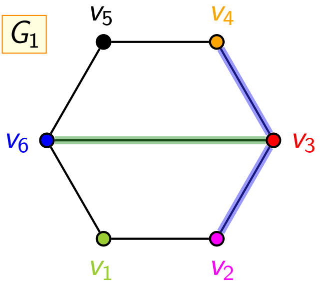


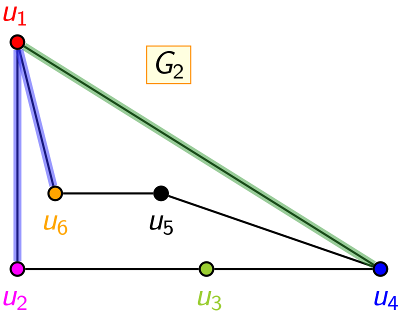
G_1 i G_2 su izomorfni grafovi.

$$\begin{array}{cccccc}
 & \boxed{u_2,\, u_6} & \boxed{u_1,\, u_4} & \boxed{u_2,\, u_6} & & \boxed{u_1,\, u_4} \\
 & \uparrow & \uparrow & \uparrow & & \uparrow \\
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
u_3 & u_2 & u_1 & u_6 & u_5 & u_4
\end{array}$$

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

Rješenje

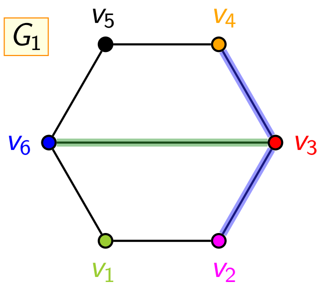


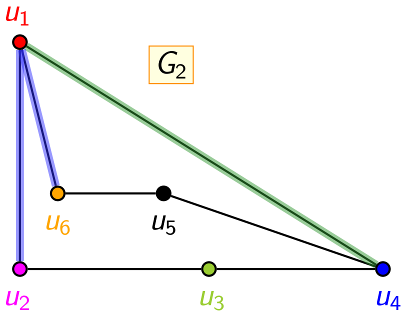


G_1 i G_2 su izomorfni grafovi.

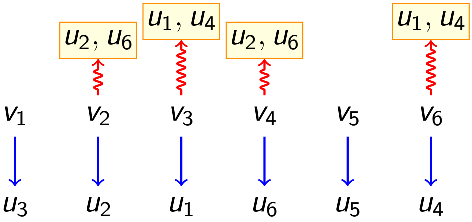
$$\begin{array}{cccccc}
 & \boxed{u_2,\, u_6} & \boxed{u_1,\, u_4} & \boxed{u_2,\, u_6} & & \boxed{u_1,\, u_4} \\
 & \uparrow & \uparrow & \uparrow & & \uparrow \\
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
u_3 & u_2 & u_1 & u_6 & u_5 & u_4
\end{array}
\qquad
P = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & \\
 & & & & & \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}$$

Rješenje



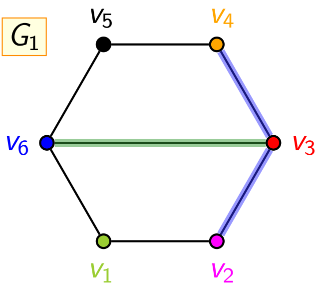


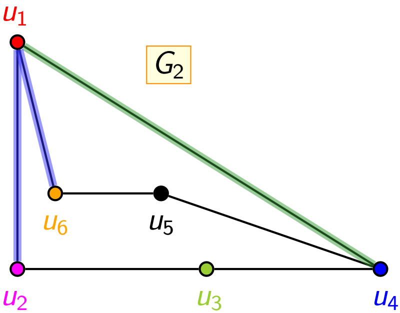
G_1 i G_2 su izomorfni grafovi.



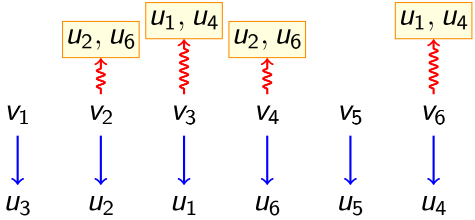
$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Rješenje





G_1 i G_2 su izomorfni grafovi.



$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A_2 = PA_1P^T$$

$$\begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ u_3 & u_2 & u_1 & u_6 & u_5 & u_4 \end{array}$$